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Self-consistent relativistic random-phase approximation with vacuum polarization

A. Haga,[1,2,*] H. Toki,[1,†] S. Tamenaga,[1,‡] Y. Horikawa,[3,§] and H. L. Yadav[4,¶]
[1]*Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan*
[2]*Department of Engineering Physics, Electronics, and Mechanics, Nagoya Institute of Technology, Gokiso, Nagoya 466-8555, Japan*
[3]*Department of Physics, Juntendo University, Inba-gun, Chiba 270-1695, Japan*
[4]*Physics Department, Rajasthan University, Jaipur-302004, India*
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We present a theoretical formulation for the description of nuclear excitations within the framework of a relativistic random-phase approximation whereby the vacuum polarization arising from nucleon-antinucleon fields is duly accounted for. The vacuum contribution to the Lagrangian is explicitly described as extra new terms of interacting mesons by means of the derivative expansion of the effective action. It is shown that the self-consistent calculation yields zero eigenvalue for the spurious isoscalar-dipole state and also conserves the vector-current density.

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Relativistic field theory based on quantum hadrodynamics (QHD) [1] has been very successful in describing nuclear properties not only for ground states but also for excited states. Although the response of a system to an external field has already been investigated in the 1980s by using the relativistic random-phase approximation (RRPA), in the relativistic mean-field (RMF) basis [2–8], self-consistent methods with a nonlinear effective Lagrangian for a quantitative description of excited states have been developed only during the past few years [9–13]. However, in particular, it is important to emphasize here that the negative-energy RMF states contribute essentially to current conservation of RRPA eigenstates and the decoupling of the spurious state. In our recent study, it has been shown that the negative-energy RRPA eigenstates generated from the RRPA equation with a fully consistent basis play a significant role in gauge invariance of the electromagnetic response [14].

The basis set used for the RRPA calculation is usually obtained from the RMF theory wherein only positive-energy nucleons are taken into account and the Dirac sea is always regarded as unoccupied (sometimes called the *no-sea approximation* [2,8]). This approximation is very convenient because we do not have to worry about a renormalization procedure in the calculation of the basis set under the full one-nucleon-loop contribution, which we refer to as the relativistic Hartree approximation (RHA), nor in the RRPA calculation in which the Feynman part is essentially divergent. However, the RHA + RRPA calculations in finite nuclei have been performed in Refs. [4–7] with the inclusion of vacuum polarization by employing the local density approximation. For all these RHA + RRPA calculations, this approximation implies that the renomalization of the one-nucleon loop in the RHA calculation and that of the Feynman part in the polarization function have been carried out for nuclear matter, and the results thus obtained have been used in the case of finite nuclei. However, because such a RRPA calculation violates self-consistency for finite nuclei, the spurious isoscalar-dipole strength associated with the uniform translation of the center-of-mass does not get shifted all the way down to zero excitation energy [6].

The main aim of this article is to demonstrate how such a deficiency in the previous RHA + RRPA calculations is removed by employing the derivative-expansion method to estimate the vacuum contribution. We verify the consistency by calculating explicitly the spurious state in the isoscalar-dipole mode and the current conservation of transition densities.

In the following we work out the vacuum-polarization effects using the Lagrangian density of the Walecka σ-ω model, which is given by

$$\begin{aligned}\mathcal{L}_N = {}& \bar{\psi}_N(i\gamma_\mu\partial^\mu - m_N + g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu)\psi_N + \tfrac{1}{2}(\partial_\mu\sigma)^2 \\ & - \tfrac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \tfrac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)^2 \\ & + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \delta\mathcal{L},\end{aligned} \tag{1}$$

