Spin structure of spin-1/2 baryon and spinless meson production amplitudes in photonic and hadronic reactions

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The most general spin structures of the spin-1/2 baryon and spinless meson production operator for both photon and nucleon induced reactions are derived from the partial-wave expansions of these reaction amplitudes. The present method provides the coefficients multiplying each spin operator in terms of the partial-wave matrix elements. The result should be useful in studies of these reactions based on partial-wave analyses, especially, when spin observables are considered.

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In the present work, we derive the most general spin structure of the reaction amplitude for both positive and negative parity spin-1/2 baryon production in nucleon-nucleon (*NN*) collisions and also in photon induced processes on nucleons. Knowing the spin structure of the transition operator is of particular importance in analyses of spin observables. The method used here to extract the spin structure is a generalization of the partial-wave expansion of the *NN* amplitude following Ref. [1]. It is quite general and, in principle can be applied to any reaction process in a systematic way. Usually, the structure of a transition operator is derived based solely on symmetry principles. The usefulness of the present method is that it also provides explicit formulas for the coefficients multiplying each spin structure in terms of the partial-wave matrix elements and this should be particularly useful in model-independent analyses based on the partial-wave expansion of the reaction amplitude.

As an example of application of the present method, we investigate the possibility of determining the parity of a narrow resonance in both photon and nucleon induced reactions. The search for new resonances, especially the so-called missing resonances and exotic resonances, is receiving increased attention [2]. Apart from establishing their existence, the determination of their basic properties are of extreme importance. Among these properties, the parity is of particular interest in connection with the substructure of these resonances [3]. However, it is often the case that no theoretical predictions can provide a conclusive result for the parity and other basic properties. A recent example of this situation is provided by the pentaquark Θ^+. The existence of this exotic baryon and the determination of its spin and parity quantum numbers have been under an intensive investigation both experimentally and theoretically over the past couple of years [4].[1] It is, therefore, very important to find a way of determining these properties in a model independent way.

In the past few years a considerable amount of data for meson production in *NN* collisions have been obtained (see Ref. [6] for a review). In particular there are data, not only for cross sections but, in the case of pion production in $NN \to NN\pi$, a large set of spin observable data [7]. In addition, the databases for the production of other mesons, such as η and η', are growing rapidly [6,8]. Also, reactions involving the production of particles containing strange quarks such as the $NN \to YNK$ reaction where Y stands for a hyperon, are receiving increased attention [6,9]. With these (high-precision) data becoming available, there is a demand for more thorough and detailed theoretical analyses of these reactions. Therefore, in the present work, we also derive the most general spin-isospin structure of the $NN \to NB'M$ transition operator, where M stands for a spinless (scalar or pseudoscalar) meson and, B', for a spin-1/2 baryon with positive-parity. The spin structure of the $NN \to NN\pi$ transition operator has been derived in Ref. [10] and applied to neutral pion production. Here, we also consider charged meson production.

The present paper is organized as follows. In Sec. I, the general spin structure of the transition operator is derived for the reaction $\gamma + N \to M + B$, where B stands for a spin-1/2 and either a positive or negative parity baryon. In Sec. II the results derived in Sec. I are illustrated by applying them to the near-threshold kinematic regime. Sections III and IV are devoted to the derivation of the most general spin structure of the reaction $N + N \to B' + B$, where B' stands for a spin-1/2 and positive-parity baryon, and to its application near-threshold, respectively. The reaction $N + N \to M + B' + N$ is considered in Secs. V and VI. A summary is given in Sec. VII. Appendices A–D contain some details of the derivation of the spin structure of the transition operators.

I. THE REACTION $\gamma + N \to M + B$

We start by making a partial-wave expansion of the $\gamma + N \to M + B$ reaction amplitude. Here M stands for a pseudoscalar meson and B, a spin-1/2 baryon. We, then, have

$$\langle \frac{1}{2}m' | \hat{M}(\vec{q},\vec{k}) | S M_S \rangle = \sum i^{L-L'} (S M_S L M_L | J M_J) \times (\frac{1}{2} m' L' M_{L'} | J M_J) M^{JS}_{L'L}(q,k) \times Y_{L'M_{L'}}(\hat{q}) Y^*_{L M_L}(\hat{k}), \quad (1)$$

[1]The existence of the Θ^+ is still controversial. While this resonance has been seen in a number of experiments, it has not been observed in a similar number of experiments, including a very recent experiment [5] with much better statistics.

where S, L, J stand for the total spin, total orbital angular momentum, and the total angular momentum, respectively, of the initial γN state. M_S, M_L, and M_J denote the corresponding projection quantum numbers. The primed quantities stand for the corresponding quantum numbers of the final MB state. The summation runs over all quantum numbers not specified in the left-hand side (l.h.s.) of Eq. (1). $\vec{k}$ and $\vec{q}$ denote the relative momenta of the two particles in the initial and final states, respectively. The partial-wave expansion given above is, of course, related to the more commonly used electric and magnetic multipole expansion. For the present analysis, however, it is convenient to use the above expansion.

Equation (1) can be inverted to solve for the partial-wave matrix element $M^{SJ}_{L'L}(q,k)$. We have

$$M^{SJ}_{L'L}(q,k) = \sum i^{L'-L}(\tfrac{1}{2}m'L'M_{L'}|JM_J)(SM_S L0|JM_J) \times \frac{8\pi^2}{2J+1}\sqrt{\frac{2L+1}{4\pi}}\int_{-1}^{+1} d(\cos(\theta))Y^*_{L'M_{L'}}(\theta,0) \times \langle \tfrac{1}{2}m'|\hat{M}(\vec{q},\vec{k})|SM_S\rangle, \quad (2)$$

where, without loss of generality, $\vec{k}$ is chosen along the z axis and $\vec{q}$ in the xz plane; $\cos(\theta) \equiv \hat{q}\cdot\hat{k}$. The summation is over all quantum numbers not specified in the l.h.s. of the equation.

The most general spin structure of the transition operator can be extracted from Eq. (1) as

$$\hat{M}(\vec{q},\vec{k}) = \sum_{SM_S m'} |\tfrac{1}{2}m'\rangle\langle\tfrac{1}{2}m'|\hat{M}(\vec{q},\vec{k})|SM_S\rangle\langle SM_S|. \quad (3)$$

Inserting Eq. (1) into Eq. (3) and recoupling gives

$$\hat{M}(\vec{q},\vec{k}) = \sum i^{L-L'}(-)^{-J-\frac{1}{2}}[J]^2 M^{JS}_{L'L}(q,k)\sum_\alpha \begin{Bmatrix} S & L & J \\ L' & \frac{1}{2} & \alpha \end{Bmatrix} \times \left[B_S \otimes A_{\frac{1}{2}}\right]^\alpha \cdot [Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^\alpha, \quad (4)$$

where we have used the notations $B_{SM_S} \equiv (-)^{S-M_S}\langle S - M_S|$, $A_{\frac{1}{2}m'} \equiv |\frac{1}{2}m'\rangle$, and $[J] \equiv \sqrt{2J+1}$. The outer summation is over the quantum numbers S, L, L', and J. In the above equation S is either 1/2 or 3/2 so that α takes the values 0, 1, and 2, and denotes the rank of the corresponding tensor. In the above equation it should be understood that the matrix elements of the meson creation and photon annihilation operators have already been taken.

We now expand $[B_S \otimes A_{\frac{1}{2}}]^\alpha$, for each tensor of rank α, in terms of the complete set of available spin operators in the problem, i.e., the photon polarization vector $\vec{\epsilon}$ and the Pauli spin matrix $\vec{\sigma}$ together with the identity matrix. The result is

$$\left[B_S\otimes A_{\frac{1}{2}}\right]^0 = \frac{1}{\sqrt{6}}\vec{\sigma}\cdot\vec{\epsilon},$$
$$\left[B_S\otimes A_{\frac{1}{2}}\right]^1 = -\frac{1+\sqrt{2}}{\sqrt{6}}\left\{\vec{\epsilon} + \frac{i}{\sqrt{2}}\left(\frac{1-\sqrt{2}}{1+\sqrt{2}}\right)(\vec{\sigma}\times\vec{\epsilon})\right\}, \quad (5)$$
$$\left[B_S\otimes A_{\frac{1}{2}}\right]^2 = \frac{1}{\sqrt{2}}[\vec{\sigma}\otimes\vec{\epsilon}]^2,$$

where the numerical factors are uniquely determined such that the spin matrix elements of the right-hand side (r.h.s.) in the above equations equal the corresponding matrix elements of the l.h.s.

What we have done so far applies to either a negative or positive parity baryon B. Total parity conservation demands that $(-)^{L+L'} = +1$ and $(-)^{L+L'} = -1$ in the case of a positive and negative parity B, respectively. This leads to distinct spin structures of the transition operator for positive and negative parity as we shall show below. Hereafter, the superscript $\pm$ on any quantity stands for the positive (+) or negative (−) parity of B.

A. Positive parity case

For a positive parity baryon B, choosing the quantization axis $\hat{z}$ along $\hat{k}$, the quantity $[Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^\alpha$ can be expressed without loss of generality as

$$[Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^0 = (-)^L\frac{[L]}{4\pi}P_L(\hat{q}\cdot\hat{k})\delta_{L,L'},$$
$$[Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^1 = i(-)^L\frac{[L]}{4\pi}\sqrt{\frac{3}{L(L+1)}}P^1_L(\hat{q}\cdot\hat{k})\delta_{L,L'}\hat{n}_2, \quad (6)$$
$$[Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^2 = a_{L'L}[\hat{q}\otimes\hat{q}]^2 + b_{L'L}[\hat{k}\otimes\hat{k}]^2 + c_{L'L}[\hat{k}\otimes\hat{q}]^2.$$

The structure in the above equation is dictated by total parity conservation. $P_L(P^1_L)$ is the ordinary (associated) Legendre function. $\hat{n}_2 \equiv (\hat{k}\times\hat{q})/|\hat{k}\times\hat{q}|$. The coefficients $a_{L'L}, b_{L'L}$ and $c_{L'L}$ are derived explicitly in Appendix A.

Inserting Eqs. (5), (6) into Eq. (4) we have

$$\hat{M}^+(\vec{q},\vec{k}) = \mathcal{F}_1\vec{\sigma}\cdot\vec{\epsilon} + i\mathcal{F}_2\vec{\epsilon}\cdot\hat{n}_2 + \mathcal{F}_3(\vec{\sigma}\times\vec{\epsilon})\cdot\hat{n}_2 + \mathcal{F}_4[\vec{\sigma}\otimes\vec{\epsilon}]^2\cdot[\hat{q}\otimes\hat{q}]^2 + \mathcal{F}_5[\vec{\sigma}\otimes\vec{\epsilon}]^2\cdot[\hat{k}\otimes\hat{k}]^2 + \mathcal{F}_6[\vec{\sigma}\otimes\vec{\epsilon}]^2\cdot[\hat{k}\otimes\hat{q}]^2, \quad (7)$$

where

$$\mathcal{F}_1 = \frac{1}{8\pi\sqrt{3}}\sum[J]^2 M^{J\frac{1}{2}}_{LL}(q,k)P_L(\hat{k}\cdot\hat{q}),$$
$$\mathcal{F}_2 = -\frac{1}{4\pi}\left(\frac{1+\sqrt{2}}{\sqrt{2}}\right)\sum(-)^{-J-\frac{1}{2}+L}[J]^2\frac{[L]}{\sqrt{L(L+1)}} \times \begin{Bmatrix} S & L & J \\ L & \frac{1}{2} & 1 \end{Bmatrix} M^{JS}_{LL}(q,k)P^1_L(\hat{k}\cdot\hat{q}),$$
$$\mathcal{F}_3 = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\mathcal{F}_2, \quad (8)$$
$$\mathcal{F}_4 = \sqrt{\frac{1}{2}}\sum i^{L-L'}(-)^{-J-\frac{1}{2}}[J]^2\begin{Bmatrix} \frac{3}{2} & L & J \\ L' & \frac{1}{2} & 2 \end{Bmatrix} M^{J\frac{3}{2}}_{L'L}(q,k)a_{L'L},$$
$$\mathcal{F}_5 = \sqrt{\frac{1}{2}}\sum i^{L-L'}(-)^{-J-\frac{1}{2}}[J]^2\begin{Bmatrix} \frac{3}{2} & L & J \\ L' & \frac{1}{2} & 2 \end{Bmatrix} M^{J\frac{3}{2}}_{L'L}(q,k)b_{L'L},$$
$$\mathcal{F}_6 = \sqrt{\frac{1}{2}}\sum i^{L-L'}(-)^{-J-\frac{1}{2}}[J]^2\begin{Bmatrix} \frac{3}{2} & L & J \\ L' & \frac{1}{2} & 2 \end{Bmatrix} M^{J\frac{3}{2}}_{L'L}(q,k)c_{L'L}.$$

The summations are over all quantum numbers appearing explicitly in the r.h.s. of the equalities.

