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Unfolding the effects of the $T = 0$ and $T = 1$ parts of the two-body interaction on nuclear collectivity in the *f-p* shell

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Calculations of the spectra of various even-even nuclei in the *fp* shell (^{44}Ti, ^{46}Ti, ^{48}Ti, ^{48}Cr, and ^{50}Cr) are performed with two sets of two-body interaction matrix elements. The first set consists of the matrix elements of the FPD6 interaction. The second set has the same $T = 1$ two-body matrix elements as the FPD6 interaction, but all the $T = 0$ two-body matrix elements are set equal to zero (T0FPD6). Surprisingly, the T0FPD6 interaction gives a semireasonable spectrum when compared to FPD6 (or else this method would make no sense). A consistent feature for even-even nuclei, e.g., ^{44,46,48}Ti and ^{48,50}Cr, is that the reintroduction of $T = 0$ matrix elements makes the spectrum look more rotational than when the $T = 0$ matrix elements are set equal to zero. The odd-odd nucleus ^{46}V is also discussed. In general, but not always, the inclusion of $T = 0$ two-body matrix elements enhances the $B(E2)$ rates.

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I. INTRODUCTION

The study of neutron-proton pairing, especially in the $T = 0$ channel, is a particularly prominent topic these days. Although the number of journal articles are far too numerous to reference, one might begin to make some headway into the varied approaches by starting from the references found in Refs. [1–3]. In so doing one will find a field of study filled with disagreement and occasionally strife.

For example, Macchiavelli *et al.* [2] claim that some apparent indicators of $T = 0$ pairing can really be explained in terms of symmetry energies. In their abstract they say "After correcting for the energy we find that the lowest $T = 1$ state in odd-odd $N = Z$ nuclei is as bound as the ground state in the neighboring even-even nucleus, thus providing evidence for isovector np pairing. However, the $T = 0$ states in odd-odd $N = Z$ nuclei are several MeV less bound than the even-even ground states...there is no evidence for an isoscalar (deuteron like) pair condensate in $N = Z$ nuclei." We will not get involved directly in the $T = 0$ pairing problem, but instead ask the simple question "What do the $T = 0$ and $T = 1$ parts of the nucleon–nucleon interaction do in a nucleus?" We feel this is a more constructive approach.

In this work we examine the yrast spectra of the even-even (*fp*) shell nuclei ^{44}Ti, ^{46}Ti, ^{48}Ti, ^{48}Cr, and ^{50}Cr. We perform full *fp*-shell calculations and compare the spectra to experiment. For comparison purposes we also discuss the odd-odd nucleus ^{46}V.

In this work we perform the shell-model calculations using the shell-model code ANTOINE [4]. To best see the effects of the $T = 1$ and $T = 0$ interactions, we perform two sets of calculations. In the first we use the FPD6 interaction [5]. Then we do the same calculations but we set all the $T = 0$ two-body interaction matrix elements to zero. We denote this interaction as T0FPD6. It is crucial to note that this interaction is not one we are suggesting for use in realistic shell-model calculations. Rather, we feel this modification allows us to separate out the effects of the $T = 0$ portion of the interaction from the $T = 1$ portion of the interaction. It is studying this separation that is the main point of the present work. (This modification of an effective interaction is along the same lines of that used by Satula *et al.* to examine Wigner energies a few years ago [6]). We have used this modification of FPD6 in the past to study a variety of things and in particular the full *fp* spectrum of ^{44}Ti [7–10].

It should be noted that in Refs. [7–10] a wide range of topics is addressed beyond the spectra of even-even nuclei. These topics include a partial dynamical symmetry that arises when one uses the T0FPD6 interaction in a single j shell for ^{43}Sc and ^{44}Ti. Also, while using the T0FPD6 interaction, a subtle relationship between the $T = \frac{1}{2}$ states in ^{43}Sc and the $T = \frac{3}{2}$ states in ^{43}Ca can be observed, likewise between the $T = 0$ states in ^{44}Ti and $T = 2$ states in ^{44}Ca. We also considered even–odd nuclei and addressed the topic of how the $T = 0$ two-body matrix elements affect $B(M1)$ transitions—both spin and orbital components, and Gamow–Teller transitions. In many cases the transition rates were very sensitive to the presence or absence of the $T = 0$ matrix elements. This was especially the case for some orbital $B(M1)$s and the Gamow–Teller transitions.

Here things will be kept simple and we focus on the spectra and $B(E2)$s of the yrast levels of selected even-even nuclei. We examine the sensitivity of these observables on the $T = 0$ two-body interaction matrix elements by setting them to zero and comparing the results thus obtained with those when the $T = 0$ matrix elements are reintroduced.

The T0FPD6 interaction is not expected to give good binding energies—clearly the $T = 0$ two-body matrix elements are important here. Nor is it expected to give the relative energies of states of different isospins in a nucleus. This can be partially compensated by adding a two-body monopole interaction in the $T = 0$ channel $a + bt(i)t(j)$, which for $T = 0$ would be $a[1/4 - t(1)t(2)]$. Such monopole interactions have been studied in the past [11,12]. However, this interaction will not affect the energy differences of states with the same isospin and it will not affect the $B(E2)$ rates, which is our concern here.

It should be emphasized that $T = 0$ two-body matrix elements are very important for binding energies. This is especially made clear by the schematic models of Chasman where it is shown that both $T = 0$ and $T = 1$ matrix elements are important in describing the Wigner energy [13]. In this work, however, we focus on spectra.

II. DISCUSSION OF SOME PREVIOUS CALCULATIONS

Our entry to this problem considered here was to note that in a single j-shell calculation of ^{44}Ti the results for the even J states were almost the same when the $T = 0$ two-body matrix elements of the FPD6 interaction were set equal to zero as they were in a full calculation. This figure is shown in Ref. [7]. There is an offset of the odd J states. However we point out that none of the odd J states have been found experimentally.

It should be pointed out that in a single j-shell calculation (but not when more than one shell is included) setting the two-body $T = 0$ matrix elements to a constant will give the same relative spectra of $T = 0$ states in ^{44}Ti as will be obtained by setting these to zero.

In another vein we showed that when $T = 0$ two-body matrix elements are set equal to zero one gets a partial dynamical symmetry [7,8]. For $T = 0$ states of ^{44}Ti with the following angular momenta $I = 3, 7, 9, 10$, and 12, the states can be classified by the dual quantum numbers (J_p, J_n). However, for states with $I = 0, 2, 4, 5, 6$, and 8 no such symmetry exists. We were able to explain this in part by noting that this symmetry exists only for states with angular momenta which are not present for a system of identical particles, i.e., ^{44}Ca.

