Dynamical response of the nuclear "pasta" in neutron star crusts

C. J. Horowitz* and M. A. Pérez-García
Nuclear Theory Center and Department of Physics, Indiana University, Bloomington, Indiana 47405, USA

D. K. Berry†
University Information Technology Services, Indiana University, Bloomington, Indiana 47408, USA

J. Piekarewicz‡
Department of Physics, Florida State University, Tallahassee, Florida 32306, USA
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The nuclear pasta—a novel state of matter having nucleons arranged in a variety of complex shapes—is expected to be found in the crust of neutron stars and in core-collapse supernovae at subnuclear densities of about 10^{14} g/cm^3. Owing to frustration, a phenomenon that emerges from the competition between short-range nuclear attraction and long-range Coulomb repulsion, the nuclear pasta displays a preponderance of unique low-energy excitations. These excitations could have a strong impact on many transport properties, such as neutrino propagation through stellar environments. The excitation spectrum of the nuclear pasta is computed via a molecular-dynamics simulation involving up to 100,000 nucleons. The dynamic response of the pasta displays a classical plasma oscillation in the 1- to 2-MeV region. In addition, substantial strength is found at low energies. Yet this low-energy strength is missing from a simple ion model containing a single-representative heavy nucleus. The low-energy strength observed in the dynamic response of the pasta is likely to be a density wave involving the internal degrees of freedom of the clusters.

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Baryonic matter is organized as a result of short-range nuclear attraction and long-range Coulomb repulsion. Often the corresponding nuclear and atomic length scales are well separated, so nucleons bind into atomic nuclei that are themselves segregated into a crystal lattice. However, at the enormous densities present in astrophysical objects—densities that exceed that of ordinary matter by 14 orders of magnitude—these length scales become comparable and complex new phenomena emerge. Complexity arises because it is impossible for the constituents to be simultaneously correlated from nuclear attraction and anticorrelated from Coulomb repulsion. Competition among these interactions plays a fundamental role in the organization of matter and results in *Coulomb frustration*. Frustration—a ubiquitous behavior in complex systems ranging from magnetism to protein folding to neural networks—develops from the inability of a system to simultaneously satisfy all of its elementary interactions. For example, the Ising antiferromagnet on a triangular lattice is frustrated because not all of the nearest neighbor spins can be antiparallel to each other. Frustrated systems have unusual dynamics owing to the preponderance of low-energy excitations [1].

At subnuclear densities of about 10^{14} g/cm^3 (normal nuclear matter saturation density is 2.5×10^{14} g/cm^3) Coulomb frustration is expected to promote the development of complex shapes. These shapes follow from the competition between surface tension and Coulomb energies. Whereas surface tension favors spherical shapes, Coulomb interactions often favor nonspherical configurations. Therefore, a variety of complex structures with a diversity of shapes—such as spheres, cylinders, and plates—have been predicted. The many phases of nuclear matter displaying this variety of shapes are known collectively as *nuclear pasta*. The complex dynamics of the nuclear pasta is of relevance to the structure of the inner crust of neutron stars and to the dynamics of core-collapse supernovae [2].

There have been several calculations of the ground-state shapes of the nuclear pasta [3]. However, to our knowledge there have been almost no calculations of the dynamical properties of the nuclear pasta. Some dynamical aspects of nuclei in the inner crust of neutron stars were considered by Magierski and Bulgac [4] while Khan, Sandulescu, and Van Giai have calculated the excitation spectrum of pasta in a random-phase approximation (RPA) [5]. They find a low-energy collective oscillation of the neutron-rich skin of the pasta. We note, however, that their use of a spherical Wigner-Seitz unit cell may be a serious drawback in the case of long-wavelength collective modes that can extend across many unit cells. Here we are interested in the low-momentum (long-wavelength) dynamic response of the pasta, which we compute from a semiclassical molecular-dynamics simulation containing up to 100,000 nucleons. As the response of the nuclear pasta at low momentum transfers is dominated by heavy clusters, their thermal de Broglie wavelength is very small compared to the intercluster separation. This fact motivates our semiclassical approach.