The above result is the most general form of the spin structure of the transition operator consistent with symmetry principles. The coefficients $\mathcal{F}_j$ are functions of the energy of the system and scattering angle θ of the meson M relative to the photon direction. We note that the transition operator in Eq. (7) has six independent spin operators and is valid for both real and virtual photons. Using the identity

$$3[\vec{\sigma}\otimes\vec{\epsilon}]^2\cdot[\hat{a}\otimes\hat{b}]^2=\tfrac{3}{2}[\vec{\sigma}\cdot\hat{a}\vec{\epsilon}\cdot\hat{b}+\vec{\sigma}\cdot\hat{b}\vec{\epsilon}\cdot\hat{a}]-(\hat{a}\cdot\hat{b})\vec{\sigma}\cdot\vec{\epsilon}, \tag{9}$$

where $\hat{a}$ and $\hat{b}$ stand for arbitrary unit vectors, Eq. (7) can be rewritten as

$$\hat{M}^+(\vec{q},\vec{k}) = F_1\vec{\sigma}\cdot\vec{\epsilon}+iF_2\vec{\epsilon}\cdot\hat{n}_2+F_3\vec{\sigma}\cdot\hat{k}\vec{\epsilon}\cdot\hat{q}+F_4\vec{\sigma}\cdot\hat{q}\vec{\epsilon}\cdot\hat{q} + F_5\vec{\sigma}\cdot\hat{k}\vec{\epsilon}\cdot\hat{k}+F_6\vec{\sigma}\cdot\hat{q}\vec{\epsilon}\cdot\hat{k}, \tag{10}$$

where

$$\begin{aligned} F_1 &= \mathcal{F}_1-\frac{1}{3}[\mathcal{F}_4+\mathcal{F}_5+(\hat{q}\cdot\hat{k})\mathcal{F}_6],\quad F_2=\mathcal{F}_2,\\ F_3 &= \frac{1}{|\hat{k}\times\hat{q}|}\mathcal{F}_3+\frac{1}{2}\mathcal{F}_6,\quad F_4=\mathcal{F}_4,\quad F_5=\mathcal{F}_5,\\ F_6 &= -\frac{1}{|\hat{k}\times\hat{q}|}\mathcal{F}_3+\frac{1}{2}\mathcal{F}_6. \end{aligned} \tag{11}$$

It should be noted that Eq. (10) is equivalent to that of Ref. [11]. In fact, apart from an irrelevant overall factor of i, the coefficients $f_j\equiv\mathcal{F}_j^{V(\pm,0)}, (j=1,\ldots,6)$ in Ref. [11] are related to those in Eq. (10) by

$$\begin{aligned} &F_1=f_1-(\hat{q}\cdot\hat{k})f_2,\quad F_2=|\hat{k}\times\hat{q}|f_2,\quad F_3=(f_2+f_3),\\ &F_4=f_4,\quad F_5=-f_1-(\hat{q}\cdot\hat{k})f_3-\frac{k^2}{k_0}f_5,\\ &F_6=-(\hat{q}\cdot\hat{k})f_4-\frac{k^2}{k_0}f_6. \end{aligned} \tag{12}$$

In the case of a real photon (photoproduction amplitude), the number of independent spin operators in Eq. (10) reduces to four due to the transversality condition, i.e., the terms F_5 and F_6 are absent. Equation (10) then becomes equivalent to that of Ref. [12]. The coefficients $f_j, (j=1,\ldots,4)$ in Ref. [12] are related to the corresponding coefficients F_j in Eq. (10) as given by Eq. (12) with $k^2=\vec{\epsilon}\cdot\hat{k}=0$.

One difference between our work [Eq. (10)] and Refs. [11,12] is that in the present work these coefficients are related explicitly [Eq. (8)] to the partial-wave matrix elements introduced in Eq. (1).

B. Negative parity case

For a negative parity baryon B, choosing the quantization axis $\hat{z}$ along $\hat{k}$, $[Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^\alpha$ can be expressed as

$$\begin{aligned} &[Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^0=0,\\ &[Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^1=\frac{[LL']}{4\pi}\Bigg[\sqrt{\frac{2}{L'(L'+1)}}(L0L'1|11)\\ &\qquad\times P^1_{L'}(\hat{k}\cdot\hat{q})\hat{n}_1+(L0L'0|10)P_{L'}(\hat{k}\cdot\hat{q})\hat{k}\Bigg],\\ &[Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^2=a'_{L'L}[\hat{k}\otimes\hat{n}_2]^2+b'_{L'L}[\hat{q}\otimes\hat{n}_2]^2, \end{aligned} \tag{13}$$

where $\hat{n}_1\equiv[(\hat{k}\times\hat{q})\times\hat{k}]/|\hat{k}\times\hat{q}|$. The coefficients $a'_{L'L}$ and $b'_{L'L}$ are calculated explicitly in Appendix A.

Inserting Eqs. (5), (13) into Eq. (4) we have

$$\begin{aligned} \hat{M}^-(\vec{q},\vec{k}) &= i\mathcal{G}_1\vec{\epsilon}\cdot\hat{n}_1+\mathcal{G}_2\vec{\sigma}\cdot(\vec{\epsilon}\times\hat{n}_1)+\mathcal{G}_3\vec{\sigma}\cdot(\vec{\epsilon}\times\hat{k})\\ &\quad+\mathcal{G}_4[\vec{\sigma}\otimes\vec{\epsilon}]^2\cdot[\hat{k}\otimes\hat{n}_2]^2+\mathcal{G}_5[\vec{\sigma}\otimes\vec{\epsilon}]^2\cdot[\hat{q}\otimes\hat{n}_2]^2\\ &\quad+i\mathcal{G}_6\vec{\epsilon}\cdot\hat{k}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} \mathcal{G}_1 &= \frac{1}{4\pi}\left(\frac{1+\sqrt{2}}{\sqrt{3}}\right)\sum i^{L-L'+1}(-)^{-J-\frac{1}{2}}[J]^2\frac{[LL']}{\sqrt{L'(L'+1)}}\\ &\quad\times(L0L'1|11)\begin{Bmatrix} S & L & J\\ L' & \frac{1}{2} & 1\end{Bmatrix}M^{JS}_{L'L}(q,k)P^1_{L'}(\hat{k}\cdot\hat{q}),\\ \mathcal{G}_2 &= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\mathcal{G}_1,\\ \mathcal{G}_3 &= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\mathcal{G}_6,\\ \mathcal{G}_4 &= \sqrt{\frac{1}{2}}\sum i^{L-L'}(-)^{-J-\frac{1}{2}}[J]^2\begin{Bmatrix} \frac{3}{2} & L & J\\ L' & \frac{1}{2} & 2\end{Bmatrix}M^{J\frac{3}{2}}_{L'L}(q,k)a'_{L'L}\\ \mathcal{G}_5 &= \sqrt{\frac{1}{2}}\sum i^{L-L'}(-)^{-J-\frac{1}{2}}[J]^2\begin{Bmatrix} \frac{3}{2} & L & J\\ L' & \frac{1}{2} & 2\end{Bmatrix}M^{J\frac{3}{2}}_{L'L}(q,k)b'_{L'L}\\ \mathcal{G}_6 &= \frac{1}{4\pi}\left(\frac{1+\sqrt{2}}{\sqrt{6}}\right)\sum i^{L-L'+1}(-)^{-J-\frac{1}{2}}[LL'][J]^2\\ &\quad\times(L0L'0|10)\begin{Bmatrix} S & L & J\\ L' & \frac{1}{2} & 1\end{Bmatrix}M^{JS}_{L'L}(q,k)P_{L'}(\hat{k}\cdot\hat{q}). \end{aligned} \tag{15}$$

The summations are over all quantum numbers appearing explicitly in the r.h.s. of the equalities.

The above result is the most general form of the spin structure of the transition operator for a negative parity baryon B. As in the positive parity case, we note that the amplitude in Eq. (14) has six independent spin operators and is valid for both real and virtual photons. Using Eq. (9), it can be rewritten as

$$\begin{aligned} \hat{M}^-(\vec{q},\vec{k}) &= iG_1\vec{\epsilon}\cdot\hat{q}+G_2\vec{\sigma}\cdot(\vec{\epsilon}\times\hat{q})+G_3\vec{\sigma}\cdot(\vec{\epsilon}\times\hat{k})\\ &\quad+G_4[\vec{\sigma}\cdot\hat{k}\vec{\epsilon}\cdot\hat{n}_2+\vec{\sigma}\cdot\hat{n}_2\vec{\epsilon}\cdot\hat{k}]+G_5[\vec{\sigma}\cdot\hat{q}\vec{\epsilon}\cdot\hat{n}_2\\ &\quad+\vec{\sigma}\cdot\hat{n}_2\vec{\epsilon}\cdot\hat{q}]+iG_6\vec{\epsilon}\cdot\hat{k}, \end{aligned} \tag{16}$$

where

$$\begin{aligned} &G_1=\frac{1}{|\hat{k}\times\hat{q}|}\mathcal{G}_1,\quad G_2=\frac{1}{|\hat{k}\times\hat{q}|}\mathcal{G}_2,\quad G_3=\mathcal{G}_3-(\hat{k}\cdot\hat{q})G_2,\\ &G_4=\tfrac{1}{2}\mathcal{G}_4,\quad G_5=\tfrac{1}{2}\mathcal{G}_5,\quad G_6=\mathcal{G}_6-(\hat{k}\cdot\hat{q})G_2. \end{aligned} \tag{17}$$

Quite recently, Zhao and Al-Khalili [13] have also given the spin structure of the photoproduction amplitude in connection to the reaction $\gamma N\to\bar{K}\Theta^+$ for the case of negative parity Θ^+. The structure given in Eq. (16) with $\vec{\epsilon}\cdot\hat{k}=0$ is equivalent to that of Eq. (18) in Ref. [13], except for the term $\vec{\sigma}\cdot\hat{n}_2\vec{\epsilon}\cdot\hat{q}$ which has not been included in Ref. [13] on the grounds that it is a higher-order contribution. However, this term and the $\vec{\sigma}\cdot\hat{q}\vec{\epsilon}\cdot\hat{n}_2$ term contribute with the same coefficient G_5.

We also note that the recent model-independent analysis of the Θ^+ photoproduction [14] has been based on the present results and, in particular, on Eqs. (10), (16) with $\vec{\epsilon}\cdot\hat{k}=0$.

II. APPLICATION: NEAR THRESHOLD AMPLITUDE IN $\gamma N \rightarrow MB$

As an application of the present results, we consider the reaction $\gamma N \rightarrow MB$ in the near-thresholds kinematics. The complete transition amplitude is given by Eq. (10) for a positive parity baryon B and by Eq. (16) for a negative baryon B, respectively, with $\vec{\epsilon}\cdot\hat{k}=0$. In the near-threshold energy region, the final MB is mainly in relative S and P waves. Then, considering only $L'=0,1$, there are seven partial-wave amplitudes. For positive parity B, they are

$$\begin{aligned} &{}^1S_1 \rightarrow S_1, \quad {}^3D_1 \rightarrow S_1, \quad {}^{3,1}P_1 \rightarrow P_1,\\ &{}^{3,1}P_3 \rightarrow P_3, \quad {}^3F_3 \rightarrow P_3, \end{aligned} \tag{18}$$

where we have used the notation ${}^{2S}L_{2J} \rightarrow L'_{2J}$. For these amplitudes, the coefficients F_i, $(i=1,\ldots,4)$ in Eq. (10) exhibit the following angular and energy (due to $q^{L'}$) dependences:

$$\begin{aligned} &F_1 = A_0 + \left(\frac{q}{\Lambda}\right) A_1\cos(\theta), \quad F_2 = \left(\frac{q}{\Lambda}\right) B_1 \sin(\theta),\\ &F_3 = \left(\frac{q}{\Lambda}\right) C_1, \quad F_4 = 0. \end{aligned} \tag{19}$$

In the above equation, $A_{L'}$, $B_{L'}$ and $C_{L'}$ denote the linear combinations of the partial-wave matrix elements $M^{JS}_{L'L}$ resulting from Eqs. (8), (11) for those states specified in Eq. (18) having orbital angular momentum L'. They are given explicitly in Appendix B. Note that the factor $(q/\Lambda)^{L'}$ contained in $M^{JS}_{L'L}$ due to the centrifugal barrier has been displayed explicitly in the above equation. Λ is a typical scale of the problem which may be taken to be the four-momentum transfer at threshold, $\Lambda^2 \sim -t = [m_B - m_N^2/(m_B+m_M)]m_M$, where m_B, m_M, and m_N denote the masses of the baryon B, meson M, and nucleon, respectively. Therefore, near threshold, higher partial-wave contributions will be suppressed by the factor $(q/\Lambda)^{L'}$ if heavy particles are produced in the final state. Moreover, for short-range processes, the coefficients $A_{L'}$, $B_{L'}$, and $C_{L'}$ are nearly constant independent of energy.

Analogously, for a negative parity baryon B, the possible partial-wave amplitudes are

$$\begin{aligned} &{}^{3,1}P_1 \rightarrow S_1, \quad {}^1S_1 \rightarrow P_1, \quad {}^3D_1 \rightarrow P_1,\\ &{}^3S_3 \rightarrow P_3, \quad {}^{3,1}D_3 \rightarrow P_3. \end{aligned} \tag{20}$$

With these amplitudes, the coefficients G_i in Eq. (16) exhibit the following angular and energy (due to $q^{L'}$) dependences

$$\begin{aligned} &G_1 = \left(\frac{q}{\Lambda}\right) A'_1, \quad G_2 = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) G_1,\\ &G_3 = B'_0 + \left(\frac{q}{\Lambda}\right) B'_1\cos(\theta),\\ &G_4 = \left(\frac{q}{\Lambda}\right) C'_1 \sin(\theta), \quad G_5 = 0. \end{aligned} \tag{21}$$

In the above equation, $A'_{L'}$, $B'_{L'}$, and $C'_{L'}$ denote the linear combinations of the partial-wave matrix elements $M^{JS}_{L'L}$ resulting from Eqs. (15), (17) for those states specified in Eq. (20) having orbital angular momentum L'. They are given explicitly in Appendix B. For a short-range process, $A'_{L'}$, $B'_{L'}$, and $C'_{L'}$ are nearly constant independent of energy.