where $U(\sigma) = \frac{1}{3!}g_2\sigma^3 + \frac{1}{4!}g_3\sigma^4$ denotes the self-interaction terms of scalar meson, and $\delta\mathcal{L} = -\frac{1}{4}\zeta_\omega(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)^2 + \frac{1}{2}\zeta_\sigma(\partial_\mu\sigma)^2 + \sum_{i=1}^{4}(\alpha_i/i!)\sigma^i$ represents the counterterms to regularize the nucleon self-energy. The renormalization procedure in a finite nuclear system requires considerable effort even at the mean-field level [15,16]. The explicit calculation of the vacuum polarization for a finite system has been performed recently by Haga *et al.* [15], who found the density variation to be substantial. The use of the effective action developed in Ref. [17], however, provides a direct and simple approach to estimate the vacuum corrections. It is interesting to observe that the lowest order of derivative expansion for the one-baryon-loop correction agrees with the rigorous results [15]. In the derivative expansion, the contribution from the Dirac sea to the Lagrangian density, which is expressed by the trace and the logarithm of the inverse of the Dirac Green

*Electronic address: haga.akihiro@nitech.ac.jp
†Electronic address: toki@rcnp.osaka-u.ac.jp
‡Electronic address: stame@rcnp.osaka-u.ac.jp
§Electronic address: horikawa@sakura.juntendo.ac.jp
¶Electronic address: hlyadav@sancharnet.in

function, is expanded by derivatives of the meson fields as expressed by

$$\int \frac{d^4p}{(2\pi)^4}[\mathrm{Tr}\ln(\gamma_\mu p^\mu - m_N + g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu) - \mathrm{Tr}\ln(\gamma_\mu p^\mu - m_N)] - \delta\mathcal{L} = -V_F(\sigma)+\frac{1}{2}Z_F^\sigma(\sigma)(\partial_\mu\sigma)^2+\frac{1}{4}Z_F^\omega(\sigma)(\partial_\mu\omega_\nu-\partial_\nu\omega_\mu)^2+\cdots. \tag{2}$$

Each term of the right-hand side of Eq. (2) is finite, and therefore we can treat them as ordinary potential terms. Here, we need to use a standard renormalization technique to obtain explicit forms of $V_F(\sigma)$, $Z_F^\sigma(\sigma)$, and $Z_F^\omega(\sigma)$. The method of calculating them has been discussed by many authors [18–20] and it has also been verified that the expansion converges quite rapidly within a mean-field approximation [15,16,21]. In the present calculation, we employ only the first three terms on the right-hand side of Eq. (2).

We proceed now to describe the calculational details of the RRPA based on the leading-order terms of the derivative expansion. In the σ-ω model, we know that the effective Lagrangian density is given by

$$\begin{aligned}\mathcal{L}_{(1)}^{\mathrm{ren}} = {}& \bar\psi_N^+(i\gamma_\mu\partial^\mu - m_N + g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu)\psi_N^+ + \tfrac{1}{2}(\partial_\mu\sigma)^2 \\ & - \tfrac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \tfrac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)^2 + \tfrac{1}{2}m_\omega^2(\omega_\mu)^2 \\ & - V_F(\sigma) + \tfrac{1}{2}Z_F^\sigma(\sigma)(\partial_\mu\sigma)^2 + \tfrac{1}{4}Z_F^\omega(\sigma)(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)^2,\end{aligned} \tag{3}$$

where superscript + in the nucleon-field operators means that only the positive-energy states are to be treated explicitly. Then, in a stationary, spherical system, the ground-state expectation values of the σ and ω_0, which are written as ϕ and V_0, satisfy the following coupled equations:

$$\begin{aligned}\left(\partial_\mu\partial^\mu + m_\sigma^2\right)\phi = {}& g_\sigma\langle\bar\psi^+\psi^+\rangle - U'(\phi) - V_F'(\phi) \\ & + \tfrac{1}{2}Z_F^{\sigma\prime}(\phi)(\partial_\mu\phi)^2 - \partial_\mu\left(Z_F^\sigma(\phi)\partial^\mu\phi\right) \\ & + \tfrac{1}{4}Z_F^{\omega\prime}(\phi)(\partial_\mu V_0)^2,\end{aligned} \tag{4}$$

