Tests of the fission-evaporation competition in the deexcitation of heavy nuclei

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In order to verify methods of calculating the fission-evaporation competition in reactions used to synthesize new super-heavy nuclei in "cold" ($1n$) and "hot" ($3n$, $4n$) fusion reactions, we present an analysis of existing experimental data on the evaporation-residue cross sections in two selected reactions, ^{208}Pb(^{16}O,xn) and ^{236}U(^{12}C,xn), for which complementary experimental information necessary to unambiguously calculate the survival probabilities is available: precisely measured fusion excitation functions and saddle-point energies of the fissioning nuclei, deduced from experiments. Standard statistical model calculations, with shell effects accounted for by the Ignatyuk formula, were carried out assuming the ground state shell corrections of Möller *et al.*, and zero shell correction at the saddle configuration (resulting from the presented systematics). Good agreement of the calculated evaporation-residue cross sections with experimental data for different xn reaction channels at low excitation energies leaves no room for modifications of the conventional way of calculating the Γ_n/Γ_f ratio, particularly for including into this ratio an additional preexponential factor (such as the Kramers fission hindrance factor or an effective collective factor) significantly different from 1.

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I. INTRODUCTION

Calculation of the "survival probability," P_{surv}, in deexcitation of heavy compound nuclei formed in nucleus-nucleus fusion reactions is a basic component of theoretical models aimed to predict the production cross sections of super-heavy nuclei. Some selected papers representing different theoretical approaches to this question are listed in Refs. [1–7]. There are several unresolved questions in statistical-model calculations which leave a large margin of uncertainty in estimates of the survival probability. To reduce these uncertainties we have analyzed experimental data on the evaporation-residue cross sections in two selected reactions, ^{208}Pb(^{16}O,xn) [8] and ^{236}U(^{12}C,xn) [9], for which absolute values of the *fusion* cross section have been measured in separate experiments [10,11], and moreover, the saddle-point energies of the fissioning nuclei are known from experimentally determined fission barriers (see Ref. [12] and the compilation by Smirenkin [13], updated in Ref. [14]). This unique set of experimental information imposes sufficiently strong constraints to verify the way of calculating the survival probabilities, particularly to answer the question whether or not conventional expressions for evaporation and fission widths indeed have to be modified by rather arbitrary collective factors and/or the dissipative Kramers factor—as suggested in some of the papers listed in Refs. [1–6].

II. SURVIVAL PROBABILITY

The survival probability $P_{\text{surv}}(Z, A, E, J)$ is the probability for the compound nucleus of atomic and mass numbers Z, and A, the excitation energy E, and spin J to decay to the ground state of a final residual nucleus via evaporation of light particles and γ rays thus surviving fission. In practical applications, when the cross section for production of a selected evaporation-residue nucleus is measured, for example in one of the (fusion,xn) reaction channels, the survival probability refers to the formation of this particular final nucleus (Z, $A - xn$) in its ground state.

A. One-neutron-out reaction

In the simplest case of one-neutron-out reaction, (fusion, $1n$), the survival probability is given by

$$P_{\text{surv}}(1n) = \frac{\Gamma_n}{\Gamma_{\text{tot}}}(1 - p''), \tag{1}$$

where Γ_n is the partial width for neutron emission, and Γ_{tot} is the total decay width consisting of the partial width for fission Γ_f and the sum of all evaporation widths Γ_i(evap), including Γ_n,

$$\Gamma_{\text{tot}} = \Gamma_f + \sum \Gamma_i(\text{evap}). \tag{2}$$

The quantity p'' denotes the probability that after the neutron emission the decaying nucleus will undergo second-chance fission or emit another light particle. Otherwise it will reach the ground state (by emitting γ rays) with the probability $(1 - p'')$. In Ref. [7] simple formulas for calculating $P_{\text{surv}}(1n)$ are given.

B. General case

At higher excitation energies, successive emission of more particles is energetically possible. The competition between different evaporation channels and fission has to be then decided at every consecutive stage of the deexcitation cascade, proportionally to the widths Γ_f and Γ_i(evap). The ultimate population of a given final evaporation-residue nucleus of the mass and atomic numbers A_f, Z_f, relative to the population of all other evaporation and fission reaction channels, is the survival probability for this selected reaction channel:

$$P_{\text{surv}}(A, Z \Rightarrow A_f, Z_f) = \frac{N(A_f, Z_f)}{N(\text{fission}) + \sum_j N(A_j, Z_j)}, \quad (3)$$

where $N(A_f, Z_f)$ is the number of cascades leading to formation of a selected nucleus A_f, Z_f in its ground state, while the number in the denominator of Eq. (3) is the total number of deexcitation cascades which end by fission (at any stage of the cascade) or by formation of any evaporation-residue nucleus in the ground state.

Due to the complexity of multiparticle cascades, Monte Carlo methods are especially suitable to evaluate the final distribution of the evaporation-residue and fission events in Eq. (3). In short, the following scheme of the Monte Carlo calculations is used: At the beginning of each cascade, a value of the angular momentum l is drawn out of the distribution assumed to be proportional to $2l + 1$. The information on the angular momentum is kept throughout the deexcitation cascade in order to calculate the rotational and thermal components of the excitation energy at each stage of the cascade. A given deexcitation cascade is generated by drawing first the decay mode (i.e., the evaporation of a specific particle i or fission—proportionally to the partial width $\Gamma_i/\Gamma_{\text{tot}}$), and then by drawing a value of the kinetic energy of the selected (evaporated) particle—assuming its kinetic energy spectrum to be Maxwellian. Apart from neutrons, several other light particles, ranging from protons to Li isotopes, are accounted for as candidates for evaporation throughout the deexcitation cascade. A reasonably accurate sharp cut-off approximation describing the competition of γ decay with other decay modes is applied in our calculations. Namely, the γ-decay width is always neglected when evaporation of light particles or above-the-barrier fission is possible energetically. Otherwise, i.e., at sub-fission-barrier excitation energies, the γ decay prevails over fission.