Following Ref. [14] we now introduce $\vec{\epsilon}_\perp \equiv \hat{y}$ and $\vec{\epsilon}_\parallel \equiv \hat{x}$ denoting the photon polarization perpendicular to and lying in the reaction plane (xz plane), respectively. Recall that the reaction plane is defined as the plane containing the vectors $\vec{k}$ (in the $+z$-direction) and $\vec{q}$ and that $\vec{k}\times\vec{q}$ is along the $+y$-direction, in which case, $\hat{n}_1=\hat{x}$ and $\hat{n}_2=\hat{y}$. Then, from Eqs. (10), (19)

$$\begin{aligned} \hat{M}^{+\perp} &= \left[A_0 + \left(\frac{q}{\Lambda}\right) A_1\cos(\theta)\right]\sigma_y + i\left(\frac{q}{\Lambda}\right) B_1\sin(\theta),\\ \hat{M}^{+\parallel} &= \left[A_0 + \left(\frac{q}{\Lambda}\right) A_1\cos(\theta)\right]\sigma_x + \left(\frac{q}{\Lambda}\right) C_1\sin(\theta)\sigma_z. \end{aligned} \tag{22}$$

Similarly, from Eqs. (16), (21)

$$\begin{aligned} \hat{M}^{-\perp} &= \left[B'_0 + \left(\frac{q}{\Lambda}\right) \bar{B}'_1\cos(\theta)\right]\sigma_x + \left(\frac{q}{\Lambda}\right) \bar{C}'_1\sin(\theta)\sigma_z,\\ \hat{M}^{-\parallel} &= -\left[B'_0 + \left(\frac{q}{\Lambda}\right) \bar{B}'_1\cos(\theta)\right]\sigma_y + i\left(\frac{q}{\Lambda}\right) A'_1\sin(\theta). \end{aligned} \tag{23}$$

In the above equation, $\bar{B}'_1 \equiv B'_1 + \bar{A}'_1$ and $\bar{C}'_1 \equiv C'_1 - \bar{A}'_1$, with $\bar{A}'_1 \equiv \frac{1}{\sqrt{2}}(\frac{\sqrt{2}-1}{\sqrt{2}+1})A'_1$.

Following Ref. [14], any observable in the $\gamma N \rightarrow MB$ reaction can be readily calculated from Eqs. (22), (23) for both positive and negative parity baryons B. It is then straightforward to show that no observable in this reaction is able to distinguish between positive and negative parity B, unless one measures the polarization of B. In particular, neither the energy dependence nor the angular distribution exhibits features sufficiently distinct to determine the parity of the baryon B unambiguously. As has been shown in Ref. [15] in connection with $NN \rightarrow Y\Theta^+$, the situation is quite different in the $NN \rightarrow B'B$ reaction. We shall discuss this latter reaction in the following two sections.

III. THE REACTION $N+N \rightarrow B'+B$

We now consider the process $N+N \rightarrow B'+B$, where B' stands for a positive parity spin-1/2 baryon and B, either a positive or negative parity spin-1/2 baryon. The partial-wave expansion of the corresponding reaction amplitude is

$$\begin{aligned} \langle S'M_{S'}|\hat{M}(\vec{p}\,',\vec{p})|SM_S\rangle = \sum i^{L-L'}&(S'M_{S'}L'M_{L'}|JM_J)\\ &\times (SM_SLM_L|JM_J)M^{S'SJ}_{L'L}(p',p)\\ &\times Y_{L'M_{L'}}(\hat{p}')Y^*_{LM_L}(\hat{p}), \end{aligned} \tag{24}$$

where S, L, J stand for the total spin, total orbital angular momentum, and the total angular momentum, respectively, of the initial NN state. M_S, M_L, and M_J denote the corresponding projection quantum numbers. The primed quantities stand for the corresponding quantum numbers of the final $B'B$ state. The summation runs over all quantum numbers not specified in the l.h.s. of Eq. (24). $\vec{p}$ and $\vec{p}\,'$ denote the relative momenta of the two particles in the initial and final states, respectively. We note that, in Eq. (24), in addition to the restrictions on the quantum numbers encoded in the geometrical factors, total parity conservation imposes that $(-)^{L+L'} = +1$ and $(-)^{L+L'} = -1$ for positive and negative parity B, respectively.

Equation (24) can be inverted to solve for the partial-wave matrix element $M^{S'SJ}_{L'L}(p', p)$. We have

$$M^{S'SJ}_{L'L}(p', p) = \sum i^{L'-L}(S'M_{S'}L'M_{L'}|JM_J) \times (SM_S L0|JM_J)\frac{8\pi^2}{2J+1}\sqrt{\frac{2L+1}{4\pi}} \times \int_{-1}^{+1} d(\cos(\theta))Y^*_{L'M_{L'}}(\theta, 0) \times \langle S'M_{S'}|\hat{M}(\vec{p}', \vec{p})|SM_S\rangle, \quad (25)$$

where, without loss of generality, the z axis is chosen along $\vec{p}$ and $\vec{p}'$ in the xz plane; $\cos(\theta) = \hat{p}' \cdot \hat{p}$. The summation is over all quantum numbers not specified in the l.h.s. of the equation.

The most general spin structure of the transition operator can be obtained from Eq. (24) as

$$\hat{M}(\vec{p}', \vec{p}) = \sum_{S'SM_SM_{S'}} |S'M_{S'}\rangle\langle S'M_{S'}|\hat{M}(\vec{p}', \vec{p})|SM_S\rangle\langle SM_S|. \quad (26)$$

Inserting Eq. (24) into Eq. (26) and recoupling gives

$$\hat{M}(\vec{p}', \vec{p}) = \sum i^{L-L'}(-)^{J+S'}[J]^2 M^{S'SJ}_{L'L}(p', p)\sum_\alpha \begin{Bmatrix} S & L & J \\ L' & S' & \alpha \end{Bmatrix} \times [A_{S'} \otimes B_S]^\alpha \cdot [Y_{L'}(\hat{p}') \otimes Y_L(\hat{p})]^\alpha, \quad (27)$$

where we have used the notations $B_{SM_S} \equiv (-)^{S-M_S}\langle S - M_S|$ and $A_{S'M_{S'}} \equiv |S'M_{S'}\rangle$.

We now expand $[A_{S'} \otimes B_S]^\alpha$, for each tensor of rank α, in terms of the complete set of available spin operators in the problem, i.e., the Pauli spin matrices $\vec{\sigma}_1$ and $\vec{\sigma}_2$, corresponding to the interacting particles 1 and 2, together with the identity matrix. Then, α takes the values 0, 1, and 2, and denotes the rank of the corresponding (spin) tensor. There are six cases to be considered:

$$S = S' = 0, \alpha = 0: \quad [A_{S'} \otimes B_S]^0 = |00\rangle\langle 00| \equiv P_{S=0},$$

$$S = S' = 1, \alpha = 0: \quad [A_{S'} \otimes B_S]^0 = \frac{1}{\sqrt{3}}\sum_{M_S}|1M_S\rangle\langle 1M_S| \equiv \frac{1}{\sqrt{3}}P_{S=1},$$

$$S = S' = 1, \alpha = 1: \quad [A_{S'} \otimes B_S]^1 = \frac{1}{2\sqrt{2}}(\vec{\sigma}_1 + \vec{\sigma}_2), \quad (28)$$

$$S = 0, S' = 1, \alpha = 1: \quad [A_{S'} \otimes B_S]^1 = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)P_{S=0},$$

$$S = 1, S' = 0, \alpha = 1: \quad [A_{S'} \otimes B_S]^1 = -\frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)P_{S=1},$$

$$S = S' = 1, \alpha = 2: \quad [A_{S'} \otimes B_S]^2 = \frac{1}{\sqrt{2}}[\vec{\sigma}_1 \otimes \vec{\sigma}_2]^2,$$

where P_S stands for the spin projection operator onto the (initial) spin singlet and triplet states as $S = 0$ and 1, respectively. In terms of the Pauli spin matrices we have $P_{S=0} = (1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2)/4$ and $P_{S=1} = (3 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)/4$. Also, $(\vec{\sigma}_1 - \vec{\sigma}_2)P_S/2 = [(\vec{\sigma}_1 - \vec{\sigma}_2) + (-)^S i(\vec{\sigma}_1 \times \vec{\sigma}_2)]/4$.

In the following we shall consider the case of a positive and a negative parity baryon B separately.

A. Positive parity case

For a positive parity baryon B, the quantity $[Y_{L'}(\hat{p}') \otimes Y_L(\hat{p})]^\alpha$ in Eq. (27) can be read off from Eq. (6). Note that $[Y_{L'}(\hat{p}') \otimes Y_L(\hat{p})]^\alpha = (-)^\alpha[Y_L(\hat{p}) \otimes Y_{L'}(\hat{p}')]^\alpha$ for the positive parity case. Inserting this and Eq. (28) into Eq. (27), we have

$$\hat{M}^+(\vec{p}', \vec{p}) = \mathcal{D}_1 P_{S=0} + \mathcal{D}_2 P_{S=1} + i\mathcal{D}_3(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\hat{n}_2 + i\mathcal{D}_4(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\hat{n}_2 P_{S=0} + i\mathcal{D}_5(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\hat{n}_2 P_{S=1} + \mathcal{D}_6[\vec{\sigma}_1 \otimes \vec{\sigma}_2]^2 \cdot [\hat{p} \otimes \hat{p}]^2 + \mathcal{D}_7[\vec{\sigma}_1 \otimes \vec{\sigma}_2]^2 \cdot [\hat{p}' \otimes \hat{p}']^2 + \mathcal{D}_8[\vec{\sigma}_1 \otimes \vec{\sigma}_2]^2 \cdot [\hat{p} \otimes \hat{p}']^2, \quad (29)$$

where

$$\mathcal{D}_1 = \frac{1}{4\pi}\sum [L]^2 M^{00L}_{LL}(p', p)P_L(\hat{p}' \cdot \hat{p}),$$

$$\mathcal{D}_2 = \frac{1}{4\pi}\frac{1}{3}\sum [J]^2 M^{11J}_{LL}(p', p)P_L(\hat{p}' \cdot \hat{p}),$$

$$\mathcal{D}_3 = -\frac{1}{4\pi}\frac{1}{8}\sum [J]^2\left(1 + \frac{2 - J(J+1)}{L(L+1)}\right)M^{11J}_{LL}(p', p)P^1_L(\hat{p}' \cdot \hat{p}),$$

$$\mathcal{D}_4 = -\frac{1}{4\pi}\frac{1}{2}\sum \frac{[L]^2}{\sqrt{L(L+1)}}M^{10L}_{LL}(p', p)P^1_L(\hat{p}' \cdot \hat{p}), \quad (30)$$

$$\mathcal{D}_5 = \frac{1}{4\pi}\frac{1}{2}\sum \frac{[L]^2}{\sqrt{L(L+1)}}M^{01L}_{LL}(p', p)P^1_L(\hat{p}' \cdot \hat{p}),$$

$$\mathcal{D}_6 = \frac{1}{2}\sum i^{L-L'}(-)^{J+1}[J]^2 \begin{Bmatrix} 1 & L & J \\ L' & 1 & 2 \end{Bmatrix} M^{11J}_{L'L}(p', p)a_{L'L},$$

$$\mathcal{D}_7 = \frac{1}{2}\sum i^{L-L'}(-)^{J+1}[J]^2 \begin{Bmatrix} 1 & L & J \\ L' & 1 & 2 \end{Bmatrix} M^{11J}_{L'L}(p', p)b_{L'L},$$

$$\mathcal{D}_8 = \frac{1}{2}\sum i^{L-L'}(-)^{J+1}[J]^2 \begin{Bmatrix} 1 & L & J \\ L' & 1 & 2 \end{Bmatrix} M^{11J}_{L'L}(p', p)c_{L'L}.$$

The coefficients $a_{L'L}$, $b_{L'L}$, and $c_{L'L}$ are given by Eqs. (A4), (A5) with the replacements $\hat{k} \to \hat{p}$ and $\hat{q} \to \hat{p}'$; these same replacements are needed to calculate $\hat{n}_2$ in Eq. (29).

Equation (29) is the most general spin structure of the transition operator for a positive parity baryon B, consistent with symmetry principles. It contains eight independent spin structures. The first two terms are the central spin singlet and triplet interactions, respectively. The third term is the spin-orbit interaction. The fourth and fifth terms describe the spin singlet→triplet and triplet→singlet transitions, respectively. The last three terms are the tensor interactions of rank 2. Apart from the fourth and fifth terms, all the other terms conserve the total spin in the transition.

It should be mentioned that in the case of identical particles, i.e., $NN \to NN$, the structure $(\vec{\sigma}_1 - \vec{\sigma}_2)$ is not allowed in the scattering amplitude, which reduces the total number of independent spin structures in Eq. (29) to six [1]. Furthermore, for elastic scattering, $\mathcal{D}_6 = \mathcal{D}_7$, as a consequence of time reversal invariance [1].

