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Is a physically observable tetraneutron resonance compatible with realistic nuclear interactions?

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The possible existence of four-neutron resonances close to the physical energy region is explored. Faddeev-Yakubovsky equations have been solved in configuration space using realistic nucleon-nucleon interaction models. Complex scaling and analytical continuation in the coupling constant methods were used to follow the resonance pole trajectories, which emerge out of artificially bound tetraneutron states. The final pole positions for four-neutron states lie in the third energy quadrant with negative real energy parts and should thus not be physically observable.

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I. INTRODUCTION

The existence of pure neutron systems could have far-reaching implication in nuclear physics [1]. However, being fed by a long series of controversial predictions and measurements, the question of multineutron existence is far from being cut-and-dried, both in theory as well as in experiment. Recently, much attention has been paid to the possible existence of bound tetraneutron (four-neutron system). This interest has been triggered by the experimental observation of a few events in the ^{14}Be break-up reaction [2]. On the other hand, such a prospect rises serious objections from the point of view of nuclear interaction theory. It has been shown by several groups [3–6] that realistic nuclear Hamiltonians exclude the existence of bound $3n$, $4n$ and even larger neutron clusters. In fact, the most favorable mechanism to construct tetraneutron would be by putting together two virtual (almost bound) dineutron pairs. However, in order to force the binding of virtual dineutrons, one has to have very strong neutron-neutron attraction in P and/or higher partial waves, which is not compatible with our present understanding of the nuclear interaction.

Nevertheless the possible existence of resonant states in pure neutron systems having observable effects in nuclear reactions, could not be eliminated. Such a scenario is evoked in a recent analysis of ^{8}He(d, ^{6}Li)$4n$ reaction: some excess of counts has been observed in the $4n$ energy spectrum slightly above the threshold, which cannot be explained by phase space analysis involving both four free neutrons and two noncorrelated dineutron pairs in the final state [7]. Furthermore the authors of Ref. [2], in their very recent study [8], agreed that the previously observed signal could result from the existence of near-threshold four-neutron resonance, without involving bound tetraneutron. The aim of this study is to clarify whether or not the existence of resonant tetraneutrons can be tolerated by modern nuclear interaction models and thus if these experimental claims can be supported in a theoretical ground. This work is a natural extension of our preceding work [9], in which we have demonstrated that realistic nuclear Hamiltonians exclude the existence of physically observable three-neutron resonances.

No proper *ab initio* calculations of the resonant tetraneutron with realistic *nn* forces are known to the authors. Some conclusions were drawn in favor of its existence based on calculations of a tetraneutron bound in an external well [3]; furthermore it was suggested that these resonances could have rather large widths. The only rigorous study of tetraneutron resonances was accomplished in Ref. [10] using the simplistic MT I-III *nn* interaction, which contains only S waves. Unfortunately, no observable resonances have been found there and only the existence of some broad subthreshold structures (S-matrix poles with negative real energy parts) was pointed out. The same authors remarked however that the positions of these subthreshold states strongly depend on the details of the *nn* interaction. Realistic nucleon-nucleon (NN) models contain indeed interactions in higher partial waves and are therefore better suited to accommodate tetraneutron and push its resonant states out of the subthreshold region.

II. THEORETICAL BACKGROUND

Although many of the nuclear excited states are resonances, they are seldom considered in theoretical nuclear structure calculations due to the huge technical difficulties of describing the continuum states in many-body systems. These states are often treated as being bound, but such a procedure is justified only for very narrow resonances and is not appropriate in our case. Resonant tetraneutron, if existing at all, will probably have a rather large width. The problem we are dealing with represents therefore a double challenge: first it is a four-particle problem and second, being a continuum

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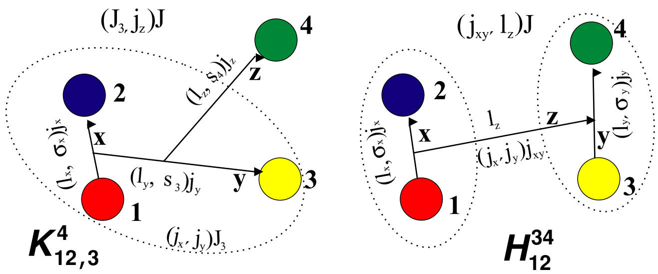

FIG. 1. (Color online) Faddeev-Yakubovsky components K and H. Asymptotically, as $z \to \infty$, components K describe $3+1$ particle channels, whereas components H contain asymptotic states of $2+2$ channels.

state, it has an exponentially diverging wave function. We will present, in what follows, the equations allowing one to solve the four-particle problem in a mathematically rigorous way and will describe the methods allowing for the treatment of resonant states.