But even with a full interaction, i.e., when the $T = 0$ matrix elements are present, the $T = 0$ interaction appears to be weak for the states $I = 3, \ldots, 12$ for which the dynamical symmetry exists. For example, the wave function of the $J = 3^+, T = 0$ state in an MBZ [14] calculation is as follows:

$$\Psi = \sum_{J_P, J_N} D^I(J_P J_N)[(j^2)^{J_P}(j^2)^{J_N}]^I \tag{1}$$

J_P	J_N	$3_1^+ T = 0$	$3_2^+ T = 0$
2	2	0.0000	0.0000
2	4	0.6968	−0.1202
4	2	−0.6968	0.1202
4	4	0.0000	0.0000
4	6	0.1202	0.6968
6	4	−0.1202	−0.6968
6	6	0.0000	0.0000

The point is that, even with the $T = 0$ interaction present, (J_p, J_n) are almost good quantum numbers. The 3_1^+ state consists mostly of the (J_p, J_n) of (24) and (42); the (46) and (64) amplitudes are only 0.1202. The 3_2^+ state is mainly (46) and (64). When the $T = 0$ matrix elements are turned off, the wave functions collapse to the following:

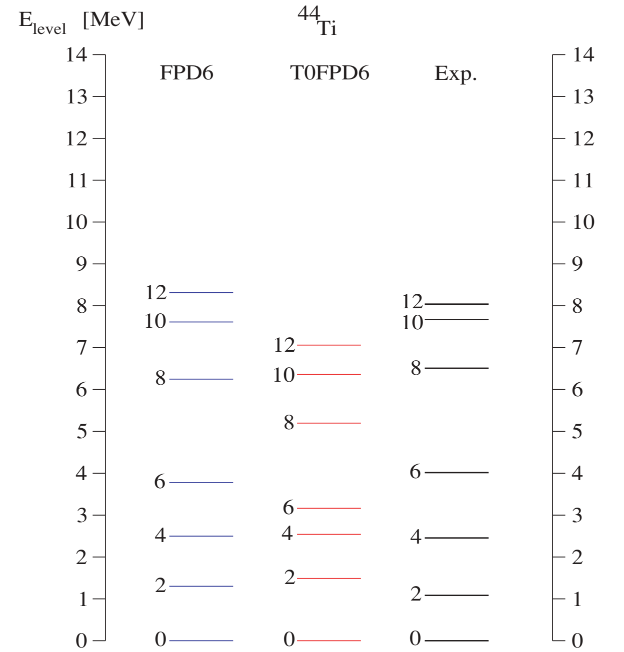

FIG. 1. (Color online) Full fp space calculations of even J $T = 0$ states in ^{44}Ti and comparison with experiment.

$$3_1^+ = \frac{1}{\sqrt{2}}[(2,4) + (4,2)] \tag{2}$$

$$3_2^+ = \frac{1}{\sqrt{2}}[(6,4) + (4,6)] \tag{3}$$

Of course intrinsically the $T = 0$ interaction is not weak but it appears to act weak in certain cases.

III. RESULTS

A. The even-even isotopes ^{44,46,48}Ti and ^{48,50}Cr even J states

In Figs. 1 to 5 we show the $T = T_{\min} = |N - Z|/2$ even J states of ^{44,46,48}Ti and ^{48,50}Cr. In the first column, we have the full fp-shell calculation using FPD6. In the second column, we have T0FPD6, which signifies that the $T = 0$ two-body matrix elements have been set to zero. In the third column, experimental yrast levels are shown [15]. We show separately a comparison of the odd J states in the Ti and Cr isotopes for FPD6 and T0FPD6 in Figs. 6 to 10. These figures show experimental odd J levels, although very little is known about the odd J states in these nuclei. Hopefully this article serves as an impetus to search for such states.

We now make some broad remarks about the results. The first point to be made is that with the full FPD6 interaction one gets a very good overall fit to the experimental spectrum. This should not come as a surprise. They were designed to do so.

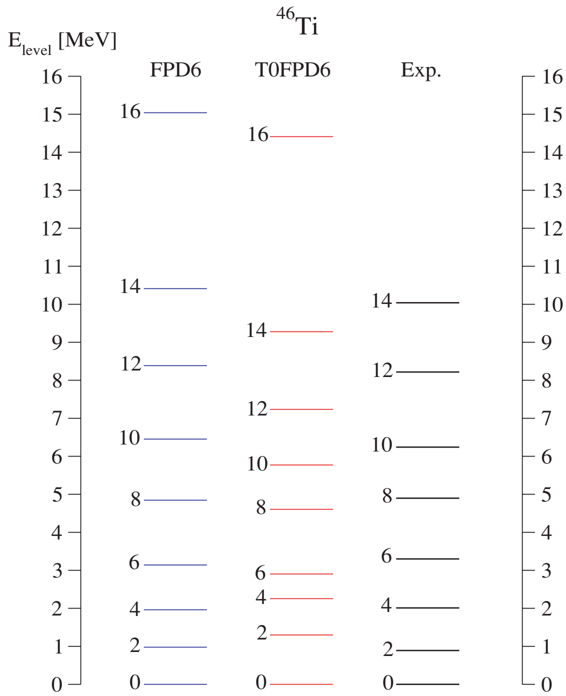

FIG. 2. (Color online) Full *fp* space calculations of even J $T = 1$ states in ^{46}Ti and comparison with experiment.

What is surprising is that, when we set all $T = 0$ matrix elements to zero (T0FPD6), we get a semireasonable spectrum when compared to that calculated with the full FPD6 interaction. If this were not the case, then what we are doing would make no sense. It would appear that the $T = 1$ two-body matrix elements, acting alone, gave us the "spine" of the spectrum. The addition of the $T = 0$ matrix elements gives a needed overall improvement, but as we will show later, there are still some discrepancies even with the full interaction.

It should be pointed out that if we had reversed the procedure and set all the $T = 1$ matrix elements to zero and kept the $T = 0$ matrix elements as they are, we would get an unrecognizably bad spectrum [9].

We next take a closer look at the two calculated spectra. We see that with the full FPD6 in ^{44}Ti, the spectrum for $J = 0, 2, 4, 6,$ and 8 looks somewhat more rotational than with T0FPD6. This is consistent with the knowledge that the $T = 0$ n-p interaction enhances the nuclear collectivity. In the rotational limit the spectrum would be of the form $J(J + 1)$, whereas in the simple vibrational limit one gets equally spaced levels. Experiment resides between these two limits.

Comparing FPD6 to T0FPD6, we find a closer agreement for the even J spectra for ^{46}Ti than for ^{44}Ti. Indeed the closeness in ^{46}Ti is remarkable. It could be that ^{44}Ti is relatively more rotational than ^{46}Ti and hence the $T = 0$ interaction plays a more important role. This could also be a measure of the relative numbers of $T = 0$ pairs in a $T = 0$ nucleus as opposed to a $T = 1$ nucleus.

