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Mirror energy differences in the $A = 31$ mirror nuclei, ^{31}S and ^{31}P, and their significance in electromagnetic spin-orbit splitting

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Excited states in ^{31}S and ^{31}P were populated in the ^{12}C(^{20}Ne,n) and ^{12}C(^{20}Ne,p) reactions, respectively, at a beam energy of 32 MeV. High spin states of positive and negative parity have been observed in ^{31}S for the first time, and the yrast scheme of ^{31}P has been extended. Large mirror energy differences between the first $9/2^-$ and $13/2^-$ states were observed, but only small differences for the first $7/2^-$ and $11/2^-$ levels. The significance of these observations is discussed in relation to the electromagnetic spin-orbit effect and the relative binding energy of the levels.

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If nuclear forces were charge symmetric and neutrons and protons had equal mass, charge, and magnetic moments, the spectrum of states in mirror pairs of nuclei should be identical. In fact, the spectra are indeed similar, and deviations between mirror partners provide a sensitive probe of our knowledge of nuclear structure. The absolute offset of ground-state binding energies of mirror pairs, the Coulomb energy differences (CEDs), has been of interest since the 1930s and is still difficult to account for exactly, as has been discussed by Nolen and Schiffer [1]. If the position of each level relative to its ground state is compared, much of the monopole shift, arising from proton and neutron mass difference and Coulomb displacement, is removed and subtle differences in structure emerge. These differences are called mirror energy differences (MEDs), with the difference $\Delta E = E(J, T_z = -1/2) - E(J, T_z = +1/2)$, where J is the total angular momentum and T_z is the projection of the isospin. In general these MEDs are small, on the order of 50 keV. However, attention has recently been drawn to some significantly larger energy differences between excited states. Ekman *et al.* have studied the $T = 1/2$, $A = 35$ mirror pair ^{35}Ar and ^{35}Cl and have also reviewed $T = 1/2$ data from $A = 33$ to $A = 39$ [2]. Many interesting effects were observed, especially for very pure, single-amplitude, shell-model configurations. Here, MEDs of several hundred keV were found. These shifts were interpreted by means of modern large-basis shell-model calculations [3], which provide an excellent description of the wave functions and allow detailed investigation of charge-symmetry breaking in the interactions. A key inference was the claim relating to the observation of the electromagnetic spin-orbit interaction on the uncoupled last nucleon; the effect of the interaction between its magnetic moment and the magnetic field induced by motion in the Coulomb field of the nucleus. This effect is completely analogous to the spin-orbit interaction felt by atomic electrons. In this Rapid Communication we present new measurements on ^{31}S that allow the extension of these investigations to the $A = 31$ mirror pair ^{31}S and ^{31}P. We find similar effects to those previously reported [2] but suggest that differences in the relative binding energy of states may make a major contribution to these shifts.

^{31}P is a stable nucleus and has been extensively studied [4]. In contrast, much less is known about ^{31}S. Several studies have identified excited states using one- and two-nucleon transfer reactions [5–8], but γ-ray spectroscopy has been limited to a few decays between low-lying levels [8,9]. In the present work, ^{31}P and ^{31}S were produced at the same time through the ^{12}C(^{20}Ne,p) and ^{12}C(^{20}Ne,n) reactions using a 32-MeV beam from the ATLAS accelerator at Argonne National Laboratory. The 90 μg/cm^2 target was mounted on a rotating target wheel and bombarded with a beam of up to 40 pnA for four days. The target was surrounded by Gammasphere [10], an array of 100 large, Compton-suppressed germanium detectors covering $>90\%$ of 4π and having an efficiency of $\sim 8.9(2)\%$ for 1.33-MeV photons. For the first half of the experiment, Gammasphere was triggered by a zero-degree mass spectrometer, with the fragment mass analyzer (FMA) [11] set to accept only $A = 31$, $q = 10^+$ ions through use of mass-defining slits. Z separation was achieved using a standard 30-cm-deep ΔE-ΔE-E ion chamber at the

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TABLE I. Spectroscopic information for ^{31}P from the present work including energies and intensities of γ rays. Angular correlation ratios are given along with the assignment.