In order to solve the four-body problem, we decompose the wave function into a sum of 18 Faddeev-Yakubovsky (FY) components, see Fig. 1, and rewrite the Schrödinger equation as a set of coupled FY equations [11]. If all four particles are identical, only two of the 18 FY components are independent, which we denote by K and H. These components are furthermore related by two integrodifferential equations:

$$\begin{aligned} (E - H_0 - V)\,K &= V(P^+ + P^-)\,[(1+Q)K + H]\,, \\ (E - H_0 - V)\,H &= V\tilde{P}\,[(1+Q)K + H]\,, \end{aligned} \tag{1}$$

where P^+, P^-, $\tilde{P}$, and Q are particle permutation operators:

$$\begin{aligned} P^+ &= (P^-)^- = P_{23}P_{12}; \quad Q = \varepsilon P_{34}; \\ \tilde{P} &= P_{13}P_{24} = P_{24}P_{13}, \end{aligned} \tag{2}$$

and ε is a Pauli factor related to the exchange of two identical particles: $\varepsilon = -1$ in case of fermions. Using these notations, the four-body wave function is given by

$$\begin{aligned} \Psi = {} & [1 + (1 + P^+ + P^-)Q](1 + P^+ + P^-)K \\ & + (1 + P^+ + P^-)(1 + \tilde{P})H. \end{aligned} \tag{3}$$

Each FY component $F = (K, H)$ is considered as a function of a proper Jacobi coordinate set $(\vec{x}, \vec{y}, \vec{z})$, defined, respectively, by

$$\begin{aligned} \vec{x}_K &= \vec{r}_2 - \vec{r}_1, & \vec{x}_H &= \vec{r}_2 - \vec{r}_1, \\ \vec{y}_K &= \sqrt{\frac{4}{3}}\left(\vec{r}_3 - \frac{\vec{r}_1 + \vec{r}_2}{2}\right), & \vec{y}_H &= \vec{r}_4 - \vec{r}_3, \\ \vec{z}_K &= \sqrt{\frac{3}{2}}\left(\vec{r}_4 - \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3}\right), & & \\ \vec{z}_H &= \sqrt{2}\left(\frac{\vec{r}_3 + \vec{r}_4}{2} - \frac{\vec{r}_1 + \vec{r}_2}{2}\right). & & \end{aligned} \tag{4}$$

The angular, spin, and isospin dependence of these components is expanded using tripolar harmonics $\mathcal{Y}_\alpha(\hat{x}, \hat{y}, \hat{z})$, i.e.,

$$\langle \vec{x}\vec{y}\vec{z}|F\rangle = \sum_\alpha \frac{F_\alpha(x, y, z)}{xyz}\,\mathcal{Y}_\alpha(\hat{x}, \hat{y}, \hat{z}). \tag{5}$$

The quantities $F_\alpha(x, y, z)$ are called regularized FY amplitudes, and the label α holds for the set of ten intermediate quantum numbers describing a $(J^\pi, T = 2, T_z = 2)$ state. When describing tetraneutron, the isospin dependence of the FY amplitudes is trivial and can be omitted. The set of quantum numbers α reduces to eight elements. We use the $j - j$ scheme for the intermediate coupling of FY amplitudes, defined as

$$K \equiv \left\{\left[\left(l_x(s_1s_2)_{\sigma_x}\right)_{j_x}(l_ys_3)_{j_y}\right]_{J_3}(l_zs_4)_{j_z}\right\}_{J^\pi}, \tag{6}$$

$$H \equiv \left\{\left[\left(l_x(s_1s_2)_{\sigma_x}\right)_{j_x}\left(l_y(s_3s_4)_{\sigma_y}\right)_{j_y}\right]_{j_{xy}} l_z\right\}_{J^\pi}, \tag{7}$$

where $s_i = 1/2$ is the spin of the individual particle and J^π the total angular momentum of the four-particle system. Each of the $N_c = N_K + N_H$ amplitudes in the expansion (5) is further conditioned by the antisymmetry properties $(-)^{\sigma_x + l_x + 1} = \varepsilon$ for K and $(-)^{\sigma_x + l_x + 1} = (-)^{\sigma_y + l_y + 1} = \varepsilon$ for H. FY components K and H are regular at the origin, and it can be shown that for a bound state problem, they decrease exponentially outside the interaction domain. In this case, one can impose these functions to vanish on the borders of a compact box:

$$F_\alpha(x = x_{\max}, y = y_{\max}, z = z_{\max}) = 0. \tag{8}$$

Equations (2)–(8) are sufficient to solve the bound state problem.