With one notable exception, the spectrum of ^{48}Ti is as good as that of ^{46}Ti when the $T = 0$ matrix elements are set to zero. The exception concerns the $(J = 6, J = 4)$ splitting that is too small when we remove the $T = 0$ matrix elements. This could be connected with the near degeneracy of the two lowest 6^+ levels in ^{48}Ti, a problem that we previously addressed in Ref. [16]. In FPD6 these levels are separated by 0.08 MeV and in T0FPD6 by 0.23 MeV. In the single j-shell model, the two $J = 6^+$ states have opposite signatures—this might explain

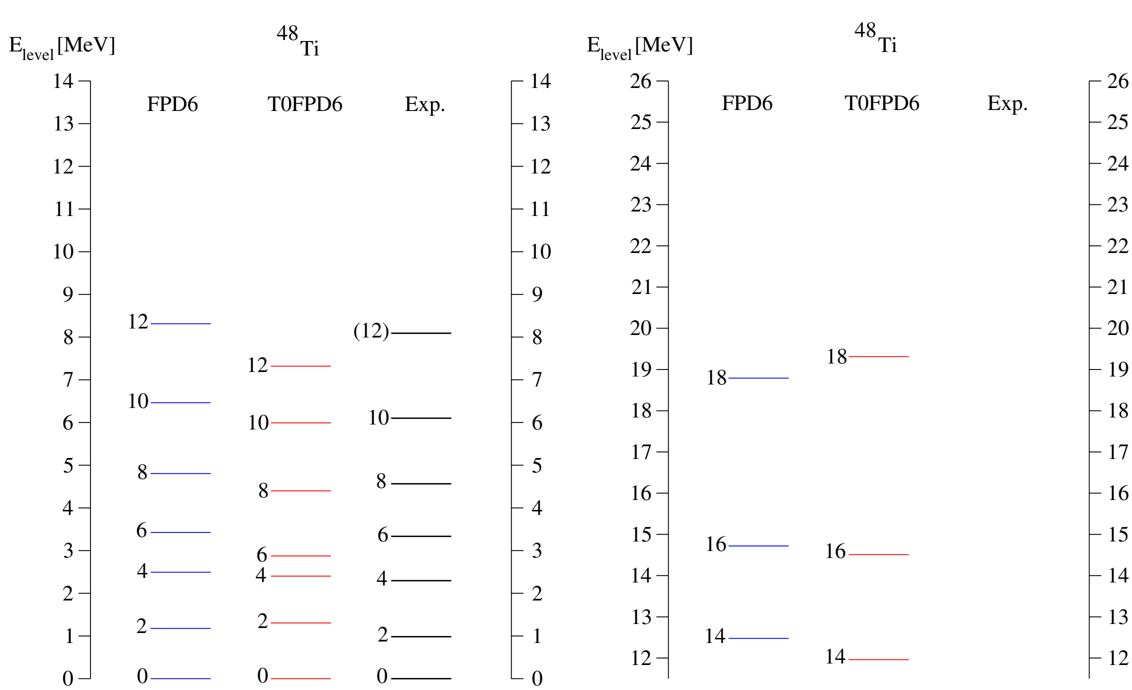

FIG. 3. (Color online) Full *fp* calculations of even J $T = 2$ states in ^{48}Ti and comparison with experiment.

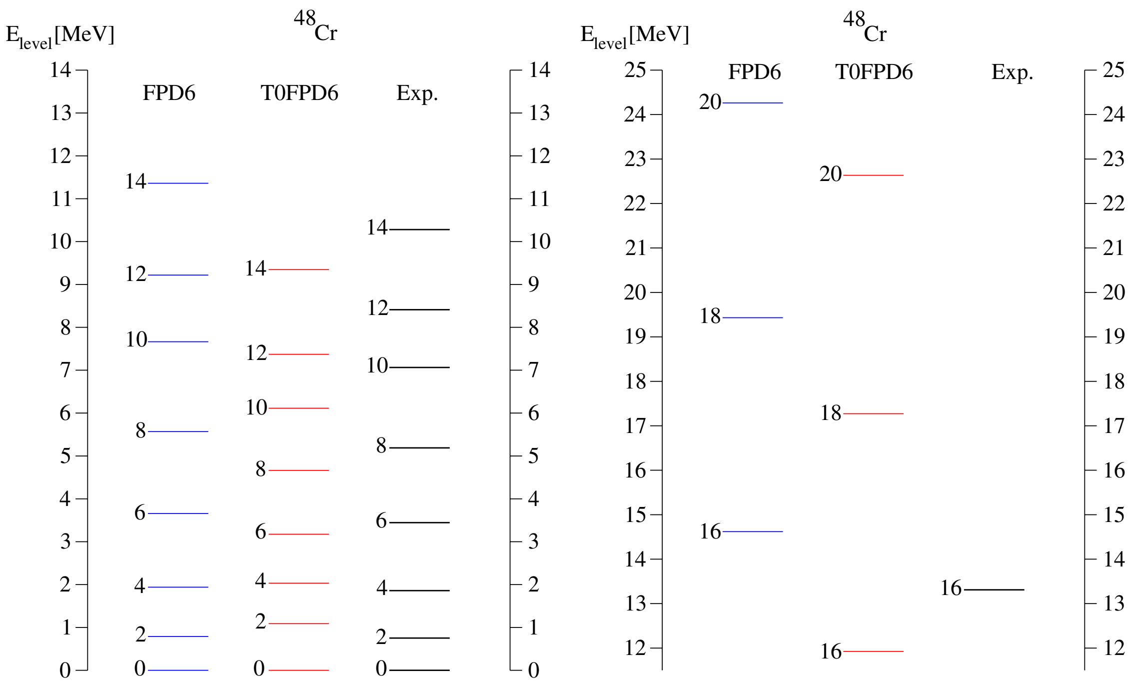

FIG. 4. (Color online) Full *fp* calculations of even J $T=0$ states in ^{48}Cr and comparison with experiment.

in part why there is not a lot of level repulsion between both $J=6^+$ states.

For the even J states of the $N=Z$ nucleus ^{48}Cr, the low spin spectrum ($J=0$, 2, 4, and 6) is more in the direction of a rotational spectrum with FPD6 than it is with T0FPD6. At higher spins the FPD6 states are at a higher energy than those of T0FPD6. For example, there is a substantial difference—almost 2 MeV for the $J=14^+$ state. Similar results hold for $J=8$, 10, 12, and 16. For ^{50}Cr there is a similar story but the differences are not so pronounced.