C. The particle emission width Γ_i(evap)

The partial width Γ_i for emission of a particle i from a compound nucleus with the excitation energy E_0 is given by Weisskopf formula [15]

$$\Gamma_i = \frac{g_i\, m_i\, \sigma_i}{\pi^2 \hbar^2 \rho_0(E_0)} \int_0^{E_i^{\max}} \rho_i\left(E_i^{\max} - \epsilon_i\right) \epsilon_i\, d\epsilon_i, \quad (4)$$

where m_i and g_i are the mass and spin degeneracy of the emitted particle, ϵ_i is its kinetic energy, σ_i is the cross section for the formation of the decaying nucleus in the inverse process, $\rho_0(E_0)$ is the level density of the parent nucleus at the excitation energy E_0, and $\rho_i(E_i^{\max} - \epsilon_i)$ the level density of the daughter nucleus after emission of the particle i. The energy $E_i^{\max}$ is the upper limit of the final-state excitation energy after emission of the particle i.

To a good approximation only thermal excitation energies U can be taken in the *ratio* of the level densities of the daughter and parent nuclei, ρ_i and ρ_0, respectively. In this approximation the Fermi-gas-model expression, $\rho = \text{const} \cdot \exp 2\sqrt{aU}$, can be used. Thus we obtain

$$\Gamma_i = \frac{g_i\, m_i\, \sigma_i\, U_i^{\max}}{\pi^2 \hbar^2 a_i} \exp\left(2\sqrt{a_i U_i^{\max}} - 2\sqrt{a_0 U_0}\right), \quad (5)$$

where a_0 is the level density parameter of the parent nucleus at the thermal excitation energy corrected for its pairing energy P_0,

$$U_0 = E_0 - E_0^{\text{rot}} - P_0, \quad (6)$$

and a_i is the level density parameter of the final nucleus (after emission of the particle i) at the upper limit of the final-state thermal excitation energy

$$U_i^{\max} = E_0 - E_i^{\text{rot}} - S_i - B_i - P_i, \quad (7)$$

where S_i is the separation energy of the emitted particle, B_i is the asymptotic Coulomb interaction energy of this particle with the daughter nucleus, and P_i is the pairing energy of the final nucleus. The rotational energies of the nuclei before and after emission of the particle i, E_0^{rot} and E_i^{rot}, respectively, are calculated assuming the rigid-body moments of inertia and ground-state deformations as predicted in Ref. [21]. Pairing energies are calculated using conventional parametrization: $P = 0$ for odd-odd nuclei, $P = 12\ \text{MeV}/\sqrt{A}$ for odd-A nuclei, and $P = 24\ \text{MeV}/\sqrt{A}$ for even-even nuclei.

D. The fission width Γ_f

To describe the competition between the particle evaporation and fission we need to evaluate also the fission width Γ_f for the parent nucleus at the excitation energy E_0. The fission width can be expressed in terms of the transition state theory [16,17] as

$$\Gamma_f = \frac{1}{2\pi \rho_0(E_0)} \int_0^{E_f^{\max}} \rho_f\left(E_f^{\max} - K\right) dK, \quad (8)$$

where the level density of the fissioning nucleus at the saddle configuration, ρ_f is integrated over the kinetic energy K in the fission degree of freedom. Here $E_f^{\max}$ denotes the upper limit of the excitation energy at the saddle.

Again, by taking only the thermal excitation energies in the ratio of ρ_f and ρ_0, and assuming the Fermi-gas-model expression for the level density, we obtain

$$\Gamma_f = \frac{2\sqrt{a_f U_f^{\max}} - 1}{4\pi\, a_f} \exp\left(2\sqrt{a_f U_f^{\max}} - 2\sqrt{a_0 U_0}\right), \quad (9)$$

where a_0 and U_0 mean the same as in Eq. (5), a_f is the level density parameter of the fissioning nucleus at the saddle configuration, and $U_f^{\max}$ denotes the upper limit of the thermal excitation energy at the saddle:

$$U_f^{\max} = E(\text{saddle}) - E^{\text{rot}}(\text{saddle}) - P(\text{saddle}). \quad (10)$$

Here E(saddle), E^{rot}(saddle), and P (saddle) denote, respectively, the total excitation energy, the rotational energy, and the pairing energy—all taken at the saddle configuration. The saddle-point pairing energy is assumed to be the same as at equilibrium, $P(\text{saddle}) = P_0$, and the rotational energy is calculated assuming the rigid-body moment of inertia for the saddle-point shape.

E. Dependence of a_i and a_f on shell effects

Equations (5) and (9) are the basis for calculating the survival probability. It is essential in these calculations to use realistic values of the level density parameters a_i and a_f for evaporation and fission channels. In the present work we use the well tested parametrization proposed by Reisdorf [18], accounting for the volume, surface, and curvature dependence of the single-particle level density at the Fermi surface, combined with the Ignatyuk formula [19] for shell effects. Thus the *smooth*, shell-independent level-density parameter is given [18] by the following expression:

$$\tilde{a} = 0.04543 r_0^3 A + 0.1355\, r_0^2 A^{2/3} B_S + 0.1426\, r_0 A^{1/3} B_K, \tag{11}$$

where B_S and B_K are the surface and curvature factors defined in the droplet model [20], and r_0 is the radius parameter found to be $r_0 = 1.15$ fm [18]. For spherical nuclei $B_S = B_K = 1$. Values of B_S and B_K for deformed shapes (most important for the saddle configuration in fission channel) have been tabulated in Ref. [20].

