Axial-vector mesons in a relativistic point-form approach

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The Poincaré invariant coupled-channel formalism for two-particle systems interacting via one-particle exchange, which has been developed and applied to vector mesons, is now applied to axial vector mesons. We thereby extend the previous study of a dynamical treatment of the Goldstone-boson exchange by comparison with the commonly used instantaneous approximation to the case of orbital angular momentum $l = 1$. Effects in the mass shifts show more variations than for the vector-meson case. Results for the decay widths are sizable, but comparison with sparse experimental data is inconclusive.

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During the past years, constituent quark models have been used successfully to calculate spectra and properties of hadrons. In the course of the development of these models, relativity has emerged as a key ingredient in the light-quark sector. This has found early consideration; e.g., the well-known works of Godfrey and Isgur [1] and Capstick and Isgur [2] introduced relativistic corrections in a nonrelativistic potential model; Feynman *et al.* [3] based their model on a relativistic harmonic oscillator; and Carlson *et al.* [4] used relativistic kinetic energies plus flux-tube motivated potential terms. In the particular case of the Goldstone-boson-exchange (GBE) constituent-quark model (CQM) [5], the early nonrelativistic formulation Refs. [6,7] showed severe inconsistencies and was soon superseded by a "semirelativistic" version [8]. In the semirelativistic treatment, the Hamiltonian of the model contains the relativistic kinetic energy plus potential terms, and it can in fact be reinterpreted as the mass operator of a Poincaré invariant model as long as the potential terms are rotationally invariant and do not depend on the total momentum of the bound state [9]. In this form, the GBE CQM has been applied to spectroscopy of the light and strange baryon sector with wide success; however, an analogous calculation for mesons seemed to indicate failure of the model in this sector [10]. Only recently [11] has a Poincaré invariant coupled-channel (CC) formalism for confined two-particle systems interacting via one-particle exchange been shown to elucidate the relevance and applicability of the GBE CQM for vector mesons.

Aside from spectroscopy, the semirelativistic GBE CQM has also been used to determine hadronic decay widths of baryons using perturbative calculations employing elementary emission [12–14] (without introducing additional parameters) and also pair creation (one additional parameter) models [15]. At that stage, a satisfactory description of experimental data was impossible. A relativistic treatment of hadronic decays in an elementary-emission-type model seemed natural [16], and recently a Lorentz invariant decay model along the point-form spectator approximation has been suggested in Refs. [17,18]. It was observed that, in general, the theoretical results considerably underestimated the experimental data. In light of these results, the next natural step is to couple the decay channels explicitly to the qqq (for baryons) and $\bar{q}q$ (for mesons) channels. In Refs. [11,19,20], a study of vector mesons has been done to investigate the effects of such a dynamical treatment of the exchange particle in the GBE in a Poincaré invariant framework both in terms of mass shifts *and* decay widths. The semirelativistic form of the model produces a much too large mass splitting of ϱ and ω. This flaw is removed in the CC treatment, leading to small mass shifts from the GBE in vector mesons, which confirms the expectation that this type of interaction should not contribute much to the binding of such states. Regarding hadronic decays, the situation is difficult to judge with regard to comparison with experimental data, since for only one branching ratio, which can be calculated in the model [11], are data actually available [21].

In the present work, we follow Ref. [11], which contains the details of all aspects of the formalism and model used here, applying those to axial-vector mesons with quantum numbers $J^{PC} = 1^{++}$ and $J^{PC} = 1^{+-}$ as well as the strange sector via mixing of the states with the respective quantum numbers. The results identify dynamical effects for $l = 1$ and enable the calculation of more hadronic decay widths within the restrictions of the states contained in the model. While the formalism could be applied to other mesons with $l = 1$ as well, axial vectors are most suitable for this study: their ground- and first-excited-state masses are best matched by the model's confinement piece, and the structure of the hyperfine interaction remains rather simple.

We will briefly review the main ingredients of the model, present and discuss the results, and propose conclusions.

The central point of the CC treatment described in Ref. [11] is the CC mass operator, which is defined on the little Hilbert space of the direct sum of $\mathcal{H}_{\bar{q}q}$ (quark-antiquark) and $\mathcal{H}_{\bar{q}q\Pi}$ (quark-antiquark-pseudoscalar meson) as

$$M = M_c + M_I = \begin{pmatrix} \mathcal{D}^c_{\bar{q}q} & 0 \\ 0 & \mathcal{D}^c_{\bar{q}q\Pi} \end{pmatrix} + \begin{pmatrix} 0 & K^\dagger \\ K & 0 \end{pmatrix}. \quad (1)$$