For the purpose of calculating the observables directly from the transition operator of Eq. (29), it is convenient to reexpress it in the form

$$\hat{M}^+ = \sum_{\lambda=1}^{9}\sum_{n,n'=0}^{3} M^\lambda_{nn'}\sigma_n(1)\sigma_{n'}(2), \quad (31)$$

where $\sigma_0(i) \equiv 1, \sigma_1(i) \equiv \sigma_x(i)$, etc., for ith nucleon. The coefficients $M^{\lambda}_{nn'}$ are linear combinations of the coefficients appearing in Eq. (30). Explicitly, we have

$$\begin{aligned}
M^1_{nn'} &= \tfrac{1}{4}\left[3\mathcal{D}_2 + \mathcal{D}_1\right]\delta_{n,0}\delta_{n',0},\\
M^2_{nn'} &= i\left[\mathcal{D}_3 + \tfrac{1}{2}(\mathcal{D}_4 + \mathcal{D}_5)\right]\hat{n}_{2n}(1-\delta_{n,0})\delta_{n',0},\\
M^3_{nn'} &= i\left[\mathcal{D}_3 - \tfrac{1}{2}(\mathcal{D}_4 + \mathcal{D}_5)\right]\hat{n}_{2n'}\delta_{n,0}(1-\delta_{n',0}),\\
M^4_{nn'} &= \tfrac{1}{2}\left[\mathcal{D}_5 - \mathcal{D}_4\right]\varepsilon_{nn'k}\hat{n}_{2k}(1-\delta_{n,0})(1-\delta_{n',0}),\\
M^5_{nn'} &= \left[\tfrac{1}{4}(\mathcal{D}_2-\mathcal{D}_1) - \tfrac{1}{3}(\mathcal{D}_6+\mathcal{D}_7+(\hat{p}\cdot\hat{p}')\mathcal{D}_8)\right]\delta_{n,n'}(1-\delta_{n,0}),\\
M^6_{nn'} &= \mathcal{D}_6\hat{p}_n\hat{p}_{n'}(1-\delta_{n,0})(1-\delta_{n',0}),\\
M^7_{nn'} &= \mathcal{D}_7\hat{p}'_n\hat{p}'_{n'}(1-\delta_{n,0})(1-\delta_{n',0}),\\
M^8_{nn'} &= \tfrac{1}{2}\mathcal{D}_8\hat{p}_n\hat{p}'_{n'}(1-\delta_{n,0})(1-\delta_{n',0}),\\
M^9_{nn'} &= \tfrac{1}{2}\mathcal{D}_8\hat{p}'_n\hat{p}_{n'}(1-\delta_{n,0})(1-\delta_{n',0}),
\end{aligned} \tag{32}$$

where we have used the notation $\hat{a}_n$ for the nth component of an arbitrary unit vector $\hat{a}$; $\varepsilon_{nn'k}$ denotes the antisymmetric Levi-Civita tensor and λ is an index for the type of term (operator) being considered. In obtaining Eq. (32), we have made use of the identity

$$\begin{aligned}
&3[\vec{\sigma}_1\otimes\vec{\sigma}_2]^2\cdot[\hat{a}\otimes\hat{b}]^2\\
&\quad= \tfrac{3}{2}[\vec{\sigma}_1\cdot\hat{a}\vec{\sigma}_2\cdot\hat{b} + \vec{\sigma}_1\cdot\hat{b}\vec{\sigma}_2\cdot\hat{a}] - (\hat{a}\cdot\hat{b})\vec{\sigma}_1\cdot\vec{\sigma}_2,
\end{aligned} \tag{33}$$

where $\hat{a}$ and $\hat{b}$ denote arbitrary unit vectors.

With the transition operator in the form of Eq. (31), it is straightforward to calculate any observable of interest. Of course, it can also be expressed in terms of the matrix elements of total spin of Eq. (24); for convenience some observables are given in Appendix D in both representations.

B. Negative parity case

For a negative parity baryon B, the quantity $[Y_{L'}(\hat{p}')\otimes Y_L(\hat{p})]^\alpha$ in Eq. (27) can be read off from Eq. (13). For this case $[Y_{L'}(\hat{p}')\otimes Y_L(\hat{p})]^\alpha = (-)^{\alpha+1}[Y_L(\hat{p})\otimes Y_{L'}(\hat{p}')]^\alpha$. Inserting this and Eq. (28) into Eq. (27), we have

$$\begin{aligned}
\hat{M}^-(\vec{p}',\vec{p}) = {}& i(\mathcal{H}_1\hat{n}_1 + \mathcal{H}_2\hat{p})\cdot(\vec{\sigma}_1+\vec{\sigma}_2)\\
&+ i(\mathcal{H}_3\hat{n}_1 + \mathcal{H}_4\hat{p})\cdot(\vec{\sigma}_1-\vec{\sigma}_2)P_{S=0}\\
&+ i(\mathcal{H}_5\hat{n}_1 + \mathcal{H}_6\hat{p})\cdot(\vec{\sigma}_1-\vec{\sigma}_2)P_{S=1}\\
&+ \mathcal{H}_7[\vec{\sigma}_1\otimes\vec{\sigma}_2]^2\cdot[\hat{p}\otimes\hat{n}_2]^2\\
&+ \mathcal{H}_8[\vec{\sigma}_1\otimes\vec{\sigma}_2]^2\cdot[\hat{p}'\otimes\hat{n}_2]^2,
\end{aligned} \tag{34}$$

where

$$\begin{aligned}
\mathcal{H}_1 &= \frac{1}{4\pi}\frac{1}{2}\sum i^{L-L'+1}(-)^J[J]^2\frac{[LL']}{\sqrt{L'(L'+1)}}(L0L'1|11)\\
&\quad\times\begin{Bmatrix}1 & L & J\\ L' & 1 & 1\end{Bmatrix}M^{11J}_{L'L}(p',p)P^1_{L'}(\hat{p}'\cdot\hat{p}),\\
\mathcal{H}_2 &= \frac{1}{4\pi}\frac{1}{2\sqrt{2}}\sum i^{L-L'+1}(-)^J[LL'][J]^2(L0L'0|10)\\
&\quad\times\begin{Bmatrix}1 & L & J\\ L' & 1 & 1\end{Bmatrix}M^{11J}_{L'L}(p',p)P_{L'}(\hat{p}'\cdot\hat{p}),\\
\mathcal{H}_3 &= \frac{1}{4\pi}\frac{1}{\sqrt{6}}\sum i^{L-L'+1}(-)^{L'+1}\frac{[L'][L]^2}{\sqrt{L'(L'+1)}}(L0L'1|11)\\
&\quad\times M^{10L}_{L'L}(p',p)P^1_{L'}(\hat{p}'\cdot\hat{p}),\\
\mathcal{H}_4 &= \frac{1}{4\pi}\frac{1}{2\sqrt{3}}\sum i^{L-L'+1}(-)^{L'+1}[L'][L]^2(L0L'0|10)\\
&\quad\times M^{10L}_{L'L}(p',p)P_{L'}(\hat{p}'\cdot\hat{p}),\\
\mathcal{H}_5 &= \frac{1}{4\pi}\frac{1}{\sqrt{6}}\sum i^{L-L'+1}(-)^L\frac{[L][L']^2}{\sqrt{L'(L'+1)}}(L0L'1|11)\\
&\quad\times M^{01L'}_{L'L}(p',p)P^1_{L'}(\hat{p}'\cdot\hat{p}),\\
\mathcal{H}_6 &= \frac{1}{4\pi}\frac{1}{2\sqrt{3}}\sum i^{L-L'+1}(-)^L[L][L']^2(L0L'0|10)\\
&\quad\times M^{01L'}_{L'L}(p',p)P_{L'}(\hat{p}'\cdot\hat{p}),\\
\mathcal{H}_7 &= \frac{1}{2}\sum i^{L-L'}(-)^J[J]^2\begin{Bmatrix}1 & L & J\\ L' & 1 & 2\end{Bmatrix}M^{11J}_{L'L}(p',p)a'_{L'L},\\
\mathcal{H}_8 &= \frac{1}{2}\sum i^{L-L'}(-)^J[J]^2\begin{Bmatrix}1 & L & J\\ L' & 1 & 2\end{Bmatrix}M^{11J}_{L'L}(p',p)b'_{L'L}.
\end{aligned} \tag{35}$$

The coefficients $a'_{L'L}$ and $b'_{L'L}$ are given by Eqs. (A9), (A10) with the replacements $\hat{k}\to\hat{p}$ and $\hat{q}\to\hat{p}'$. The same replacements are also required to calculate $\hat{n}_2$.

Equation (34) is the most general spin structure of the transition operator for a negative parity baryon B, consistent with symmetry principles. It also contains eight independent spin structures, but no central interaction is present in this case.

Analogous to the positive parity case, Eq. (34) can be reexpressed in the form

$$\hat{M}^- = \sum_{\lambda=1}^{10}\sum_{n,n'=0}^{3} M^{\lambda}_{nn'}\sigma_n(1)\sigma_{n'}(2), \tag{36}$$

where the coefficients $M^{\lambda}_{nn'}$ are given by

$$\begin{aligned}
M^1_{nn'} &= i\left[\mathcal{H}_1 + \tfrac{1}{2}(\mathcal{H}_3+\mathcal{H}_5)\right]\hat{n}_{1n}(1-\delta_{n,0})\delta_{n',0},\\
M^2_{nn'} &= i\left[\mathcal{H}_2 + \tfrac{1}{2}(\mathcal{H}_4+\mathcal{H}_6)\right]\hat{p}_n(1-\delta_{n,0})\delta_{n',0},\\
M^3_{nn'} &= i\left[\mathcal{H}_1 - \tfrac{1}{2}(\mathcal{H}_3+\mathcal{H}_5)\right]\hat{n}_{1n'}\delta_{n,0}(1-\delta_{n',0}),\\
M^4_{nn'} &= i\left[\mathcal{H}_2 - \tfrac{1}{2}(\mathcal{H}_4+\mathcal{H}_6)\right]\hat{p}_{n'}\delta_{n,0}(1-\delta_{n',0}),\\
M^5_{nn'} &= i\tfrac{1}{2}\left[\mathcal{H}_3 - \mathcal{H}_5\right]\varepsilon_{nn'k}\hat{n}_{1k}(1-\delta_{n,0})(1-\delta_{n',0}),\\
M^6_{nn'} &= i\tfrac{1}{2}\left[\mathcal{H}_4 - \mathcal{H}_6\right]\varepsilon_{nn'k}\hat{p}_k(1-\delta_{n,0})(1-\delta_{n',0}),\\
M^7_{nn'} &= \tfrac{1}{2}\mathcal{H}_7\hat{p}_n\hat{n}_{2n'}(1-\delta_{n,0})(1-\delta_{n',0}),\\
M^8_{nn'} &= \tfrac{1}{2}\mathcal{H}_7\hat{p}_{n'}\hat{n}_{2n}(1-\delta_{n,0})(1-\delta_{n',0}),\\
M^9_{nn'} &= \tfrac{1}{2}\mathcal{H}_8\hat{p}'_n\hat{n}_{2n'}(1-\delta_{n,0})(1-\delta_{n',0}),\\
M^{10}_{nn'} &= \tfrac{1}{2}\mathcal{H}_8\hat{p}'_{n'}\hat{n}_{2n}(1-\delta_{n,0})(1-\delta_{n',0}).
\end{aligned} \tag{37}$$

IV. APPLICATION: NEAR THRESHOLD AMPLITUDE IN $pp \to B'B$

As an application of the results of the previous section, we consider the reaction $pp \to B'B$ in the near-threshold energy region. We restrict to S and P waves in the final state, i.e., $L' = 0, 1$. In contrast to the photoproduction reaction discussed in Sec. II, here the Pauli principle restricts the initial pp state to $(-)^{S+L} = +1$.

For a positive parity baryon B, we then have six partial-wave states

$$^1S_0 \to {}^1S_0, \quad {}^3P_1 \to {}^1P_1, \\ {}^3P_{0,1,2} \to {}^3P_{0,1,2}, \quad {}^3F_2 \to {}^3P_2. \tag{38}$$

Here we use the notation $^{2S+1}L_J \to {}^{2S'+1}L'_J$. Then, the coefficients $\mathcal{D}_i$ in Eq. (30) reduce to

$$\begin{aligned} \mathcal{D}_1 &= A_0^0, \quad \mathcal{D}_2 = \left(\frac{p'}{\Lambda}\right) A_1^1 \cos(\theta), \\ \mathcal{D}_3 &= \left(\frac{p'}{\Lambda}\right) B_1^1 \sin(\theta), \quad \mathcal{D}_4 = 0, \\ \mathcal{D}_5 &= \left(\frac{p'}{\Lambda}\right) C_1^1 \sin(\theta), \quad \mathcal{D}_6 = 0, \\ \mathcal{D}_7 &= \left(\frac{p'}{\Lambda}\right) D_1^1 \cos(\theta), \quad \mathcal{D}_8 = \left(\frac{p'}{\Lambda}\right) E_1^1, \end{aligned} \tag{39}$$

where the quantities $A_{L'}^S$, $B_{L'}^S$, etc. are given in Appendix C. Here, again, the $(p'/\Lambda)^{L'}$ dependence of the partial-wave matrix elements due to the centrifugal barrier is displayed explicitly. As in the photoproduction reaction discussed in Sec. II, the scale Λ may be taken to be the four-momentum transfer at threshold, $\Lambda^2 \sim -t = m_B m_{B'} - m_N^2$, where m_B denotes the mass of the baryon B and $m_{B'}$, the mass of the positive parity baryon B'.

For a negative parity baryon B, the possible partial-wave transitions are

$$^3P_0 \to {}^1S_0, \quad {}^3P_1 \to {}^3S_1, \quad {}^1S_0 \to {}^3P_0, \quad {}^1D_2 \to {}^3P_2. \tag{40}$$

With these partial-wave states, Eq. (35) reduces to

$$\begin{aligned} \mathcal{H}_1 &= 0, \quad \mathcal{H}_2 = A_0^{\prime 11}, \quad \mathcal{H}_3 = \left(\frac{p'}{\Lambda}\right) B_1^{\prime 10} \sin(\theta), \\ \mathcal{H}_4 &= \left(\frac{p'}{\Lambda}\right) C_1^{\prime 10} \cos(\theta), \quad \mathcal{H}_5 = 0, \\ \mathcal{H}_6 &= C_0^{\prime 01}, \quad \mathcal{H}_7 = 0, \quad \mathcal{H}_8 = 0, \end{aligned} \tag{41}$$

where the quantities $A_{L'}^{\prime S'S}$, $B_{L'}^{\prime S'S}$, etc. are given in Appendix C.