E_γ(keV)	I_γ(%)	R_{DCO}	Assignment
745.8(2)	3.1(6)	1.61(12)	$13/2^- \to 13/2^-$
776.7(3)	3.0(5)	0.99(15)	$17/2^- \to 15/2^{(-)}$
988.9(3)	4.0(4)	1.68(16)	$11/2^+ \to 11/2^+$
1016.4(1)	3.1(5)	2.00(15)	$7/2^- \to 7/2^+$
1042.4(2)	3.4(3)	0.96(7)	$13/2^- \to 13/2^+$
1061.6(2)	11.1(9)	1.41(4)	$5/2^+ \to 5/2^+$
1070.7(4)	2.4(2)	—	$15/2^{(-)} \to 13/2^-$
1080.9(6)	2.2(3)	—	$17/2^- \to 13/2^-$
1110.1(3)	5.5(5)	1.32(9)	$11/2^+ \to 9/2^+$
1135.6(2)	23.4(1.5)	0.89(1)	$7/2^- \to 5/2^+$
1180.9(2)	2.6(4)	—	$7/2^+ \to 5/2^+$
1219.4(1)	8.7(6)	1.18(4)	$7/2^+ \to 7/2^+$
1266.1(1)	162(2)	0.86(1)	$3/2^+ \to 1/2^+$
1339.6(3)	8.8(7)	1.23(7)	$7/2^+ \to 5/2^+$
1445.0(3)	2.9(3)	—	$9/2^+ \to 7/2^+$
1480.8(2)	25.7(8)	0.85(1)	$11/2^- \to 9/2^+$
1733.4(7)	2.9(6)	1.04(16)	$13/2^+ \to 11/2^+$
1815.9(4)	4.2(4)	0.99(9)	$15/2^{(-)} \to 13/2^-$
1883.7(4)	12.7(1.2)	1.15(9)	$13/2^- \to 11/2^-$
1928.0(1)	27.3(1.6)	0.82(1)	$9/2^+ \to 7/2^+$
2028.8(2)	38.8(2.1)	1.21(2)	$5/2^+ \to 3/2^+$
2071.5(2)	9.5(8)	2.01(9)	$9/2^- \to 7/2^-$
2148.4(1)	100	1.57(2)	$7/2^+ \to 3/2^+$
2197.0(2)	36.8(1.2)	0.96(8)	$7/2^- \to 5/2^+$
2205.3(15)	0.6(1)	—	$13/2^- \to 9/2^-$
2233.6(1)	48.4(3.0)	—	$5/2^+ \to 1/2^+$
2394.0(2)	22.6(1.5)	1.53(18)	$11/2^- \to 7/2^-$
2628.8(9)	<1	1.04(9)	$13/2^- \to 11/2^-$
2723.5(8)	23.2(1.2)	0.83(2)	$13/2^+ \to 11/2^+$
2994.6(24)	1.7(2)	—	$(15/2^+) \to 13/2^+$
3039.7(4)	55.7(2.5)	1.60(4)	$11/2^+ \to 7/2^+$
3110.9(6)	2.8(3)	—	$9/2^+ \to 5/2^+$
3392.3(7)	6.5(5)	1.12(12)	$13/2^- \to 11/2^-$
3844.6(1)	6.9(6)	—	$9/2^+ \to 5/2^+$
4027.8(3)	11.0(9)	1.57(8)	$11/2^+ \to 7/2^+$

focal plane filled with isobutane at a pressure of 8 torr. The residues from the reaction were slow, ~0.65 MeV/u, despite the inverse kinematics, and so complete Z separation could not be achieved. Nevertheless, the loci of the dE/dx stopping data were sufficiently different that $Z = 16$ and $Z = 15$ γ-ray spectra could be formed after some manipulation. The time of flight of the ions through the 8.2-m flight path of the FMA was used to create an Et^2 spectrum (proportional to mass), gating on which further improved the quality of the $Z = 16$ spectrum. Known transitions in ^{31}P were used to monitor the cleanliness of this event selection (Table I). The $Z = 16$ spectrum revealed many new transitions associated with ^{31}S. In the latter half of the experiment, Gammasphere was operated in a stand-alone mode with a trigger requirement of two coincident γ rays. These coincidence data were sorted off-line into a γ-γ matrix and a γ-γ-γ cube. Spectra were obtained from these matrices for coincidences with transitions positively identified as associated with ^{31}P and ^{31}S in the analysis of the FMA data. These were used to develop level

TABLE II. Spectroscopic information for ^{31}S from the present work including energies and intensities of γ rays. Angular correlation ratios are given along with the assignment.