Resonance wave functions are however divergent and cannot be described by the boundary conditions (8). In order to solve the resonance problem, we make use of two different methods, successfully applied in Ref. [9] to treat the three-neutron system. The implementation of these techniques in the four-body FY equations is analogous to the three-body case. Therefore we only briefly discuss them here and the interested reader can refer to Ref. [9] for technical aspects.

The method of Analytical Continuation in the Coupling Constant (ACCC), proposed by Kukulin *et al.* [12], is based on the observation that a resonant state commonly arises from a bound state one when the interaction between the particles is made less attractive. The corresponding eigenenergy is considered as an analytical function of a coupling constant λ, which determines the strength of the attractive part of the potential. Therefore, one can try to analytically continue the energy of the bound state as a function of the strength λ to the complex plane and obtain this way the width and the position of the resonance. It can be shown moreover that close to the threshold, where bound state turns into the resonance, the momenta $k = \sqrt{E - E_0}$ is proportional to

$$k \sim x \equiv \begin{cases} \lambda - \lambda_0 & \text{for virtual state} \\ \sqrt{\lambda - \lambda_0} & \text{for resonant state,} \end{cases} \tag{9}$$

where λ_0 is a critical value of the coupling constant and $E_0 = E(\lambda_0)$ is the threshold energy. If the multiparticle system does not possess bound states in its subsystems, as is a case for multineutron, then $E_0 = E(\lambda_0) = 0$.

It turns out that using an analytical continuation of $k(x)$ in terms of a simple polynomial, the expansion converges very slowly and that the Padé expansion in terms of rational functions of order $[N, M]$

$$k^{n,m}(x) = \frac{a_1 x + a_2 x^2 + \ldots + a_N x^N}{1 + b_1 x + b_2 x^2 + \ldots + b_M x^M} \qquad (10)$$

is more appropriate.

It is quite simple to put the ACCC method into practice. One should artificially bind the tetraneutron by adding some attractive interaction to the system's Hamiltonian $H = H_0 + \lambda V_{\rm att}$. Then, the critical value of the coupling constant (λ_0) is determined and several eigenenergies $E(\lambda_i)$ are calculated for $\lambda_i > \lambda_0$; these values are used to fix the Padé expansion (10) coefficients. However, to make this extrapolation efficient, one should provide rather accurate binding energies $E(\lambda_i)$ and an especially precise λ_0 value as input. While only the few lowest order terms in Padé expansion are enough to determine the positions of narrow nearthreshold resonances, the description of deep resonances requires several terms and very accurate input of $E(\lambda_i)$. The determination of high order Padé expansion terms requires at least five digit accuracy in the binding energies.

The other method we use, namely, complex scaling (CS) [13], can be applied to calculate resonance positions directly. This method makes use of the similarity transform

$$\widehat{S} = e^{i\theta r \frac{\partial}{\partial r}}, \qquad (11)$$

applied to the Hamiltonian of the system, i.e.,

$$(\widehat{S}\widehat{H}\,\widehat{S}^{-1})(\widehat{S}\Psi_{\rm res}) = E_{\rm res}(\widehat{S}\Psi_{\rm res}). \qquad (12)$$

Such transformation does not affect the eigenvalue ($E_{\rm res}$) spectra. However, if the scaling angle is large enough—$\theta > \frac{1}{2}|\arg E_{\rm res}|$—the modified resonance eigenfunctions ($\widehat{S}\Psi_{\rm res}$) become square integrable. Evidently, the CS method can be applied to FY equation. By this transformation all the radial variables $r \equiv (x, y, z)$ in Eq. (2) are replaced by $re^{i\theta} \equiv (xe^{i\theta}, ye^{i\theta}, ze^{i\theta})$. The problem becomes analogous to a bound state one with complex variable and transformed FY amplitudes ($\widehat{S}F$), which—unlike resonance eigenfunctions $\Psi_{\rm res}$ or the non transformed FY amplitudes—are in the Hilbert space.

CS transformation requires the analytical continuation of the potential $V(x)$ into the complex plane $V(xe^{i\theta})$. This turns out to be a weak point of this method when applied to nuclear systems. As discussed in Ref. [9], nuclear potentials have mischievous analytical properties: they become strongly oscillating and even divergent already for relatively small transformation angles $\theta > 30^\circ$. This fact limits the applicability of CS method to narrow resonances, with $\mathrm{Im}(-E_{\rm res}) < 2\mathrm{Re}(E_{\rm res})$ values.

