# Mass of the nucleon in a chiral quark-diquark model

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The mass of the nucleon is studied in a chiral quark-diquark model. Both scalar and axial-vector diquarks are taken into account for the construction of the nucleon state. After the hadronization procedure is used to obtain an effective meson-baryon Lagrangian, the quark-diquark self-energy is calculated to generate the baryon kinetic term as well as determine the mass of the nucleon. It turns out that both the scalar and axial-vector parts of the self-energy are attractive for the mass of the nucleon. We investigate the range of parameters that can reproduce the mass of the nucleon.

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## I. INTRODUCTION

An effective Lagrangian approach is an useful method for the description of hadron properties at low energies. Such a Lagrangian contains various terms and parameters expressing not only structures of mesons and baryons but also their interactions. A microscopic description for such terms is desired, especially when we consider, for instance, character changes of hadrons at finite temperatures and densities, which is one of the interesting topics of current hadron physics.

Eventually, QCD should address this issue, but the present situation is not very satisfactory. If we start, however, from an intermediate QCD-oriented theory, we can make reasonably good progress. One such approach is the Nambu-Jona-Lasinio (NJL) model [1–3] for mesons and the quark diquark model for mesons and baryons [4,5]. The models have been tested to a great extent for the description of various meson and baryon properties. It has been shown that the hadronization method based on the path-integral formalism is useful, because it can incorporate hadron structure in terms of quarks and diquarks while respecting important symmetries such as the gauge and chiral symmetries. This idea was first investigated by Cahill [6] and Reinhardt [7], and then by Ebert and Jurke in a simplified framework [4], which was later elaborated upon by Abu-Raddad et al. [5]. Recently, the method was applied also to the nuclear force by the present authors [8]. Nonetheless, these previous studies were done only with the scalar diquark, though the construction of the baryon requires two types of diquarks: scalar and axial-vector ones. The inclusion of the axial-vector diquarks is crucially important for the description of spin-isospin quantities, such as the axial coupling constant  $g_A$  and isovector magnetic moment  $\mu$  of the nucleon, and also the nuclear force.

In this paper, we extend our previous study and calculate the nucleon mass with the inclusion of the axial-vector diquark. This is a necessary step to complete the program of the hadronization method. It is shown that, by choosing suitable parameters, the mass of the nucleon is reproduced with the same significant amount of the axial-vector diquark component, which will help improve observables such as  $g_A$ and isovector magnetic moment. The paper is organized as follows. In Sec. II, we construct a microscopic (quark-diquark) Lagrangian and derive the macroscopic (meson-baryon) Lagrangian through the hadronization of the microscopic Lagrangian. In Sec. III, we study the quark-diquark self-energy and calculate the mass of the nucleon. In Sec. IV, we present numerical results. The final section is devoted to summary and conclusions.

#### **II. LAGRANGIAN**

We briefly review the method to derive the effective mesonbaryon Lagrangian following the work of Abu-Raddad *et al.* [5]. Let us start from the  $SU(2)_L \times SU(2)_R$  NJL Lagrangian

$$\mathcal{L}_{\text{NJL}} = \bar{q}(i\partial - m_0)q + \frac{G}{2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2], \quad (2.1)$$

where q is the current quark field,  $\tau_a(a = 1, 2, 3)$  is the flavor Pauli matrices, G is the NJL coupling constant with dimension of (mass)<sup>-2</sup>, and  $m_0$  is the current quark mass. In this paper, we set  $m_0 = 0$  for simplicity. The NJL Lagrangian is bosonized by introducing collective meson fields as auxiliary fields in the path-integral method. As an intermediate step, we find the following Lagrangian:

$$\mathcal{L}'_{q\sigma\pi} = \bar{q}(i\partial \!\!\!/ - g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}))q - \frac{g^2}{2G}(\sigma^2 + \vec{\pi}^2). \quad (2.2)$$

Here  $\sigma$  and  $\vec{\pi}$  are properly normalized scalar-isoscalar sigma and pseudoscalar-isovector pion fields as generated from  $\sigma \sim \bar{q}q$  and  $\vec{\pi} \sim i\bar{q}\vec{\tau}\gamma_5 q$ , respectively, and g is a meson-quark coupling constant.

