Reliable potential for studying fusion of weakly bound nuclei

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We propose that the parameter-free São Paulo potential be applied in studies of fusion involving weakly bound nuclei in situations in which experimental barrier distributions cannot be obtained. If one aims to study the effect of the breakup process on the fusion cross section, comparison of data with theoretical calculations requires a realistic and trustworthy bare potential. We show that the São Paulo potential has these characteristics.

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I. INTRODUCTION

Large theoretical and experimental efforts have been concentrated on studies of the influence of the breakup process on the fusion cross section for systems involving weakly bound nuclei (see, e.g., Refs. [1–4]). The suitable stable weakly bound nuclei for this sort of investigation are ^{6}Li, ^{7}Li, and ^{9}Be, which have small threshold energies against breakup, from 1.4 to 2.5 MeV. The main radioactive nuclei studied so far are ^{6}He, ^{11}Be, and ^{17}F, with even smaller threshold energies. Theorists have been facing conflicting ideas about whether the fusion of weakly bound nuclei is enhanced or hindered owing to the strong coupling to the breakup channel [5–10]. To illuminate this question, experimental fusion cross sections, both above and below the Coulomb barrier, are usually compared with results of theoretical coupled-channel (CC) calculations, in which the predictions do not take into account the breakup process or continuum discretized CC calculations are performed, including continuum-continuum couplings. A critical question when one wants to compare fusion data with theory is the choice of the bare potential to be adopted. The conclusions concerning the effect of the breakup on the fusion, depending on its characteristics, may be rather different. In this sense, it is useful to experimentally derive the fusion barrier distribution (BD) that acts as a strong constraint to the potential parameters, as the potential should match the barrier height and shape of the BD. When the BD is not available, a "reasonable" potential is often used, which however, might not be appropriate.

Formally, when dealing with heavy-ion nuclear reactions, one should solve a large number of coupled equations to take into account all reaction channels. In practical terms, some degree of simplification is always adopted in this kind of analysis. One can perform CC calculations with only a few channels and, in this case, the optical potential involved in the analysis would be the bare potential added to the polarization-potential contributions from the other channels. If all relevant channels were included in the CC calculations, with appropriate form factors, deformation parameters, etc., then the polarization potential would be negligible and the optical potential would be very similar to the bare potential. In many works the data analyses involve several adjustable parameters used to account for the data. In this case, however, one should verify whether the resulting parameters are within acceptable ranges defined from fundamental grounds. Otherwise, this approach has not much physical significance and the procedure would be only an alternative representation of the data set itself. Therefore, if a reliable estimate for the bare potential is available, the data-extracted parameters of the potential should be in reasonable agreement with the bare potential. In this paper, we propose adopting the São Paulo interaction [11,12] as the bare potential for weakly bound systems.

II. SÃO PAULO POTENTIAL

The São Paulo potential (SPP) is a model for the real part of the interaction of heavy-ion systems, which is based on the effects of Pauli nonlocality [12–14]. Apart from theoretical considerations, experimental evidence strongly supports the model. Indeed, the SPP has been successfully used to describe several reaction mechanisms, including peripheral channels and the fusion process, for a large number of systems and in a very wide energy range [11,13,15–29]. Within this model the nuclear interaction is connected with the folding potential V_F through [12]

$$V_N(R) = V_F(R)e^{-4v^2(R)/c^2}, \tag{1}$$

where c is the speed of light and $v(R)$ is the local relative velocity between the two nuclei. The velocity dependence of the potential is fundamental to accounting for the data from subbarrier to intermediate energies, about 200 MeV/nucleon.

With the aim of providing a parameter-free description of the nuclear interaction, the SPP model includes an extensive systematics of nuclear densities [12]. The two-parameter Fermi (2pF) distribution is assumed to be a good approximation to describe the densities. The radii of the 2pF distributions are well represented by

$$R_0 = 1.31A^{1/3} - 0.84 \text{ fm}, \tag{2}$$

where A is the number of nucleons of the nucleus. The values obtained for the diffuseness of the matter distributions are similar throughout the periodic table and present small variations around the average value $a = 0.56$ fm. Within the context of realistic systematics, the SPP does not contain any adjustable parameter. This is in fact an essential feature, because the lack of adjustable parameters characterizes the

model as a powerful tool for making predictions for quite different systems and energies.