The numerical solution of FY equations is performed by expanding $F_\alpha(xyz)$ on a basis of three-dimensional piecewise Hermite polynomials and projecting equation (1) with boundary conditions (8) onto tripolar harmonics. In this way, the integro-differential FY equations are converted into a linear algebra problem:

$$AX = E_{\rm res} BX$$

with A and B being large square matrices, whereas $E_{\rm res}$ and X are, respectively, the eigenvalue and the eigenvectors to be determined. The reader interested in a detailed discussion on the formalism and the numerical methods used should refer to Ref. [6].

III. RESULTS AND DISCUSSION

The results presented below have been obtained by using the charge-symmetry breaking Reid 93 potential to describe the *nn* interaction. This choice is dictated by purely practical reasons: as discussed in Refs. [9,14], the Reid 93 model has better analytical properties to perform the complex scaling operation (11) than its coordinate-space modern concurrents. We would like to remark however that other realistic NN interaction—namely AV14, AV18, and Nijm II—provide very similar results for two- and three-neutron systems and exhibit also a similar behavior for the—artificially bound—tetraneutron. These facts let us believe that no qualitative changes in the four-neutron resonance can emerge from the properties of a particular model. All the calculations presented in what follows use the value $\frac{\hbar^2}{m_n} = 41.44$ MeV·fm^2 as an input for the neutron mass.

In a way similar to the one used in the study of the three-neutron system [9], we introduce an additional attractive four-nucleon (4*N*) force to analyze the tetraneutron resonance trajectories in a systematic way. We have chosen the form

$$V_{4n} = -W\rho e^{-\frac{\rho}{\rho_0}}, \qquad (13)$$

where W and ρ_0 are, respectively, the strength and range parameters of the potential, and the hyperradius $\rho = \sqrt{x^2 + y^2 + z^2}$ is an invariant quantity with respect to the permutation operators (3). Such kind of force is easy to implement in FY equations (2). In our previous work [9] devoted to the three-neutron system, we used a 3*n* force having the standard Yukawa form. However we have found the functional form (13) more appropriate for studying artificially bound tetraneutron. This form does not diverge as $\rho \to 0$ and thus avoids a rapid shrinking of the bound structures generated.

As has been already remarked in Refs. [3,5,6], an extremely strong additional interaction is required to force tetraneutron binding. As a consequence, the generated bound system is a very compact object making its physical existence unlikely. On the other hand, resonances are extended structures. In order to ease the transition from bound to resonant tetraneutron, we have fixed a rather large value for the range parameter ρ_0 in Eq. (13) and taken $\rho_0 = 2.5$ fm, a value considerably larger than the one we could expect for a realistic 4*N* interaction.

Our strategy in studying 4*n* resonances is to vary the strength of the potential W and trace the resonance energy-trajectory $E_{\rm res}(W)$. The final resonance positions, which correspond to realistic nuclear interaction, are eventually reached at $E_{\rm res}(W = 0)$.

When applying ACCC method, the parameter λ given in Eq. (9) is identified to the 4*N* force strength $\lambda \equiv W$. We determine several auxiliary values of $E_i(\lambda_i)$ in the bound tetraneutron region $\lambda_i > \lambda_0$. These values are later used as an input to determine the Padé expansion coefficients (a_i, b_j) of Eq. (10). Several calculations are performed in the low energy region as well, in order to determine the critical value $W_0 \equiv \lambda_0$ for which tetraneutron is bound with zero energy

TABLE I. Critical strengths W_0 (MeV fm^{-1}) of the phenomenological force (13) required to bind four neutrons in different states. The range parameter ρ_0 of this force was fixed to 2.5 fm. W' are the strength values at which tetraneutron becomes subthreshold, i.e., $\mathrm{Re}(E_{\mathrm{res}}) = 0$. In the last row of the table the positions of the physical resonances ($W = 0$) are given.

J^π	0^-	1^-	2^-	0^+	1^+	2^+
W_0	38.70	38.67	38.68	22.90	22.92	40.38
W'	3.0	3.2	3.9	3.5	3.6	4.1
$E_{\mathrm{res}}(W=0)$	$-1.0-9.9i$	$-1.1-9.8i$	$-1.4-9.7i$	$-1.1-6.3i$	$-1.1-6.5i$	$-1.4-10.9i$

$E_{\mathrm{res}}(W_0) = 0$. Once the threshold value λ_0 and the coefficients (a_i, b_j) of expansion (10) are known, we use this latter equation to analytically continue the $E_{\mathrm{res}}(W)$ curve in the resonance region $W < W_0$.