For our purpose, it is convenient to work in the nonlinear basis [4,9,10]. First, the meson fields are expressed as

$$\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi} = f \exp\left(-\frac{i}{F_\pi}\gamma_5 \vec{\tau} \cdot \vec{\Phi}\right), \qquad (2.3)$$

where f and  $\Phi$  are new meson fields in the nonlinear basis and  $f^2 = \sigma^2 + \vec{\pi}^2$ . Spontaneous breaking of chiral symmetry is realized when f takes a nonzero vacuum expectation value  $\langle f \rangle = F_{\pi}$ , which is identified with the pion decay constant ~93 MeV, generating the constituent quark mass dynamically,  $m_q = g F_{\pi}$  [11]. The nonlinear Lagrangian is, then, achieved by chiral rotation from the current (q) to constituent ( $\chi$ ) quark fields:

$$\chi = \xi_5^{\dagger} q, \quad \xi_5 = \exp\left(\frac{i}{2F_{\pi}} \gamma_5 \vec{\tau} \cdot \vec{\Phi}\right). \tag{2.4}$$

Thus we find

$$\mathcal{L}'_{\chi\sigma\pi} = \bar{\chi}(i\partial - m_q - \partial - \partial \gamma_5)\chi - \frac{1}{2G}f^2, \qquad (2.5)$$

where

$$v_{\mu} = \frac{1}{2i} (\xi^{\dagger} \partial_{\mu} \xi + \xi \partial_{\mu} \xi^{\dagger}), \quad a_{\mu} = \frac{1}{2i} (\xi^{\dagger} \partial_{\mu} \xi - \xi \partial_{\mu} \xi^{\dagger}) \quad (2.6)$$

are the vector and axial-vector currents written in terms of the chiral field

$$\xi = \exp\left(\frac{i}{2F_{\pi}}\vec{\tau}\cdot\vec{\Phi}\right). \tag{2.7}$$

The Lagrangian (2.5) describes not only the kinetic term of the quark but also quark-meson interactions such as the Yukawa and the Weinberg-Tomozawa types among others.

For the description of baryons, we introduce diquarks and their interactions with quarks. We assume local interactions between a quark and a diquark to generate the nucleon field. As suggested previously [12], we consider two diquarks; one is a Lorentz scalar, of isoscalar color  $\overline{3}$ , denoted by D, and the other is an axial-vector, of isovector color  $\overline{3}$ ,  $D_{\mu}$ . The ground-state nucleon is then described as a superposition of the bound state of a quark and scalar diquark ( $\equiv$  scalar channel) and the bound state of a quark and axial-vector diquark ( $\equiv$ axial-vector channel). Hence, our microscopic Lagrangian for quarks, diquarks, and mesons is given by [5]

$$\mathcal{L} = \bar{\chi} (i\partial - m_q - \psi - \phi \gamma_5) \chi - \frac{g^2}{2G} f^2 + D^{\dagger} (\partial^2 + M_S^2) D + \vec{D}^{\dagger \mu} \left[ (\partial^2 + M_A^2) g_{\mu\nu} - \partial_{\mu} \partial_{\nu} \right] \vec{D}^{\nu} + \tilde{G} (\sin \theta \bar{\chi} \gamma^{\mu} \gamma^5 \vec{\tau} \cdot \vec{D}_{\mu}^{\dagger} + \cos \theta \bar{\chi} D^{\dagger}) \times (\sin \theta \vec{D}_{\nu} \cdot \vec{\tau} \gamma^{\nu} \gamma^5 \chi + \cos \theta D \chi).$$
(2.8)

In the last term  $\tilde{G}$  is a coupling constant for the quark-diquark interaction and an angle  $\theta$  controls the mixing ratio of the scalar and axial-vector channels in the nucleon wave function. In this construction, we have assumed a local interaction between the quark and diquarks. This stems from, for instance, the static limit of a quark exchange between a quark and a diquark as shown in Fig. 1. In this case, owing to the spin-flavor-color structure, the interactions become attractive both for the scalar and axial-vector diquark channels. In Eq. (2.8), a positive



 $\tilde{G}$  guarantees an attractive interaction, which is the case we consider.