As demonstrated by Ignatyuk [19], the level densities determined experimentally can be well reproduced assuming that the smooth value of the level density parameter $\tilde{a}$, Eq. (11), is modified due to shell effects according to the formula

$$a = \tilde{a}\left[1 + \frac{\delta_{\text{shell}}}{U}(1 - \exp(-U/E_D))\right], \tag{12}$$

where δ_{shell} is the shell correction energy, and E_D is a parameter determining the damping of shell effects with the increasing excitation energy U. According to Ref. [18] $E_D = 18.5$ MeV. Values of the shell correction energies δ_{shell}, calculated with the Strutinsky method for a wide range of nuclei, can be found in Ref. [21].

Mughabghab and Dunford [22] proposed slightly modified parametrization of $\tilde{a}$, with the volume, surface and curvature coefficients in Eq. (11) determined as free parameters. They used also a modified expression for the damping of shell effects in Eq. (12). This alternative way of calculating the level density parameters gives results essentially very similar to those resulting from the Reisdorf-Ignatyuk prescription of Eqs. (11) and (12) because *ratios* of the level densities are not sensitive to minor changes in parametrization of a. In our calculations we consequently keep the Reisdorf parametrization, Eq. (11), that contains only one empirically adjusted parameter r_0.

It is clear that the Ignatyuk formula, Eq. (12), has to be applied for calculating shell effects in both a_i and a_f. While shell corrections at the equilibrium configuration (determining a_i) are known and tabulated (see, e.g., Ref. [21]), shell corrections at the saddle configuration, necessary to calculate a_f, are not known so precisely. However, from analysis of the fissionability data in the actinide region, Reisdorf [18] concluded that the saddle-point shell corrections are small, of the order of 0.5 MeV or even less. Similar conclusion was reached by Myers and Świątecki [23] from a comparison of experimental fission barriers [13] with those predicted within the macroscopic Thomas-Fermi model, modified only by the ground-state shell correction (i.e., assuming the saddle-point shell corrections equal to zero). In the Appendix we present systematics of the saddle-point shell corrections deduced from experimental fission barriers compiled in Refs. [13,14] for a wide range of nuclei of $88 \leqslant Z \leqslant 100$. It is seen from our systematics that the saddle-point shell corrections, δ_{shell}(saddle), indeed are close to zero. Therefore in the case when no information on δ_{shell}(saddle) is available, one can quite safely assume in Eq. (12) that $\delta_{\text{shell}}(\text{saddle}) = 0$.

III. VERIFICATION OF THE MODEL WITH EXPERIMENTAL DATA

To test the method of calculating the survival probability in reactions leading to the synthesis of super-heavy nuclei we selected a set of data for two lighter systems, $^{16}O + ^{208}Pb$ and $^{12}C + ^{236}U$, for which along with the evaporation-residue (xn) cross sections, also *fusion* excitation functions were measured in separate experiments [10,11]. Still more importantly, *experimental* values of the saddle-point energy (obtained from experimentally determined $l = 0$ fission barriers [13,14]) in various isotopes of Th and Cf, produced during deexcitation of the compound nucleus in these two reactions, are known or can be quite precisely estimated. In addition, both $^{16}O + ^{208}Pb$ and $^{12}C + ^{236}U$ systems are *asymmetric* enough to guarantee that the compound nucleus formation is not affected by the dynamical fusion hindrance factor [7], known to reduce fusion cross sections even by orders of magnitude for heavier and more symmetric systems. Thus, the unique set of experimental data available for the $^{16}O + ^{208}Pb$ and $^{12}C + ^{236}U$ systems imposes sufficiently strong constraints to unambiguously verify the way of calculating the survival probabilities.

The completeness of the $^{16}O + ^{208}Pb$ and $^{12}C + ^{236}U$ data is of special importance in the situation when the standard way of calculating P_{surv} with the conventional expressions given by Eqs. (5) and (9) has been modified in some models [1–6] by rather arbitrarily introduced preexponential collective factors [24,25], and/or the Kramers factor (to replace the conventional Bohr-Wheeler fission width with the Kramers formula [26]).

Another question to be verified is the validity of the "symmetric formula" for Γ_i/Γ_f proposed by Swiatecki [27]. In this approach Γ_i is evaluated the same way as Γ_f, i.e., without singling out the two translational degrees of freedom transverse to the direction of the emitted particle i. This also leads to a preexponential factor in the branching ratio Γ_i/Γ_f, which is different from that resulting from conventional Eqs. (5) and (9).