When applying the CS method, we perform a series of direct resonance calculations for several decreasing values of $W < W_0$, until the calculations become unstable due both to the large size of the resonance widths and to the necessity of using ever increasing scaling parameter θ. This method is used as well in the near threshold region—$W \lessapprox W_0$ and $\mathrm{Im}(-E_{\mathrm{res}}) \ll \mathrm{Re}(E_{\mathrm{res}})$—where it is very helpful to improve the accuracy of λ_0 used in ACCC calculations.

In Table I, the critical strengths W_0 required to bind the tetraneutron in states with different J^π quantum numbers are summarized (second row). Even though we have taken a 4N force with a rather long range, the critical strength values W_0 are still considerable. Another noticeable feature is that these critical values are almost equal for all negative parity tetraneutron states $J^\pi = 0^-$, 1^-, and 2^- we have considered. The reason for such a degeneracy is that tetraneutron binding energies were found to be insensitive to *nn* interaction in *P* and higher partial waves. Actually, their values remain unchanged up to three to four digits if these $l_{nn} \geqslant 1$ interaction terms are switched off. Tensor coupling is present only in $l_{nn} \geqslant 1$ partial waves and has very small impact on these states. As a consequence the total spin ($S = 1$) and angular momenta ($L = 1$) are separately conserved.

A similar situation is observed for positive parity states $J^\pi = 0^+$ and 1^+, which are also almost degenerate. These states are dominated by the FY amplitude of type K with $l_x = l_y = l_z = 0$ intermediate quantum numbers, i.e., they are almost pure $L = 0$ states. On the other hand, they differ by their total spin ($S = 0$ for $J^\pi = 0^+$ and $S = 1$ for $J^\pi = 1^+$). Unlike expected, the $J^\pi = 0^+$ state has a structure dominated by a "*nn*+*n*+*n*" type configuration and not by a "*nn*+*nn*" one, i.e., containing only a single $^1S_0$ dineutron pair and not two. For $J^\pi = 2^+$, W_0 is considerably larger. This state must have a total spin $S = 2$ to be realized with $L = 0$, i.e., all neutron spins pointing in the same direction and thus $^1S_0$ dineutron pairs—with antiparallel neutron spins—being absent. The corresponding large W_0 value can therefore be understood as the price to pay for breaking the remaining dineutron pair. $J^\pi = 0^+$ and 1^+ states remain also unchanged if *nn* *P*-wave interaction is switched off. A very strong enhancement of these waves is required, as much as creating a dineutron resonance, in order to see their effect in the binding energies. The only state sensible to *nn* *P* waves is thus the $J^\pi = 2^+$.

In Figs. 2 and 3 are displayed the tetraneutron resonance trajectories for the same negative and positive parity states we have considered in Table I. In both figures, CS results are indicated using empty symbols (square, circle, and triangle) which correspond to different values of the 4NF strength parameter W. ACCC trajectories for different states are depicted by solid, dashed, and dotted lines. They have overimposed star-like $\times, *, +$ symbols which correspond to the same W values than those used in CS calculations. In order to compare quantitatively the agreement between both methods, the numerical values of some resonance positions are also given in Table II. This agreement is rather good for narrow resonances. For wider resonances, small discrepancies appear, which are due to the drawbacks present in the CS method described above. Sizable differences appear for resonances with $\mathrm{Re}(E_{\mathrm{res}}) < -\mathrm{Im}(E_{\mathrm{res}})$, which is the limit of applicability of CS transformation.

In this region, ACCC results are still rather well converged (better than 5%) with respect to the Padé expansion. However, this convergence is getting worse when going away further and further from the bound state region. The accuracy of Padé expansion is 20% near the subthreshold region, where resonance trajectory moves into the third energy quadrant. The accuracy of the physical resonance positions, when the additional interaction is fully removed ($W = 0$), deteriorates to 50%.