The hadronization procedure is straightforward: First, a baryon field is introduced as an auxiliary field  $B \sim \sin\theta D_{\nu} \cdot \vec{\tau} \gamma^{\nu} \gamma_5 \chi + \cos\theta D \chi$ , then the quark and diquark fields in Eq. (2.8) are eliminated. The final result is written in a compact form as [5]

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2G} f^2 - i \operatorname{tr} \ln(i\partial - m_q - \psi - \phi \gamma_5) - \frac{1}{\tilde{G}} \bar{B}B + i \operatorname{tr} \ln(1 - \Box), \qquad (2.9)$$

where traces are taken over space-time, color, flavor, and Lorentz indices, and the operator  $\Box$  is defined by

$$\Box = \begin{pmatrix} \mathcal{A} & \mathcal{F}_2 \\ \mathcal{F}_1 & \mathcal{S} \end{pmatrix}, \qquad (2.10)$$

with

$$\mathcal{A}^{\mu i, \nu j} = \sin^2 \theta \, \bar{B} \, \gamma_{\rho} \gamma^5 \, \tau_k \, \tilde{\Delta}^{\rho k, \, \mu i} \, S \tau^j \gamma^{\nu} \gamma^5 B, \qquad (2.11a)$$

$$S = \cos^2\theta \ B \Delta SB, \tag{2.11b}$$

$$(\mathcal{F}_1)^{\mu i} = \sin\theta \cos\theta \ \bar{B} \Delta S t^{\rho} \gamma \gamma B, \qquad (2.11c)$$
$$(\mathcal{F}_2)^{\mu i} = \sin\theta \cos\theta \ \bar{B} \ \tilde{\Delta}^{\rho k,\mu i} \gamma_{\rho} \gamma^5 \tau_{\nu} SB, \qquad (2.11d)$$

$$(\mathcal{F}_2)^{\mu\nu} = \sin\theta\cos\theta \ B \ \Delta^{\mu\nu,\mu\nu} \ \gamma_{\rho}\gamma^{\sigma}\tau_k SB. \tag{2.11d}$$

The *S*,  $\Delta$ , and  $\tilde{\Delta}$  are the quark, scalar diquark, and axial-vector diquark propagators, respectively. The tr log can be expanded as

$$\operatorname{tr}\ln(1-\Box) = -\operatorname{tr}\left(\Box + \frac{\Box^2}{2} + \cdots\right). \tag{2.12}$$

The first term on the right-hand side describes one-particle properties of the nucleon, as it contains the nucleon bilinear form  $\bar{B}\Gamma B$ ; higher order terms describe interactions for two, three, and more nucleons.

Finally, we comment on the properties of the nucleon field. Since we take the nonlinear representation, the transformation properties of baryons under chiral  $SU(2)_L \times SU(2)_R$  are simple. Baryons transform in the same way as quarks do:

$$\chi \to \chi'(x) = h(x)\chi(x), \quad B(x) \to B'(x) = h(x)B(x),$$
(2.13)

where h(x) is the nonlinear function of the chiral transformations and of the chiral field at a point x [10]. Here we note that the baryon field B(x), in terms of quarks and diquarks, is related to the nucleon wave function in the constituent quark model by way of

$$D\chi = 2\phi_{\rho}\chi_{\rho}, \qquad (2.14)$$

$$D_{\nu} \cdot \vec{\tau} \gamma^{\nu} \gamma_5 \chi = 6 \phi_{\lambda} \chi_{\lambda}, \qquad (2.15)$$

in the nonrelativistic limit, where  $\phi_{\rho}$ ,  $\phi_{\lambda}$  and  $\chi_{\rho}$ ,  $\chi_{\lambda}$  are the standard three-quark spin and isospin wave functions [10]. If we take  $\tan \theta = 1/3$ , we realize the SU(4) spin-flavor symmetry of the constituent quark model.

FIG. 1. The quark exchange diagram (left) in the Faddeev approach [13,14] and its static limit (right).



FIG. 2. A diagrammatic representation of the quark-diquark selfenergy. The single, double, and triple lines represent the quark, diquark, and nucleon, respectively.