Here M_c represents the diagonal part of M, which includes the confinement interaction such that in both the $\bar{q}q$ and $\bar{q}q\Pi$ channels the $\bar{q}q$ pair is confined. M_I contains a vertex piece K and its Hermitian adjoint as defined in Eqs. (31) and (32) of Ref. [11]. This mass operator constitutes a quantum mechanical setup of a model with a finite number of degrees of freedom. Further channels (e.g., featuring $\mathcal{D}_{\bar{q}q\Pi\Pi}$) could in principle be included but are not for the sake of simplicity. This is a valid approach, since contributions from the hyperfine interaction resulting from the coupling K are small with respect

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to those from the confinement piece. When the eigenvalue equation for M is reduced to the $\bar{q}q$ channel, one obtains the effective interaction term on the right-hand side of

$$\left(\mathcal{D}^c_{\bar{q}q} - m\right)|\Psi_{\bar{q}q}\rangle = K^\dagger\left(\mathcal{D}^c_{\bar{q}q\Pi} - m\right)^{-1} K|\Psi_{\bar{q}q}\rangle. \quad (2)$$

In this equation, m is the eigenvalue and appears also in the effective interaction term. We note here that the interaction contains terms that correspond to the exchange of a pseudoscalar meson Π inside the $\bar{q}q$ pair and others, in which the pseudoscalar meson Π couples to the same constituent twice. We will refer to the latter as "loop terms." The particular form and choices for the operators in Eq. (2) and their matrix elements are described in detail in the appendix of Ref. [11]. Extending the model with the parameters established in the vector-meson sector to the axial vectors gives rise to the question of the need for readjustment of some of the parameters. Already for the vector-meson calculation, the parameters used in the exchange part of the interaction were taken without change from the semirelativistic GBE CQM in [8], and we have kept them the same here as well. This includes the definition and parameters of the quark-meson vertex form factors which determine the range of the Π exchange. For the confinement piece, a harmonic oscillator (HO) model was used in M^2 for two reasons: first, the HO mass operator's eigenvalues and eigensolutions are analytically known, which facilitates the calculations; second, it mimicks the spectrum of a linear confinement potential for M. The actual form used in Ref. [11] is

$$\mathcal{D}^c_{\bar{q}q} \to M_{nl} = \sqrt{8\,a^2\,(2n + l + 3/2) + V_0 + 4\bar{m}^2}, \quad (3)$$

where $\bar{m}^2$ is determined by the constituent quark masses, and n and l are the radial and orbital angular momentum quantum numbers, respectively. a is the confinement strength and V_0 a constant used to fix the mass of the ϱ ground state to its physical value. a was adjusted such that the splitting of the ϱ ground and first excited states was reproduced. It should be noted here that this is not the best possible choice for $\mathcal{D}^c_{\bar{q}q}$ in terms of an accurate fit to experimental data for higher excited states. However, the main emphasis of our studies still lies on the effects of a dynamical treatment of the one-particle exchange as compared to an instantaneous approximation (IA). Therefore, $\mathcal{D}^c_{\bar{q}q}$ is sufficient for the present purpose. In principle, one could choose any confinement operator that is diagonal in the basis of Eq. (2), satisfies the (in our case point-form) Bakamjian-Thomas requirements [9], and has known solutions and can therefore be used to discretize the problem.

In an attempt to extend the basic HO piece of the model beyond the vector-meson sector, one can use the concept of Regge trajectories [22]. This approach has already been used in Ref. [3] and recently in the context of relativistic Hamiltonian dynamics [23]. If one uses such a trajectory, which contains the ϱ ground state, to determine the parameters a and V_0, one finds that the parameter set established for the vector mesons does not need to be changed. The parameters used in all calculations presented here are thus $a = 312$ MeV and $V_0 = -1.04115$ GeV^2. This completes the summary of the model definitions.

In the calculations we make two approximations. First, we do not treat the loop contributions in the effective interaction explicitly. This is motivated by the assumption that their effects can be accounted for via a change in the constituent-quark mass. An explicit treatment of these terms is an extension of the model which will be incorporated in future studies. Second, some of the matrix elements occurring in Eq. (2) contain Wigner rotations, which come from the overlap of the various sets of basis states used in the computation of the effective interaction, see Eq. (A16) in Ref. [11]. We neglect these rotations because, while the numerical effort to include them is considerable, their effects have been found to be small compared to boost effects in calculations of electromagnetic form factors of the nucleon [24].