The nuclear pasta is a unique hybrid state consisting of both atomic and nuclear matter. Therefore, the excitation spectrum involves both atomic and nuclear modes that may occur at similar energies. This allows for a unique mixing

*E-mail: horowit@indiana.edu
†E-mail: dkberry@indiana.edu
‡E-mail: jorgep@csit.fsu.edu

between atomic and nuclear excitations. For example, a dense system of charged particles displays plasma oscillations whereas nuclei exhibit collective density oscillations known as giant resonances. Such a novel excitation spectrum may strongly impact on a variety of transport properties, such as thermal conductivity, viscosity, diffusion, and opacity. These are all important for many neutron-star observables and could influence the dynamics of core-collapse supernovae. Indeed, in core-collapse supernovae—the giant explosions of massive stars—99% of the energy of the explosion is radiated in the form of neutrinos. In a previous work [6,7] we have computed the *static structure factor* of the pasta and the resulting neutrino mean free path that is dominated by coherent scattering from the various pasta shapes. Here we extend our previous work to study the excitation spectrum of the pasta by computing its *dynamical response*. This may influence the energy equilibration between matter and μ and τ neutrinos. We note that many complex fluids, such as polymers, colloids, water-surfactant-oil solutions, microemulsions, and liquid crystals, display similar complex shapes (see, for example [8]). Neutron and x-ray scattering from these systems probe these complex shapes in a manner that is analogous to neutrino scattering from nuclear pasta.

The nuclear pasta is described through a simple semiclassical model that we have used earlier to compute its static structure factor [6,7]. Here we are interested in computing its dynamical response to neutrino scattering at low momentum transfers. We model the nuclear pasta as a charge-neutral system of neutrons, protons, and electrons. At the relevant densities and temperatures of our simulations, the electrons form a degenerate, relativistic Fermi gas that is not modeled explicitly. Rather, the electrons modify the Coulomb interaction between the protons through a screening length λ. Neutrons and protons are described by the following semiclassical Hamiltonian: $H = K + \sum_{i<j} v_{ij}$, where K represents the kinetic energy for nucleons of mass m and the two-body potential is given by

$$v_{ij}(r) = a\mathrm{e}^{-r^2/\Lambda} + b_{ij}\mathrm{e}^{-r^2/2\Lambda} + e_i e_j \frac{\mathrm{e}^{-r/\lambda}}{r} \, . \tag{1}$$

Here e_i is the electric charge of the ith nucleon and the parameters of the model are as follow: $a = 110$ MeV, $b_{ij} = -2$ MeV for the interaction between either two neutrons or two protons, $b_{ij} = -50$ MeV for the interaction between a neutron and a proton, and $\Lambda = 1.25$ fm^2. These parameters have been fitted so that molecular dynamics simulations at a temperature of 1 MeV reproduce a saturation density $\rho = 0.16$ fm^{-3} and a binding energy near -16 MeV [6]. These are appropriate values for symmetric nuclear matter near zero temperature. This is enough to reproduce the main features of pasta formation, that matter clumps into clusters of appropriate density. However, the clusters cannot be too large because of the Coulomb interaction. Note that our semiclassical model is not directly applicable at zero temperature. We keep the value of the screening length fixed at $\lambda = 10$ fm to compare with our earlier calculations. (This value is slightly smaller than the Thomas-Fermi screening length of the electron gas.) Watanabe and collaborators have computed static properties of the nuclear pasta by performing quantum-molecular-dynamics simulations with a more complicated interaction [9]. Our aim here is to employ a "minimal model" that, by incorporating both nuclear saturation and Coulomb repulsion, may capture the essential features of frustration in a transparent fashion.

The differential cross section for neutrino scattering from the nuclear pasta may be written as follows [6]:

$$\frac{d\sigma}{d\Omega dE} = \frac{G_F^2 E_\nu^2}{4\pi^2}\left[c_a^2(3-x)S_A(q,\omega) + c_v^2(1+x)S_V(q,\omega)\right], \tag{2}$$

where G_F is the Fermi constant, E_ν is the neutrino energy, $x = \cos\theta$ (with θ the scattering angle), and the weak vector charge of a nucleon is $c_v = -1/2$ for a neutron and $c_v \approx 0$ for a proton. Further, the dynamic response is probed at a momentum transfer q and at an energy transfer ω. The axial term involving $c_a = \pm 1.26/2$ and the dynamical spin response $S_A(q, w)$ will be discussed in a later work. Here we focus exclusively on the vector (density) response $S(q,\omega) \equiv S_V(q,\omega)$, which should be greatly enhanced by coherent effects.