E_γ(keV)	I_γ(%)	R_{DCO}	Assignment
909.4(5)	2.4(3)	—	$11/2^+ \to 11/2^+$
1050.4(2)	9.8(7)	1.19(5)	$5/2^+ \to 5/2^+$
1090.7(10)	3.6(4)	—	$11/2^+ \to 9/2^+$
1166.2(3)	35.7(1.4)	0.88(8)	$7/2^- \to 5/2^+$
1233.8(5)	12.7(6)	—	$7/2^+ \to 7/2^+$
1248.9(1)	155(2)	0.90(3)	$3/2^+ \to 1/2^+$
1299.1(2)	7.6(5)	1.21(15)	$7/2^+ \to 5/2^+$
1393.9(6)	1.3(2)	—	$(9/2^+) \to 7/2^+$
1532.2(2)	12.8(6)	0.94(7)	$11/2^- \to 9/2^+$
1628.2(4)	3.3(3)	—	$(13/2^-) \to 11/2^-$
1852.1(14)	<1	—	$13/2^+ \to 11/2^+$
1926.0(3)	10.2(5)	0.44(6)	$9/2^- \to 7/2^-$
1949.2(2)	33.5(1.6)	0.73(3)	$9/2^+ \to 7/2^+$
2035.8(2)	27.3(1.3)	0.48(3)	$5/2^+ \to 3/2^+$
2084.4(11)	<1	—	$(13/2^-) \to 9/2^-$
2102.4(2)	100	—	$7/2^+ \to 3/2^+$
2236.1(5)	18.3(6)	—	$5/2^+ \to 1/2^+$
2382.8(3)	33.1(1.4)	1.68(6)	$11/2^- \to 7/2^-$
2760.7(11)	12.4(7)	1.27(15)	$13/2^+ \to 11/2^+$
3042.4(4)	29.9(1.5)	1.58(13)	$11/2^+ \to 7/2^+$
3285.3(11)	6.5(5)	—	$5/2^+ \to 1/2^+$
3312.9(10)	7.4(6)	—	$(13/2^-) \to 11/2^-$
3952.7(6)	12.7(9)	1.69(15)	$11/2^+ \to 7/2^+$

schemes for the two nuclei (see Fig. 1). An angular correlation analysis was performed for the strongest γ rays observed in the present work. The ratio R_{DCO} was defined as the intensity of a γ ray observed at forward (32° and 37°) or backward (143° and 148°) angles to those at 90°. Under this geometry, a stretched quadrupole transition was expected to have a ratio of 1.6(1), whereas a stretched dipole transition was found to have a ratio of 0.90(5).

The data on ^{31}P allowed the positive-parity yrast line to be extended from $J^\pi = 11/2^+$ [3] to a probable $J^\pi = 15/2^+$ level at 12.17 MeV in excitation. More importantly for the issue of electromagnetic spin-orbit coupling, many new negative-parity states were found above 4.4 MeV, the minimum energy to promote a particle from the sd to f shell. Once this excitation is allowed, the negative-parity states become energetically favored and many levels are found extending to a probable spin of $J^\pi = 17/2^-$. In a lowest seniority shell-model picture, with two quasiparticles in the upper sd shell, and one in the $f_{7/2}$ level, the maximum spin that can be generated is $J^\pi = 13/2^-$, with a proton and neutron in the $d_{3/2}$ orbit coupled to $J^\pi = 3^+$. In shell-model terms, this is a special and rather pure configuration, as it is the only one that can generate so large an angular momentum and is critical in the search for electromagnetic spin-orbit coupling. A synopsis of results is given in Table I.

Almost all of the decay scheme for ^{31}S is new. With the ^{31}P template in hand, rapid progress is straightforward, as in nearly all cases there is a 1:1 matching of levels, intensities, and similar branching ratios. Of course, it is the differences between the two schemes that are interesting. The results for ^{31}S are given in Table II.