An interesting feature in Eq. (39) is that the amplitudes with spin-triplet initial states depend linearly on (p'/Λ) (i.e., the final state is in a P wave) if the parity of the baryon B is positive, whereas if the parity of B is negative [Eq. (41)], they do not depend on (p'/Λ) (i.e., the final state is an S state). This energy dependence is interchanged for amplitudes with spin-singlet initial states. The feature just mentioned is a direct consequence of the Pauli principle and total parity conservation,

$$(-)^{S+L'+T} = \mp 1, \tag{42}$$

where the $(-)$ and $(+)$ signs refer to the positive and negative parity baryon B, respectively. T denotes the total isospin ($T = 1$ for pp). In fact, as pointed out in Ref. [15], it follows immediately from Eq. (42) that the energy dependence of a partial-wave amplitude with spin-triplet initial state ($S = 1$) is given by an odd power of (p'/Λ) if the parity of B is positive since in this case the final state must be in an odd orbital angular momentum L', whereas it is given by an even power of (p'/Λ) if the parity is negative since L' is even in this case. For completeness, we note that for the $pn \to B'B$ reaction, this energy dependence is interchanged [15].

With the coefficients $\mathcal{D}_i$ and $\mathcal{H}_i$ given by Eqs. (39), (41), it is straightforward to calculate any observable of interest in the $pp \to B'B$ reaction using Eqs. (31), (32), (36), (37) and the method given in Appendix D. Alternatively, one can calculate the observables using the matrix elements given by Eq. (24), in terms of which, some of the observables are also given in Appendix D. In particular, the cross section with the spin-triplet initial state $3\sigma_\Sigma$, as defined by Eq. (D10) in Appendix D, can be expressed as

$$\left(\frac{p'}{\Lambda}\right)^{-1} \frac{d(^3\sigma_\Sigma^+)}{d\Omega} = \left[\beta_0 + \beta_1 \cos^2(\theta)\right] \left(\frac{p'}{\Lambda}\right)^2, \tag{43}$$

for a positive parity baryon B and,

$$\left(\frac{p'}{\Lambda}\right)^{-1} \frac{d(^3\sigma_\Sigma^-)}{d\Omega} = \beta_0', \tag{44}$$

for a negative parity baryon B. Note that we have divided out the cross section by a factor of (p'/Λ) due to the final state phase space which introduces an extra p' dependence into the cross section. In the above equations

$$\begin{aligned} \beta_0 &\equiv 2\left|B_1^1\right|^2 + \tfrac{1}{2}\left(\left|C_1^1\right|^2 + \left|E_1^1\right|^2\right) - \left|U_1^1\right|^2, \\ \beta_1 &\equiv -\beta_0 + \tfrac{3}{4}\left|A_1^1\right|^2 + \tfrac{2}{3}\left|D_1^1 + E_1^1\right|^2 - 2\left|W_1^1\right|^2, \\ \beta_0' &\equiv 2\left|A_0^{\prime 11}\right|^2 + \left|C_0^{\prime 01}\right|^2, \end{aligned} \tag{45}$$

where U_1^1 and W_1^1 are defined in Appendix C.

Equations (43), (44) show that, apart from the p' dependence due to phase space, the spin-triplet cross section scales quadratically in p' (or equivalently, linearly in excess energy Q since $\sqrt{Q} \propto p'$), if the parity of B is positive, whereas it is constant if the parity of B is negative. This is the principal result of Ref. [15], where the reaction $NN \to Y\Theta^+$ has been investigated. Furthermore, the spin-triplet cross section exhibits a $\cos^2(\theta)$ angular dependence in the case of a positive parity B, whereas it is isotropic in the case of a negative parity B. Therefore, an observation of a strong $\cos^2(\theta)$ dependence in the measured angular distribution near threshold would imply a positive parity baryon B. An isotropic angular distribution, on the other hand, would be inconclusive about the parity of B.

The spin-triplet cross section can be related to the spin correlation coefficients A_{ii} as (see Appendix D)

$$\frac{d(^3\sigma_\Sigma)}{d\Omega} = \frac{1}{4}\frac{d\sigma}{d\Omega}(2 + A_{xx} + A_{yy}), \tag{46}$$

with $d\sigma/d\Omega$ denoting the unpolarized cross section and, A_{ii}, the spin correlation coefficient with the spin orientations along the i-axis. Therefore, in general, $^3\sigma_\Sigma$ can be extracted experimentally by measuring the spin correlation coefficients A_{xx} and A_{yy} in conjunction with the unpolarized cross section. Furthermore, it is immediate from Eq. (D7) that, at threshold, as pointed out by Rekalo and Tomasi-Gustafsson in Ref. [16] in connection to the reaction $pp \to \Sigma^+\Theta^+$, $A_{xx} = A_{yy} = -1$ for a positive parity baryon B while $A_{xx} = A_{yy} \geqslant 0$ for a negative parity B.

V. THE REACTION $N+N \to M+B'+N$

We now focus on the process $N + N \to M + B' + N$, where M stands for a spinless (scalar or pseudoscalar) meson and, B' for a spin-1/2 and positive-parity baryon. We start by making a partial-wave expansion of the corresponding reaction amplitude

$$\begin{aligned}\langle S'M_{S'}|\hat{M}(\vec{q},\vec{p}';\vec{p})|SM_S\rangle &= \sum i^{L-L'-l}(lm_l J'M_{J'}|JM_J)(S'M_{S'}L'M_{L'}|J'M_{J'})\\ &\times (SM_S LM_L|JM_J)M^{S'J'SJ}_{lL'L}(q,p';p)\\ &\times Y_{lm_l}(\hat{q})Y_{L'M_{L'}}(\hat{p}')Y^*_{LM_L}(\hat{p}), \end{aligned} \tag{47}$$

where S, L, J stand for the total spin, total orbital angular momentum, and the total angular momentum, respectively, of the initial NN state. M_S, M_L, and M_J denote the corresponding projection quantum numbers. The primed quantities stand for the corresponding quantum numbers of the final $B'N$ state. l and m_l denote the orbital angular momentum of the emitted meson and its projection, respectively, relative to the center of mass of the final two-baryon system. The summation runs over all quantum numbers not specified in the l.h.s. of Eq. (47). $\vec{p}$ and $\vec{p}'$ denote the relative momenta of the two baryons in the initial and final states, respectively. $\vec{q}$ denotes the momentum of the emitted meson with respect to the center of mass of the two baryons in the final state. We note that, in Eq. (47), apart from the restrictions on the quantum numbers encoded in the geometrical factors, total parity conservation imposes that $(-)^{l+L+L'} = -1$ in the case of pseudoscalar meson production and $(-)^{l+L+L'} = +1$ in the case of scalar meson production.

Equation (47) can be inverted to solve for the partial-wave matrix element $M^{S'J'SJ}_{lL'L}(q,p';p)$. We have

$$\begin{aligned}M^{S'J'SJ}_{lL'L}(q,p';p) &= \sum i^{L'+l-L}(lm_l J'M_{J'}|JM_J)\\ &\times (S'M_{S'}L'M_{L'}|J'M_{J'})(SM_S L0|JM_J)\\ &\times \frac{8\pi^2}{2J+1}\sqrt{\frac{2L+1}{4\pi}}\int d\Omega_{p'}Y^*_{L'M_{L'}}(\hat{p}')\\ &\times \int_{-1}^{+1} d(\cos(\theta_q))Y^*_{lm_l}(\theta_q,0)\\ &\times \langle S'M_{S'}|\hat{M}(\vec{q},\vec{p}';\vec{p})|SM_S\rangle, \end{aligned} \tag{48}$$

where, without loss of generality, the z axis is chosen along $\vec{p}$ and $\vec{q}$ in the xz plane; $\cos(\theta_q) = \hat{q}\cdot\hat{p}$. The summation is over all quantum numbers not specified in the l.h.s. of the equation.

The most general spin structure of the transition operator can be extracted from Eq. (47) as

$$\hat{M}(\vec{q},\vec{p}';\vec{p}) = \sum_{S'SM_SM_{S'}} |S'M_{S'}\rangle\langle S'M_{S'}|\hat{M}(\vec{q},\vec{p}';\vec{p})|SM_S\rangle\langle SM_S|. \tag{49}$$

Inserting Eq. (47) into Eq. (49) and recoupling gives

$$\begin{aligned}\hat{M}(\vec{q},\vec{p}';\vec{p}) &= \sum i^{L-L'-l}(-)^{L-J+J'+l+S'+S}[J'][J]^2\\ &\times M^{S'J'SJ}_{lL'L}(q,p';p)\sum_{\alpha\beta}\begin{Bmatrix} L' & J' & S' \\ J & \beta & l\end{Bmatrix}\begin{Bmatrix} S' & \beta & J \\ L & S & \alpha\end{Bmatrix}\\ &\times [A_{S'}\otimes B_S]^\alpha\cdot[X_{(lL')\beta L}]^\alpha, \end{aligned} \tag{50}$$

where $[X_{(lL')\beta L}]^\alpha$ is defined as

$$[X_{(lL')\beta L}]^\alpha \equiv [[Y_l(\hat{q})\otimes Y_{L'}(\hat{p}')]^\beta \otimes Y_L(\hat{p})]^\alpha, \tag{51}$$

and contains all the information on the angular dependence of the transition operator.

We now expand $[A_{S'}\otimes B_S]^\alpha$, for each tensor rank α, in terms of the complete set of available spin operators in the problem. Since the meson M produced in the final state is a spinless meson, the expansion of $[A_{S'}\otimes B_S]^\alpha$ is exactly the same as that for the $N + N \to B' + B$ reaction discussed in Sec. III and is given by Eq. (28).

Inserting Eq. (28) into Eq. (50) yields

$$\begin{aligned}\hat{M}(\vec{q},\vec{p}';\vec{p}) &= \mathcal{R}_0 P_{S=0} + \mathcal{R}_1 P_{S=1} + \vec{\mathcal{R}}_2\cdot(\vec{\sigma}_1+\vec{\sigma}_2)\\ &+ \vec{\mathcal{R}}_3\cdot(\vec{\sigma}_1-\vec{\sigma}_2)P_{S=0} + \vec{\mathcal{R}}_4\cdot(\vec{\sigma}_1-\vec{\sigma}_2)P_{S=1}\\ &+ \mathcal{R}^2_5\cdot[\vec{\sigma}_1\otimes\vec{\sigma}_2]^2. \end{aligned} \tag{52}$$

The above result is the most general spin structure of the spinless meson production operator in NN collisions consistent with symmetry principles. The first two terms are the central spin singlet and triplet interactions, respectively. The third term is a tensor of rank 1 and corresponds to the usual spin-orbit interaction. The fourth and fifth terms are also tensors of rank 1 but they describe the spin singlet→triplet and triplet→singlet transitions, respectively. The last term corresponds to the tensor interaction of rank 2. Apart from the fourth and fifth terms, all the other terms conserve the total spin in the transition. The coefficients multiplying the spin operators in the above equation contain the dynamics of the reaction process and are given by

$$\begin{aligned}\mathcal{R}_0 &= \sum i^{L-L'-l}(-)^L[L]M^{0L'0L}_{lL'L}(q,p';p)[X_{(lL')LL}]^0,\\ \mathcal{R}_1 &= \frac{1}{3}\sum i^{L-L'-l}(-)^{J'}[J'][J]^2\begin{Bmatrix} L' & J' & 1 \\ J & L & l\end{Bmatrix}\\ &\times M^{1J'1J}_{lL'L}(q,p';p)[X_{(lL')LL}]^0,\\ \vec{\mathcal{R}}_2 &= \frac{1}{2\sqrt{2}}\sum i^{L-L'-l}(-)^{L-J+J'+1}[J'][J]^2M^{1J'1J}_{lL'L}(q,p';p)\\ &\times \sum_\beta[\beta]\begin{Bmatrix} L' & J' & 1 \\ J & \beta & l\end{Bmatrix}\begin{Bmatrix} 1 & \beta & J \\ L & 1 & 1\end{Bmatrix}[X_{(lL')\beta L}]^1,\\ \vec{\mathcal{R}}_3 &= \frac{1}{2\sqrt{3}}\sum i^{L-L'-l}(-)^{L+J'}[L][J']M^{1J'0L}_{lL'L}(q,p';p)\\ &\times \sum_\beta(-)^\beta[\beta]\begin{Bmatrix} L' & J' & 1 \\ L & \beta & l\end{Bmatrix}[X_{(lL')\beta L}]^1,\\ \vec{\mathcal{R}}_4 &= \frac{1}{2\sqrt{3}}\sum i^{L-L'-l}(-)^{L'-J+1}[J]M^{0L'1J}_{lL'L}(q,p';p)\\ &\times [X_{(lL')JL}]^1,\\ \mathcal{R}^2_5 &= \frac{1}{2}\sum i^{L-L'-l}(-)^{L-J+J'+1}[J'][J]^2M^{1J'1J}_{lL'L}(q,p';p)\\ &\times \sum_\beta[\beta]\begin{Bmatrix} L' & J' & 1 \\ J & \beta & l\end{Bmatrix}\begin{Bmatrix} 1 & \beta & J \\ L & 1 & 2\end{Bmatrix}[X_{(lL')\beta L}]^2. \end{aligned} \tag{53}$$

Note that the summations over the quantum numbers in the above equation are restricted by total parity conservation and the Pauli principle:

$$(-)^{l+L+L'} = \pm 1, \quad (-)^{L+S+T} = -1, \tag{54}$$

where the $\pm$ signs refer to scalar (+) and pseudoscalar (−) mesons, respectively; T denotes the total isospin of the two interacting nucleons in the initial state. If the baryon B in the final state is a nucleon, the summations in Eq. (53) are further restricted by the Pauli principle

$$(-)^{L'+S'+T'} = -1, \tag{55}$$

where T' denotes the total isospin of the two interacting nucleons in the final state.