The dynamical response of the system to a density perturbation is given by

$$S(q,\omega) = \frac{1}{\pi}\int_0^{T_{\rm max}} S(q,t)\,\cos(\omega t)dt. \tag{3}$$

Here $S(q,t)$ represents the ensemble average of the density-density correlation function that is computed as the following time average:

$$S(q,t) = \frac{1}{N}\frac{1}{T_{\rm ave}}\int_0^{T_{\rm ave}} \rho(\mathbf{q}, t+s)\rho(-\mathbf{q}, s)ds. \tag{4}$$

In this expression N is the number of neutrons in the system and we discuss choices for $T_{\rm max}$ and $T_{\rm ave}$ in the following. Note that to improve statistics, an angle average of Eq. (4) over the direction of $\mathbf{q}$ has been performed. Finally, the one-body neutron density is given by $\rho(\mathbf{q},t) = \sum_{i=1}^{N}\exp[i\mathbf{q}\cdot\mathbf{r}_i(t)]$ with $\mathbf{r}_i(t)$ the position of the ith neutron at time t. Note that because $c_v \approx 0$ for protons, the sum over i runs only over neutrons. Further, the static structure factor computed in Ref. [7] is easily recovered from $S(q) \equiv S(q, t=0) = \int_0^\infty S(q,\omega)d\omega$.

$S(q,\omega)$ is now computed for conditions studied in an earlier publication [7]. These include a fixed temperature of $T = 1$ MeV, a proton-to-baryon fraction of $Y_p = 0.2$, and baryon densities of $\rho = 0.01$ fm^{-3} and $\rho = 0.05$ fm^{-3} (these represent about 1/15 and 1/3 of normal nuclear density). Note that during core-collapse supernovae, the proton fraction starts near 1/2 and drops to about 0.1 owing to electron capture, so $Y_p = 0.2$ is a representative value. To fit a 10-MeV neutrino with a 120-fm wavelength into the simulation volume, we must include up to 100,000 nucleons in our molecular-dynamics simulations. We start the first simulation by distributing 80,000 neutrons and 20,000 protons at random within the simulation volume and with their velocities selected according to a Maxwell-Boltzmann distribution at a temperature of $T = 1$ MeV. At each time step of $\Delta t = 1$–2 fm/c, Newton's equations of motion are integrated using a standard velocity

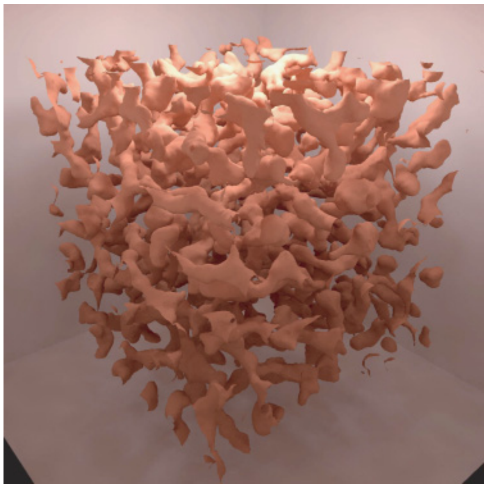

FIG. 1. (Color online) The 0.03 fm^{-3} proton density isosurface for one configuration of 100,000 nucleons at a baryon density of 0.05 fm^{-3}. The simulation volume is a cube of 126 fm on a side.

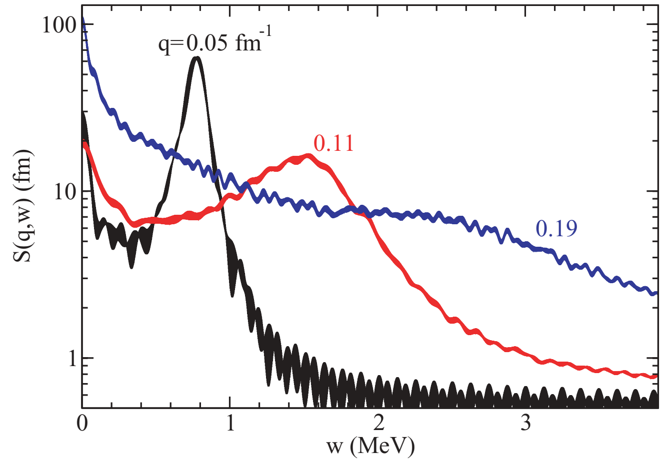

FIG. 2. (Color online) The dynamical response function $S(q,\omega)$ versus excitation energy ω at a density of $\rho = 0.05$ fm^{-3} and momentum transfers of $q = 0.05$, 0.11, and 0.19 fm^{-1}.