As an overview, if we look at the results for all the even-even nuclei, we find that the full FPD6 interaction somewhat goes

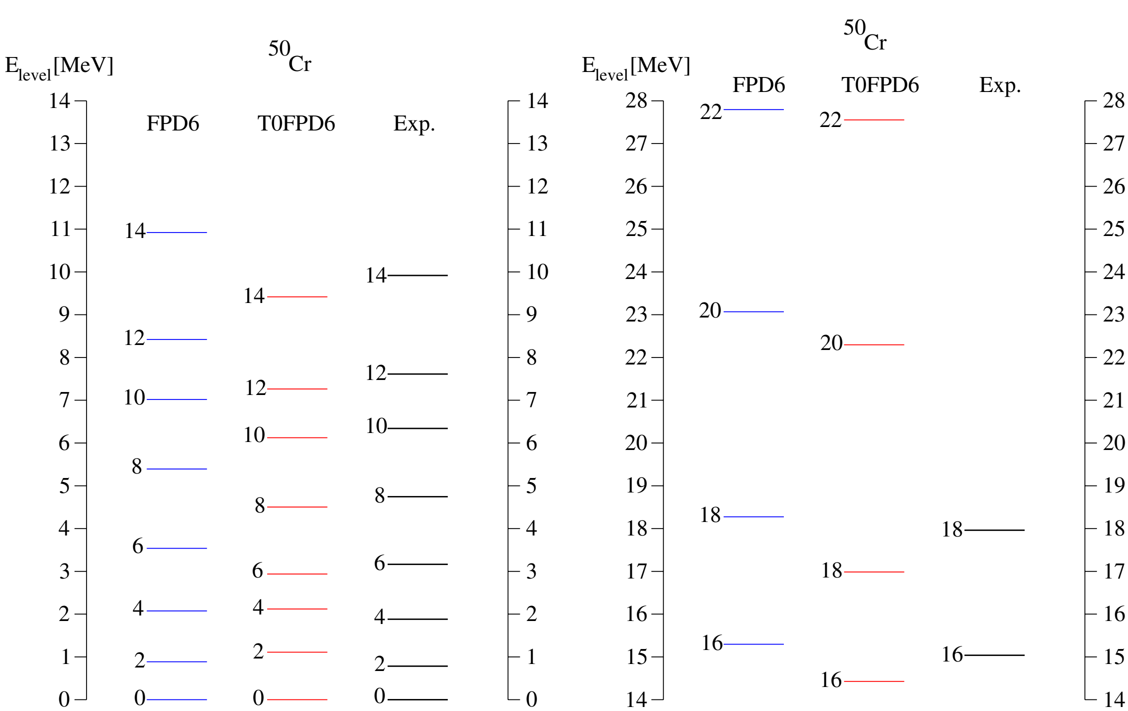

FIG. 5. (Color online) Full *fp* calculations of even J $T=1$ states in ^{50}Cr and comparison with experiment.

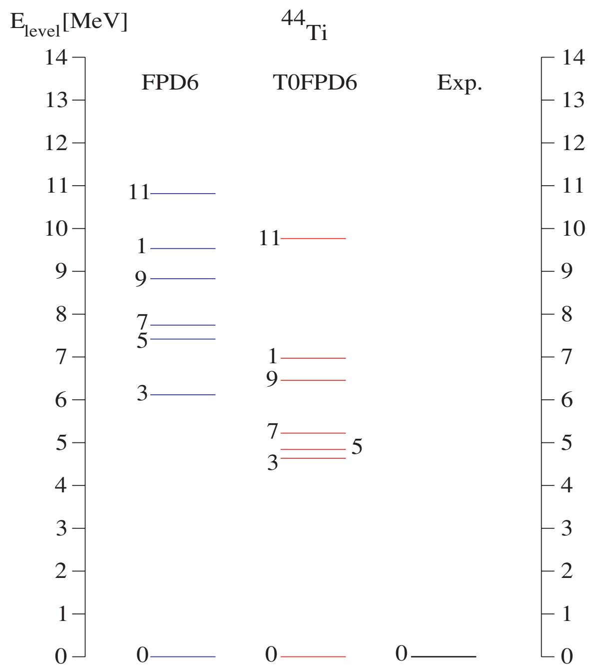

FIG. 6. (Color online) Full *fp* space calculations of odd J $T = 0$ states in ^{44}Ti.

too far in the description of rotational motion, but T0FPD6 does not go far enough. This is especially evident by looking at the high spin states, which, on the average, are too high with FPD6 but too low with T0FPD6.

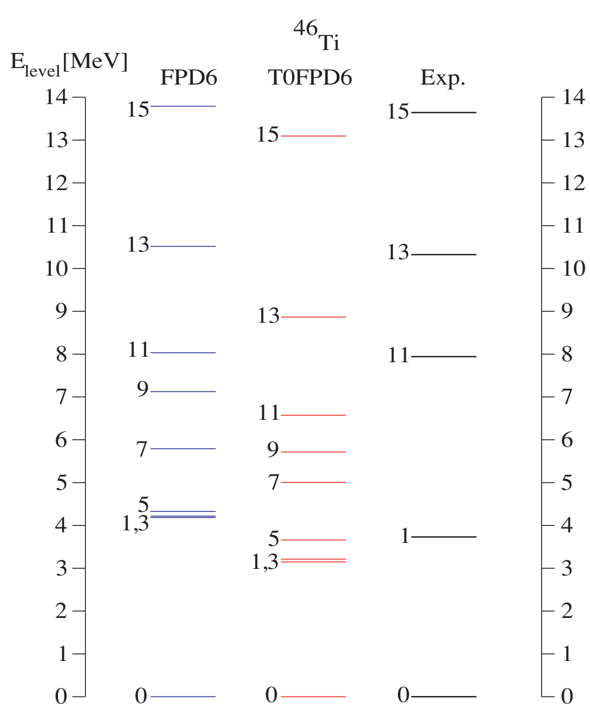

FIG. 7. (Color online) Full *fp* space calculations of odd J $T = 1$ states in ^{46}Ti and comparison with experiment.

B. Odd *J* states in even-even Ti and Cr isotopes

We show a comparison between FPD6 and T0FPD6 in Figs. 6 to 10 for the odd J^+ excitation energies in ^{44}Ti, ^{46}Ti, ^{48}Ti, ^{48}Cr, and ^{50}Cr. We note that the experimental data on odd J are very sparse. In ^{44}Ti there are no odd J, $T = 0$ states identified. There is a known 1^+ state at 7216 keV but this state has isospin 1 and has been associated with the scissors mode state. In ^{46}Ti there are two nearly degenerate 1^+ $T = 1$ states at 3731 and 3872 MeV. In the *fp*-shell-model space one can get only one 1^+ state at this energy; one of these must be an intruder state. In general, as stated above, there are not too many odd J, $T = |N - Z|/2$ states known in the even-even Ti and Cr isotopes. We show in the relevant figures the few that are known.