### III. THE QUARK-DIQUARK SELF-ENERGY

In the first-order term with respect to  $\Box$  of Eq. (2.12), the quark-diquark self-energy corresponding to Fig. 2 is given by

$$\mathcal{L}_0 = -i \operatorname{tr} \left[ \mathcal{S} + \mathcal{A}_{ij}^{\mu\nu} \right] - \frac{1}{\tilde{G}} \bar{B} B.$$
(3.1)

Using the interaction terms in Eqs. (2.11), we obtain

$$\mathcal{L}_{0} = \cos^{2}\theta \bar{B}(p)\Sigma_{S}(p)B(p) + \sin^{2}\theta \bar{B}(p)\Sigma_{A}(p)B(p) - \frac{1}{\tilde{G}}\bar{B}B, \qquad (3.2)$$

where  $\Sigma_S(p)$  and  $\Sigma_A(p)$  are the nucleon self-energies corresponding to the scalar and axial-vector diquark channels, respectively,

$$\Sigma_{S}(p) = -iN_{c} \int \frac{d^{4}k}{(2\pi)^{4}} \frac{1}{k^{2} - M_{S}^{2}} \frac{\not p - \not k + m_{q}}{(p - k)^{2} - m_{q}^{2}}, \quad (3.3a)$$

$$\Sigma_{A}(p) = -iN_{c} \int \frac{d^{4}k}{(2\pi)^{4}} \frac{k^{\mu}k^{\nu}/M_{A}^{2} - g^{\mu\nu}}{k^{2} - M_{A}^{2}}$$

$$\times \delta_{ij}\gamma_{\nu}\gamma_{5}\tau_{j}\frac{\not p - \not k + m_{q}}{(p - k)^{2} - m_{q}^{2}}\tau_{i}\gamma_{\mu}\gamma_{5}. \quad (3.3b)$$

These are divergent;  $\Sigma_S(p)$  is logarithmically and  $\Sigma_A(p)$  quadratically divergent. In the previous works, we employed the Pauli-Villars regularization to keep the divergences finite. In the present work, however, we shall employ the three-momentum cutoff method, since the Pauli-Villars method is not appropriate to regularize the quadratic divergence in  $\Sigma_A$ . The quadratic nature necessarily requires two independent cutoff parameters in the Pauli-Villars method, whereas it is sufficient to introduce a single cutoff parameter in the three-momentum cutoff scheme.

In the rest frame of the nucleon [i.e.,  $p = (p_0, \vec{0})$ ], the selfenergies as functions of  $p_0$  can be written as, after integrating Eqs. (3.3) over  $k_0$ ,

$$\Sigma_{S}(p_{0}) = \Sigma_{S}^{0}(p_{0}) + \Sigma_{S}^{1}(p_{0})\gamma_{0},$$
  

$$\Sigma_{A}(p_{0}) = \Sigma_{A}^{0}(p_{0}) + \Sigma_{A}^{1}(p_{0})\gamma_{0},$$
(3.4)

where the coefficients are given as

$$\Sigma_{S}^{0}(p_{0}) = -N_{c} \int_{0}^{\Lambda} \frac{k^{2} dk}{2\pi^{2}} \left\{ \frac{m_{q}}{2e_{1} [(p_{0} + e_{1})^{2} - e_{2}^{2}]} + \frac{m_{q}}{2e_{2} [(p_{0} - e_{2})^{2} - e_{1}^{2}]} \right\},$$

$$\Sigma_{S}^{1}(p_{0}) = -N_{c} \int_{0}^{\Lambda} \frac{k^{2} dk}{2\pi^{2}} \left\{ \frac{-e_{1}}{2e_{1} [(p_{0} + e_{1})^{2} - e_{2}^{2}]} + \frac{(p_{0} - e_{2})}{2e_{2} [(p_{0} - e_{2})^{2} - e_{1}^{2}]} \right\}$$
(3.5)

and

$$\begin{split} \Sigma_{A}^{0}(p_{0}) &= -\frac{N_{c}\vec{\tau}^{2}}{M_{A}^{2}} \int_{0}^{\Lambda} \frac{k^{2}dk}{2\pi^{2}} \Biggl\{ \frac{3m_{q}M_{A}^{2}}{2e_{3}[(e_{3}-p_{0})^{2}-e_{1}^{2}]} \\ &+ \frac{4m_{q}M_{A}^{2}-m_{q}[(p_{0}+e_{1})^{2}-\vec{k}^{2}]}{2e_{1}[(p_{0}+e_{1})^{2}-e_{3}^{2}]} \Biggr\}, \end{split}$$
(3.6)  
$$\Sigma_{A}^{1}(p_{0}) &= -\frac{N_{c}\vec{\tau}^{2}}{M_{A}^{2}} \int_{0}^{\Lambda} \frac{k^{2}dk}{2\pi^{2}} \Biggl\{ \frac{-3e_{3}M_{A}^{2}+p_{0}M_{A}^{2}+2e_{3}^{2}p_{0}}{2e_{3}[(e_{3}-p_{0})^{2}-e_{1}^{2}]} \\ &+ \frac{-2e_{1}M_{A}^{2}-e_{1}(p_{0}+e_{1})^{2}+(2p_{0}+e_{1})\vec{k}^{2}}{2e_{1}[(p_{0}+e_{1})^{2}-e_{3}^{2}]} \Biggr\}. \end{split}$$