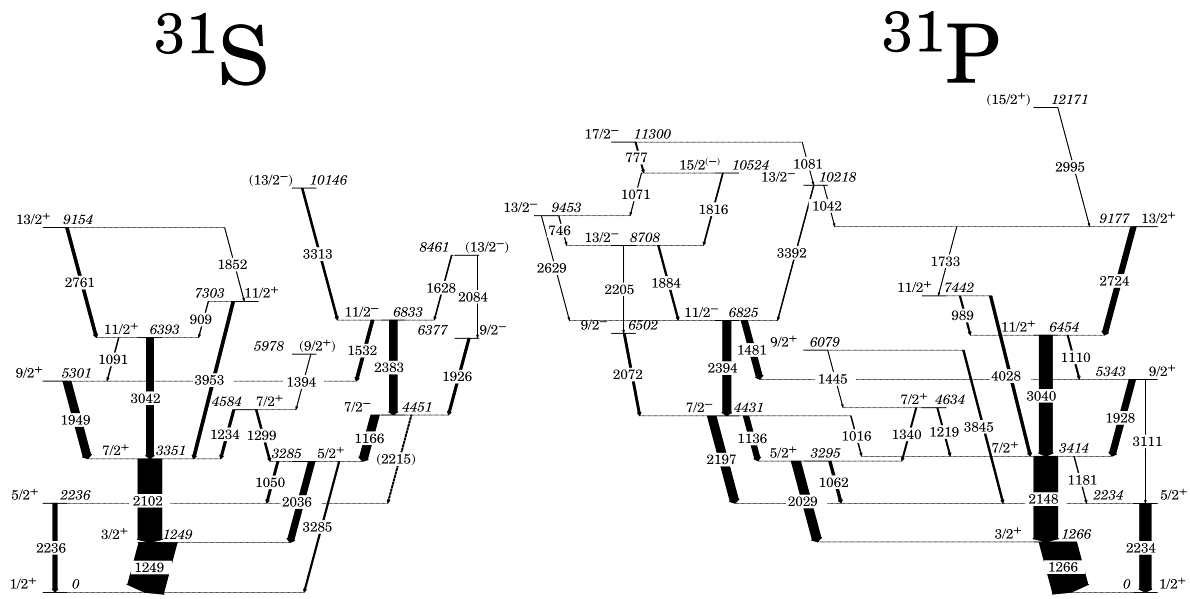

FIG. 1. Comparison of the high spin level schemes for ^{31}S (left) and ^{31}P (right). Transition and level energies are in keV. The width of the arrows represents the relative intensity of the transitions in each nucleus.

Figure 2 shows the inferred MEDs for the $A = 31$, $T = 1/2$ pair. Among the lowest positive-parity states of each spin, the MEDs are small, with a mean value of -33 keV and showing a very slight tendency to increase with spin. This trend probably reflects the slow increase in occupancy of higher angular momentum orbits that have slightly larger mean radii, and thus lower Coulomb energy. The second positive-parity states of each spin show a more dramatic change in MED, which is more difficult to account for. It is the negative-parity states, however, that show a big variation of shifts, varying from $+8$ to -247 keV. The largest shifts are of -125 keV for the $J^\pi = 9/2^-$ states and -247 keV for the $J^\pi = 13/2^-$ levels (see Fig. 3). Both are expected to have rather simple shell-model configurations.

It is interesting to attempt a naive quantification of the electromagnetic spin-orbit effect [1,12]. In natural nuclear physics units, we can write

$$V_{\text{eso}} = 0.0112\,(g_s - g_l)\left\langle \frac{1}{r}\frac{dV_c(r)}{dr}\right\rangle \langle \vec{l}\cdot\vec{s}\rangle, \qquad (1)$$

$$\langle \vec{l}\cdot\vec{s}\rangle = \tfrac{1}{2}[j(j+1) - l(l+1) - s(s+1)], \qquad (2)$$

where $V_c(r)$ and V_{eso} are in MeV, r is in fm, $g_s = 5.586$, $g_l = 1.0$ for protons, $g_s = -3.828$, $g_l = 0.0$ for neutrons, and $j, l,$