In ^{44}Ti the ordering of odd J states is the same for T0FPD6 as it is for FPD6. However, there is a large overall downward shift. This can be taken care of by a one-body field. When this is done the comparison is fairly good but there are some deviations. The splitting of the $J = 1^+$ and 11^+ states (neither of these states is present in the $f_{7/2}$ model space) is much larger for T0FPD6 than for FPD6. There is more sensitivity in the odd J spectrum to the $T = 0$ two-body matrix elements than for even J. It would therefore be worthwhile to devise experiments that can find these odd J states.

In ^{46}Ti and ^{48}Ti the deviations between T0FPD6 and FPD6 are not as large as for ^{44}Ti, but there are overall one-body shifts to be taken into account. This is explored in a later section.

It should be noted that there is a simplicity in the spectrum of the odd J states. In all three Ti isotopes, we find that, except for the $J = 1^+$ state, there is a sequential ordering $J = 3^+, 5^+, 7^+, 9^+, 11^+, 13^+$, and 15^+, which suggests a band structure that should be investigated.

For the odd J states of ^{48}Cr, there is a downward shift in the energies of the states calculated with T0FPD6. For $J = 7$, the difference is about 2 MeV.

For the odd J states of ^{50}Cr, there is also a downward shift of the energies when T0FPD6 is used as compared to the full FPD6 interaction. With the full interaction, there is better agreement for the $J = 1^+$ state, but not so for the other known states $J = 5, 11, 13, 15$, and 17.

One purpose of this article is to point out that the data on odd J, $T = T_{\text{min}}$ states in even-even nuclei is very sparse and it would be of interest to device means, perhaps with radioactive beams and projectiles that have nonzero spin, of exciting such states and unfolding their systematics.

C. The $T = 0$ and $T = 1$ spectra of ^{46}V

Recent studies and calculations for ^{46}V have been performed by Möller *et al.* [17] and Brandolini *et al.* [18].

In Fig. 11 we show a full *fp* calculation for the odd-odd $N = Z$ nucleus ^{46}V. We show both the $T = 0$ and $T = 1$ states. The calculated spectrum of $T = 1$ states in ^{46}V is identical to that of ^{46}Ti because we are using charge-independent interactions. To not clutter things up, we show only the even J $T = 1$ states of ^{46}V. That serves as an orientation for where the $T = 0$ states are located.

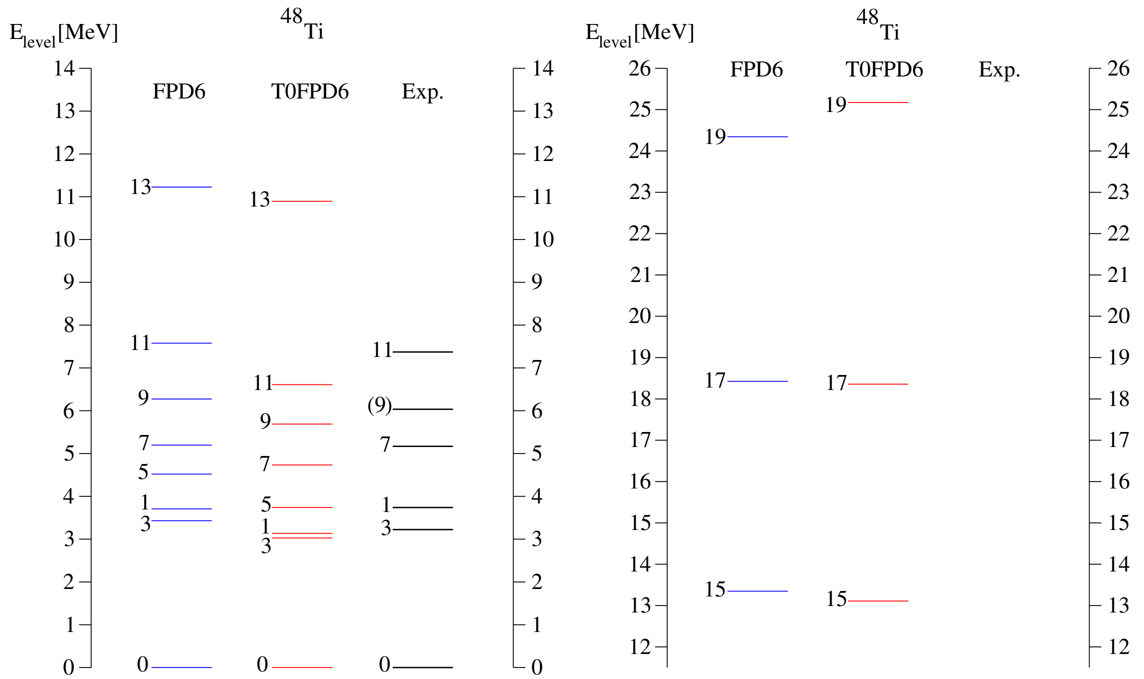

FIG. 8. (Color online) Full *fp* space calculations of odd J $T = 2$ states in ^{48}Ti and comparison with experiment.

The full FPD6 fit to experiment for the $T = 0$ states is very good. We now compare FPD6 with T0FPD6. Clearly the $T = 0$ states as a whole are shifted up with T0FPD6. This can be resolved by adding the $T = 0$ monopole interaction $a[\frac{1}{4} - t(1)\,t(2)]$. A downward shift of about 2 MeV will make the comparison with FPD6 much better.

IV. THE MONOPOLE SHIFT

As noted above in several places, the binding energies, or absolute energies of the T0FPD6 spectra are not expected to be in agreement with experimental data or the full FPD6 calculation. The surprising thing is how well the structure of