In these equations  $N_c$  is the number of colors, and  $e_1 = \sqrt{\vec{k}^2 + m_q^2}$ ,  $e_2 = \sqrt{\vec{k}^2 + M_S^2}$ ,  $e_3 = \sqrt{\vec{k}^2 + M_A^2}$ .

Physical nucleon fields are defined such that the self-energy becomes the nucleon propagator on the nucleon mass shell. This condition is implemented by expanding the self-energy around  $p_0 = M_N$ :

$$\bar{B}\left(\cos^{2}\theta\Sigma_{S}(p_{0}) + \sin^{2}\theta\Sigma_{A}(p_{0}) - \frac{1}{\tilde{G}}\right)B$$
$$= Z^{-1}\bar{B}(p_{0}\gamma^{0} - M_{N})B$$
$$= \bar{B}_{phys}(p_{0}\gamma^{0} - M_{N})B_{phys}, \qquad (3.7)$$

where  $B_{\text{phys}} = \sqrt{Z^{-1}}B$  is the properly normalized physical nucleon field. The parameters *Z* and *M<sub>N</sub>* are the wave-function renormalization constant and the mass of the nucleon. The parameters *Z*, *M<sub>N</sub>*, and  $\tilde{G}$  are determined by the following conditions:

$$Z^{-1} = \frac{\partial \Sigma(p_0)}{\partial p_0} \bigg|_{p_0 \to M_N}$$
  
=  $\cos^2 \theta \frac{\partial \Sigma_S(M_N)}{\partial p_0} + \sin^2 \theta \frac{\partial \Sigma_A(M_N)}{\partial p_0},$  (3.8)

$$\tilde{G} = (\cos^2 \theta \Sigma_S(M_N) + \sin^2 \theta \Sigma_A(M_N))^{-1}.$$
 (3.9)

Therefore, we obtain the mass of the physical nucleon by solving Eqs. (3.8) and (3.9).

### **IV. RESULTS**

To begin, we briefly discuss our parameters, which are listed in Table I. We use the values in Ref. [5] for the mass of the constituent quarks,  $m_q$ , the NJL coupling constant G, and the cutoff mass  $\Lambda$ . Then  $m_q$ , G, and  $\Lambda$  are determined self-consistently in the NJL model by solving the gap equation and reproducing the pion decay constant  $f_{\pi} = 93$  MeV [2,11,15]. The masses of the scalar and axial-vector diquarks,  $M_S$  and  $M_A$ , may be determined, for instance, in the NJL

TABLE I. Model parameters. All parameters are in units of GeV.

$\overline{m_q}$	$M_S$	$M_A$	Λ
0.39	0.60	1.05	0.6

model by solving the Bethe-Salpeter (BS) equation in the corresponding diquark channels [3,16]. These masses have been also calculated by QCD oriented methods [17–19]. Results are, however, somewhat dependent on the methods. Here, instead of solving the BS equation rigorously, we simply choose a reasonable set of diquark masses. These parameters can reproduce, for instance, the mass splitting between the nucleon and delta [3,17].

In Fig. 3, we plot the real parts of the self-energies, Eqs. (3.5) and (3.6), as functions of  $p_0$ .

We find that both scalar and axial-vector channels are positive, meaning that both channels contribute to the mass of the nucleon attractively, or decrease the mass of the nucleon. Obviously the contributions of the axial-vector diquark part is considerably larger than that of the scalar diquark part, reflecting the stronger (quadratic) divergence of the former. In Fig. 3 we also explicitly show a threshold  $m_q + M_s$ . Above the threshold, the mass of the nucleon takes a complex value, which is not a physical consequence due to the absence of confinement in the present model. Recently, several studies including the mimic effects of the confinement have been made [20–22]. Although we continue this simple treatment, we should confine this model in the region  $M_N < m_q + M_s$ .