The quantity $[X_{(lL')\beta L}]^{\alpha}$ in the above equation can be most easily evaluated by choosing the z axis along the relative momentum $\vec{p}$ of the two nucleons in the initial state. We then have

$$\begin{aligned}[X_{(lL')\beta L}]^{\alpha}_{M_\alpha} &= \frac{[LL']}{\sqrt{4\pi}}(\beta M_\alpha L0|\alpha M_\alpha)[Y_l(\hat{q}) \otimes Y_{L'}(\hat{p}')]^{\beta} \\ &= \frac{[LL']}{\sqrt{4\pi}}(\beta M_\alpha L0|\alpha M_\alpha) \\ &\quad \times \sum_{m_l, M_{L'}} (l m_l L' M_{L'}|\beta M_\alpha) Y_{l m_l}(\hat{q}) Y_{L' M_{L'}}(\hat{p}'). \end{aligned} \tag{56}$$

The evaluation of $[X_{(lL')\beta L}]^{\alpha}$ can be further simplified if we choose the relative momentum $\vec{p}'$ of the two nucleons in the final state in the xz-plane, in which case, $Y_{L'M_{L'}}(\hat{p}') = (-)^{M_{L'}} \sqrt{(L'-M_{L'})!/(L'+M_{L'})!}[(2L'+1)/4\pi]^{1/2} P_{L'}^{M_{L'}}(\hat{p}' \cdot \hat{p})$.

What we have done so far is completely general and applies to any spinless meson production in *NN* collisions. In the following, we treat the production of neutral and charged spinless mesons separately in the $NN \to MNN$ reaction.

A. Neutral meson production amplitude in $N + N \to M + N + N$

Since the only difference between the scalar and pseudoscalar meson production amplitude is that the sum $l + L + L'$ be even (scalar meson) or odd (pseudoscalar meson) as expressed in Eq. (54), we restrict the following considerations to *pseudoscalar* meson production. The scalar meson production amplitude is easily obtained from the pseudoscalar meson production amplitude by changing the restriction on the orbital angular momenta from $(-)^{l+L+L'} = -1$ to $(-)^{l+L+L'} = +1$.

In the case of neutral meson production, the available isospin operators in the problem are the usual isospin operators $\vec{\tau}_1$ and $\vec{\tau}_2$ together with the identity operator acting on the nucleon sector. We need to construct a scalar from these available operators. Since the total isospin of the two interacting nucleons is conserved in the production of a neutral meson, the isospin structure of the reaction amplitude is most simply expressed in terms of the isospin projection operator P_T for the isosinglet and isotriplet transitions as $T = 0$ and 1, respectively. Explicitly, $P_{T=0} = (1 - \vec{\tau}_1 \cdot \vec{\tau}_2)/4$ and $P_{T=1} = (3 + \vec{\tau}_1 \cdot \vec{\tau}_2)/4$.

We now note that, since the total isospin is conserved in the neutral meson production process, total parity conservation together with Pauli principle [Eqs. (54), (55)] demands that $(-)^{l+S+S'} = -1$ in the case of pseudoscalar meson production. This implies that the coefficients $\vec{\mathcal{R}}_3$ and $\vec{\mathcal{R}}_4$ in Eq. (53) involve the summation over even l only while all other coefficients in Eq. (53) involve summation over odd l only. In what follows we will use the superscript (e) or (o) in the coefficients in Eq. (53) to indicate the restricted summation to even or odd l, respectively.

With the above considerations, the (pseudoscalar) neutral meson production operator for a transition with a given isospin T is given by

$$\begin{aligned}\hat{M}_T(\vec{q}, \vec{p}'; \vec{p}) = \{&\mathcal{R}_0^{(o)} P_{S=0} + \mathcal{R}_1^{(o)} P_{S=1} + \vec{\mathcal{R}}_2^{(o)} \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) \\ &+ \vec{\mathcal{R}}_3^{(e)} \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) P_{S=0} + \vec{\mathcal{R}}_4^{(e)} \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \\ &\times P_{S=1} + \mathcal{R}_5^{2(o)} \cdot [\vec{\sigma}_1 \otimes \vec{\sigma}_2]^2\} P_T. \end{aligned} \tag{57}$$

The above result is the most general spin-isospin structure of the neutral pseudoscalar meson production operator in *NN* collisions consistent with symmetry principles.

In the total isospin basis, the *pp* and *pn* states are expressed as

$$\begin{aligned}|pp\rangle &= |T = 1, M_T = 1\rangle, \\ |pn\rangle &= \tfrac{1}{\sqrt{2}}[|T = 1, M_T = 0\rangle + |T = 0, M_T = 0\rangle], \end{aligned} \tag{58}$$

so that, for *pp* collisions, we have

$$\hat{M}_{ppM}(\vec{q}, \vec{p}'; \vec{p}) = \hat{M}_{T=1}(\vec{q}, \vec{p}'; \vec{p}), \tag{59}$$

while for *pn* collisions,

$$\hat{M}_{pnM}(\vec{q}, \vec{p}'; \vec{p}) = \frac{1}{2} \sum_{T=0,1} \hat{M}_T(\vec{q}, \vec{p}'; \vec{p}). \tag{60}$$

The transition operator for neutral *scalar* meson production is also given by Eq. (57), except that the superscripts (e) and (o) in the quantities appearing in Eq. (57) should be interchanged.

B. Charged meson production amplitude in $N + N \to M + N + N$

For charged meson production, the available nucleon isospin operators are the same as those for neutral meson production as discussed in the previous subsection. In addition, there is the isospin creation operator $\hat{\pi}_m^{\dagger}$ in the meson sector [$|1m> = \hat{\pi}_m^{\dagger}|0>$], where the subscript m stands for the charge of the produced meson ($m = +1, 0, -1$). Again, we form all possible scalars from these operators with $\hat{\pi}_m$ appearing once in each term. The isospin structure of the transition operator is then of the form $\hat{\pi} \cdot \hat{O}$, where $\hat{O}$ stands for an isospin operator of rank 1 in the two nucleon isospin sector. Three independent structures are possible for $\hat{O}$:

$$\begin{aligned} T = T' = 1: &\quad \hat{O} = (\vec{\tau}_1 + \vec{\tau}_2), \\ T = 0, T' = 1: &\quad \hat{O} = (\vec{\tau}_1 - \vec{\tau}_2)\, P_{T=0}, \\ T = 1, T' = 0: &\quad \hat{O} = (\vec{\tau}_1 - \vec{\tau}_2)\, P_{T=1}. \end{aligned} \tag{61}$$

In the isospin singlet→triplet and triplet→singlet transitions, total parity conservation and the Pauli principle [Eqs. (54), (55)] demand that $(-)^{l+S+S'} = +1$ in the case of pseudoscalar meson production. This condition is just opposite to that for either the isospin singlet-singlet or triplet-triplet transitions as discussed in the previous subsection. Taking into account this restriction together with Eq. (61), we have for a given transition $T \to T'$

$$\begin{aligned}\hat{M}_{T'T}(\vec{q}, \vec{p}'; \vec{p}) = \{&\mathcal{R}_0^{(o)} P_{S=0} + \mathcal{R}_1^{(o)} P_{S=1} + \vec{\mathcal{R}}_2^{(o)} \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) \\ &+ \vec{\mathcal{R}}_3^{(e)} \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) P_{S=0} + \vec{\mathcal{R}}_4^{(e)} \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \\ &\times P_{S=1} + \mathcal{R}_5^{2(o)} \cdot [\vec{\sigma}_1 \otimes \vec{\sigma}_2]^2\} \hat{\pi} \cdot (\vec{\tau}_1 + \vec{\tau}_2) \\ &+ \{\mathcal{R}_0^{(e)} P_{S=0} + \mathcal{R}_1^{(e)} P_{S=1} + \vec{\mathcal{R}}_2^{(e)} \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) \\ &+ \vec{\mathcal{R}}_3^{(o)} \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) P_{S=0} + \vec{\mathcal{R}}_4^{(o)} \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \\ &\times P_{S=1} + \mathcal{R}_5^{2(e)} \cdot [\vec{\sigma}_1 \otimes \vec{\sigma}_2]^2\} \hat{\pi} \cdot (\vec{\tau}_1 - \vec{\tau}_2) P_T.\end{aligned} \tag{62}$$

Note that the coefficients of the spin operators in the two-nucleon isospin nonflip term have different restrictions on the angular momentum of the emitted meson relative to the corresponding coefficients in the two-nucleon isospin flip term. The above result is the most general spin-isospin structure of the charged pseudoscalar meson production operator in *NN* collisions consistent with symmetry principles. The transition operator for a specific reaction can be trivially obtained from Eq. (62) using Eq. (58).

The transition operator for charged scalar meson production is also given by Eq. (62), except that the superscripts (e) and (o) in the quantities appearing in Eq. (62) should be interchanged.

VI. NEAR-THRESHOLD $pp \to ppM$ REACTION

In any particle production reaction, the near-threshold energy region is of particular interest due to the limited number of relevant partial-wave amplitudes. In this section, as an example, we consider the $pp \to ppM$ reaction near threshold, where the produced pseudoscalar meson M is primarily in an s-wave state. Then, considering only $l = 0$, the transition operators of the previous sections take particularly simple forms. Indeed, Eq. (59) becomes

$$\hat{M}_{ppM}(\vec{q}, \vec{p}'; \vec{p}) = \vec{\mathcal{R}}_3^{(e)} \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) P_{S=0} + \vec{\mathcal{R}}_4^{(e)} \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) P_{S=1}, \tag{63}$$

where the isopin matrix element has been taken. From Eq. (53), the coefficients $\vec{\mathcal{R}}_3^{(e)}$ and $\vec{\mathcal{R}}_4^{(e)}$ reduce to

$$\begin{aligned}\vec{\mathcal{R}}_3^{(e)} &= \frac{1}{2\sqrt{3}} \sum i^{L-L'} (-)^{L'} [L] M_{0L'L}^{1L0L}(q, p'; p) [X_{(0L')L'L}]^1, \\ \vec{\mathcal{R}}_4^{(e)} &= -\frac{1}{2\sqrt{3}} \sum i^{L-L'} [L'] M_{0L'L}^{0L'1L'}(q, p'; p) [X_{(0L')L'L}]^1,\end{aligned} \tag{64}$$

with

$$\begin{aligned}[X_{(0L')L'L}]^1 = \frac{[LL']}{(4\pi)^{\frac{3}{2}}} \Bigg[&\sqrt{\frac{2}{L'(L'+1)}} (L'1L0|11) P_{L'}^1(\hat{p} \cdot \hat{p}') \hat{n}_1 \\ &+ (L'0L0|10) P_{L'}(\hat{p} \cdot \hat{p}') \hat{p} \Bigg].\end{aligned} \tag{65}$$

In the above equation, $\hat{n}_1 \equiv [(\vec{p} \times \vec{p}') \times \vec{p}]/|\vec{p} \times \vec{p}'|$.

Restricting the final two proton states to S and P waves, Eq. (63) reduces further to

$$\begin{aligned}\hat{M}_{ppM}(\vec{q}, \vec{p}'; \vec{p}) = i\{&[\beta_2 \hat{p}' - \beta_3(\hat{p}' - 3\hat{p} \cdot \hat{p}'\hat{p})] \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \\ &\times P_{S=0} - \beta_1 \hat{p} \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) P_{S=1}\},\end{aligned} \tag{66}$$

where

$$\begin{aligned}\beta_1 &\equiv \frac{1}{2(4\pi)^{3/2}} M_{001}^{1101}(q, p'; p), \\ \beta_2 &\equiv \frac{1}{2(4\pi)^{3/2}} M_{010}^{0111}(q, p'; p), \\ \beta_3 &\equiv \frac{1}{2(4\pi)^{3/2}} \sqrt{\frac{5}{2}} M_{012}^{0111}(q, p'; p),\end{aligned} \tag{67}$$

are proportional to the partial-wave matrix elements corresponding to ${}^3P_0 \to {}^1S_0 s$, ${}^1S_0 \to {}^3P_0 s$, and ${}^1D_2 \to {}^3P_2 s$ transitions, respectively [17]. Equation (66) agrees with the structure derived in Ref. [18] based directly on symmetry considerations. We also note that the higher partial-wave contributions in $pp \to pp\eta$ reported in Ref. [18] have been calculated based on the present method.