Various theoretical models [1–7] have been applied so far to reproduce cross sections for the production of super-heavy compound-residue nuclei. However the interplay of several unknown factors such as experimentally unknown saddle-point energies, unmeasured fusion- or capture cross sections, effects of the dynamical fusion hindrance, shell effects, and finally the collective and/or Kramers factors applied in some of these models—make the problem too complex to unravel the

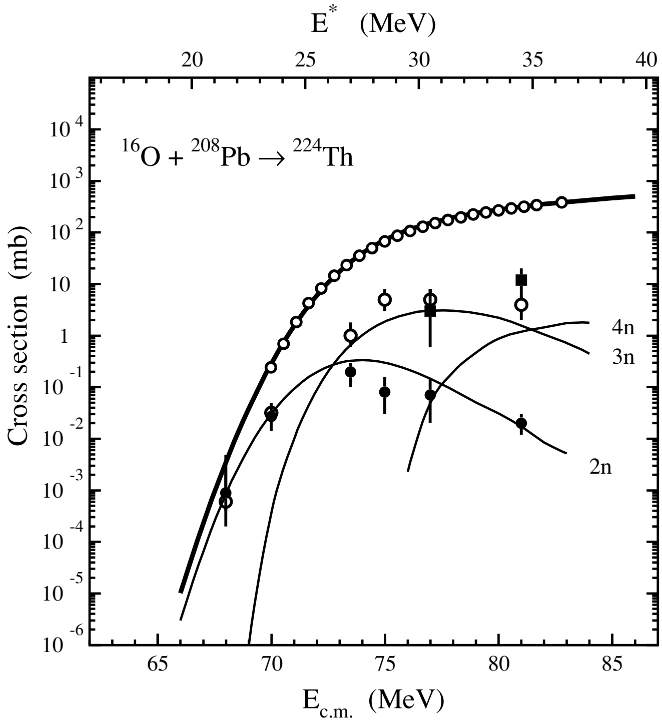

FIG. 1. Evaporation residue cross sections for $2n, 3n$, and $4n$ reaction channels (full circles, open circles, and full squares, respectively) in the $^{16}O + ^{208}Pb$ reaction, measured by Sagaidak *et al.* [6,8], and fusion excitation function for this reaction measured by Morton *et al.* [10]. Theoretical predictions based on standard statistical model calculations, with shell effects taken as described in the text, are shown by solid lines.

individual components. In the selected two "clean" cases of the $^{16}O + ^{208}Pb$ and $^{12}C + ^{236}U$ systems we are going to resolve the essential question whether the branching ratios Γ_i/Γ_f are consistent with the conventional formulas (6) and (9) or we really need to account for the mentioned above effects influencing the preexponential factor in the Γ_i/Γ_f ratios.

A. The $^{16}O + ^{208}Pb$ reaction

In Fig. 1 we present the available experimental information on the $^{16}O + ^{208}Pb$ reaction combined with results of our calculations. The fusion excitation function for this reaction was measured very precisely by Morton *et al.* [10]. The experimental data (small open circles in Fig. 1) are fitted with the "diffused barrier formula," Eq. (6) of Ref. [28], that gives a reliable extrapolation at sub-barrier energies. The evaporation residue cross sections for $2n$, $3n$, and $4n$ reaction channels, reported in Ref. [8], are also shown in Fig. 1. Thin solid lines represent the calculated cross sections for these reaction channels obtained assuming the branching ratios in the evaporation cascade as given by Eqs. (5) and (9).

Regarding precision of the theoretical predictions, it is essential to know the saddle-point energies of the fissioning nuclei very accurately. It is therefore a great advantage that for Th isotopes produced during the deexcitation cascade following the $^{16}O + ^{208}Pb$ reaction, the experimental fission barriers [12] could be used for determination of the saddle-point energies. The following values of the fission barrier: 6.8, 6.7, 6.7, 6.9, and 6.7 MeV have been adopted for $^{220-224}$Th nuclei, respectively [14]. The unmeasured barrier for ^{224}Th was adopted on the basis of the systematics for adjacent nuclei (see Table 2 of Ref. [14]). It should be pointed out that the experimental fission barriers quoted in Refs. [13,14] usually originate from analysis of the near-barrier dependence of the first-chance fission probability in neutron or charged-particle induced fission as well as in electromagnetic fission [12]. At these low excitation energies the deduced fission barriers are determined quite precisely, with an accuracy of $\pm$0.2–0.3 MeV.

As seen from Fig. 1, the evaporation residue cross sections calculated with the adopted experimental fission barriers agree with the experimental data quite satisfactorily. It was checked in our calculations that the uncertainty of the height of the fission barrier $\pm$0.2 MeV results in a change of the cross section at the maximum of the excitation function for $2n$ and $3n$ channels by a factor of about 1.5 and 2.5, respectively. This demonstrates how critical is the precise knowledge of the saddle-point energy. The observed discrepancies between calculated and experimental maximum values of the cross section for $2n$ and $3n$ channels are of this order. There is an indication of a larger discrepancy for the $4n$ channel at the highest studied excitation energies. This effect will be discussed in Sec. III B.

We emphasize again the role of shell corrections in both, the compound residue nucleus after neutron emission (determining Γ_n), and in the saddle configuration (determining Γ_f). Following arguments given in Sec. II E, in the analysis of the $^{16}O + ^{208}Pb$ reaction we assumed the saddle-point shell corrections in the fissioning $^{224-221}$Th nuclei to be zero, $\delta_{\rm shell}({\rm saddle}) = 0$. The resulting dependence of the a_f/a_n ratio on the excitation energy of a given decaying nucleus, calculated for the ground-state shell corrections $\delta_{\rm shell}$(g.s.) taken from Ref. [21], is illustrated in Fig. 2. As it follows from Eqs. (11) and (12), for typical values of B_S and B_K characterizing the equilibrium and saddle configuration shapes and for usually negative values of $\delta_{\rm shell}$(g.s.), the ratio a_f/a_n is always larger than 1, with the tendency to increase with the decreasing excitation energy. It is seen from Fig. 2 that for different Th isotopes a_f exceeds a_n by about 10%.