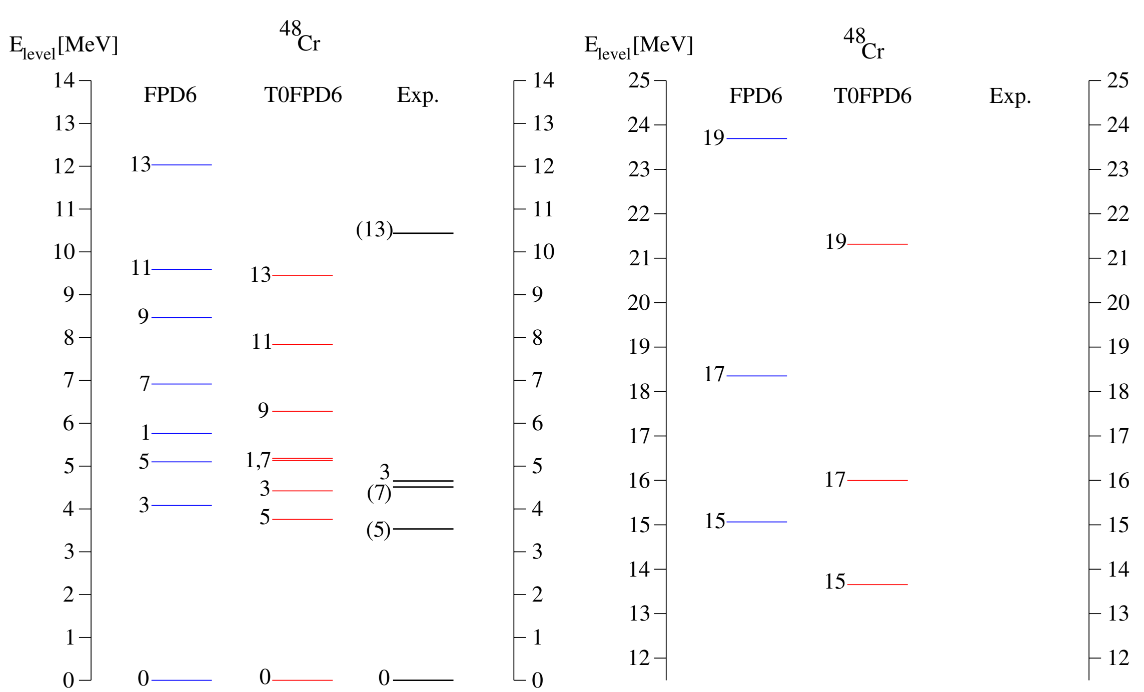

FIG. 9. (Color online) Full *fp* space calculations of odd J $T = 0$ states in ^{48}Cr and comparison with experiment.

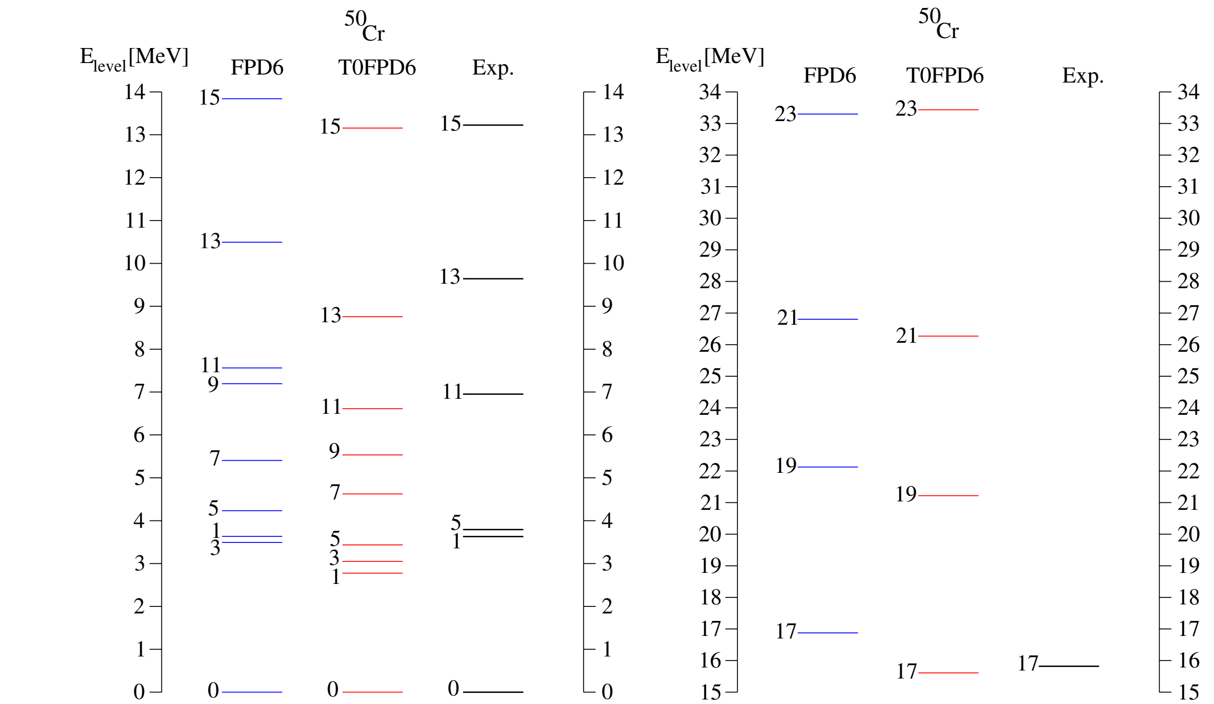

FIG. 10. (Color online) Full *fp* space calculations of odd J $T = 1$ states in ^{50}Cr and comparison with experiment.

these calculations agree if only the excitation energies are considered.

A simple downward shift of the energies obtained using T0FPD6 using a monopole-monopole interaction $a[\frac{1}{4} - t(1)t(2)]$ can adjust the T0FPD6 spectra so that the binding energies are also brought into agreement.

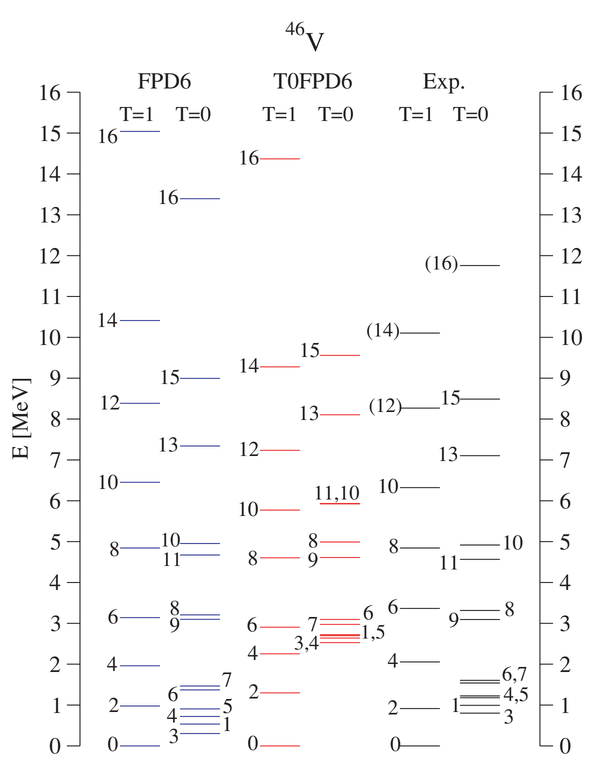

FIG. 11. (Color online) Full *fp* calculation and experimental results for $T = 0$ and 1 states in ^{46}V.

The expectation value of the monopole interaction for A nucleons in a system with total isospin T is given by $a[(\frac{A^2}{8} + \frac{A}{4} - \frac{1}{2}T(T+1)]$, which is the same as a times the number of $T = 0$ pairs.