The transition operator given by Eqs. (57), (59) can be reexpressed in the form

$$\hat{M}_{ppM} = \sum_{\lambda=1}^{6} \sum_{n,n'=0}^{3} M_{nn'}^{\lambda} \sigma_n(1) \sigma_{n'}(2), \tag{68}$$

where

$$\begin{aligned}M_{nn'}^1 &= \frac{1}{4}(3\mathcal{R}_1^{(o)} + \mathcal{R}_0^{(o)}) \delta_{n,0} \delta_{n',0}, \\ M_{nn'}^2 &= \left\{\frac{1}{4}(\mathcal{R}_1^{(o)} - \mathcal{R}_0^{(o)}) + \tilde{\mathcal{R}}_{a5}^{2(o)}\right\} (1 - \delta_{n,0}) \delta_{n,n'}, \\ M_{nn'}^3 &= \left[\vec{\mathcal{R}}_2^{(o)} + \frac{1}{2}(\vec{\mathcal{R}}_3^{(e)} + \vec{\mathcal{R}}_4^{(e)})\right]_n (1 - \delta_{n,0}) \delta_{n',0}, \\ M_{nn'}^4 &= \left[\vec{\mathcal{R}}_2^{(o)} - \frac{1}{2}(\vec{\mathcal{R}}_3^{(e)} + \vec{\mathcal{R}}_4^{(e)})\right]_{n'} (1 - \delta_{n',0}) \delta_{n,0}, \\ M_{nn'}^5 &= i\frac{1}{2} \sum_k \varepsilon_{nn'k} (\vec{\mathcal{R}}_3^{(e)} - \vec{\mathcal{R}}_4^{(e)})_k (1 - \delta_{n,0})(1 - \delta_{n',0}), \\ M_{nn'}^6 &= \tilde{\mathcal{R}}_{b5}^{2(o)} (1 - \delta_{n,0})(1 - \delta_{n',0}),\end{aligned} \tag{69}$$

with

$$\begin{aligned}\tilde{\mathcal{R}}_{a5}^{2(o)} \equiv &\sum_{m_1 m_2 m} (-)^m \mathcal{R}_{5m}^{2(o)} (1m_1 1m_2|2-m) \\ &\times \Bigg\{\frac{m_1 m_2}{2}(\delta_{n,1} - m_1 m_2 \delta_{n,2}) + (1 - |m_1|) \\ &\times (1 - |m_2|) \delta_{n,3}\Bigg\}, \\ \tilde{\mathcal{R}}_{b5}^{2(o)} \equiv &\sum_{m_1 m_2 m} (-)^m \mathcal{R}_{5m}^{2(o)} (1m_1 1m_2|2-m) \\ &\times \Bigg\{i\frac{m_1 m_2}{2}(m_2 \delta_{n,1} \delta_{n',2} + m_1 \delta_{n,2} \delta_{n',1}) - \frac{m_1}{\sqrt{2}} \\ &\times (1 - |m_2|)(\delta_{n,1} + i m_1 \delta_{n,2}) \delta_{n',3} \\ &- \frac{m_2}{\sqrt{2}} (1 - |m_1|) \delta_{n,3} (\delta_{n',1} + i m_2 \delta_{n',2})\Bigg\}.\end{aligned} \tag{70}$$

Any observable of interest can then be calculated either from Eq. (68) or from Eq. (47) as given in Appendix D.

In near-threshold kinematics with only s-wave mesons, we see from Eqs. (63), (69) that the nonvanishing coefficients are $M_{nn'}^{\lambda=3,4,5}$. If, in addition, the final two nucleon states are restricted to S and P waves, we have

$$\begin{aligned} M^3_{nn'} &= i\frac{1}{2}\Big\{(\beta_2-\beta_3)\hat{p}'_n + [3\beta_3(\hat{p}\cdot\hat{p}') - \beta_1]\hat{p}_n\Big\} \\ &\quad \times (1-\delta_{n,0})\delta_{n',0}, \\ M^4_{nn'} &= -i\frac{1}{2}\Big\{(\beta_2-\beta_3)\hat{p}'_{n'} + [3\beta_3(\hat{p}\cdot\hat{p}') - \beta_1]\hat{p}_{n'}\Big\} \\ &\quad \times \delta_{n,0}(1-\delta_{n',0}), \\ M^5_{nn'} &= -\frac{1}{2}\sum_k \varepsilon_{nn'k}\Big\{(\beta_2-\beta_3)\hat{p}'_k + [3\beta_3(\hat{p}\cdot\hat{p}') + \beta_1]\hat{p}_k\Big\} \\ &\quad \times (1-\delta_{n,0})(1-\delta_{n',0}), \end{aligned} \tag{71}$$

where β_i's are defined in Eq. (67).

Inserting the above results into Eqs. (D2)–(D4) [or alternatively using Eq. (47) and Eqs. (D5)–(D7)] yields

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= |\beta_1|^2 + |\beta_2|^2 + (3\cos^2\theta+1)|\beta_3|^2 \\ &\quad + 2(3\cos^2\theta - 1)\Re[\beta_2\beta_3^*], \\ \frac{d\sigma}{d\Omega}A_i &= 0, \\ \frac{d\sigma}{d\Omega}A_{xx} &= \frac{d\sigma}{d\Omega}A_{yy} \\ &= |\beta_1|^2 - |\beta_2|^2 - (3\cos^2\theta+1)|\beta_3|^2 \\ &\quad - 2(3\cos^2\theta - 1)\Re[\beta_2\beta_3^*], \end{aligned} \tag{72}$$

which are equivalent to the results derived directly from Eq. (66) in Ref. [18]. From the above results, the final state (NN) S-wave contribution can be isolated via the combination

$$\frac{1}{2}\frac{d\sigma}{d\Omega}(1 + A_{xx}) = \frac{d(^3\sigma)}{d\Omega} = |\beta_1|^2, \tag{73}$$

which is nothing other than a consequence of the Pauli principle and parity conservation as discussed in Sec. IV.

VII. SUMMARY

Based on the partial-wave expansion of the reaction amplitude, we have derived the most general spin structure of the transition operator for the reactions $\gamma + N \to M + B$ and $N + N \to B' + B$. Also, we have derived the most general spin structure of the spinless meson production operator for the $N + N \to M + B' + N$ reaction. The present method used to extract the spin structure of the transition operator is quite general and, in principle, can be applied to any reaction process in a systematic way. The advantage of this method is that it relates the coefficients multiplying each spin operator directly to the partial-wave matrix elements to any desired order of the corresponding expansion.

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APPENDIX A

In this appendix we will determine the coefficients $a_{L'L}, b_{L'L}$, and $c_{L'L}$ in Eq. (6) as well as $a'_{L'L}$ and $b'_{L'L}$ in Eq. (13).

Taking the scalar product of the last equality in Eq. (6) with $[\hat{q}\otimes\hat{q}]^2$, $[\hat{k}\otimes\hat{k}]^2$, and $[\hat{k}\otimes\hat{q}]^2$, respectively, we have

$$\begin{aligned} 3u &= 2a_{L'L} + (3\cos^2\theta - 1)b_{L'L} + 2\cos\theta c_{L'L}, \\ 3v &= (3\cos^2\theta - 1)a_{L'L} + 2b_{L'L} + 2\cos\theta c_{L'L}, \\ 3w &= 2\cos\theta a_{L'L} + 2\cos\theta b_{L'L} + \tfrac{1}{2}(3+\cos^2\theta)c_{L'L}, \end{aligned} \tag{A1}$$

where $\cos\theta \equiv \hat{k}\cdot\hat{q}$ and

$$\begin{aligned} u &\equiv [Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^2\cdot[\hat{q}\otimes\hat{q}]^2, \\ v &\equiv [Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^2\cdot[\hat{k}\otimes\hat{k}]^2, \\ w &\equiv [Y_L(\hat{k})\otimes Y_{L'}(\hat{q})]^2\cdot[\hat{k}\otimes\hat{q}]^2. \end{aligned} \tag{A2}$$

In order to arrive at Eq. (A1), we have also made use of the results

$$\begin{aligned} [\hat{q}\otimes\hat{q}]^2\cdot[\hat{q}\otimes\hat{q}]^2 &= \tfrac{2}{3}, \\ [\hat{k}\otimes\hat{k}]^2\cdot[\hat{q}\otimes\hat{q}]^2 &= \tfrac{1}{3}(3\cos^2\theta - 1), \\ [\hat{k}\otimes\hat{q}]^2\cdot[\hat{q}\otimes\hat{q}]^2 &= \tfrac{2}{3}\cos\theta, \\ [\hat{k}\otimes\hat{q}]^2\cdot[\hat{k}\otimes\hat{q}]^2 &= \tfrac{1}{6}(3+\cos^2\theta). \end{aligned} \tag{A3}$$

Equation (A1) can be readily inverted to yield

$$\begin{aligned} a_{L'L} &= \frac{1}{\sin^4\theta}[2u + (1+\cos^2\theta)v - 4\cos\theta w], \\ b_{L'L} &= \frac{1}{\sin^4\theta}[(1+\cos^2\theta)u + 2v - 4\cos\theta w], \\ c_{L'L} &= \frac{2}{\sin^4\theta}[-2\cos\theta(u+v) + (3\cos^2\theta+1)w]. \end{aligned} \tag{A4}$$

Choosing the quantization axis $\hat{z}$ along $\hat{k}$, the quantities u, v, and w defined in Eq. (A2) can be expressed without loss of generality as

$$\begin{aligned} u &= \frac{1}{4\pi}\sqrt{\frac{2}{3}}[LL'](L0L'0|20)P_L(\hat{k}\cdot\hat{q}), \\ v &= \frac{1}{4\pi}\sqrt{\frac{2}{3}}[LL'](L0L'0|20)P_{L'}(\hat{k}\cdot\hat{q}), \\ w &= \frac{[LL']}{\sqrt{4\pi}}\sum_l\frac{1}{[l]}(L'010|l0)\sum_{M,m_l}(L0L'M|2M) \\ &\quad \times (101M|2M)(L'M1M|lm_l)Y_{lm_l}(\theta,0). \end{aligned} \tag{A5}$$

Similarly to what has been done above, taking the scalar product of the last equality in Eq. (13) with $[\hat{k}\otimes\hat{n}_2]^2$ and $[\hat{q}\otimes\hat{n}_2]^2$, respectively, we have

$$\begin{aligned} 2r &= a'_{L'L} + \cos\theta b'_{L'L}, \\ 2t &= \cos\theta a'_{L'L} + b'_{L'L}, \end{aligned} \tag{A6}$$

where

$$r \equiv [Y_L(\hat{k}) \otimes Y_{L'}(\hat{q})]^2 \cdot [\hat{k} \otimes \hat{n}_2]^2,$$
$$t \equiv [Y_L(\hat{k}) \otimes Y_{L'}(\hat{q})]^2 \cdot [\hat{q} \otimes \hat{n}_2]^2. \tag{A7}$$

In order to arrive at Eq. (A6), we have also made use of the results

$$[\hat{k} \otimes \hat{n}_2]^2 \cdot [\hat{k} \otimes \hat{n}_2]^2 = \tfrac{1}{2},$$
$$[\hat{k} \otimes \hat{n}_2]^2 \cdot [\hat{q} \otimes \hat{n}_2]^2 = \tfrac{1}{2}\cos\theta. \tag{A8}$$

Equation (A6) can be readily inverted to yield

$$a'_{L'L} = \frac{2}{\sin^2\theta}\,(r - t\cos\theta)\,,$$
$$b'_{L'L} = \frac{2}{\sin^2\theta}\,(t - r\cos\theta)\,. \tag{A9}$$

Choosing the quantization axis $\hat{z}$ along $\hat{k}$, the quantities r and t defined in Eq. (A7) can be expressed as

$$r = -i\frac{1}{4\pi}\frac{[LL']}{\sqrt{L'(L'+1)}}(L0L'1|21)P^1_{L'}(\hat{k}\cdot\hat{q}),$$
$$t = -i\frac{[LL']}{\sqrt{2\pi}}\sum_l \frac{1}{[l]}(L'010|l0)\sum_{M,m's}(-)^M(L0L'M|2M)$$
$$\times (L'M1m|lm_l)(1m11|2-M)Y_{lm_l}(\theta,0). \tag{A10}$$

APPENDIX B

In this appendix we will give the explicit expression for the quantities $A_{L'}$, $B_{L'}$, and $C_{L'}$ in Eq. (19) and $A'_{L'}$, $B'_{L'}$, and $C'_{L'}$ in Eq. (21).