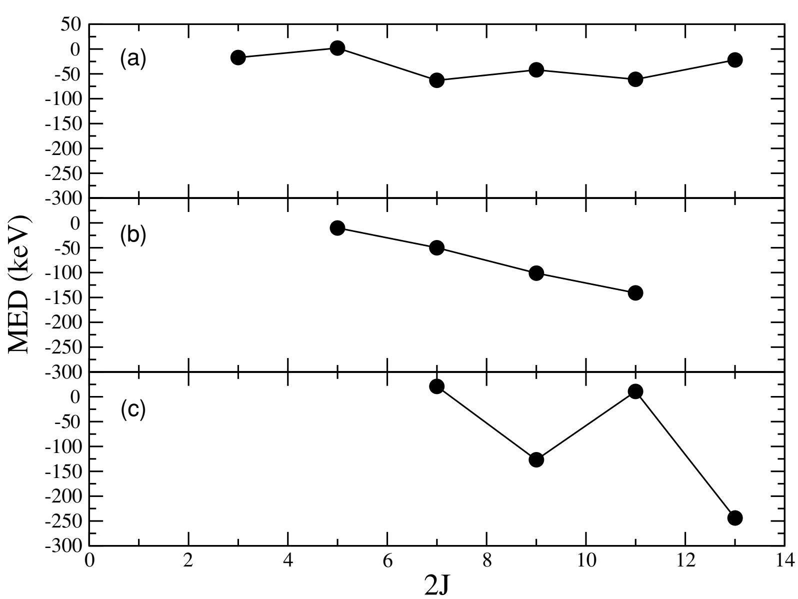

FIG. 2. Mirror energy differences between states in ^{31}S and ^{31}P as a function of spin: (a) yrast states, (b) positive-parity yrare states, (c) negative-parity sequence built on $7/2^-$ state.

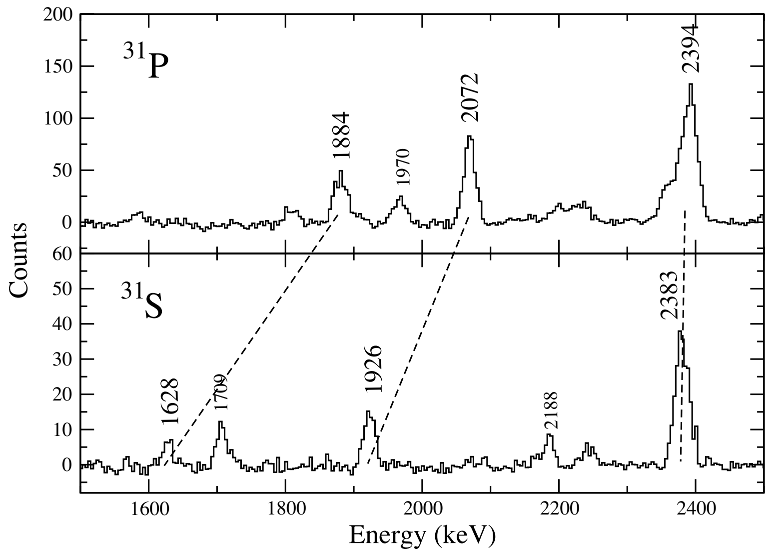

FIG. 3. γ-ray spectra illustrating the large mirror shifts associated with the $9/2^-$ and $13/2^-$ between ^{31}P and ^{31}S. The top spectrum shows transitions above the $7/2^-$ level in ^{31}P and is double-gated by the 1136- and 2029-keV transitions in the γ-γ-γ cube. The bottom spectrum contains transitions above the $7/2^-$ level in ^{31}S and is double-gated by the 1166- and 2036-keV transitions in the γ-γ-γ cube. Mirror transitions of interest are labeled in large type and connected with dashed lines as a guide to the eye.

and s are integers. $V(r)$ can be approximated by a uniformly charged sphere of radius $R = 1.2\ A^{1/3}$ fm and the radial function averaged over the interior of the nucleus:

$$V_c(r) = 1.44\frac{Z}{R}\left[3/2 - 1/2\left(\frac{r}{R}\right)^2\right], \qquad r < R, \qquad (3)$$

$$1.44\frac{Z}{r}, \qquad r > R. \qquad (4)$$

Damgard has generalized this approach for deformed nuclei [13]. The derivative of the potential is always negative, averaging −0.44 MeV. The sign of the electromagnetic spin-orbit splitting is unlike either the atomic or nuclear case, as it does not solely depend on the stretched or folded nature of the configurations, but on the product of the angular momentum coupling multiplied by the gyromagnetic ratio, and so has opposite sign for protons and neutrons. By evaluating $V_{\rm eso}$, the single-particle shifts, for the $A = 31$ mirror pair, the ^{31}S $f_{7/2}$ level (a proton coupled to the ^{30}P core) is lowered by −68 keV and the ^{31}P $f_{7/2}$ level (a neutron coupled to the ^{30}P core) is raised by +57 keV. For the relevant positive-parity states, the $d_{3/2}$ level shifts are numerically equal to the $f_{7/2}$ levels, but with the opposite sign, whereas the $s_{1/2}$ level has no orbital angular momentum, so shows no effect.