In this paper, the mass of the nucleon,  $M_N$ , is treated as a function of  $\tilde{G}$  and the mixing angle  $\theta$ . In Fig. 4, we show the contour plot for the nucleon mass as a function of  $\tilde{G}$  and  $\theta$ . One finds that both of the scalar and axial-vector parts of the self-energy contribute attractively to the mass of the nucleon and that the attraction from the axial part is larger than that from the scalar part. At  $\theta = 0$ , where only  $\Sigma_S$  contributes to the mass of the nucleon, the experimental value  $M_N = 0.94$  GeV is obtained when  $\tilde{G} \sim 17$ . In making comparisons, we note that in the present calculation the pion cloud effect is not included, its inclusion might make a substantial contribution to nucleon properties at the quantitative level [23–25]. Nevertheless, for the qualitative discussions in the present paper, we simply compare the results with experiments directly.



FIG. 4. Contour plot of the nucleon mass  $M_N$  as a function of the coupling constant  $\tilde{G}$  and the mixing angle  $\theta$ . Lines are for  $M_N = 0.94$ , 0.90, 0.85 GeV from bottom to top.

At  $\theta = 30$  degrees, for instance, the mass of the nucleon is reproduced when  $\tilde{G} \sim 9.5$ . As the mixing angle  $\theta$  increases, or the axial-vector component in the nucleon wave function becomes larger, the mass of the nucleon decreases. This behavior is also shown in Fig. 5, where  $\theta$  dependence is shown for fixed values of  $\tilde{G}$ . One sees that, for larger values of  $\theta$ ,  $M_N$ is reproduced for smaller values of  $\tilde{G}$ , showing once again that the attraction from the axial-vector part is larger than the attraction in the scalar part. Although we do not discuss it in this paper, a finite value of  $\theta$  is favored when explaining the isovector magnetic moments  $\mu$  and isovector axial-vector coupling constant  $g_A$ .

#### V. SUMMARY

In this paper, we have studied the nucleon state in terms of a microscopic model for hadrons, namely, a chiral quark-diquark model. The nucleon was constructed as a superposition of the two quark-diquark channels including the scalar and axial-vector diquarks. The quark-diquark model was then hadronized in the path-integral method to obtain an effective Lagrangian for the mesons and nucleon. The present work is an extension of previous ones including only a scalar diquark channel. Here, to test the validity of the method and to ascertain the role the axial-vector diquark channel plays, we investigated the mass of the nucleon, which was calculated through the renormalization conditions of the nucleon self-energies.



FIG. 3. Real parts of the self-energies  $\Sigma_{S,A}^1$  (left) and  $\Sigma_{S,A}^0$  (right). The vertical dashed lines represent the threshold (=  $m_q + M_S$ ) for the quark and scalar diquark channel.



FIG. 5. The mixing angle  $\theta$  dependence of the mass of the nucleon  $M_N$  for several values of the quark-diquark coupling constant  $\tilde{G}$ . The three curves are for  $\tilde{G} = 20$ , 17, 10 GeV<sup>-1</sup> from bottom to top as indicated. The dashed horizontal line is the threshold =  $m_q + M_s$ .

We found that the mass of the nucleon is reproduced by choosing the mixing angle  $\theta$  and the coupling constant  $\tilde{G}$  appropriately. Our result is consistent with previous work that involved solving the Faddeev equations for the three-quark system in the NJL model [14].

The present result suggests that determining a solution to the quark-diquark model is simpler than solving the threequark system directly; it is also a practically useful model for the description of the nucleon. As advocated previously, an advantage of the present method is to be able to work out to a large extent in an analytic way preserving important symmetries such as gauge and chiral symmetries.

Naturally, it is a further extension to apply the present method to various hadronic properties such as electromagnetic couplings and the nuclear force. For some quantities such as isovector magnetic moments and axial-vector coupling constants, it is expected that the axial-vector channel plays an important role [26,27]. Furthermore, this is necessary to describe the octet and decuplet baryons. In addition, the axial-vector channel may play another role that we did not consider explicitly in the present work, for only a single state of the nucleon was constructed. If both channels are treated as independent degrees of freedom, then the two-nucleon states may be described as bound states of the quark and diquarks. This is investigated in a separate paper [28].

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