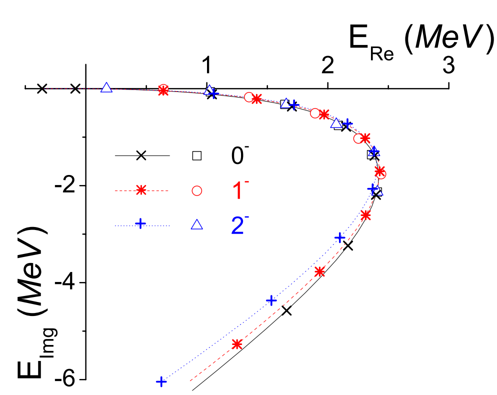

FIG. 2. (Color online) Negative parity tetraneutron resonance trajectories parameterized by the strength W of the phenomenological 4NF. ACCC results are denoted by lines with overimposed $\times, *, +$ symbols. They correspond to W values separated by 4 MeV fm^{-1} steps, starting from 38 MeV fm^{-1} for $J^\pi = 0^-$ and 2^- states and from 36 MeV fm^{-1} for 1^- CS results are represented by circles, squares, and triangles.

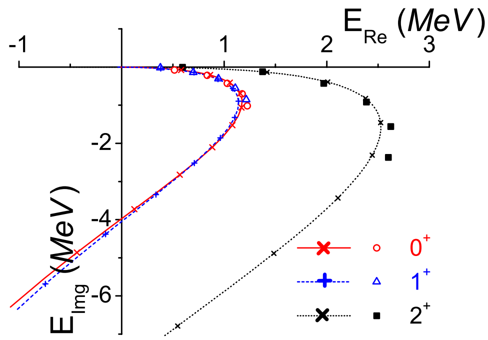

FIG. 3. (Color online) The same as in Fig. 2 for positive parity states. W values are reduced from 20 MeV fm^{-1} with steps of 2 MeV fm^{-1} for $J^\pi = 0^+$, from 21 MeV fm^{-1} in steps of 2 MeV fm^{-1} for 1^+ and from 38 MeV fm^{-1} in steps of 4 MeV fm^{-1} for 2^+.

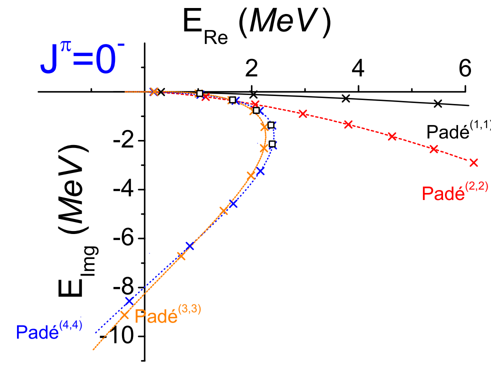

FIG. 4. (Color online) Convergence of ACCC method with respect to the order $[M, N]$ of Padé expansion for $J^\pi = 0^-$ tetraneutron state. ACCC curves are followed by star-like points indicating the resonance positions for W values decreasing from 38 MeV fm^{-1} by steps of 4 MeV fm^{-1}. CS results are presented by full circles and correspond to W values from 38 to 18 MeV fm^{-1}.

These limitations in ACCC accuracy are due to the increasing size of the Padé expansion argument $x = \sqrt{\lambda - \lambda_0}$, which implies to take into account higher order terms. The precise determination of high order Padé coefficients fails due to the severe accuracy criteria it imposes to the input. As an example, we have illustrated in Fig. 4, the Padé expansion convergence for $J^\pi = 0^-$. We can see that the shapes for $[N, M]$ = [3,3] and [4,4] order Padé expansion curves are already very close to each other. However the separation between the energies corresponding to the same W values still exists and it increases when one departs from the bound state region.

The uncertainty in determining the final resonance positions is also manifested in Fig. 5. In this figure, we compare the resonance trajectories for $J^\pi = 2^-$ tetraneutron state obtained with 4NF of Eq. (13) having different values for the range parameter ρ_0. In a Padé expansion not suffering from numerical errors, the final position of all the trajectories should coincide. However, as can be seen in this figure, the trajectory corresponding to $\rho_0 = 2.5$ fm ended at $E_{\rm res} = -1.4$–$9.7i$ MeV, while for $\rho_0 = 2$ fm it ends at $E_{\rm res} = -2.5$–$12.5i$ MeV.

As has been discussed above, tetraneutron negative parity states on one hand and $J^\pi = 0^+$ and 1^+ ones on the other hand, are almost degenerate in energy. This degeneracy is also reflected in the corresponding resonance trajectories, which superimpose close to the threshold. Notice however that the small difference—not exceeding several keV—in the binding energies, results into an increasing separation of these curves. This demonstrates the necessity of providing very accurate inputs in the Padé extrapolation and the difficulty in describing broad resonances with the ACCC method.

