Improved quark mass density-dependent model with quark and nonlinear scalar field coupling

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An improved quark mass density-dependent model that includes the coupling between the quarks and a nonlinear scalar field is presented. A numerical analysis of solutions of the model is performed over a wide range of parameters. The wave functions of the ground state and the lowest one-particle excited states with even and odd parities are given. The root-mean-squared radius, the magnetic moment, and the ratio between the axial-vector and the vector β-decay coupling constants of the nucleon are calculated. We found that the presented model is successful in describing the properties of the nucleon.

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I. INTRODUCTION

Since the conjecture of Witten [1] that strange quark matter (SQM) would be more stable than normal nuclear matter, much theoretical effort has been directed toward the investigation of its properties and applications [2–13]. Because of the well-known difficulty of QCD in the nonperturbative domain, many effective models reflecting the characteristics of the strong interaction are used to study SQM. They include the MIT bag model [2–4], the quark-meson coupling (QMC) model [5], the Friedberg-Lee (FL) soliton bag model [6], the chiral SU(3) quark model [7], the quark mass density-dependent (QMDD) model [8–11], the quark mass density- and temperature-dependent model [12,13], etc. In this paper we will focus our attention on the QMDD model.

The QMDD model was first suggested by G. N. Fowler, S. Raha, and R. M. Weiner. According to this model, the masses of u, d quarks and strange quarks (and the corresponding antiquarks) are given by

$$m_q = \frac{B}{3n_B}(i = u, d, \bar{u}, \bar{d}), \quad (1)$$

$$m_{s,\bar{s}} = m_{s0} + \frac{B}{3n_B}, \quad (2)$$

where m_{s0} is the current mass of the strange quark, B is the vacuum energy density inside the bag, and $n_B = \frac{1}{3}(n_u + n_d + n_s)$ is the baryon number density, with n_u, n_d, n_s representing the density of the u quark, d quark, and s quark. The basic hypothesis, Eqs. (1) and (2), in the QMDD model can easily be understand from the quark confinement mechanism. A confinement potential kr (r^2) must be added to a quark system in the phenomenological effective models because the perturbative QCD cannot give us the confinement solutions of quarks. The confinement potential prevents the quark from going to infinity or to very large regions. The large volume means that the density is small. This mechanism of confinement can be mimicked through the requirement that the mass of an isolated quark become infinitely large so that the vacuum is unable to support it [11,12]. This is just the physical picture given by Eqs. (1) and (2). In fact, this confinement mechanism is very similar to that of the MIT bag model. But the advantage of the QMDD model is that it does not need to introduce a quark confined boundary condition like that of the MIT bag model.

Although the QMDD model is successful for describing the properties of SQM [8–11], it is still an ideal quark gas model. Compared with the usual ideal quark gas model, the basic improvements of the QMDD model are that the quark masses depend on density and that the quark confinement mechanism is mimicked. Obviously, if we hope to investigate the physical properties of nucleons and hyperons by means of the QMDD model, the quark-quark interactions must be considered. Following the line of the QMC model, we will introduce quark and scalar σ field nonlinear coupling self-consistently to improve the QMDD model in this paper. As a first step, we ignore the s quark and consider the coupling of u and d quarks to a nonlinear scalar field only.

We hope to emphasize that there are two basic differences between our improved quark mass density-dependent (IQMDD) model and the usual QMC model suggested first by Guichon [14] and developed by Saito and Thomas [15] and by Jin and Jennings [16]. First, unlike in the QMC model, we do not need an MIT bag for the nucleon in the IQMDD model. The constraint of the MIT bag boundary condition disappears in our formulas because the quark confinement mechanism has been established in Eqs. (1) and (2). This is very important because it should provide a reasonable starting point for many-body calculations. Second, the interaction between the quark and the scalar meson is limited in the bag regions for the QMC model, but for the IQMDD model this interaction is extended to the whole free space. In fact, our model is similar to that of the FL model, but instead of massless quarks in the FL model, the masses of u and d quarks in our model are given by Eqs. (1) and (2).

This paper is organized as follows. The main formulas of the IQMDD model will be given in the next section. The numerical

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results for the ground state and a number of physical quantities, namely, the root-mean-squared (rms) charge radius r_p, the magnetic moment μ_p, and the ratio between the axial-vector and the vector β-decay coupling constants of the nucleon g_A/g_V, will be presented in Sec. III. In Sec. IV we will study the lowest one-particle excited states with even or odd parity. The last section is a summary and discussion.