The SPP has been used to describe fusion cross-section data for hundreds of systems, besides elastic scattering and peripheral reaction channels, within the context of the unidimensional barrier-penetration model (BPM) [19,23,24]. Although the above barrier data were accounted for within about 10% precision, the theoretical BPM cross sections underestimate the subbarrier data by several orders of magnitude, mainly for heavy systems. Of course, as expected, for heavy and very deformed systems, in which the couplings are very strong, the SPP cannot account for coupling effects, such as large enhancements of subbarrier fusion cross sections, in the context of the BPM. On the other hand, for light systems with weak couplings, the use of the parameter-free SPP within the BPM produces quite good predictions for fusion cross sections also at subbarrier energies [23].

In the context of parameter-free systematics for densities, a systematization of the imaginary part of the optical potential was also obtained:

$$W(R,E) = N_I V_N(R,E), \tag{3}$$

where $V_N(R,E)$, whose energy dependence results from the local equivalence of the otherwise nonlocal interaction [12–14], is obtained by Eq. (1). For numerous systems, elastic-scattering angular distributions and total reaction cross sections over wide energy ranges were simultaneously well fitted with an optical potential defined by Eqs. (1) and (3), with $N_I = 0.78$ [11].

Of course, the model has limitations. As already stated, the SPP cannot account for strong couplings within a simple BPM prescription. In these cases, CC calculations are obviously required. In addition, if all relevant channels are not included in the CC calculations, significant contributions of the polarization potential must be added to the SPP to obtain the optical potential. Furthermore, as extensively discussed in Ref. [12] and already commented on here, the model is based on systematics of nuclear densities, and it is only within this context that the SPP has no adjustable parameters. Small deviations of the density distribution parameters R_0 and a from the average values are expected along the table of nuclides because of structure effects. These deviations are reflected also in the potential strengths. Therefore there is room for small variations of the potential not only because of the spread of the density parameters but also because of possible contributions of the polarization. In Ref. [12] it is estimated that, because of these possible density parameter variations, the potential strength at the barrier radius could vary by about 20%.

III. TESTING THE STANDARD SÃO PAULO POTENTIAL

The SPP has already been successfully applied in the description of elastic scattering for systems involving exotic nuclei [18,22]. However, so far it has not been used in the investigation of the effect of breakup on the fusion cross section, except through analyses of the elastic-scattering threshold anomaly for a few systems involving weakly bound nuclei [25,27]. Therefore we decided to test its consistency by comparing the corresponding results with those obtained through adjusted potentials derived in previous works, in which experimental BDs were obtained.

We tested the SPP with earlier results for ^{6}Li + ^{209}Bi [30,31], ^{7}Li + ^{209}Bi [30,31], ^{9}Be + ^{208}Pb [31,32], and the tightly bound ^{16}O + ^{144}Sm [33]. Woods-Saxon (WS) shape potentials were assumed in those works, with parameters adjusted such that the corresponding barrier heights (V_B) matched those measured. CC calculations were performed and, for the three weakly bound systems, suppression factors (SFs) of the complete fusion cross sections were observed at energies above the barrier.

In the present analyses, we have used the CCFULL code [34] that requires WS potentials. Thus we have assumed WS potentials that are very similar to the standard SPP at the surface region, around the s-wave barrier radius (R_B). By standard SPP we mean the context of the systematics of nuclear densities, in which the average values for radius and diffuseness are adopted in the folding calculations. Thus the standard SPP does not contain any adjustable parameter. The parameters of the equivalent WS potentials are shown in Table I, as well as the corresponding barrier heights [V_B(SSPP) in Table I]. For comparison purposes, the table

TABLE I. Values of the parameters of WS potentials, which are equivalent to the corresponding SPP in the surface region, for the systems studied in the present work. Also shown is the barrier height obtained from the standard SPP, V_B(SSPP), and from experimental BDs, V_B(BD). The table includes the renormalization factors (N_R) applied to the SPP. SFs for the fusion cross section obtained in earlier works [SF(EW)] and in the present work with the standard [SF(SSPP)] or renormalized [SF(RSPP)] SPP are presented in the table. The V_B values are quoted in mega-electron-volts.

System	^{6}Li + ^{209}Bi	^{7}Li + ^{209}Bi	^{9}Be + ^{208}Pb	^{16}O + ^{144}Sm
V_0 (MeV)	150	160	180	280
r_0 (fm)	1.06	1.06	1.06	1.06
a (fm)	0.71	0.71	0.71	0.75
N_R	0.83	0.78	0.67	1.00
V_B(BD)	30.1 ± 0.3	29.7 ± 0.2	38.3 ± 0.6	61.10 ± 0.05
V_B(SSPP)	29.8	29.4	38.5	61.04
SF(EW)	0.65 ± 0.05	0.73 ± 0.03	0.68 ± 0.08	
SF(SSPP)	0.61	0.71	0.62	
SF(RSPP)	0.63	0.74	0.73	

TABLE II. Coupling parameters assumed in the CC calculations.