To evaluate the MED for the $J^\pi = 9/2^-$ pair and the $J^\pi = 13/2^-$ pair, we need to now carefully evaluate the shifts of each of the three contributing particle levels. The relevant core is ^{30}P, which has a $J = 1, T = 0$ ground state and a $J = 3, T = 0$ state at 1973 keV. For the case of the $J^\pi = 9/2^-$ states, with a configuration of $f_{7/2}$ coupled to a proton and neutron in $s_{1/2}$ levels making a $J^\pi = 1^+$ core, the overall shift comes the different contributions from the valence proton and neutron $f_{7/2}$ levels, as the core states have no shift. Thus, the MED between $J^\pi = 9/2^-$ states in ^{31}S and ^{31}P is estimated to be $-68 - (+57) = -125$ keV. In the case of the $J^\pi = 13/2^-$ configuration, the core $d_{3/2}$ proton state moves up by 68 keV and the neutron state moves down by −57 keV, so the net effect is only +11 keV. However, it is exactly the same for ^{31}S and ^{31}P, so even this small effect has no role in the MED, which again only depends on the relative shifts of $f_{7/2}$ levels, so has MED ($J^\pi = 13/2^-$) = −125 keV, exactly as for the $J^\pi = 9/2^-$ state. Our experimental observations match this simple estimate surprisingly well. The exact agreement between the calculation and measurement for the $J^\pi = 9/2^-$ states is undoubtedly fortuitous, though in this case it is worth noting that the C_{ll} term discussed by Ekman *et al.* [2] should play no role for s states. For the $J^\pi = 13/2^-$ case, the measured MED is twice our estimate, indicating the splitting is only partly arising from the electromagnetic spin-orbit effect. We note that in our estimate (and those of Ekman) we used bare nucleon g factors, which are empirically known always to be too large; the quenched values (and smaller shifts) would be more realistic. We thus infer that the observed shifts are only partially due to the electromagnetic spin-orbit effect.

Beyond $V_{\rm eso}$, another critical factor in determining the MEDs is the binding energy of the states. Poorly bound (or unbound) states have more extended wave functions and consequently lower Coulomb displacements, as on average the protons are slightly farther apart. This type of effect was not discussed in the work of Ekman [2], although it appears to us at least as significant as $V_{\rm eso}$ in discussing MEDs. The size of this effect can be estimated using a Woods-Saxon potential of fixed radius and diffuseness, with a well depth adjusted to reproduce the experimentally known neutron separation energy for each state in the $T_z = +1/2$ nucleus. Repeating the calculation for the $T_z = -1/2$ partner without changing parameters yields a prediction of the equivalent proton state, and hence the binding energy contribution to the MED can be deduced. The effect seems to track the electromagnetic spin-orbit effect closely, and it difficult to isolate either one as they are both always in play. A quantitative analysis of this binding energy effect is in progress, but in the $A = 31$ case it appears to of similar magnitude to $V_{\rm eso}$.

To evaluate the MED of more complicated states requires a good knowledge of the structure of the wave functions of relevant states, particularly the isospin couplings involved, as contributions from protons and neutrons will tend to cancel. Without a detailed shell-model calculation only the simplest states can be estimated. In this respect, the $J^\pi = 13/2^-$ states