II. IMPROVED QUARK MASS DENSITY-DEPENDENT MODEL

We now briefly outline the model and its pertinent features below. The Hamiltonian density of the IQMDD model reads as

$$H = \psi^+ \left[\frac{1}{i}\overrightarrow{\alpha} \cdot \overrightarrow{\nabla} + \beta(m_q + f\sigma)\right]\psi + \frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\sigma)^2 + U(\sigma), \quad (3)$$

where $m_q = B/3n_B$ is the mass of the $u(d)$ quark, $\overrightarrow{\alpha}$ and β are the standard Dirac matrices, ψ represents the quark quantum field (color and flavor indices suppressed) satisfying the canonical anticommutation relations

$$\{\bar{\psi}(\overrightarrow{r}, t), \psi(\overrightarrow{r}', t)\} = \delta^3(\overrightarrow{r} - \overrightarrow{r}'), \quad (4)$$

and f is the coupling constant between the quark field ψ and the meson field σ. The σ field is considered independent of time, and consequently the commutator

$$[\pi(\overrightarrow{r}), \sigma(\overrightarrow{r}')] = 0, \quad (5)$$

where π is the conjugate field of the scalar meson field. Hence σ is treated as a classical field. In Eq. (3) $U(\sigma)$ is the self-interaction potential for the σ field, which has the phenomenological form

$$U(\sigma) = \frac{c_2}{2}\sigma^2 + \frac{c_3}{6}\sigma^3 + \frac{c_4}{24}\sigma^4 + B, \quad (6)$$

$$c_3^2 > 3c_2c_4, \quad (7)$$

to ensure that the absolute minimum of $U(\sigma)$ is at $\sigma = \sigma_{\text{vac}} \neq 0$. The bag constant B is introduced in order that

$$U(\sigma_{\text{vac}}) = 0, \quad U(0) = B. \quad (8)$$

Without any lose of generality, we may choose $c_3 < 0$, and therefore $\sigma_{\text{vac}} > 0$:

$$\sigma_{\text{vac}} = \frac{3}{2c_4}\left[-c_3 + \left(c_3^2 - \frac{8}{3}c_2c_4\right)^{1/2}\right]. \quad (9)$$

One can construct a Fock space of quark states and expand the operator ψ in terms of annihilation and creation operators on this space with c-number spinor functions $\varphi_n^{(\pm)}$, which satisfies the Dirac equation

$$\left[\frac{1}{i}\overrightarrow{\alpha}\cdot\overrightarrow{\nabla} + \beta(m_q + f\sigma)\right]\varphi_n^{(\pm)} = \pm\epsilon_i\varphi_n^{(\pm)}, \quad (10)$$

with superscripts $\pm$ denoting the positive and the negative energy solutions, respectively. The spinor functions φ_n are normalized according to

$$\int \varphi_n^+\varphi_n d^3r = 1. \quad (11)$$

The total energy of the quark-scalar field system is given by

$$E(\sigma) = \sum_n \epsilon_n + \int \left[\frac{1}{2}(\nabla\sigma)^2 + U(\sigma)\right] d^3r. \quad (12)$$

The minimum of $E(\sigma)$ occurs when σ is the solution of

$$-\nabla^2\sigma + \frac{dU(\sigma)}{d\sigma} = -f\sum_n \bar{\varphi}_n\varphi_n. \quad (13)$$

The equations of the quark field for the ground state and the excited states will be given in Secs. III and IV, respectively.

III. GROUND STATE SOLUTION

We discuss the ground state solution of the system now. Define φ as the lowest positive energy wave function. It can be expressed as

$$\varphi = \begin{pmatrix} u \\ i\left(\frac{\overrightarrow{\sigma}\cdot\overrightarrow{r}}{r}\right)v \end{pmatrix}\chi_m, \quad (14)$$

where $\overrightarrow{\sigma}$ are the Pauli matrices,

$$\chi_m = \begin{pmatrix}1\\0\end{pmatrix} \quad \text{or} \quad \begin{pmatrix}0\\1\end{pmatrix}.$$

The radial functions $u(r)$ and $v(r)$ satisfy

$$\frac{du(r)}{dr} = -[\epsilon + m_q + f\sigma(r)]v(r), \quad (15)$$

$$\frac{dv(r)}{dr} = -2\frac{v(r)}{r} + [\epsilon - m_q - f\sigma(r)]u(r). \quad (16)$$

The normalized condition reads as

$$4\pi\int_0^\infty [u^2(r) + v^2(r)]r^2dr = 1, \quad (17)$$

and the equation of motion of the σ field, Eq. (13), becomes

$$\frac{d^2\sigma(r)}{dr^2} + \frac{2}{r}\frac{\sigma(r)}{dr} = U'(\sigma(r)) + 3f[u^2(r) - v^2(r)]. \quad (18)$$

Equations (15), (16), and (18) can be solved with the boundary conditions

$$v(r=0) = 0, \quad u(r=\infty) = 0, \quad v(r=\infty) = 0,$$
$$\sigma(r=\infty) = \sigma_V, \quad \sigma'(r=0) = 0. \quad (19)$$