In Table I we list difference in the binding energies for the valence nucleons between FPD6 and T0FPD6, the number of $T = 0$ pairs, and a for the nucleus in question. For ^{46}V, the given energy difference is for the $J^\pi = 3^+$ state as it is the lowest calculated state in both FPD6 and T0FPD6.

We note first that a majority of the binding energy comes from the $T = 1$ interaction. This is in part because, except for ^{44}Ti there are more $T = 1$ pairs than $T = 0$ pairs. The total number of pairs is given by $(A)(A-1)/2$. Hence for ^{44}Ti there are 3 $T = 0$ pairs and 3 $T = 1$ pairs, whereas for ^{48}Cr there are 18 $T = 1$ pairs and 10 $T = 0$ pairs. Another reason is that the $J = \ \ T = 1$ state is somewhat below the $J = 1$ $T = 0$ state in ^{42}Sc.

TABLE I. Monopole shift to align ground states in full FPD6 and T0FPD6.

Nucleus	BE(FPD6)-BE(T0FPD6)	Num. of $T = 0$ pairs	a
^{44}Ti	7.73	3	2.57
^{46}Ti	10.86	5	2.17
^{48}Ti	12.53	7	1.79
^{48}Cr	19.95	10	1.99
^{50}Cr	24.32	14	1.74
^{46}V	13.09	6	2.18

TABLE II. ^{44}Ti yrast $B(E2)$ values [e^{2}fm^4] in full FPD6 and T0FPD6.

Transition	FPD6	T0FPD6	Ratio
$0 \to 2$	607.24	375.09	0.618
$2 \to 4$	297.71	146.18	0.491
$4 \to 6$	202.05	61.164	0.303
$6 \to 8$	127.20	65.242	0.513
$8 \to 10$	117.50	78.088	0.665
$10 \to 12$	65.501	47.968	0.732

V. $B(E2)$ RATES

Nuclei in the region are of interest because they display behavior between vibrational and rotational. Such nuclei are hard to handle in a purely collective fashion. They are, however, well suited to the shell model that has no difficulty in describing such mixed behavior. For this reason, it should be of interest to compare the results of the FPD6 interaction with respect to the T0FPD6 interaction.

The calculated $B(E2)$ rates in the full *fp* space for ^{44}Ti, ^{46}Ti, ^{48}Ti, ^{48}Cr, and ^{50}Cr are listed in Tables II to VII. The effective charges used are the standard $1.5e$ for the proton and $0.5e$ for the neutron. The difference in the effective charges from 1 and 0 is intended to take care of the fact that the $\Delta N = 2$ and higher excitations are not present in this model space. The results for FPD6 and T0FPD6 are shown. We also display the ratios of the results for the two interactions.

For ^{46}Ti the reintroduction of the $T = 0$ two-body matrix elements causes an increase (relative to T0FPD6) of a factor of 2 or more for all the transitions considered. So there is evidence here that the $T = 0$ matrix elements contribute to the collectivity.

The behavior of ^{48}Ti is very similar to that of ^{46}Ti with two exceptions. The $B(E2)$ for the transition $4 \to 6$ is clearly peculiar in its behavior, as are the transitions involving the $J = 12$ yrast state. Although the reason for this behavior of the $J = 12$ state is not yet clear, the $J = 6$ states of ^{48}Ti have been studied before. The existence of two close lying 6^+ states require us to examine this closer. In Table V we examine the yrast transitions for these close lying states, finding that it is only for the $4 \to 6_1$ transition that we get a strong enhancement when removing the $T = 0$ matrix elements.

TABLE III. ^{46}Ti yrast $B(E2)$ values [e^{2}fm^4] in full FPD6 and T0FPD6.

Transition	FPD6	T0FPD6	Ratio
$0 \to 2$	682.06	432.81	0.635
$2 \to 4$	349.03	179.18	0.513
$4 \to 6$	273.85	92.867	0.339
$6 \to 8$	218.61	82.478	0.377
$8 \to 10$	157.63	75.154	0.477
$10 \to 12$	56.441	29.610	0.525
$12 \to 14$	39.923	18.930	0.474
$14 \to 16$	1.1333	0.4274	0.377

TABLE IV. ^{48}Ti yrast $B(E2)$ values [e^{2}fm^4] in full FPD6 and T0FPD6.

Transition	FPD6	T0FPD6	Ratio
$0 \to 2$	560.78	401.97	0.717
$2 \to 4$	306.35	171.89	0.561
$4 \to 6$	64.147	76.029	1.185
$6 \to 8$	79.337	26.664	0.336
$8 \to 10$	75.571	39.341	0.521
$10 \to 12$	30.055	29.710	0.988
$12 \to 14$	5.0445	3.4293	0.680
$14 \to 16$	42.526	11.608	0.273
$16 \to 18$	0.9308	0.3383	0.363

In the ^{48}Cr and ^{50}Cr (Tables VI and VII), for the most part, the $B(E2)$s are larger when the $T = 0$ two-body matrix elements are reintroduced, but there are some notable exceptions. In ^{48}Cr the $14^+ \to 16^+$ transition is larger for T0FPD6 than for FPD6, the ratio being 1.029. In ^{50}Cr the ratios for $8^+ \to 10^+$, $10^+ \to 12^+$, and $12^+ \to 14^+$ are, respectively, 2.174, 1.271, and 1.116. It was previously noted by Zheng and Zamick [19] that the 10^+ state in ^{50}Cr is not consistent with being a member of the $K = 0$ ground-state band; rather, it looked like a $K = 10$ state, as noted by Zamick, Zheng, and Fayache [20]. This is in agreement with the experimental results of Brandolini *et al*. [21].

VI. POSSIBLE EXPLANATIONS

To partially explain why one gets a semireasonable spectrum with T0FPD6, we can look at the spectrum of ^{42}Sc, which consists of one proton and one neutron beyond the closed shell ^{40}Ca. The energy levels have been used to get a single j-shell two-body effective interaction in the $f_{7/2}$ shell, i.e., taking matrix elements from experiment. In this simplified procedure, one makes the association $\langle (j^2)^J V (j^2)^J \rangle = E(J) + \text{constant}$. Note that the constant will not affect the excitation energies or wave functions in this model. Thus, for example, the excitation energy of the $J = 6_1^+, T = 1$ state relative to the $J = 0^+, T = 1$ state is 3.122 MeV. So we have $\langle (j^2)^6 V (j^2)^6 \rangle = 3.122\ \text{MeV} + \text{constant}$, and so on.

Setting the $J = 0, T = 1$ energy to zero in ^{42}Sc, the remaining states have the following excitation energies (in MeV):

$T = 1$		$T = 0$	
J	Energy	J	Energy
2	1.613	1	0.611
4	2.815	3	1.490
6	3.122	5	1.510
		7	0.616

Note that the total spread of the $T = 1$ states [$(E(6) - E(0)$] is 3.122 MeV. More than three times the spread of the

TABLE V. ^{48}Ti yrast $B(E2)$ values [e^2fm^4] in full FPD6 and T0FPD6 for 6^+ states.