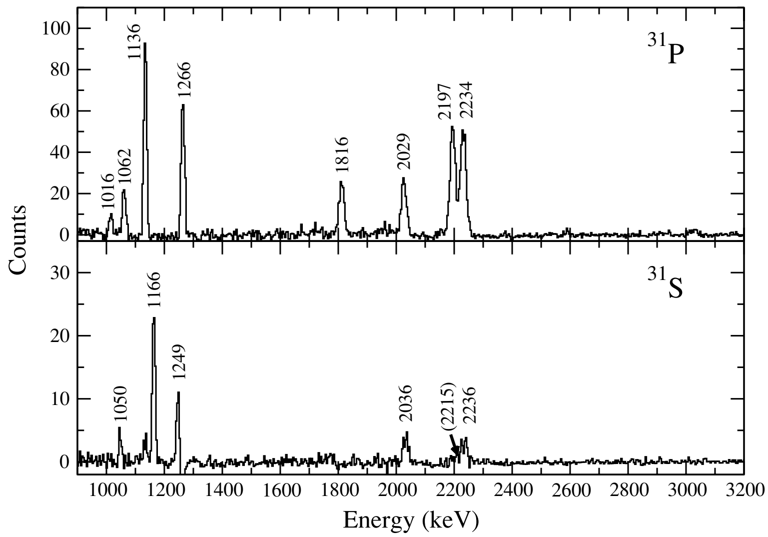

FIG. 4. The top spectrum shows transitions above the $7/2^-$ level in ^{31}P and is double-gated by the 1136- and 2029-keV transitions in the γ-γ-γ cube. The bottom spectrum contains transitions above the $7/2^-$ level in ^{31}S and is double-gated by the 1166- and 2036-keV transitions in the γ-γ-γ cube. Mirror transitions of interest are labeled with their energy in keV.

stand out as they are dominated by coupling the low-lying $T = 0$, $J^\pi = 3^+$ core state to an $f_{7/2}$ particle. The situation for the $J^\pi = 9/2^-$ level, arising from coupling the core $T = 0$, $J^\pi = 1^+$ levels to $f_{7/2}$ particles, is similar (although there are two low-lying $J^\pi = 1^+$ states in ^{30}P). In contrast, the $J^\pi = 7/2^-$ and $J^\pi = 11/2^-$ states have more complex wave functions. The $J^\pi = 7/2^-$ and $11/2^-$ states experimentally do not show effects of the same type. However, this situation is similar to that in $A = 35$ [2], where the small shifts were accounted for by the suggestion that only fully aligned configurations have simple wave functions (and big shifts), whereas the states that are not fully stretched in angular momentum have more mixing of proton and neutron states, which tend to compete and cancel the effect. We note that our estimates of V_{eso} are for pure three-particle configurations, which could be disturbed by mixing with other states. The presence of three relatively close-lying $J^\pi = 13/2^-$ states in ^{31}P might imply that these estimates are excessively naive, and indeed, a better understanding can only come from large-basis shell-model calculations.

A further feature of the mirror symmetry between ^{31}S and ^{31}P is worthy of comment, namely, the very different pattern in the decay of the lowest $7/2^-$ level in the respective nuclei. In ^{31}P, the $7/2^- \rightarrow 5/2^-_2$ transition is of comparable strength to the $7/2^- \rightarrow 5/2^-_1$ transition, whereas in ^{31}S the analogous transition is essentially absent (see Fig. 4). Very similar behavior for the decay of the $7/2^-$ level was seen in the $A = 35$ mirror pair [2]. $E1$ transitions are purely isovector in nature. Under the assumption of isospin purity, the matrix elements for the corresponding $E1$ transitions in mirror nuclei should be equal and opposite in sign, since they are proportional to the isospin projection, T_z. There are two ways in which differences in transition strengths could be obtained. The first scenario, as proposed by Ekman *et al.* [2], would be isospin mixing, that is, the presence of small $T = 3/2$ components in the wave functions. A second way in which interference in the transition strengths could be introduced would be to have $M2$ admixtures in the transitions since these may have a strong isoscalar component in their matrix element. To understand the problem in detail, it will be necessary to compare transition strengths by measuring lifetimes.

In conclusion, we have extended the decay scheme for ^{31}P and developed an extensive new scheme for ^{31}S. In the lowest positive-parity states the mirror energy differences are small, but the lowest negative-parity states show some significant shifts. The $J^\pi = 9/2^-$ and $13/2^-$ levels have shifts of -125 and -247 keV, respectively. These shifts can be partially attributed to the electromagnetic spin-orbit effect, but differences in the binding energies of states may also be important. To understand the relative contributions of these effects requires a systematic investigation of all the data on MEDs, which is ongoing.

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