Before numerical calculation, we address the parameters of the model first. There are four free parameters, namely, c_2, c_3, c_4, and f in the IQMDD model. The parameters c_2, c_3, and c_4 fix the interaction potential U, while f measures the coupling between the quark and the scalar field. The set of equations (15), (16), and (18) should be solved alternately until consistency is obtained by using the iterative method [17]. Once the solutions of the above equations are obtained, one can calculate a number of physical quantities pertaining to the three-quark system, which have been measured experimentally. Let r_p, μ_p,

TABLE I. Variation of the properties with increasing parameters for the $B^{1/4} = 145$ MeV bag for the two values of f.

f	c_4	ϵ	E	μ_p	g_A/g_V
30	8×10^4	1.54	6.54	2.39	0.904
	1×10^5	1.56	6.56	2.38	0.929
	4×10^5	1.74	6.94	2.25	1.06
	8×10^5	1.85	7.26	2.15	1.11
200	8×10^4		No solution		
	1×10^5		No solution		
	4×10^5	1.24	5.23	2.64	0.603
	8×10^5	1.22	5.21	2.63	0.618

and g_A/g_V be, respectively, the rms charge radius, the magnetic moment, and the ratio between the axial-vector and the vector β-decay coupling constants of the nucleon. They satisfy [18]

$$\langle r_p^2 \rangle = \int \varphi^+ \varphi r^2 d^3r \Big/ \int \varphi^+ \varphi \, d^3r, \tag{20}$$

$$\mu_p = \frac{1}{2} \left(\int \vec{r} \times \varphi^+ \vec{\alpha} \varphi d^3r \right)_z \Big/ \int \varphi^+ \varphi d^3r, \tag{21}$$

$$g_A/g_V = \frac{5}{3} \int \varphi^+ \sigma_z \varphi d^3r \Big/ \int \varphi^+ \varphi d^3r. \tag{22}$$

By using Eq. (14), we find that

$$\langle r_p^2 \rangle = 4\pi \int_0^\infty (u^2 + v^2) r^4 \, dr, \tag{23}$$

$$\mu_p = \frac{8\pi}{3} \int_0^\infty r^3 uv \, dr, \tag{24}$$

$$g_A/g_V = \frac{20\pi}{3} \int_0^\infty r^2 \left(u^2 - \frac{1}{3} v^2 \right) dr. \tag{25}$$

Since in the present form the model is flavor independent, the charge radius of the neutron in the model is

$$\langle r_n^2 \rangle \equiv 0, \tag{26}$$

and the neutron magnetic moment is

$$\mu_n = -\tfrac{2}{3} \mu_p, \tag{27}$$

as given by $SU(6)$ algebra. Corrections to these relations arise only when QCD effects are included.

For comparison, let us remind ourselves that the experimental values of the proton mass $E = 4.69$ fm^{-1}, the proton magnetic moment $\mu = 2.79$ nuclear magnetons, $g_A/g_V = 1.25$, and the charge radius of proton $r_p = 0.83$ fm. To do the numerical calculation, we follow Refs. [17,18]. We fix $r_p = 0.83$ fm first. Once this value is chosen, there are only three free parameters. We have studied mostly two families of parameters for the ground state system. These are the cases $c_2 = 0$ and $B^{1/4} = 145$ MeV. Each choice fixes one other parameter by a specific requirement for the shape of the potential. Therefore it is sufficient to label our results with the coupling constant f and constant c_4.

Our results for the two families ($c_2 = 0$, $B^{1/4} = 145$ MeV) are summarized in three tables. Throughout these tables we use the units $\hbar = c = 1$ and use fomtometers as the fundamental unit for length.

We consider the case $B^{1/4} = 145$ MeV first. The bag constant B corresponds to the difference between two minima of the $U(\sigma)$. In Table I we list the bag properties as a function of the parameter c_4 for two values of the coupling constant f. The variation of bag properties with the coupling constant f for several values of c_4 is given in Table II. Several features emerge from these calculations. First, we note that an increase of the coupling constant produces a continuous change from a volume quark distribution for small f to a surface quark distribution for large f. This change is illustrated in Fig. 1, where we plot the quark density $u^2 - v^2$ for three values of f. It should be noted that shape of the soliton field does not change significantly with f for given c_4. The variations of the quark charge density as a function of the radius with increasing c_4 are plotted in Figs. 2 and 3. When c_4 decreases, with a fixed value of f, the quark charge distribution $u^2 - v^2$ changes slowly from surface to volume. This is illustrated in Figs. 2 and 3 for $f = 75$ and $f = 200$, respectively. The change from volume to surface quark charge density is also evident in the variations of the values of the magnetic moment μ_p and g_A/g_V. For instance, from Table II, when $c_4 = 8 \times 10^5$, the magnetic moment varies with increasing f from $1.87\mu_B$ to $2.61\mu_B$, where μ_B is the Bohr magneton of the proton. Similarly, g_A/g_V varies from 1.21 for small f to 0.618 for large f. It is also found from the tables that for some c_4 and f cases we can not find a solution for $r_p = 0.83$ fm.