Transition	FPD6	T0FPD6	Ratio
$4 \to 6_1$	64.147	76.195	1.188
$4 \to 6_2$	129.29	7.3963	0.057
$6_1 \to 8$	79.337	26.570	0.335
$6_2 \to 8$	37.301	14.443	0.387

$T = 0$ states $[E(5) - E(1)]$ of 0.899 MeV. Thus, we can say that, to a first approximation, the $T = 0$ spectrum is almost degenerate, judging by the scale set by the $T = 1$ interaction. This would then justify the starting point of setting the $T = 0$ matrix elements to a constant. It is easy to show that in this single *j*-shell model space, if one adds a constant to the $T = 0$ matrix elements, it will not affect the wave functions of the states and will not affect the excitation energies of the states that have the same isospin.

It can be seen that the two-particle $T = 1$ spectrum in ^{42}Sc is quite different from that of a pairing interaction, for which the $J = 2$, 4, and 6 states are degenerate. The fact that the excitation energy of the 6^+ state is about twice that of the 2^+ state indicates that other components of the nucleon–nucleon interaction are present, e.g., a quadrupole–quadrupole interaction. Hence, the $T = 1$ spectrum of ^{42}Sc has built into it some aspects necessary for nuclear collectivity.

The above discussion suggests that, in a full *fp* calculation, the single *j* components are sufficiently prevalent so as to get the overall pattern of the spectrum in reasonably good shape. The higher shell admixtures then readjust the spectrum so as to change from what is roughly a vibrational pattern to a rotational one, and here the $T = 0$ two-body matrix elements play an important role.

An examination of the $T = 0$ two-body matrix elements in Fig. 12 does not show any obvious simplicity. Their distribution looks just as complex as those with $T = 1$ shown in Fig. 13. If the $T = 0$ diagonal matrix elements were all constant and the off-diagonal matrix elements were zero, we could represent the results by a two-body monopole interaction as $a[1/4 - t(1)\,t(2)]$. This would be an easy explanation of the insensitivity but certainly it would not be a correct one.

TABLE VI. ^{48}Cr yrast $B(E2)$ values [e^2fm^4] in full FPD6 and T0FPD6.

Transition	FPD6	T0FPD6	Ratio
$0 \to 2$	1378.4	813.06	0.590
$2 \to 4$	692.96	376.46	0.543
$4 \to 6$	577.42	230.16	0.399
$6 \to 8$	491.87	241.58	0.491
$8 \to 10$	371.28	194.37	0.523
$10 \to 12$	157.33	123.28	0.784
$12 \to 14$	140.42	112.80	0.803
$14 \to 16$	69.141	71.157	1.029
$16 \to 18$	1.8306	1.4921	0.815
$18 \to 20$	7.5903	1.8787	0.247

TABLE VII. ^{50}Cr yrast $B(E2)$ values [e^2fm^4] in full FPD6 and T0FPD6.

Transition	FPD6	T0FPD6	Ratio
$0 \to 2$	1219.0	736.60	0.604
$2 \to 4$	636.22	341.01	0.536
$4 \to 6$	427.64	147.30	0.344
$6 \to 8$	349.16	156.47	0.448
$8 \to 10$	36.549	79.449	2.174
$10 \to 12$	48.488	61.638	1.271
$12 \to 14$	66.120	73.792	1.116
$14 \to 16$	4.3417	3.4128	0.786
$16 \to 18$	85.995	42.408	0.493
$18 \to 20$	1.8424	0.8246	0.448

VII. CLOSING REMARKS

Concerning the future of this subject, it would be of great interest to fill in the missing levels that have been shown in the tables. In particular we have noted that, although there are much data on even spins in the even-even nuclei, there is very little known about the odd *J* positive parity states. Figures 6 to 10 show some interesting band structure for odd *J* states. If the levels are found, we can put more constraints on the effective nucleon-nucleon interaction in this region.

In summary, in studying the problem of the $T = 0$ neutron–proton interaction in a nucleus, it may prove more fruitful to begin by removing this channel altogether as was done here by setting all the $T = 0$ two-body matrix elements to zero and then reintroducing them rather than adopting the more common approach of investigating the effects of a pairing interaction separated from the rest of the interaction. Although such an interaction should not be used for realistic shell-model calculations, it should show very clearly what effect the $T = 0$ portion of the nuclear interaction has on

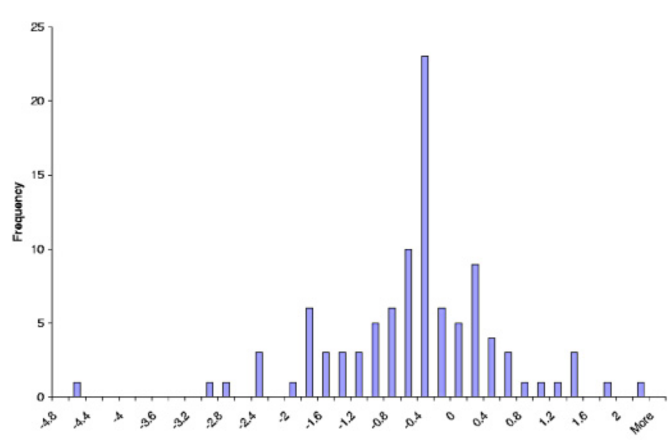

FIG. 12. (Color online) $T = 0$ two-body matrix element value distribution for FPD6.

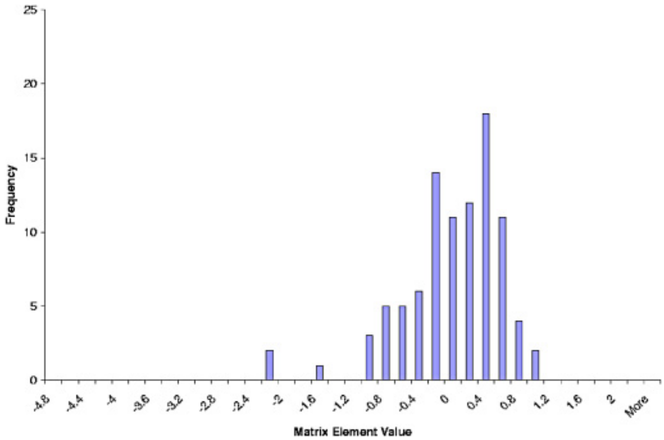

FIG. 13. (Color online) $T = 1$ two-body matrix element value distribution for FPD6.

nuclear observables. This may be especially true in the shell model as the suggestion has been made by Satula and Wyss that it may not be appropriate to separate out a pairing interaction from the rest of the Hamiltonian in a shell-model context [22].

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