Nucleus	^{6}Li	^{7}Li	^{9}Be	^{209}Bi	^{208}Pb	^{144}Sm
$E*$ (MeV)	0	0	2.40	2.62	2.61	1.66
λ/N	2/1	2/1	2/2	3/2	3/2	2/1
β	0.870	0.80	0.924	0.153	0.16	0.081
r_0 (fm)	1.20	1.20	1.20	1.06	1.06	1.06
$E*$ (MeV)				3.09	3.2	1.81
λ/N				5/1	5/1	3/1
β				0.110	0.110	0.14
r_0 (fm)				1.06	1.06	1.06

also contains the V_B values derived in the earlier works from the BD obtained experimentally [V_B(BD) in Table I]. The experimental and theoretical V_B values agree within 0.3-MeV precision, which actually is approximately equal to the experimental uncertainties.

We performed CC calculations exactly as given in the prescriptions found in the reported works [28–31]. A special version of the CCFULL code [35] was used in the calculations of the ^{9}Be + ^{208}Pb system. The coupling parameters are presented in Table II. Figure 1 shows the fusion data and theoretical predictions obtained through BPM (dotted curves) or CC (dashed curves) calculations. In the case of ^{16}O + ^{144}Sm, the agreement between data and theoretical predictions is remarkable. In the three cases of weakly bound nuclei, the predictions overestimate the data. Then we found SFs that, multiplied by the theoretical results, reproduce the average behavior of the data (solid curves in Fig. 1). As expected, the couplings almost do not have an effect on the fusion cross sections at energies above the barrier, except by small deviations in the ^{9}Be + ^{208}Pb case. Thus, if one considers only the above barrier data, the data-extracted SF values do not significantly depend on the model, BPM or CC calculations, assumed for obtaining the theoretical predictions. The SF values obtained with the standard SPP [SF(SSPP) in Table I] are in reasonable agreement with the values found in earlier works [SF(EW) in Table I]. However, a careful inspection of Fig. 1 shows that, despite the good fit to the higher-energy regions, our calculations seem to be shifted down in energy for the three weakly bound systems. This means that the calculated barrier heights do not precisely match the experimental ones. To compensate for this effect, a renormalization of the nuclear potential was already used in earlier works [31,32].

At this point, we proceeded as if the experimental fusion BDs were not extracted for these systems, as should be the case for systems with unstable nuclei. Then we renormalized the SPP by changing the V_0 parameter, to match the experimental barrier height values. We extracted the experimental and theoretical V_B values by the usual procedure of assuming a linear behavior of the higher-energy fusion cross sections against $1/E_{\rm CM}$. The normalization factors (N_R) that multiply the standard SPP in order to reproduce the experimental barriers are listed in Table I. As commented on in Sec. II, small variations of the potential, about 20%, are expected because of the spread of the density parameters around the corresponding average values. Clearly the renormalization factors found here are within this acceptable range, except maybe for the ^{9}Be + ^{208}Pb system, for which a rather small $N_R = 0.67$ was found. The upper panels of Figs. 2–4 show the fits obtained with