Regardless the convergence problems mentioned above, our results indicate that the final resonance positions will always stay in the third energy quadrant for all tetraneutron states. An accurate determination of the physical resonance position is not possible with the methods used in the present work. Nevertheless, in all calculations we have performed testing different artificially binding mechanisms, the final resonance positions were situated in the third energy quadrant (Re(E) < 0, Im(E) < 0). Their approximate values obtained with the ACCC

TABLE II. Comparison of CS and ACCC method results. Resonance positions for tetraneutron states obtained by adding phenomenological $4n$ force with strength W (in MeV fm^{-1}) and range $\rho_0 = 2.5$ fm are compared.

W	CS			ACCC		
	0^-	1^-	2^-	0^-	1^-	2^-
30.0	$1.67 - 0.33i$	$1.68 - 0.33i$	$1.67 - 0.33i$	$1.70 - 0.36i$	$1.72 - 0.35i$	$1.72 - 0.34i$
24.0	$2.24 - 1.03i$	$2.24 - 1.02i$	$2.24 - 1.02i$	$2.30 - 1.05i$	$2.31 - 1.03i$	$2.30 - 1.00i$
20.0	$2.41 - 1.67i$	$2.41 - 1.67i$	$2.41 - 1.66i$	$2.42 - 1.75i$	$2.43 - 1.72i$	$2.41 - 1.67i$
W	CS			ACCC		
	0^+	1^+	2^+	0^+	1^+	2^+
34.0			$1.37 - 0.12i$			$1.41 - 0.13i$
26.0			$2.39 - 0.92i$			$2.38 - 0.84i$
18.0	$0.84 - 0.22i$	$0.84 - 0.22i$	$2.60 - 2.46i$	$0.85 - 0.21i$	$0.84 - 0.22i$	$2.44 - 2.31i$
15.0	$1.11 - 0.56i$	$1.11 - 0.55i$		$1.12 - 0.54i$	$1.09 - 0.56i$	$2.22 - 3.12i$
13.0	$1.22 - 0.85i$	$1.21 - 0.86i$		$1.17 - 0.86i$	$1.14 - 0.89i$	$1.98 - 3.76i$

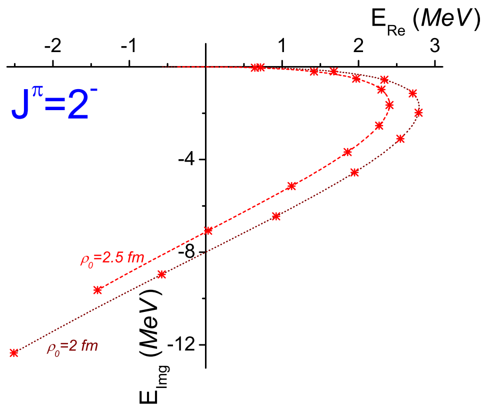

FIG. 5. (Color online) Comparison of resonance trajectories for $J^\pi = 2^-$ tetraneutron, two different curves correspond calculations with 4NF Eq. (13) having length $\rho_0 = 2.5$ (dashed curve) and 2 fm (dotted curve). The points correspond resonance positions for W being reduced from 36 in steps of 4 MeV fm^{-1} for $\rho_0 = 2.5$ fm curve and from 72 in steps of 8 MeV fm^{-1} for $\rho_0 = 2$ fm curve.

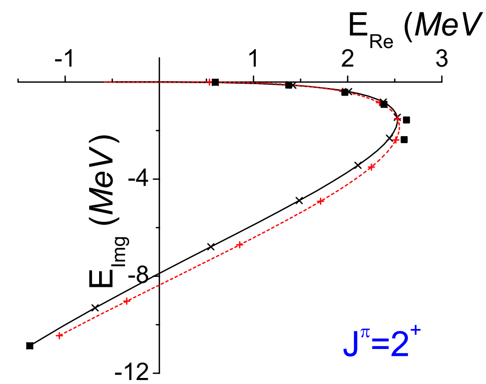

FIG. 6. (Color online) Sensibility of the 2^+ tetraneutron resonance trajectory with respect to *nn* *P* waves. Solid line correspond to the Reid 93 *nn* interaction and dashed line was obtained with *nn* *P* waves enhanced by a factor $\gamma = 1.2$.

method and 4NF of Eq. (13) are summarized in the last row of Table I. The accuracy of these results is estimated to be of 50%.