$$\left(\partial_\mu\partial^\mu + m_\omega^2\right)V_0 = g_\omega\langle\bar\psi^+\gamma^0\psi^+\rangle + \partial_\mu\left(Z_F^\omega(\phi)\partial^\mu V_0\right), \tag{5}$$

where the contributions of the negative-energy states to source terms are contained in V_F and Z_F. The first step in calculating the RRPA response is the computation of the RHA, which involves just solving these equations together with the Dirac equation under the potentials of ϕ and V_0. The potentials achieved in the mean-field calculation are then used to generate the lowest order polarization function. Thus, we solve the RRPA equation given by

$$\begin{aligned}\Pi_{\mathrm{RPA}}(\Gamma^a,\Gamma^b;\boldsymbol{p},\boldsymbol{q};E) &= \Pi_D(\Gamma^a,\Gamma^b;\boldsymbol{p},\boldsymbol{q};E) + \sum_{ij}\int d\boldsymbol{k}_1 d\boldsymbol{k}_2 \\ &\quad\times \Pi_D(\Gamma^a,\Gamma^i;\boldsymbol{p},\boldsymbol{k}_1;E)D_{ij}(\boldsymbol{k}_1,\boldsymbol{k}_2;E) \\ &\quad\times \Pi_{\mathrm{RPA}}(\Gamma^j,\Gamma^b;\boldsymbol{k}_2,\boldsymbol{q};E),\end{aligned} \tag{6}$$

where the summation is over the meson fields and Γ's are the 4×4 matrices that denote the vertex couplings. This RRPA equation has the exact same form as that of the *no-sea approximation*. However, it should be noted that there is an essential difference with regard to the employed meson propagator, D_{ij}, in which we *must* include the vacuum-polarization corrections to perform the consistent calculation. The σ-ω coupling term $Z_F^\omega(\sigma)(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)^2$ in the present Lagrangian provides the coupled equations between the σ-meson propagator and the time component of the ω-meson propagator:

$$\begin{pmatrix} D_\sigma & D_{\sigma\omega} \\ D_{\omega\sigma} & D_{\omega 00}\end{pmatrix} = \begin{pmatrix} D_\sigma^0 & 0 \\ 0 & D_{\omega 00}^0\end{pmatrix} + \begin{pmatrix} D_\sigma^0 & 0 \\ 0 & D_{\omega 00}^0\end{pmatrix} \times \begin{pmatrix} \tilde\Pi_F^{\sigma\sigma} & \tilde\Pi_F^{\sigma\omega} \\ \tilde\Pi_F^{\omega\sigma} & \tilde\Pi_F^{\omega\omega}\end{pmatrix}\begin{pmatrix} D_\sigma & D_{\sigma\omega} \\ D_{\omega\sigma} & D_{\omega 00}\end{pmatrix}. \tag{7}$$

The spatial component of the ω-meson propagator can be evaluated independently. Here, the Fourier transforms of

$$\begin{aligned}\tilde\Pi_F^{\sigma\sigma}(y,x) = {}& \delta(x-y)\left[U''(\phi) + V_F''(\phi) - \tfrac{1}{2}Z_F^{\sigma\prime\prime}(\phi)(\partial_\mu\phi)^2 \right. \\ & \left. - \tfrac{1}{2}Z_F^{\omega\prime\prime}(\phi)(\partial_\mu V_0)^2 + \left[\partial_\mu\partial^\mu Z_F^\sigma(\phi)\right]\right] \\ & + (\partial^\mu)\left[Z_F^\sigma(\phi)[\partial_\mu\delta(x-y)]\right],\end{aligned} \tag{8}$$

$$\tilde\Pi_F^{\sigma\omega}(y,x) = \partial_\mu\left[Z_F^{\omega\prime}(\phi)(\partial_\mu V_0)\delta(x-y)\right], \tag{9}$$