Verlet algorithm [10]. This time step typically conserves energy to at least one part in 10^4. The system is thermalized by evolving for a time (28,000 fm/c), during which the velocities are periodically rescaled to maintain the temperature fixed at $T = 1$ MeV. To calculate $S(q,\omega)$ the system is evolved, without any velocity rescaling [11], for a further time of $388,000$ fm/c= $T_{\rm ave} + T_{\rm max}$ [see Eq. (4) and the following for $T_{\rm max}$] during which the spatial configurations of all 100,000 nucleons are written to disk every 20 fm/c—for a total of 19,400 configurations. The simulations were performed on four special-purpose accelerated MDGRAPE-2 boards [7,12] with a combined performance of roughly 500 times that of a single conventional CPU.

At any time during the simulation one may examine the spatial correlations of the nuclear pasta. A typical proton density is displayed in Fig. 1 at a density of $\rho = 0.05$ fm^{-3}. Although protons are seen to cluster into complex elongated shapes, it is difficult to discern a single underlying structure (e.g., spheres, cylinders, etc.). Although not shown, most of the neutrons also cluster into these complex shapes. However, in addition, there is a low-density neutron gas between the clusters. The resulting dynamical response of the nuclear pasta is shown in Fig 2. The choice of $T_{\rm max}$ in Eq. (3) involves a tradeoff. $T_{\rm max}$ should be large enough to avoid truncation errors and small enough to minimize statistical errors from $S(q,t)$ at large t. A $T_{\rm max}$ of the order of 20,000 fm/c was used. We note that at low momentum transfers the dynamical response shows a peak just below $\omega = 1$ MeV. This peak becomes broader with increasing q. Further, there is substantial strength near $\omega = 0$. To interpret these peaks we calculate $S(q,\omega)$ at a lower density and compare it to the corresponding response of a simplified ion model [7].

A simulation at the lower density of $\rho = 0.01$ fm^{-3} (with $Y_p = 0.2$ and $T = 1$ MeV) reveals nucleons clustered into more conventional neutron-rich nuclei rather than in complex pasta shapes. At this density, thermalization is slower because of the Coulomb barrier; it may take a long time to add protons to a cluster. Therefore, a system of 40,000 nucleons initially distributed at random had to be evolved for the very long time of 1,287,000 fm/c. During this time the temperature of the system was first raised and then lowered to the target temperature of $T = 1$ MeV. Even so, we cannot rule out a further increase in cluster size from evolution on even longer time scales. To compute the dynamical response, the system was evolved for another 720,000 fm/c while writing configurations to disk every 20 fm/c, for a total of 36,000 configurations.

In Fig. 3 the response of the nuclear pasta is compared to that of an ion model where the composition of the system is assumed to be 28% free neutrons and $X_h = 72\%$ of a single representative heavy ion of average mass $A = 100$ and charge $Z = 28$. These numbers were obtained from counting nucleons in the different clusters using a procedure described in Ref. [7].

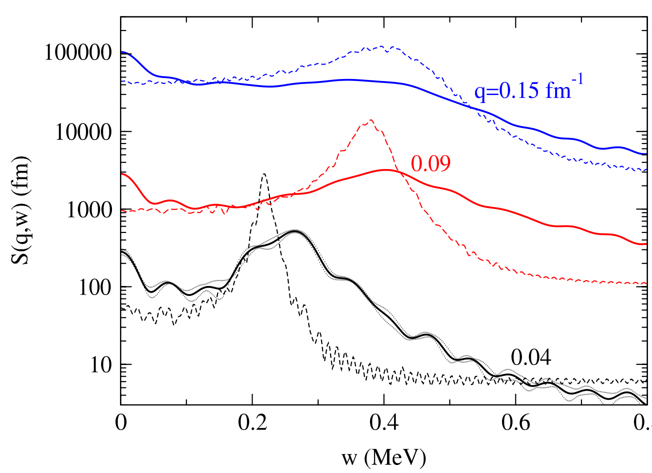

FIG. 3. (Color online) The dynamical response function $S(q,\omega)$ versus excitation energy ω at a density of $\rho = 0.01$ fm^{-3} and momentum transfers of $q = 0.04$, 0.09, and 0.15 fm^{-1}. The solid lines show results for a 40,000-nucleon simulation; the dashed lines are results for an ion model [see Eq. (5)] simulation with 288 ions. For clarity the $q = 0.09$ fm^{-1} results have been multiplied by 10 and the $q = 0.15$ fm^{-1} results by 100.