The variation of the scalar field σ as a function of the radius for $f = 50$, $c_4 = 2 \times 10^5$, and $B^{1/4} = 145$ MeV is presented in Fig. 4. The value of σ inside the hadron is very different

TABLE II. Variation of the properties as a function of f for several values of c_4 with $B^{1/4} = 145$ MeV.

c_4	f	ϵ	E	μ_p	g_A/g_V
8×10^5	15	2.18	8.16	1.87	1.21
	30	1.85	7.26	2.14	1.11
	50	1.56	6.47	2.39	0.936
	75	1.41	5.97	2.49	0.794
	100	1.24	5.70	2.55	0.720
	200	1.22	5.21	2.61	0.618
4×10^5	15	2.11	8.01	1.96	1.22
	30	1.74	6.94	2.25	1.06
	50	1.49	6.28	2.43	0.868
	75	1.36	6.07	2.55	0.743
	100	1.30	5.81	2.57	0.680
	200	1.24	5.23	2.63	0.603
1×10^5	15	1.96	7.62	2.08	1.17
	30	1.56	6.56	2.39	0.929
	50	1.41	6.09	2.48	0.765
	75	1.33	5.77	2.50	0.677
	100		No solution		
	200		No solution		
8×10^4	15	1.94	7.61	2.09	1.16
	30	1.54	6.54	2.39	0.904
	50		No solution		
	75		No solution		
	100		No solution		
	200		No solution		

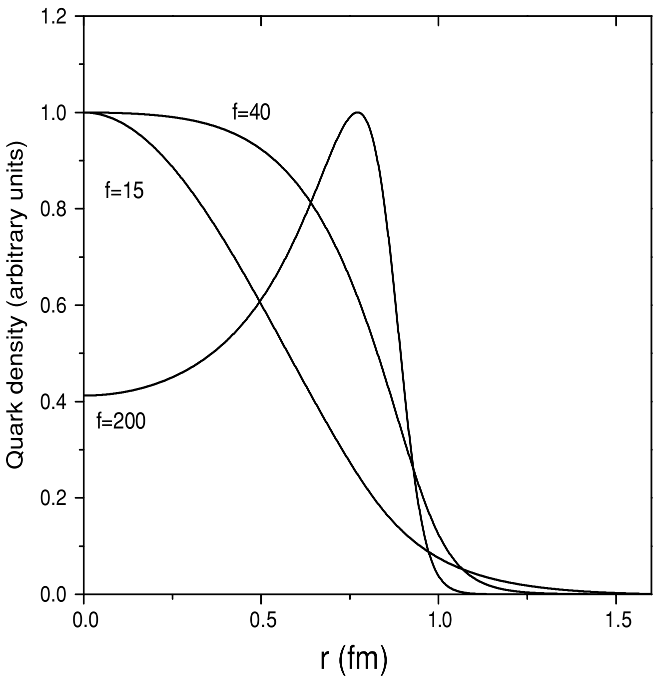

FIG. 1. Quark density $u^2 - v^2$ versus radius for $f = 15, 40, 200$ for $B^{1/4} = 145$ MeV.

from that of outside: inside σ is less than zero, but outside σ approaches $\sigma_{\rm vac}$. The abrupt transition of the scalar field through the hadron surface will contribute to the total energy remarkably, as is shown in Eqs. (11) and (12).

Another family of parameters considered is characterized by $c_2 = 0$. In this case $U(\sigma)$ has an inflection point at $\sigma = 0$, and only one minimum. We vary c_3, c_4, and f subject to the bag size constraint $r_p = 0.83$ fm. Our numerical results for this family are summarized in Table III. We find that when f increases, the character of the bag changes from volume confinement to surface confinement again. Similarly, we also find that for a given f, when c_4 increases, the magnetic moment decreases. In summary, we find the following in both cases.

(1) The quark energy ϵ and the ratio g_A/g_V decrease with increasing parameters f and c_4.
(2) On the other hand, the magnetic moment μ has just the opposite behavior: it is a decreasing function of

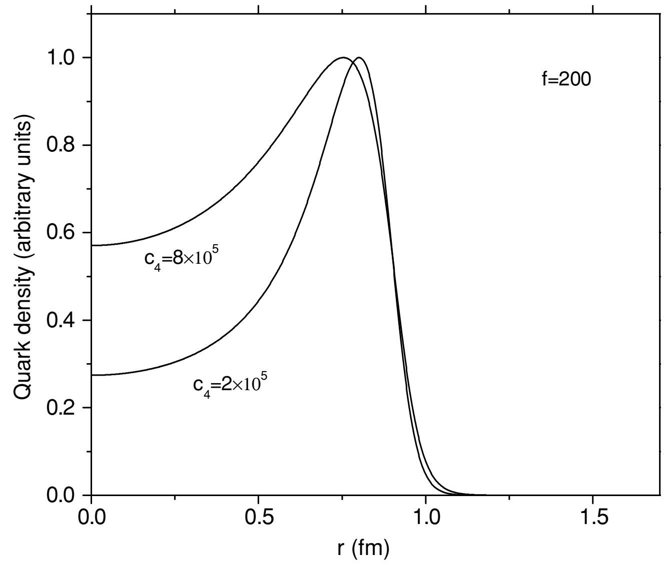

FIG. 3. Same as Fig. 2, but $f = 200$.