It should be pointed out that attempts to fit data simultaneously for different xn channels in a strong competition with the fission channel put severe constraints on a value of the Γ_n/Γ_f ratio, particularly on the preexponential factor in that ratio. To illustrate this point, we again show in Fig. 3 the data on $2n$, $3n$, and $4n$ cross sections in the $^{16}O + ^{208}Pb$ reaction, but now the data are compared to predictions based on the "symmetric formula" for the Γ_n/Γ_f ratio, as proposed by Swiatecki [27]. The symmetric formula has the same exponential term as that resulting from Eqs. (5) and (9), but it gives a preexponential factor in Γ_n/Γ_f approximately equal to $\sqrt{a_f U_n^{\rm max}/a_n U_f^{\rm max}}$, which is about 3–4 times smaller than in the conventional formula (in the studied range

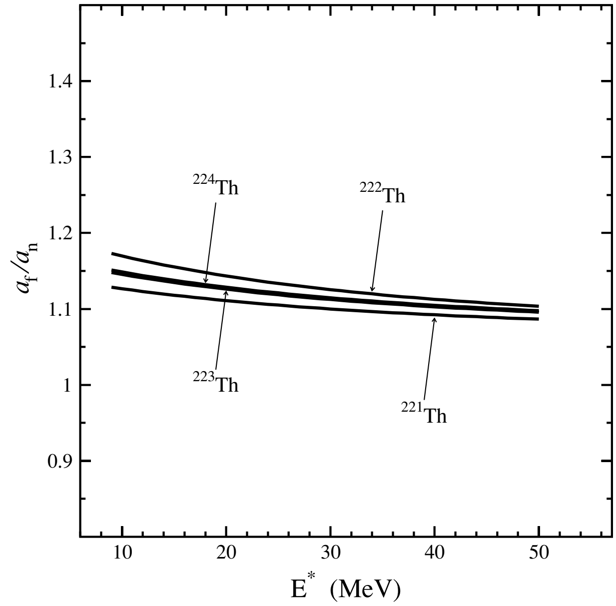

FIG. 2. Energy dependence of the a_f/a_n ratio calculated using Eqs. (11) and (12) for $^{224-221}$Th nuclei in the *xn* deexcitation cascade following the $^{16}O + ^{208}Pb$ fusion reaction. The ground-state shell corrections $\delta_{\rm shell}$(g.s.) were taken from Ref. [21], and the saddle shell corrections $\delta_{\rm shell}$(saddle) were assumed to be zero, as follows from systematics shown in Fig. 6.

of excitation energies around 25 MeV). Consequently, the predicted cross sections are clearly too small, especially for 3n and 4n channels, for which the preexponential factor acts multiplicatively 3 or 4 times, respectively. Taking into account the fact that for the studied $^{16}O + ^{208}Pb$ reaction we

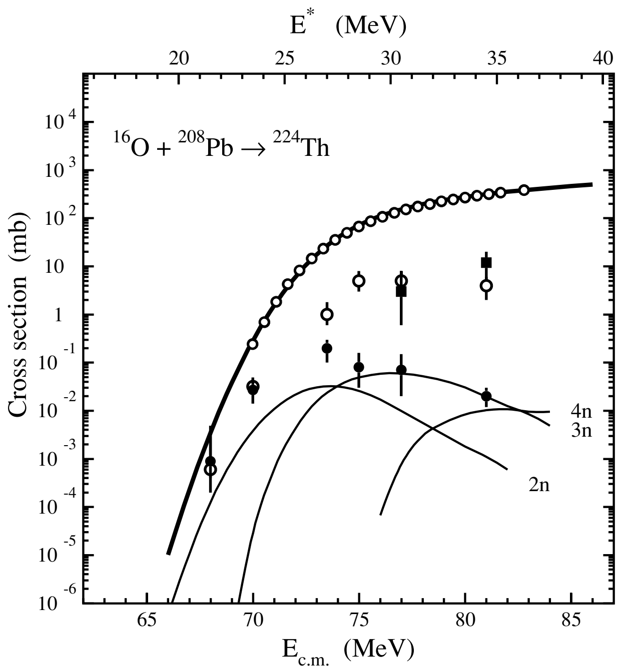

FIG. 3. Same as Fig. 1, except statistical model calculations performed assuming the "symmetric" formula for the Γ_f/Γ_n ratio [27].

know the experimental values of the ground-state fission barrier, the crucial ingredient in these calculations, and that we can safely eliminate the dynamical hindrance effects as well as the uncertainties in the fusion cross section, we infer that the results presented in Figs. 1 and 3 unambiguously demonstrate validity of the conventional formula for Γ_n/Γ_f [based on Eqs. (5) and (9)], and rather exclude validity of the symmetric formula of Ref. [27]. Moreover, these results exclude any other modification of the conventional way of calculating competition between evaporation and fission channels that would introduce into the conventional expression for Γ_n/Γ_f an additional preexponential factor significantly different from 1. However, as will be discussed in Sec. III B, at high excitation energies, approximately at $E^* = 35$ MeV or somewhat higher, an indication for the onset of the fission hindrance effects is observed.