$$\tilde\Pi_F^{\omega\sigma}(y,x) = -Z_F^{\omega\prime}(\phi)(\partial_\mu V_0)[\partial^\mu\delta(x-y)], \tag{10}$$

$$\tilde\Pi_F^{\omega\omega}(y,x) = \partial^\mu\left[Z_F^\sigma(\phi)[\partial_\mu\delta(x-y)]\right] \tag{11}$$

and the effective meson propagators D_{ij} as well as the free meson propagators D_{ij}^0 are expressed as the matrices of the momentum space. Details of the formulation on the present technique along with the comparison between the Feynman part in our calculation and that in the local-density approximation will be elucidated in a forthcoming publication.

The parameter set used in the present work is listed in Table I and has been determined to reproduce the total binding energies and charge radii of the spherical nuclei in the RHA calculation. This parameter set is similar to that introduced in Ref. [22], where the derivative expansion has been used. Small difference between the two sets stems from the fact that in the treatment of Ref. [22] the one-meson-loop corrections have been considered.

In what follows we discuss the results of our RRPA calculations. In Figs. 1(a) and 1(b) we display the distributions for the Coulomb responses of isoscalar-dipole mode in ^{16}O and ^{40}Ca at the momentum transfers of $q = 237$ and 118 MeV, respectively, as a function of the excitation energy. The most remarkable result of our RRPA calculations including the vacuum polarization is that the spurious state, which is the collective mode corresponding to the center-of-mass motion, appears at zero excitation energy in both nuclei (shown by the solid curves), as also obtained in conventional RRPA calculation without vacuum polarization (shown by the

TABLE I. Parameter set used in the present work.

	m_σ (MeV)	m_ω (MeV)	g_σ	g_2 (fm^{-1})	g_3	g_ω
RHA	458.0	814.0	7.10	24.09	−15.99	8.85

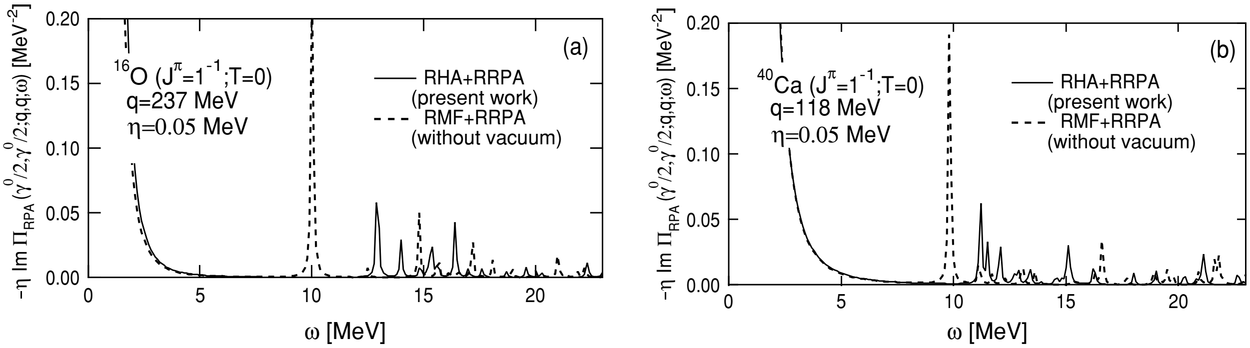

FIG. 1. Distributions of isoscalar-dipole strength (a) ^{16}O and in (b) ^{40}Ca.

dashed curves) [2]. The spurious state is clearly separated from the physical states, which are shown by using the small imaginary part of energy $\eta = 0.05$ MeV. We emphasize that, although earlier RRPA calculations with vacuum polarization have never succeeded in decoupling the spurious state [6], we are now able to achieve this by handling the Lagrangian (3) correctly.