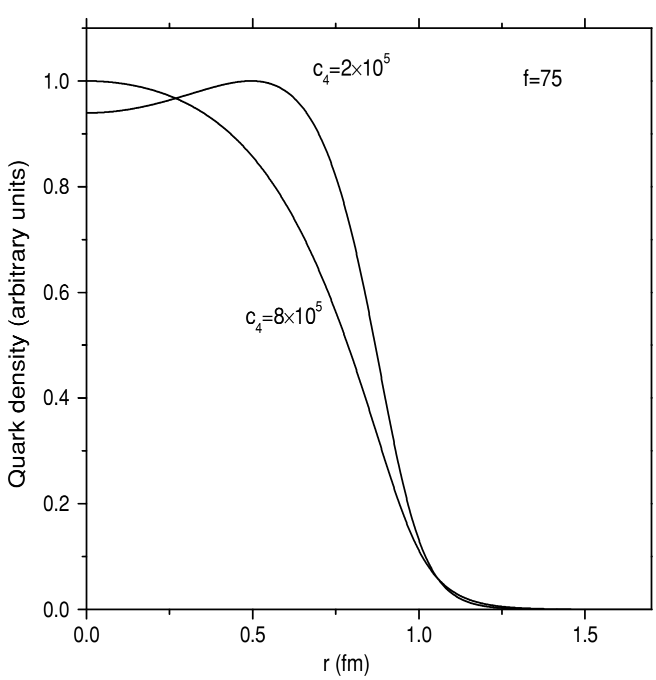

FIG. 2. Quark density $u^2 - v^2$ versus radius for $c_4 = 2 \times 10^5$, 8×10^5, $B^{1/4} = 145$ MeV, and $f = 75$.

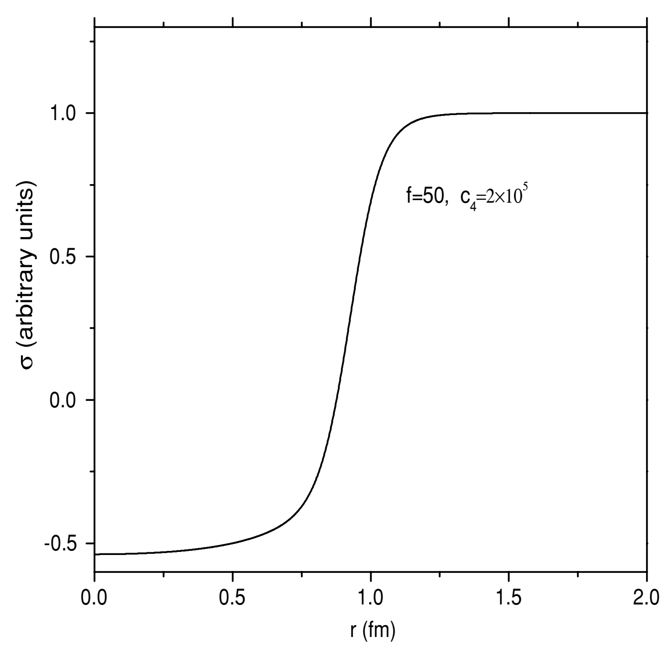

FIG. 4. Soliton field σ versus radius for $f = 50$ and $c_4 = 2 \times 10^5$, $B^{1/4} = 145$ MeV.

TABLE III. Variation of the properties as a function of f and c_4 with $c_2 = 0$.

c_4	f	ϵ	E	μ_p	g_A/g_V
1×10^4	15	1.64	7.00	2.31	1.00
	30	1.38	6.17	2.50	0.769
	50	1.29	5.77	2.57	0.667
	75	1.25	5.53	2.60	0.621
	100	1.24	5.43	2.61	0.601
	150	1.22	5.31	2.63	0.583
2×10^4	15	1.64	7.00	2.23	1.05
	30	1.38	6.17	2.48	0.810
	50	1.29	5.77	2.57	0.692
	75	1.25	5.53	2.60	0.635
	100	1.24	5.43	2.63	0.609
	150	1.22	5.31	2.63	0.587
4×10^4	15	1.83	7.36	2.16	1.12
	30	1.48	6.41	2.46	0.857
	50	1.34	5.97	2.55	0.723
	75	1.28	5.71	2.60	0.652
	100	1.25	5.57	2.63	0.620
	150	1.24	5.51	2.61	0.592

the parameter c_4 and an increasing function of the parameter f.