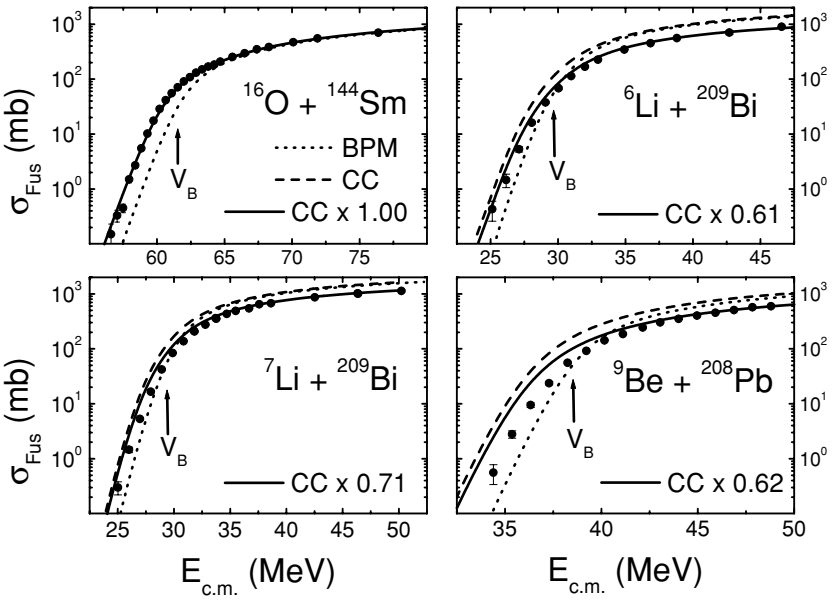

FIG. 1. Fusion cross-section data for the ^{6}Li + ^{209}Bi, ^{7}Li + ^{209}Bi, ^{9}Be + ^{208}Pb, and ^{16}O + ^{144}Sm systems. The dotted and dashed curves correspond to theoretical results obtained with the standard SPP within the BPM and CC calculations, respectively. The solid curves represent the CC results multiplied by the corresponding SF values.

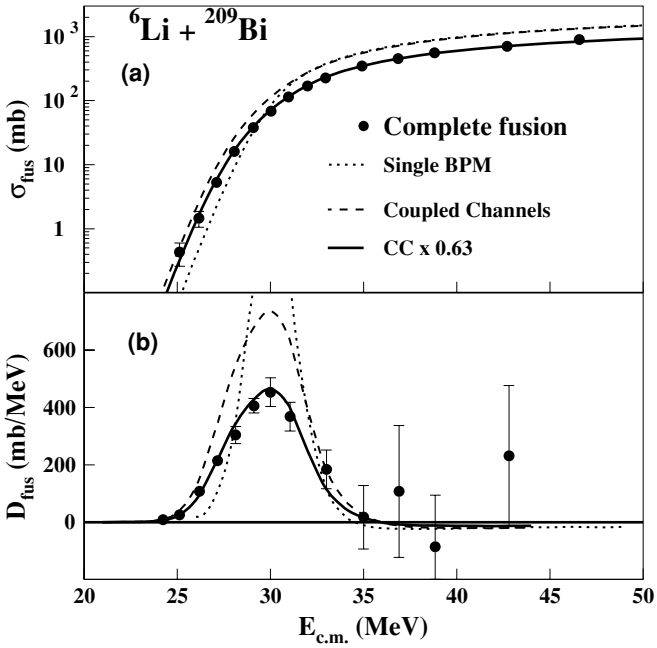

FIG. 2. (a) Fusion cross-section data and (b) experimental BD for the ^{6}Li + ^{209}Bi system. The dotted and dashed curves correspond to theoretical results obtained with the renormalized SPP within the BPM and CC calculations, respectively. The solid curves represent the CC results multiplied by the SF value.

the renormalized SPP, and again the predictions within, the BPM (dotted curves) and CC calculations (dashed curves) overestimate the data for weakly bound systems. Then we found new SF values that reproduce the average behavior of the data, which are labeled SF(RSPP) in Table I. The renormalized SPP reproduces, within the uncertainties, the SF values found in earlier works [SF(EW) in Table I]. The solid curves in the upper parts of Figs. 2–4 show the excellent fits obtained with this procedure.

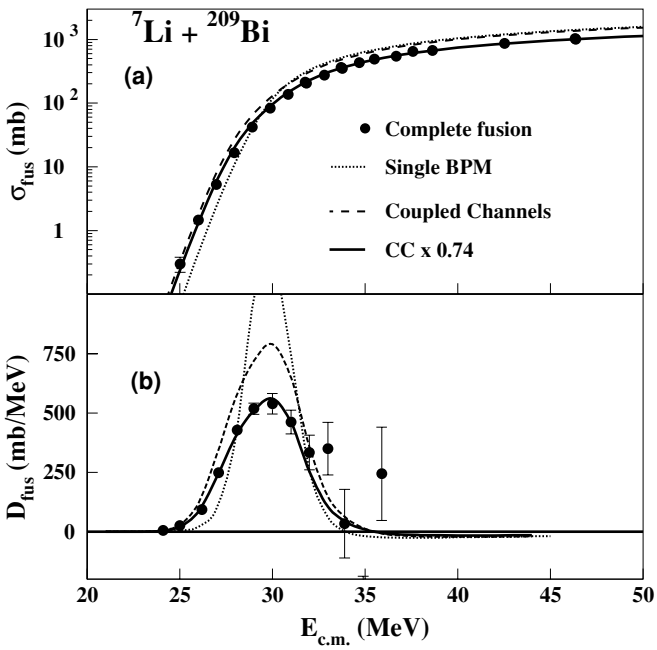

FIG. 3. The same as Fig. 2, but for the ^{7}Li + ^{209}Bi system.