B. The $^{12}C + ^{236}U$ reaction

The $^{12}C + ^{236}U$ reaction provides another almost complete set of experimental information suitable to test the way of calculating the survival probabilities. The measured fusion cross sections [11] and evaporation residue cross sections for $3n, 4n$, and $5n$ channels [9] in the $^{12}C + ^{236}U$ reaction are shown in Fig. 4. The measured fusion cross sections cover the essential range of near-barrier energies and therefore the

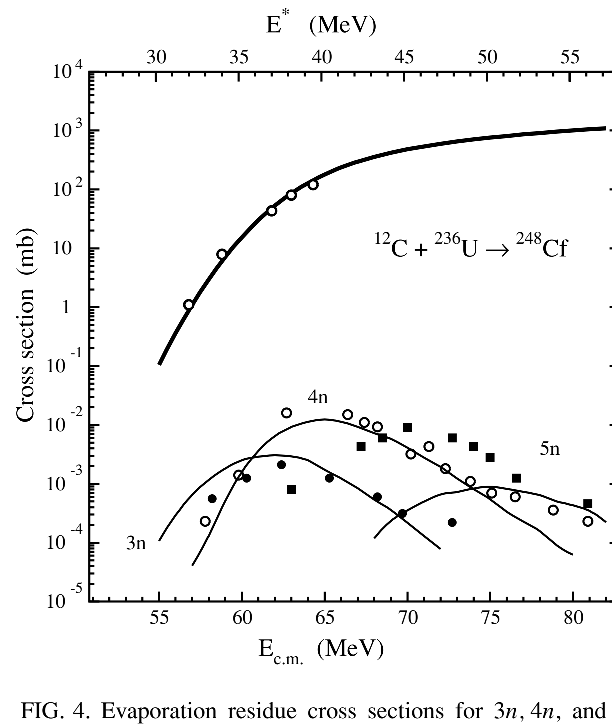

FIG. 4. Evaporation residue cross sections for $3n, 4n$, and $5n$ reaction channels (full circles, open circles, and full squares, respectively) in the $^{12}C + ^{236}U$ reaction, measured by Sikkeland *et al.* [9], and the fusion excitation function for this reaction measured by Murakami *et al.* [11]. Theoretical predictions based on standard statistical model calculations, with shell effects taken as described in the text, are shown by solid lines.

extrapolation based on fitting the diffused-barrier formula [7,28] is reliable. Moreover, the $^{12}C + ^{236}U$ system is very asymmetric. Therefore the dynamical hindrance of the fusion cross section can be safely disregarded. Unfortunately the existing experimental information on fission barriers for relevant Cf nuclei is limited only to isotopes of $A \geqslant 250$, but reasonable estimates for lighter Cf isotopes can be done on the basis of the systematics [13,14] for isobars of adjacent lighter elements. The following values of the ground-state fission barriers for a series of isotopes of Cf have been adopted: 6.3, 6.2, 6.1, 6.0, and 6.0 MeV for mass numbers 244–248, respectively.

The predicted cross sections for $3n$, $4n$, and $5n$ channels in the $^{12}C + ^{236}U$ reaction are shown in Fig. 4. Similarly as in case of the $^{16}O + ^{208}Pb$ reaction, we assumed the saddle-point shell corrections in successive nuclei formed in the xn chain to be zero, $\delta_{\text{shell}}(\text{saddle}) = 0$. The resulting dependence of the a_f/a_n ratio on the excitation energy for these Cf isotopes (for the ground-state shell corrections taken from Ref. [21]), is shown in Fig. 5.

Due to larger negative values of the ground-state shell corrections in Cf isotopes (see Fig. 6 in the Appendix), the a_f/a_n ratios in case of Cf compound nuclei (in the $^{12}C + ^{236}U$ reaction) are larger than in case of Th compound nuclei produced in the $^{16}O + ^{208}Pb$ reaction. It is clearly seen that the a_f/a_n ratio increases at small excitation energies due to the energy dependence of the shell effect for the equilibrium shape [see Eq. (12)]. A very similar energy dependence of the a_f/a_n ratio was found by Ohta [29] in his attempt to deduce the a_f/a_n ratio from a fit to experimental data for No isotopes. Our analysis demonstrates that both, a_n and a_f can be satisfactorily predicted using Eqs. (11) and (12), provided the shell effects are properly accounted for not only in the equilibrium, but also in the saddle configuration. The absolute values of the cross sections for $3n$ and $4n$ reactions are then correctly reproduced.

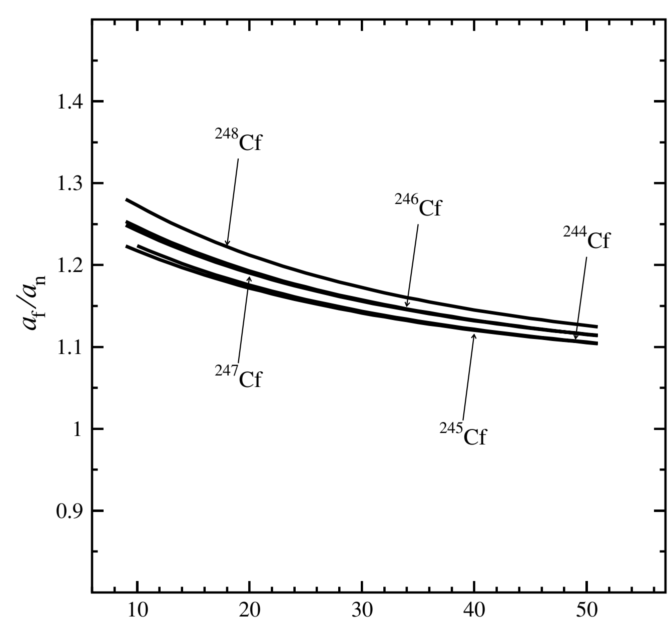

FIG. 5. Energy dependence of the a_f/a_n ratio calculated using Eqs. (11) and (12) for $^{247-243}$Cf nuclei in the xn deexcitation cascade following the $^{12}C + ^{236}U$ fusion reaction. The ground-state shell corrections $\delta_{\text{shell}}(\text{g.s.})$ were taken from Ref. [21], and the saddle shell corrections $\delta_{\text{shell}}(\text{saddle})$ were assumed to be zero, as follows from systematics shown in Fig. 6.