(3) The total energy of the ground state decreases as a function of f and increases as a function of c_4. The rise of the total energy E is caused by a contribution from 3ϵ, mainly.

(4) An increase of the coupling constant produces a continuous change from a volume quark distribution to a surface quark distribution, and an decrease of c_4 can produce the same results.

IV. ONE-PARTICLE EXCITED STATES

The one-particle excited states for the FL model were calculated by Saly and Sundaresan [19]. In this section we use their method to study the one-particle excited state for the IQMDD model.

A. First excited state with even parity

In the configuration of the first excited state with even parity, we shall consider two quarks to be in the ground state, and one quark will be placed in the first excited level ϵ_1. Then the system of equations to be solved is

$$\frac{du(r)}{dr} = -[\epsilon_0 + m_q + f\sigma(r)]v(r), \tag{28a}$$

$$\frac{dv(r)}{dr} = -2\frac{v(r)}{r} + [\epsilon_0 - m_q - f\sigma(r)]u(r), \tag{28b}$$

$$\frac{du_1(r)}{dr} = -[\epsilon_1 + m_q + f\sigma(r)]v_1(r), \tag{28c}$$

$$\frac{dv_1(r)}{dr} = -2\frac{v_1(r)}{r} + [\epsilon_1 - m_q - f\sigma(r)]u_1(r), \tag{28d}$$

$$4\pi\int_0^\infty [u^2(r) + v^2(r)]r^2 dr = 1, \tag{28e}$$

$$4\pi\int_0^\infty \left[u_1^2(r) + v_1^2(r)\right]r^2 dr = 1, \tag{28f}$$

$$\frac{d^2\sigma(r)}{dr^2} + \frac{2}{r}\frac{d\sigma(r)}{dr} = U'(\sigma) + 2f[u^2(r) - v^2(r)] + f\left[u_1^2(r) - v_1^2(r)\right]. \tag{28g}$$

The set of equations (28a)–(28g) is to be solved with the boundary conditions

$$\begin{aligned} v(r=0) &= 0, & v_1(r=0) &= 0, \\ u(r=\infty) &= 0, & v(r=\infty) &= 0, \\ v_1(r=\infty) &= 0, & u_1(r=\infty) &= 0, \\ \sigma(r=\infty) &= \sigma_V, & \sigma'(r=0) &= 0. \end{aligned} \tag{29}$$

B. Lowest-energy state with odd parity

To obtain the lowest-energy odd-parity state, we place two quarks in the ground state of even-parity and one quark in the lowest odd-parity state. In this case the system of equations is

$$\frac{d\widetilde{u}(r)}{dr} = -[\widetilde{\epsilon}_0 + m_q + f\sigma(r)]\widetilde{v}(r), \tag{30a}$$

$$\frac{d\widetilde{v}(r)}{dr} = -2\frac{\widetilde{v}(r)}{r} + [\widetilde{\epsilon}_0 - m_q - f\sigma(r)]\widetilde{u}(r), \tag{30b}$$

$$\frac{d\widetilde{v}_1(r)}{dr} = [\widetilde{\epsilon}_1 - m_q - f\sigma(r)]\widetilde{u}_1(r), \tag{30c}$$

TABLE IV. Dependence of the solutions on c_4 for $f = 30$.

Ground state			Even-parity state			Odd-parity state		
c_4	ϵ	E	ϵ_0	ϵ_1	E_+	$\widetilde{\epsilon}_0$	$\widetilde{\epsilon}_1$	E_-
8×10^5	1.85	7.26	2.02	3.92	9.11	1.87	3.10	8.70
6×10^5	1.81	7.16	1.99	3.99	9.17	1.84	3.11	8.71
4×10^5	1.76	7.09	1.95	4.12	9.29	1.80	3.11	8.74
2×10^5	1.63	6.72	1.82	4.16	9.13		No solution	
1×10^5	1.56	6.56	1.72	4.43	9.36		No solution	
8×10^4	1.54	6.54	1.58	4.41	9.84		No solution	

TABLE V. Dependence of the solutions on f for $c_4 = 4 \times 10^5$.

	Ground state		Even-parity state			Odd-parity state		
f	ϵ	E	ϵ_0	ϵ_1	E_+	$\widetilde{\epsilon}_0$	$\widetilde{\epsilon}_1$	E_-
15	2.09	7.93		No solution			No solution	
20	1.95	7.53	2.18	3.80	9.17	2.01	3.21	8.82
25	1.85	7.32	2.05	3.91	9.21	1.88	3.14	8.79
30	1.76	7.09	1.95	4.12	9.29		No solution	
35	1.66	6.79	1.80	4.32	9.29		No solution	
40	1.59	6.59	1.59	4.28	9.60		No solution	
50	1.49	6.30	1.48	4.19	9.63		No solution	

$$\frac{d\widetilde{u}_1(r)}{dr} = -2\frac{\widetilde{u}_1(r)}{r} - [\widetilde{\epsilon}_1 + m_q + f\sigma(r)]\widetilde{v}_1(r), \quad (30d)$$