Using Eqs. (8), (11) for those states specified in Eq. (18), we find

$$A_0 = \frac{1}{4\pi\sqrt{3}}\left[M_{00}^{\frac{1}{2}\frac{1}{2}} - \sqrt{2}M_{02}^{\frac{1}{2}\frac{3}{2}}\right],$$
$$\left(\frac{q}{\Lambda}\right)A_1 = \frac{1}{4\pi}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\sqrt{\frac{2}{3}}\left[M_{11}^{\frac{1}{2}\frac{1}{2}} - M_{11}^{\frac{3}{2}\frac{1}{2}} + \frac{1}{2}M_{11}^{\frac{1}{2}\frac{3}{2}} + \sqrt{\frac{5}{2}}M_{11}^{\frac{3}{2}\frac{3}{2}}\right] + \frac{1}{4\pi}\sqrt{\frac{2}{3}}\left[M_{11}^{\frac{1}{2}\frac{3}{2}} + M_{11}^{\frac{3}{2}\frac{3}{2}}\right] - \frac{1}{4\pi}\sqrt{\frac{3}{5}}M_{13}^{\frac{3}{2}\frac{3}{2}},$$
$$\left(\frac{q}{\Lambda}\right)B_1 = \frac{1}{4\pi}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\sqrt{\frac{2}{3}}\left[M_{11}^{\frac{1}{2}\frac{1}{2}} - M_{11}^{\frac{3}{2}\frac{1}{2}} + \frac{1}{2}M_{11}^{\frac{1}{2}\frac{3}{2}} + \sqrt{\frac{5}{2}}M_{11}^{\frac{3}{2}\frac{3}{2}}\right],$$
$$\left(\frac{q}{\Lambda}\right)C_1 = -\frac{1}{4\pi}\sqrt{\frac{3}{8}}\left[M_{11}^{\frac{1}{2}\frac{3}{2}} + \sqrt{2}M_{11}^{\frac{3}{2}\frac{3}{2}} - 2\sqrt{3}M_{13}^{\frac{3}{2}\frac{3}{2}}\right]. \tag{B1}$$

Similarly, using Eqs. (15), (17) for those states specified in Eq. (20), we find

$$\left(\frac{q}{\Lambda}\right)A'_1 = -\frac{1}{4\pi}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\sqrt{\frac{1}{6}}\left[M_{10}^{\frac{1}{2}\frac{1}{2}} + 2M_{12}^{\frac{1}{2}\frac{3}{2}} - 4\sqrt{\frac{2}{3}}M_{10}^{\frac{3}{2}\frac{3}{2}} - \sqrt{2}M_{12}^{\frac{3}{2}\frac{3}{2}} + 4\sqrt{\frac{1}{3}}M_{12}^{\frac{3}{2}\frac{3}{2}}\right],$$
$$B'_0 = \frac{1}{4\pi}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\sqrt{\frac{1}{3}}\left[M_{01}^{\frac{1}{2}\frac{1}{2}} - M_{01}^{\frac{1}{2}\frac{3}{2}}\right],$$
$$\left(\frac{q}{\Lambda}\right)B'_1 = -\frac{1}{4\pi}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\sqrt{\frac{1}{3}}\left[M_{10}^{\frac{1}{2}\frac{1}{2}} + M_{12}^{\frac{1}{2}\frac{3}{2}} + \sqrt{2}M_{10}^{\frac{3}{2}\frac{3}{2}} - 2M_{12}^{\frac{3}{2}\frac{1}{2}}\right],$$
$$\left(\frac{q}{\Lambda}\right)C'_1 = -\frac{1}{4\pi}\sqrt{\frac{3}{8}}\left[M_{12}^{\frac{1}{2}\frac{3}{2}} + \sqrt{\frac{1}{2}}M_{12}^{\frac{3}{2}\frac{3}{2}}\right]. \tag{B2}$$

APPENDIX C

The quantities $A^S_{L'}$, $B^S_{L'}$, etc. in Eq. (39) are given by

$$A^0_0 = \frac{1}{4\pi}M_{00}^{000},$$
$$\left(\frac{p'}{\Lambda}\right)A^1_1 = \frac{1}{4\pi}\frac{1}{3}\left[M_{11}^{110} + 3M_{11}^{111} + 5M_{11}^{112}\right],$$
$$\left(\frac{p'}{\Lambda}\right)B^1_1 = -\frac{1}{4\pi}\frac{1}{4}\left[M_{11}^{110} + \frac{3}{2}M_{11}^{111} - \frac{5}{2}M_{11}^{112}\right],$$
$$\left(\frac{p'}{\Lambda}\right)C^1_1 = \frac{1}{4\pi}\frac{3}{2\sqrt{2}}M_{11}^{011},$$
$$\left(\frac{p'}{\Lambda}\right)D^1_1 = -\frac{1}{4\pi}\sqrt{\frac{3}{2}}\frac{5}{2}M_{13}^{112},$$
$$\left(\frac{p'}{\Lambda}\right)E^1_1 = \frac{1}{4\pi}\left[\sqrt{\frac{3}{2}}M_{13}^{112} - \frac{1}{2}M_{11}^{110} + \frac{3}{4}M_{11}^{111} - \frac{1}{4}M_{11}^{112}\right]. \tag{C1}$$

For convenience we define

$$\left(\frac{p'}{\Lambda}\right)U^1_1 \equiv \frac{\sqrt{3}}{4\pi}\left[\sqrt{\frac{3}{2}}(M_{11}^{112} - M_{11}^{111}) + M_{11}^{112}\right],$$
$$\left(\frac{p'}{\Lambda}\right)W^1_1 \equiv \frac{\sqrt{3}}{4\pi}\left[\sqrt{\frac{3}{2}}(M_{11}^{112} + M_{11}^{111}) + M_{11}^{112}\right]. \tag{C2}$$

Similarly, the quantities $A'^{S'S}_{L'}$, $B'^{S'S}_{L'}$, etc. in Eq. (41) are given by

$$A'^{11}_0 = \frac{1}{4\pi}\frac{1}{2}\sqrt{\frac{3}{2}}M_{01}^{111},$$
$$\left(\frac{p'}{\Lambda}\right)B'^{10}_1 = \frac{1}{4\pi}\frac{1}{2}\left[M_{10}^{100} - \sqrt{\frac{5}{2}}M_{12}^{102}\right],$$
$$\left(\frac{p'}{\Lambda}\right)C'^{10}_1 = \frac{1}{4\pi}\frac{1}{2}\left[M_{10}^{100} + \sqrt{10}M_{12}^{102}\right],$$
$$C'^{01}_0 = \frac{1}{4\pi}\frac{1}{2}M_{01}^{010}. \tag{C3}$$

APPENDIX D

In order to calculate the observables directly from the transition operators in Eqs. (29), (34), and (52), it is convenient to express them in the form

$$\hat{M} = \sum_{\lambda} \sum_{n,n'=0}^{3} M_{nn'}^{\lambda} \sigma_n(1) \sigma_{n'}(2), \tag{D1}$$

where $\sigma_0(i) \equiv 1$, $\sigma_1(i) \equiv \sigma_x(i)$, etc., for ith nucleon.

The unpolarized cross section is then given by

$$\begin{aligned} \frac{d\sigma}{d\Omega} &\equiv \frac{1}{4} \mathrm{Tr}[\hat{M}\hat{M}^\dagger] \\ &= \sum_{\lambda,\lambda'} \sum_{n,n'=0}^{3} M_{nn'}^{\lambda} \left(M_{nn'}^{\lambda'}\right)^*. \end{aligned} \tag{D2}$$

For the analyzing power we have

$$\begin{aligned} \frac{d\sigma}{d\Omega} A_i &\equiv \frac{1}{4} \mathrm{Tr}[\hat{M}\sigma_i(1)\hat{M}^\dagger] \\ &= \sum_{\lambda,\lambda'} \sum_{n'=0}^{3} \left\{ 2\Re\left[M_{in'}^{\lambda}\left(M_{0n'}^{\lambda'}\right)^*\right] + i \sum_{k,n=1}^{3} \varepsilon_{kni} M_{nn'}^{\lambda} \left(M_{kn'}^{\lambda'}\right)^* \right\}, \end{aligned} \tag{D3}$$

where ε_{kni} denotes the Levi-Civita antisymmetric tensor. We note that, from parity conservation, $A_x = A_z = 0$ for the two-body reaction $NN \to B'B$; this result also holds for the three-body reaction $NN \to MB'N$ in the coplanar geometry.

The spin correlation coefficient A_{ij} is given by

$$\begin{aligned} \frac{d\sigma}{d\Omega} A_{ij} &\equiv \frac{1}{4} \mathrm{Tr}[\hat{M}\sigma_i(1)\sigma_j(2)\hat{M}^\dagger] \\ &= \sum_{\lambda,\lambda'} \Big\{ 2\Re\left[M_{00}^{\lambda}\left(M_{ij}^{\lambda'}\right)^* + M_{i0}^{\lambda}\left(M_{0j}^{\lambda'}\right)^*\right] \\ &\quad + \sum_{k,n=1}^{3} 2\Im\left[M_{0k}^{\lambda}\left(M_{in}^{\lambda'}\right)^* \varepsilon_{knj} + M_{k0}^{\lambda}\left(M_{nj}^{\lambda'}\right)^* \varepsilon_{kni}\right] \\ &\quad - \sum_{k,k',n,n'=1}^{3} M_{kk'}^{\lambda}(M_{nn'}^{\lambda'})^* \varepsilon_{nki} \varepsilon_{n'k'j} \Big\}. \end{aligned} \tag{D4}$$

Of course, any observable can also be expressed in terms of the spin-matrix elements given by Eqs. (24), (47). We have, e.g.,

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} \sum_{S,S',M_S,M_S'} |\langle S'M_S'|\hat{M}|SM_S\rangle|^2, \tag{D5}$$

$$\begin{aligned} \frac{d\sigma}{d\Omega} A_x &= \frac{\sqrt{2}}{4} \sum_{S,S',M_S'} \Re[\langle S'M_S'|\hat{M}|S0\rangle\langle S'M_S'|\hat{M}|1-1\rangle^* \\ &\quad + (-)^{1+S}\langle S'M_S'|\hat{M}|11\rangle\langle S'M_S'|\hat{M}|S0\rangle^*], \\ \frac{d\sigma}{d\Omega} A_y &= \frac{\sqrt{2}}{4} \sum_{S,S',M_S'} \Im[\langle S'M_S'|\hat{M}|S0\rangle\langle S'M_S'|\hat{M}|1-1\rangle^* \\ &\quad + (-)^{1+S}\langle S'M_S'|\hat{M}|11\rangle\langle S'M_S'|\hat{M}|S0\rangle^*], \\ \frac{d\sigma}{d\Omega} A_z &= \frac{1}{4} \sum_{S',M_S'} [|\langle S'M_S'|\hat{M}|11\rangle|^2 - |\langle S'M_S'|\hat{M}|1-1\rangle|^2], \end{aligned} \tag{D6}$$

$$\begin{aligned} \frac{d\sigma}{d\Omega} A_{xx} &= \frac{1}{4} \sum_{S,S',M_S,M_S'} (-)^{1+S-2M_S} \langle S'M_S'|\hat{M}|SM_S\rangle \\ &\quad \times \langle S'M_S'|\hat{M}|S-M_S\rangle^*, \\ \frac{d\sigma}{d\Omega} A_{yy} &= \frac{1}{4} \sum_{S,S',M_S,M_S'} (-)^{1+S-M_S} \langle S'M_S'|\hat{M}|SM_S\rangle \\ &\quad \times \langle S'M_S'|\hat{M}|S-M_S\rangle^*, \\ \frac{d\sigma}{d\Omega} A_{zz} &= \frac{1}{4} \sum_{S,S',M_S,M_S'} (-)^{1+M_S} |\langle S'M_S'|\hat{M}|SM_S\rangle|^2. \end{aligned} \tag{D7}$$

As has been pointed out in Ref. [19], a given initial state spin contribution to the cross section can be isolated by measuring some spin observables. In particular, using the spin-projection operator P_S as given in Sec. III in terms of the Pauli spin matrices, it is immediate that

$$\begin{aligned} \frac{d(^1\sigma)}{d\Omega} &\equiv \frac{1}{4}\mathrm{Tr}[(\hat{M}P_{S=0})(\hat{M}P_{S=0})^\dagger] = \frac{1}{4}\mathrm{Tr}[\hat{M}P_{S=0}\hat{M}^\dagger] \\ &= \frac{1}{4}\frac{d\sigma}{d\Omega}(1 - A_{xx} - A_{yy} - A_{zz}), \\ \frac{d(^3\sigma)}{d\Omega} &\equiv \frac{1}{4}\mathrm{Tr}[(\hat{M}P_{S=1})(\hat{M}P_{S=1})^\dagger] = \frac{1}{4}\mathrm{Tr}[\hat{M}P_{S=1}\hat{M}^\dagger] \\ &= \frac{1}{4}\frac{d\sigma}{d\Omega}(3 + A_{xx} + A_{yy} + A_{zz}), \end{aligned} \tag{D8}$$

where $^{2S+1}\sigma$ denotes the (initial state) spin-singlet or -triplet cross section as $S = 0$ or 1, respectively.

Also, Eq. (D7) can be inverted to solve for the spin cross sections, $d(^{2S+1}\sigma_{M_S})/d(\Omega) \equiv \sum_{S',M_S'} |\langle S'M_S'|\hat{M}|SM_S\rangle|^2/4$. We obtain [20]

$$\begin{aligned} \frac{d(^1\sigma_0)}{d\Omega} &= \frac{1}{4}\frac{d\sigma}{d\Omega}(1 - A_{xx} - A_{yy} - A_{zz}), \\ \frac{d(^3\sigma_0)}{d\Omega} &= \frac{1}{4}\frac{d\sigma}{d\Omega}(1 + A_{xx} + A_{yy} - A_{zz}), \\ \frac{d(^3\sigma_1)}{d\Omega} + \frac{d(^3\sigma_{-1})}{d\Omega} &= \frac{1}{2}\frac{d\sigma}{d\Omega}(1 + A_{zz}), \end{aligned} \tag{D9}$$

where we have made used of the relation $\sigma = {}^1\sigma_0 + {}^3\sigma_0 + {}^3\sigma_1 + {}^3\sigma_{-1}$. Note that Eq. (D9) is consistent with Eq. (D8) as ${}^1\sigma = {}^1\sigma_0$ and ${}^3\sigma = {}^3\sigma_0 + {}^3\sigma_1 + {}^3\sigma_{-1}$.

From Eq. (D9), it is immediate that the spin-triplet cross section defined as

$$\frac{d(^3\sigma_\Sigma)}{d\Omega} \equiv \frac{d(^3\sigma_0)}{d\Omega} + \frac{1}{2}\left(\frac{d(^3\sigma_1)}{d\Omega} + \frac{d(^3\sigma_{-1})}{d\Omega}\right) \tag{D10}$$

is given by Eq. (46). Note that $d(^3\sigma_1)/d\Omega = d(^3\sigma_{-1})/d\Omega$ for $NN \to B'B$ by symmetry. This holds also for $NN \to MB'N$ in the coplanar geometry or for the cross section integrated over the emission angle of the one of the three particles in the final state.

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