However, a significant disagreement is observed at higher excitation energies for the $5n$ reaction channel. (As is seen from Fig. 4, the maximum of the calculated $5n$ excitation function is lower by a factor of 10 and shifted towards higher energies by about 5 MeV, as compared with data.) We emphasize again the fact of very high sensitivity of the model calculations to the assumed hight of the saddle point energy, especially for multi-neutron emission channels. A typical error of experimentally determined or interpolated/extrapolated fission barriers of about 0.2–0.3 MeV cannot fully explain so large discrepancy. (A 0.3 MeV error in the barrier height for Cf isotopes results in a change of the cross section for the $5n$ channel by a factor of 6, but then the excitation functions for $4n$ and $3n$ channels, which are well reproduced, would also be affected.) Therefore, we rather interpret the observed behavior of the $5n$ excitation function as an effect of the fission hindrance reducing the fission width at higher excitation energies due to the dissipative phenomena [26]. Our data suggest that the fission hindrance acts only at high excitation energies, above 45 MeV. Evidence that the fission hindrance effects appear only at high excitation energies was reported also in previous studies (see Ref. [30] and references therein). We found in our calculations that the observed increase of the maximum cross section for the $5n$ channel by a factor of 10 (relative to the standard Bohr-Wheeler predictions) may be due to the Kramers factor of a value $\sqrt{1+\gamma^2} - \gamma \approx 0.1$, that is close to that expected for the mechanism of one-body dissipation. (Here γ is the dimensionless friction coefficient, having a value $\gamma \approx 5$ for one-body dissipation.)

Summarizing the results for the $^{12}C + ^{236}U$ reaction, we state that at low excitation energies corresponding to $3n$ and $4n$ channels, the conventional formula for calculating the evaporation-to-fission branching ratios based on Eqs. (5) and (9), with shell effects accounted for in the level densities, gives correct quantitative results. As shown in the previous sub-section, similar conclusions have been drawn also in case of the $^{16}O + ^{208}Pb$ reaction. Therefore our analysis clearly demonstrates that to calculate the survival probabilities in typical reactions used so far to produce new super-heavy elements—cold fusion ($1n$) or hot fusion reactions ($3n$ and $4n$)—the conventional method described in Sec. II should be used. Our analysis leaves no room for modifications of the branching ratios in form of an effective collective factor different from the conventional dependence of the level density parameter on deformation, Eq. (11), and/or Kramers factor significantly different from 1. Only in the range of very high excitation energies, exceeding some 40–45 MeV (which is irrelevant for experiments aimed to produce super-heavy elements), an indication for the presence of the fission hindrance with the strength corresponding to a value of the Kramers factor of the order of 0.1 is observed.

IV. SUMMARY AND CONCLUSIONS

In a number of publications some contradictory theoretical schemes of calculating the statistical decay of very heavy compound nuclei leading to formation of super-heavy nuclei were proposed, specifically regarding evaluation of shell effects in the fission channel, inclusion of the dissipative fission already at the lowest excitation energies, and an arbitrary treatment of nuclear collective effects. In order to clarify this situation and find the most reliable method of calculating the survival probabilities, we selected a unique set of data on two lighter but comprehensively studied systems: $^{16}O + ^{208}Pb$ and $^{12}C + ^{236}U$, for which along with the evaporation-residue (xn) cross sections, also fusion excitation functions had been measured in separate experiments. Besides, very importantly, fission barriers in nuclei produced in these two reactions (and thus their saddle-point energies) can be quite precisely estimated from experiments. Additionally, for both these very asymmetric systems one can safely assume that the compound nucleus formation is not affected by the dynamical fusion hindrance factor. Consequently, this unique set of experimental information imposes sufficiently strong constraints to unambiguously verify basic assumptions underlying calculations of the survival probabilities.

For both reactions, we have carried out standard statistical model calculations using expressions for the survival probability described in detail in Sec. II. We paid special attention to the question of shell effects which play essential role in determining effective magnitudes of the a_n/a_f ratio throughout the deexcitation cascade and thus strongly influence the survival probability. By using a compilation of experimentally determined heights of the fission barrier for nuclei in the range $88 < Z < 100$, we estimated magnitudes of the shell effect in the saddle configuration for all these nuclei, and found that the shell correction at the fission barrier, $\delta_{\text{shell}}(\text{saddle})$ is usually close to zero, and thus it can be neglected in determination of the level density parameter of the fissioning nucleus at the saddle configuration—in agreement with earlier suggestions formulated in Refs. [18,23]. Thus, by using the ground-state shell corrections from Ref. [21] to calculate respective values of a_i in the evaporation channels, and by assuming $\delta_{\text{shell}}(\text{saddle}) = 0$ in calculating the level density parameter a_f in the fission channel, one can unambiguously determine the a_i/a_f ratio at every stage of the deexcitation cascade (see Figs. 2 and 5). The resulting absolute values of the evaporation residue cross sections agree quite well with experimental data in the range of low excitation energies, $E^* < 40$ MeV, relevant for synthesis of super-heavy nuclei. Simultaneous fit to the data on different xn channels for very fissile nuclei leaves no room for modifications of the conventional way of calculating Γ_n/Γ_f, particularly for introducing into the ratio Γ_n/Γ_f an additional preexponential factor significantly different from 1. For example, our analysis rather excludes possibility of replacing the standard expression for the Γ_n/Γ_f ratio based on Eqs. (5) and (9) by the "symmetric" formula of Ref. [27] (having approximately 3–4 times smaller preexponential factor). Similarly, our analysis leaves no room for the Kramers factor (associated with the dissipative hindrance of fission) or an effective collective factor significantly different from 1.