Another important aspect for the correctness of the RPA calculations is to check whether transition charge density $\langle I'||M_\lambda(q)||I\rangle$ and current density $\langle I'||T_{\lambda L}(q)||I\rangle$, connecting the ground state I and the excited states I' for different multipolarity λ and L, satisfy the conservation law [2,8]. If we assume that ω_N denotes the excitation energy of nucleus, the conservation relation is given by

$$\omega_N\langle I'||M_\lambda(q)||I\rangle = -q\sqrt{\frac{\lambda}{2\lambda+1}}\langle I'||T_{\lambda\lambda-1}(q)||I\rangle + q\sqrt{\frac{\lambda+1}{2\lambda+1}}\langle I'||T_{\lambda\lambda+1}(q)||I\rangle. \quad (12)$$

Our results for the RRPA transition current are depicted in Fig. 2; the contributions from the left-hand side (lhs) and right-hand side (rhs) of Eq. (12) are shown separately by the dashed and dash-dotted lines, respectively. Note that for the purpose of clarity the lhs of Eq. (12) has been plotted with a negative sign. The difference of these two contributions (denoted by $\Delta\varrho$ and shown by the thick solid line in Fig. 2), represents violation of current conservation. It is gratifying to see from Fig. 2 that the RRPA transition current is sufficiently conserved.

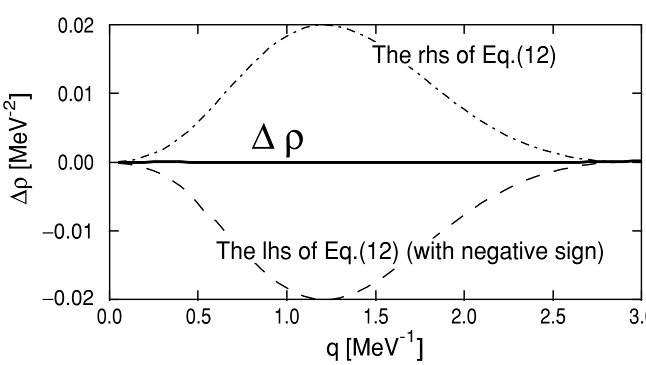

FIG. 2. RHA + RRPA results for the current conservation in the transition densities for the lowest positive-energy (12.94 MeV) isoscalar-dipole state in ^{16}O. The $\Delta\varrho$ depicted by the thick solid line shows the extent of violation of current conservation. For details refer to the text.

In summary, we have studied the self-consistent RHA + RRPA method including the vacuum-polarization contribution given by the derivative-expansion method. We stress here that the present method is able to treat the change of the negative-energy states owing to the presence of particles in the positive-energy states for the formation of the nucleus and also able to treat consistently excited states using the RRPA. In contrast to the previous RRPA calculations using the local-density approximation, the self-consistency of the model has been fulfilled so that we have obtained the desired results for decoupling the spurious isoscalar-dipole state, and for conserving the current density.

Also, it would be pertinent to take stock of the present scenario. Indeed we now have a powerful method of describing the ground state of nuclei in the relativistic mean-field approximation with the inclusion of negative-energy states that are influenced by the meson mean fields. Further, we also now have a method of calculating the excited states in nuclei whereby the excitation of nucleons from negative-energy states to positive-energy states is duly included. This makes the present treatment of excited states fully consistent with the description of the ground state. Numerically, however, the σ- and ω-mean fields obtained in the present treatment are found to be insufficient in strength, entailing a weaker spin-orbit splitting [23] which is about half that obtained in the usual RMF model [24]. There are several possibilities that can provide the missing spin-orbit splitting. One interesting idea is to explore the possibility of surface pion condensation suggested recently [25]. One could also consider the tensor coupling for the ω meson [26,27]. These extensions of the model constitute an effective field theory including vacuum polarization. Further calculations testing the RRPA with these extended RHA models for a wide variety of observables in nuclear excitations would be of immense interest.

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