$$4\pi \int_0^\infty [\widetilde{u}^2(r) + \widetilde{v}^2(r)]r^2 dr = 1, \quad (30e)$$

$$4\pi \int_0^\infty [\widetilde{u}_1^2(r) + \widetilde{v}_1^2(r)]r^2 dr = 1, \quad (30f)$$

$$\frac{d^2\sigma(r)}{dr^2} + \frac{2}{r}\frac{d\sigma(r)}{dr} = U'(\sigma) + 2f[\widetilde{u}^2(r) - \widetilde{v}^2(r)] + f[\widetilde{u}_1^2(r) - \widetilde{v}_1^2(r)], \quad (30g)$$

where $\widetilde{u}_1(r)$, $\widetilde{v}_1(r)$ are the components of wave function φ:

$$\varphi = \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{r}}{r}\widetilde{u}_1(r) \\ 0 \\ i\widetilde{v}_1(r) \\ 0 \end{pmatrix}. \quad (31)$$

The set of equations (30a)–(30g) is to be solved with the boundary conditions

$$\begin{aligned} \widetilde{v}(r=0) &= 0, & \widetilde{u}_1(r=0) &= 0, \\ \widetilde{u}(r=\infty) &= 0, & \widetilde{v}(r=\infty) &= 0, \\ \widetilde{u}_1(r=\infty) &= 0, & \widetilde{v}_1(r=\infty) &= 0, \\ \sigma(r=\infty) &= \sigma_V, & \sigma'(r=0) &= 0. \end{aligned} \quad (32)$$

For simplicity, we consider the case $B^{1/4} = 145$ MeV, $r_p = 0.83$ fm only.

Our results are summarized in Tables IV and V and Figs. 5 and 6. In Table IV we fix $r_p = 0.83$ fm, $B^{1/4} = 145$ MeV, and $f = 30$ and show the dependence of the ground state energy E, the first even-parity excited state energy E_+, and the first odd-parity state energy E_- on c_4. The dependence of E, E_+, and E_- on f for $c_4 = 4 \times 10^5$ is shown in Table V. Remember that the experimental values of the (uud) system are $E = 4.69\,\text{fm}^{-1}$, $E_+ = 7.35\,\text{fm}^{-1}$, and $E_- = 7.67\,\text{fm}^{-1}$ and compare our results with that given by Tables VI and VII of Ref. [19] for the FL model with massless quarks; we find that the agreement with experiments for the IQMDD model is better than that of FL model.

Finally, we hope to point out, as shown in Tables IV and V, that the allowed range of parameters is large for the existence of the first even-parity excited state. The only restriction is that the height of the potential well $f\sigma_V$ must be greater than the quark eigenvalues ϵ_1. But for odd-parity states, the allowed range of parameters is severely restricted. We cannot find a solution in many cases. To demonstrate this point more transparently, for

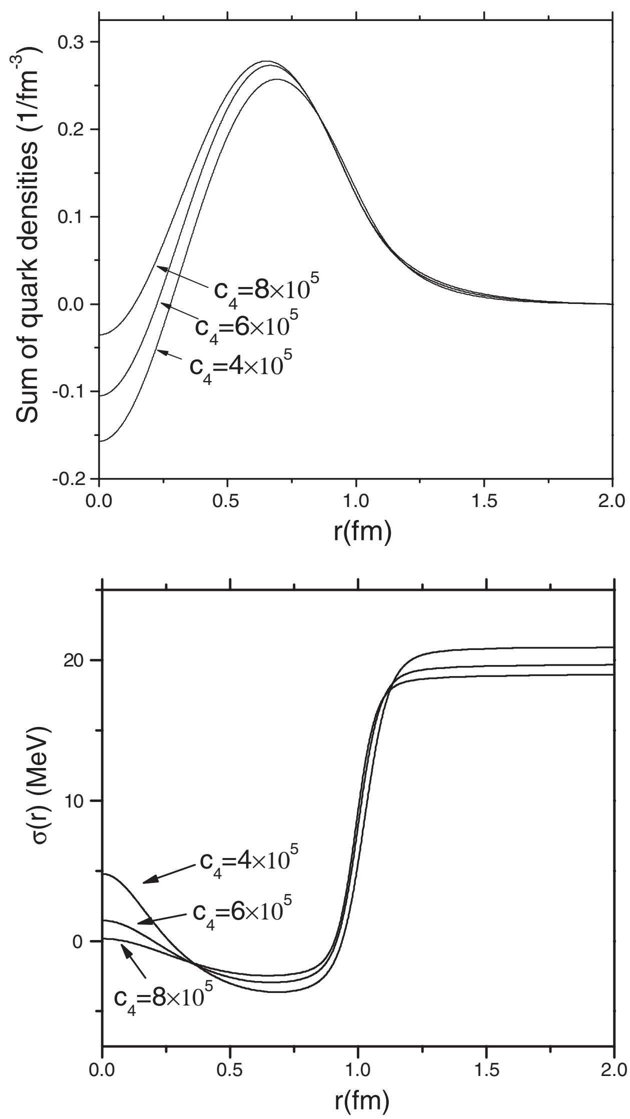

FIG. 5. Dependence of the odd-parity solutions on the parameter c_4. Other parameters are kept at $f = 30$ and $r_p = 0.83$ fm for $B^{1/4} = 145$ MeV. No solutions exist for smaller values of c_4.