The dynamical response is modeled as

$$S_{\mathrm{model}}(q,\omega) = X_h \langle N\rangle F(q)^2 S_{\mathrm{ion}}(q,\omega), \tag{5}$$

where $S_{\mathrm{ion}}(q,\omega)$ is the response of $Z = 28$ ions with only screened Coulomb interactions, $\langle N\rangle = 72$ is the average number of neutrons in a cluster, and $F(q)$ is the cluster form factor, which is approximated as a uniform density sphere of radius 6.68 fm and normalized to $F(0) = 1$ [7]. $S_{\mathrm{ion}}(q,\omega)$ is computed from a molecular-dynamics simulation for a system of 288 ions in the same volume as the 40,000-nucleon simulation. The model response in Fig. 3 shows a single peak from plasma oscillations with a width that increases with q. This peak is also visible in the full nucleon response at low q and as a broad shoulder at $q = 0.15\ \mathrm{fm}^{-1}$. The location of the peak is approximately described by the following known expression for the plasma frequency [13]:

$$\omega_{\mathrm{pl}} \approx (4\pi Z^2 e^2 \rho_i / M_i)^{1/2} (1 + (q\lambda)^{-2})^{-1/2}, \tag{6}$$

where ρ_i is the ion density and M_i is the ion mass. The second factor appearing in Eq. (6) is a crude RPA estimate of the decrease in the plasma frequency resulting from the screening length λ appearing in Eq. (1). However, note that the width of the plasma oscillation in the nucleon simulation is much larger than the width in the ion model. This may reflect a coupling of the plasma oscillation to the internal degrees of freedom of the clusters and/or failure of the single heavy-ion approximation.

In addition, the full nucleon simulation shows a peak at $\omega = 0$ that is absent from the ion model. We speculate that this mode may be associated with density fluctuations. A liquid vapor coexistence region has large density fluctuations as vapor is converted to or from liquid. Note that the energy associated with transferring a neutron from the vapor to a cluster is zero because the vapor is in equilibrium with the condensed phase. Therefore, the system can support a density wave with low excitation energy. In the nuclear pasta—a system with two conserved quantities (baryon number and electromagnetic charge)—these fluctuations are constrained at long wavelengths by charge neutrality. However, the system may still experience density fluctuations at *finite q* as nucleons condense and evaporate from the individual clusters. This density wave may represent a hallmark of frustrated, multicomponent systems having more than one conserved charge. Therefore, it is important to verify our semiclassical results with full quantum calculations. The interpretation of this $\omega = 0$ mode as a density wave should also be verified in future work. We believe our speculation is reasonable but this needs to be checked.

Finally, we compare our results at $\rho = 0.05$ and $0.01\ \mathrm{fm}^{-3}$. It is difficult to apply our ion model directly at a density of $\rho = 0.05\ \mathrm{fm}^{-3}$ because the masses and charges of the interconnected clusters are not well defined. The full simulation results at $\rho = 0.05$ have a higher frequency plasma oscillation compared to $\rho = 0.01$. Note, although the plasma frequency depends on density, it may not be strongly modified by the nonspherical cluster shapes that are present at high density. The low omega mode, which is present at $\rho = 0.01$, is more pronounced at $\rho = 0.05$.

Between densities of 0.01 and $0.05\ \mathrm{fm}^{-3}$, nonspherical shapes appear. It is natural to ask how these shapes impact the excitation spectrum. Plasma oscillations involve long-distance classical physics. The electrostatic restoring force depends only on the charge density. Therefore the plasma frequency depends on the ratio of the ion charge density to mass as indicated in Eq. (6). This ratio is independent of the shape of the clusters. Extended shapes such as long rods could also have nuclear contributions to the restoring force. This could raise the oscillation frequency. However, between 0.01 and $0.05\ \mathrm{fm}^{-3}$ we find the ratio of the frequency of the $q = 0.04\ \mathrm{fm}^{-1}$ peak in Fig. 3 to the $q = 0.05\ \mathrm{fm}^{-1}$ peak in Fig. 2 to be in good agreement with Eq. (6). This suggests that the nuclear contributions to the restoring force at a density of $0.05\ \mathrm{fm}^{-3}$ are small. Thus the plasma oscillations we find in the pasta simulation are consistent with an electrostatic restoring force only.

In conclusion, we have modeled the complex nuclear-pasta phase via a semiclassical model that reproduces nuclear saturation and includes Coulomb repulsion. The dynamical response of the nuclear pasta is computed from molecular-dynamics simulations with 40,000 and 100,000 nucleons. We find that the nuclear pasta supports a plasma oscillation with a frequency in the 1- to 2-MeV range. In addition, the dynamical response displays a substantial amount of strength at low energies, which we identify as a coherent density wave involving the internal degrees of freedom of the clusters.

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