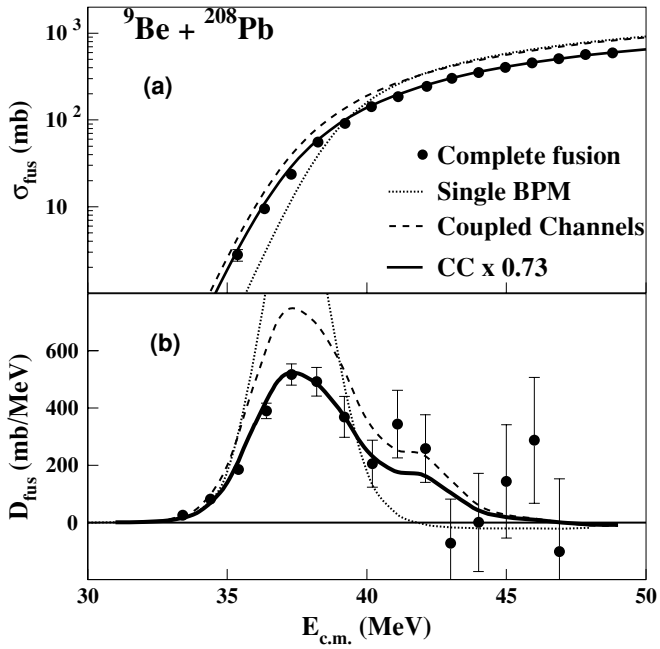

FIG. 4. The same as Fig. 2, but for the ^{9}Be + ^{208}Pb system.

IV. SÃO PAULO POTENTIAL AND BARRIER DISTRIBUTIONS

Because for the systems studied here very good fusion BDs could be extracted from the data [30–33], we used them as a further test of the calculations performed in the previous section. The bottom panels of Figs. 2–5 show the theoretical

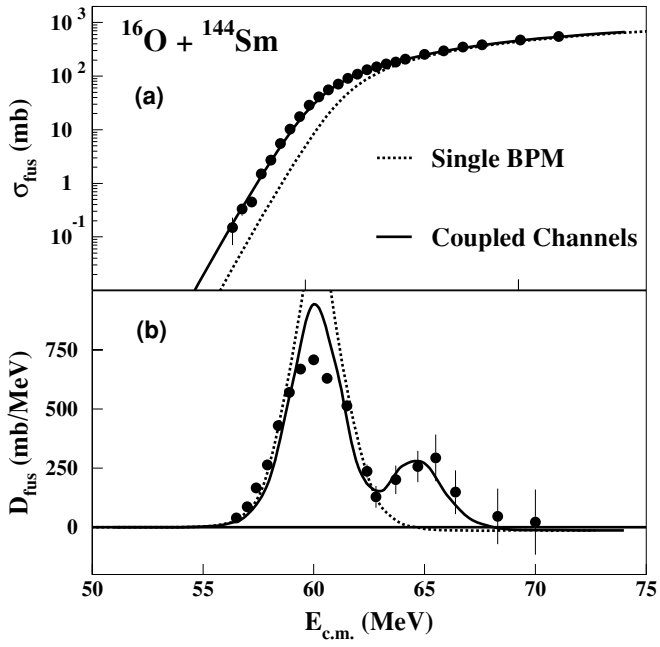

FIG. 5. The same as Fig. 2, but with the standard SPP and considering SF = 1 for the ^{16}O + ^{144}Sm system.

BDs obtained with the renormalized SPP compared with the data, except in the case of $^{16}O + ^{144}Sm$, in which $N_R = 1$ was assumed. The dotted and dashed curves correspond to the theoretical results obtained within BPM and CC calculations, respectively. Again the theoretical predictions overestimate the data for the weakly bound systems. The solid curves in the bottom panels of Figs. 2–5 correspond to the CC calculations multiplied by the SF values (SF = 1 for $^{16}O + ^{144}Sm$). Very good agreement was then obtained for all systems, giving more credence to our calculations.

V. CONCLUSIONS

In summary, we have shown that the SPP can be used as the bare potential for systems involving weakly bound nuclei. It is a trustworthy alternative to the difficult procedure of obtaining experimental BDs which requires precise and high statistical measurements. These measurements are not yet available for experiments with radioactive beams and present great experimental difficulties even when high-intensity stable beams are used. We demonstrated that, even when experimental BD is not available to constrain the parameters of the potential, the SPP model can be assumed in the theoretical calculations and it provides a good description of the reaction mechanism and an accurate estimate for the fusion SF.

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