In Table I we have also displayed the strengths W' of the $4N$ force [Eq. (13)] at which the resonance trajectories crosses the imaginary-energy axis, slipping from fourth into third energy quadrant. One can argue that these values are pretty small and that a small correction of nuclear interaction (like the presence of attractive three-nucleon force) can push tetraneutron states back to the fourth energy quadrant (with positive real energy parts). On this point we would like to mention that the smallness of W' is only apparent and entirely due to the unrealistic long range character of the 4NF we have chosen. W' value would increase drastically if the range of the potential ρ_0 is reduced to make 4NF more realistic. This fact is demonstrated in Fig. 5, where W' value for 2^- tetraneutron state increases from 3.9 to 10.8 MeV fm^{-1} when ρ_0 is reduced from 2.5 to 2 fm. This result shows that any realistic ($\rho_0 < 1.4$ fm) multineutron force should be very strong to keep a multineutron resonances in the fourth energy quadrant.

The $J^\pi = 2^+$ tetraneutron state represents an interesting case, since it shows the largest sensitivity to *nn* *P*-wave interaction. Some $3N$ and $4N$ scattering observables, which are difficult to be reproduced with the existing models, indicate a strong *nn* *P*-wave contribution. It has been suggested that these discrepancies in $3N$ and $4N$ scattering observables can be significantly improved by modifying *nn* *P* waves within 20% [15–17]. We have explored such a possibility and traced in Fig. 6 the tetraneutron resonance trajectory for the Reid 93 interaction with *nn* *P* waves enhanced by a factor $\gamma = 1.2$. By doing so a slightly weaker critical strength of 40.02 MeV fm^{-1} is required to bind tetraneutron, in comparison with 40.38 MeV fm^{-1} of the original Reid 93 force. However, apart from a small shift in the $E_{\rm res}(W)$ trajectory, such a modification of *nn* *P* waves has not changed its qualitative behavior, ending up very close to its original value, always located in the third energy quadrant. These *nn* *P* waves should be much more strongly enhanced, as much as creating dineutron resonances, to result in sizable effects in tetraneutron resonance positions.

Finally, we would like to remark that even if there was a resonance in the fourth energy quadrant having a small real energy part and a large imaginary one, it would be difficult to identify experimentally. Resonance should have a rather small width $\Gamma = -2\mathrm{Im}(E_{\rm res})$ to produce a visible effect in the experimental cross section with a centered $E = \mathrm{Re}(E_{\rm res})$ Breit-Wigner shape. At most, such a broad resonance will produce a weak enhancement in the cross section, which would be hardly discernible from the background and not centered around $E = \mathrm{Re}(E_{\rm res})$. This makes the perspective of physically observable tetraneutron resonances very doubtful. Their eventual existence would imply a too strong modification in the present nuclear Hamiltonians.

Our results are in qualitative agreement with the findings of Sofianos *et al.* [10], where authors were able to accurately determine the tetraneutron resonance positions in the third energy quadrant for positive parity states, although using *S*-wave MT I-III potential. Due to the small influence of *P* and higher *nn* partial waves on tetraneutron states, *S*-wave models become very appropriate to study this system.

IV. CONCLUSION

Configuration space Faddeev-Yakubovsky equations have been solved with the aim of determining the positions of the four-neutron resonances in the complex energy plane.

A realistic Reid 93 *nn* interaction model has been used. A systematic study of four-neutron resonances has been accomplished by first adding to the nuclear Hamiltonian an attractive four-neutron force to artificially bind tetraneutron. The trajectory of the energy eigenvalue is then traced as a function of the strength of the additional force until fully removed.

Two methods, namely, the complex scaling and analytical continuation in the coupling constant, were employed to follow these trajectories.

The low lying four-neutron resonance trajectories, corresponding to states with quantum numbers $J^{\pi} = 0^{\pm}, 1^{\pm}, 2^{\pm}$, were shown to settle in the third energy quadrant ($\mathrm{Re(E)} < 0$, $\mathrm{Im(E)} < 0$) well before the additional $4n$ force is completely removed. Furthermore, these resonances acquired a rather large imaginary energies $\Gamma = 2\mathrm{Im}(-E) \approx 15$ MeV and should hardly be experimentally observable. Tetraneutron compound—bound or resonant—can be created only in a strong external field and would disintegrate right after such a field is removed.

Finally, we have demonstrated that the four-neutron physics is entirely determined by *nn* S waves, namely $^1S_0$ one, which is controlled by the experimentally measurable *nn*-scattering length. All realistic nuclear interaction models should thus provide qualitatively identical results for tetraneutron resonances. This fact is supported by similar studies in which tetraneutron states were artificially bound by means of different mechanism and with different NN models [6].

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