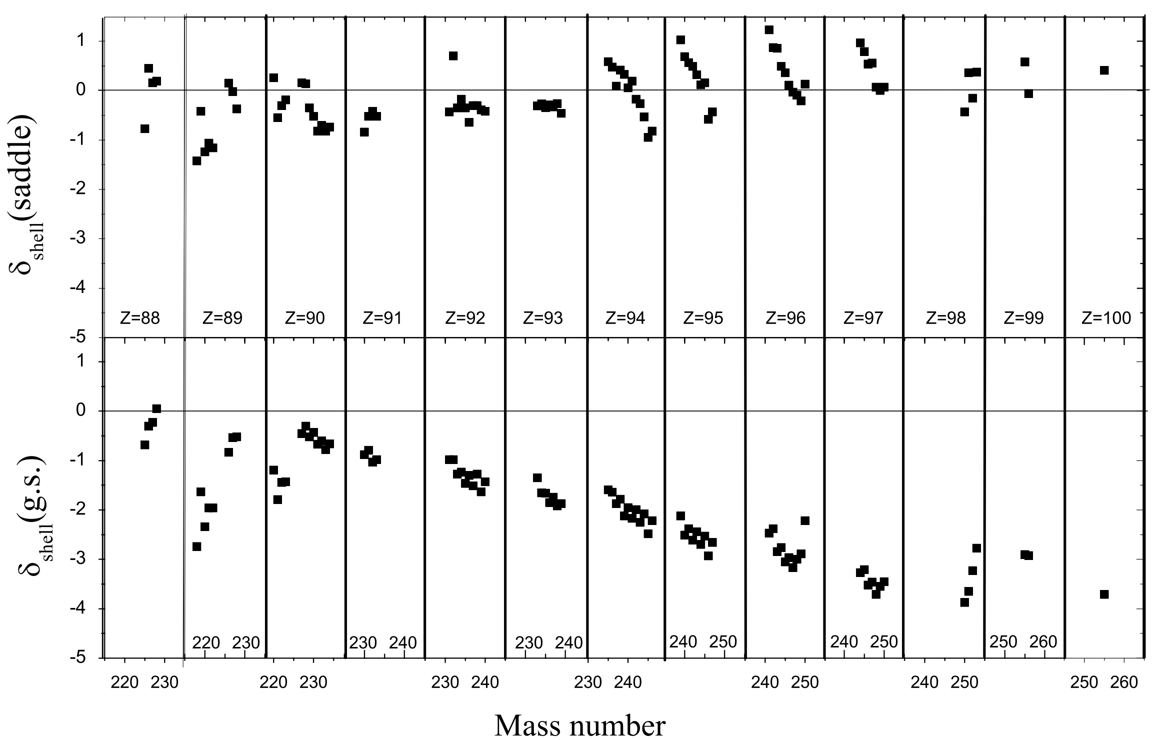

FIG. 6. Lower part: Ground state shell corrections $\delta_{\text{shell}}(\text{g.s.})$ (taken from Ref. [21] for about 90 nuclei of Z in the range $88 \leqslant Z \leqslant 100$, for which heights of the fission barrier had been determined experimentally and compiled in Refs. [13,14]. Upper part: Saddle-point shell corrections, $\delta_{\text{shell}}(\text{saddle})$, deduced from experimental B_f values by using Eq. (A1).

However at high excitation energies, above some 40–45 MeV, an indication for the presence of the fission hindrance with the strength corresponding to a value of the Kramers factor of the order of 0.1 is observed.

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APPENDIX: SHELL CORRECTIONS AT THE SADDLE CONFIGURATION

We attempt to estimate magnitudes of shell corrections at the saddle configuration on the basis of experimental values of the barrier height and theoretical shell corrections calculated for the equilibrium shape. Since the height of the fission barrier is the macroscopic value of the barrier, B_f(macro) modified by shell corrections at the saddle configuration and in the ground state, δ_{shell}(saddle) and δ_{shell}(g.s.), respectively:

$$B_f = B_f(\text{macro}) + \delta_{\text{shell}}(\text{saddle}) - \delta_{\text{shell}}(\text{g.s.}), \quad \text{(A1)}$$

we can estimate unknown values of the shell correction at the saddle, δ_{shell}(saddle), by using Eq. (A1), provided the height of the fission barrier B_f is known from experiments.

Figure 6 shows the known [21] shell corrections for the ground state δ_{shell}(g.s.), and the deduced values of the shell correction at the saddle configuration, δ_{shell}(saddle), calculated by applying Eq. (A1) for the entire set of nuclei, for which experimental values of the fission barrier B_f have been determined and compiled in Refs. [13,14]. To determine the δ_{shell}(saddle) values, the macroscopic fission barriers B_f(macro) were calculated with inclusion of finite-range effects in the nuclear surface energy, as proposed by Sierk [31]. The ground-state shell corrections, δ_{shell}(g.s.), calculated with the Strutinsky method, were taken from Ref. [21]. It is seen from Fig. 6 that shell effects indeed practically vanish at the saddle configuration.

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