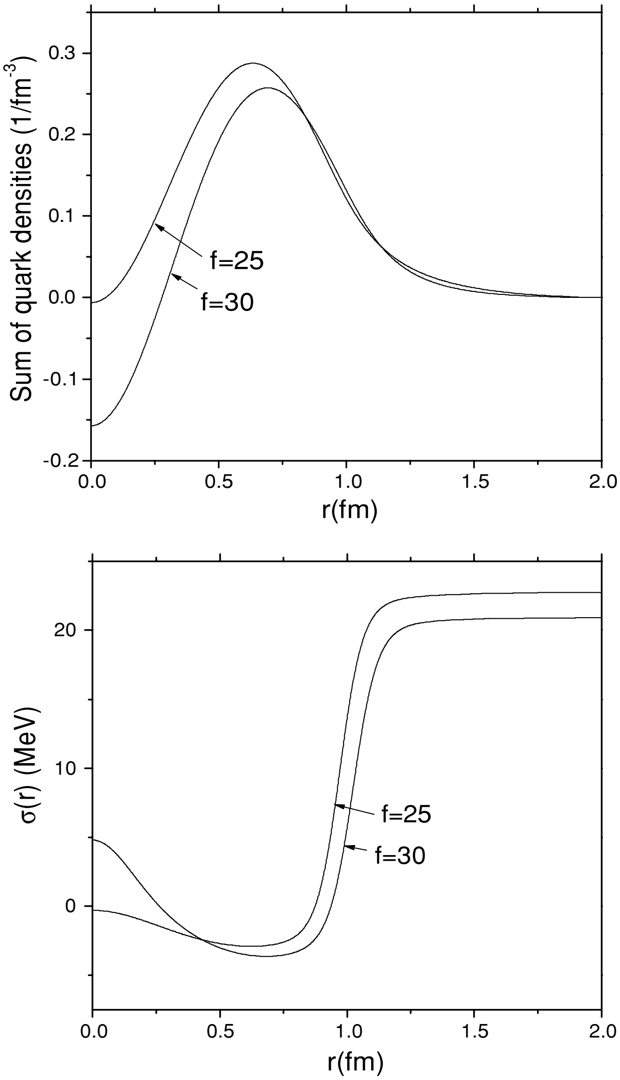

FIG. 6. Dependence of the odd-parity solutions on the parameter c_4. Other parameters are kept at $f = 30$ and $r_p = 0.83$ fm for $B^{1/4} =$ 145 MeV. No solutions exist for higher values of f.

odd-parity solutions, we fix $r_p = 0.83$ fm, $B^{1/4} = 145$ MeV, and $f = 30$ and draw the curves for total quark density $[2(u^2 - v^2) + \tilde{u_1}^2 - \tilde{v_1}^2]$ versus r and for scalar field σ versus r for with c_4 equal to 4×10^5, 6×10^5, and 8×10^5, respectively in Fig. 5. We find that no solutions exist for smaller values of c_4. The same curves for fixed $c_4 = 4 \times 10^5$ but $f = 20$ and $f = 25$ are shown in Fig. 6. We also find that no solutions exist for higher values of f.

V. SUMMARY AND DISCUSSION

After introducing the quark and nonlinear scalar field coupling, we suggest an improved quark mass density-dependent model. We obtain soliton solutions of ground state and excited states for the coupled equations for quark and scalar fields satisfying the required boundary conditions by a numerical method. Since the most interesting question is whether the results of these calculations resemble the physics that we are trying to describe, we concentrate on the dependence of the results on the phenomenological parameters introduced in this model. We present these dependence in the form of tables and figures for the ground state and low-lying excited states. The wave functions of the quark are given. By using the wave function of the ground state, we have calculated the rms charge radius, the magnetic moment, and the ratio between the axial-vector and the vector β-decay coupling constant of the nucleon and compared these values with experiment. We find that the results given by the IQMDD model are in agreement with experiment.

Since the boundary condition of the MIT bag model has been omitted in the IQMDD model, the many-body calculation beyond the mean field approximation can be easily carried out in this model.

We note that the study in this paper is still limited to zero temperature and u, d quarks. Since the spontaneous breaking symmetry of $U(\sigma)$ will be restored at finite temperature, it is of interest to extend our discussion to finite temperature and to study the effect of s quarks for hyperons. Work on this topic is in progress.

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