We have obtained results for the axial-vector states with quantum numbers $J^{PC} = 1^{++}$ and $J^{PC} = 1^{+-}$ by solving the eigenvalue equation for M numerically. For quark-model mesons, one has the relations $P = (-1)^{l+1}$ and $C = (-1)^{l+s}$, where l is the orbital angular momentum and s the total spin of the constituents. For $J^{PC} = 1^{++}$, this entails $s = 1, l = 1$; the physical states corresponding to this set of quantum numbers are the f_1 (with isospin $I = 0$), a_1 ($I = 1$), and K_{1A} ($I = 1/2$). For $J^{PC} = 1^{+-}$, one gets $s = 0, l = 1$ with the associated particles h_1 ($I = 0$), b_1 ($I = 1$), and K_{1B} ($I = 1/2$). Within the isospin 0 channels, we assume ideal mixing between the octet and singlet SU(3)-flavor configurations, meaning that the h_1 as well as the f_1 spectra each contain both pure $\bar{n}n$ and $\bar{s}s$ states (in the usual notation, n here denotes light quarks). In the strange sector, the physical states of the K_1 spectrum are mixtures of the K_{1A} and K_{1B}, since they are not charge-parity eigenstates. In our treatment of this mixing, we follow Blundell *et al.* [25]. The results are presented in Fig. 1 in six "columns" for each set of quantum numbers; the experimental values [21] are depicted in the first column (denoted by the particle names) by boxes indicating the experimental uncertainties; the second to sixth columns contain results for pure oscillator (O), CC calculation with vertex form factor (Cf), CC calculation with the form factor set = 1 (C1), an instantaneous approximation with vertex form factor (If), and the IA calculation with the form factor set = 1 (I1). These are the same categories as presented in Ref. [11]; also see this reference for details.

In the vector-meson sector [11], the $\bar{q}q$ states have mainly orbital angular momentum $l = 0$ with small admixtures of $l = 2$. There the main observation was that generally the CC treatment produces smaller mass shifts than the IA, including the prominent case of the ω ground state. For axial-vector mesons, one always has $l = 1$ and the observations about mass shifts are different, except that the main differences between CC treatment and IA can be found in the isoscalar channels; this is not surprising, because one-pion exchange is strongest in these channels and the light mass of the exchange particle plays a central role in the dynamical setup.

In general, the dependence of the mass shifts on the use of a form factor at the quark-meson vertex is smaller in the CC treatment than in the IA. This is true in particular for the case of the h_1 meson, where for the IA the shifts with and without the form factor have opposite sign. For the b_1, a different sign change appears: while the CC shifts are positive, the IA ones are negative (although in both cases the shifts are small).

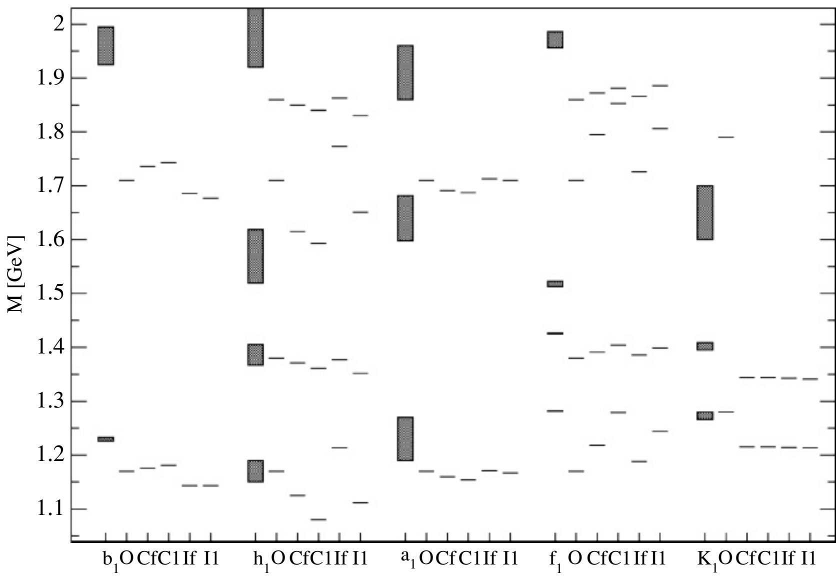

FIG. 1. Results for axial-vector meson spectra: Experimental data with uncertainties (particle name), oscillator (O), CC with vertex form factor (Cf), CC with FF = 1 (C1), instantaneous approximation with vertex FF (If), and IA with FF = 1 (I1).

In the f_1 spectrum, one observes that mass shifts using the form factor are larger for the CC than the IA results, and the same seems to apply to the results without the form factor except for the first $\bar{s}s$ state. This is opposite to the general observation for vector mesons. The reason for these variety of effects lies in the complexity of the dynamical setup for the CC formalism in connection with the wave functions for $l = 1$ states. On different ranges of the relative momentum between the q and the $\bar{q}$, these can have support of different sign, which gets modified in addition via the kinematical relations used in the calculation as apparent from Eq. (A16) in Ref. [11].

The results for the decay widths are larger than in the vector-meson case. However, similarly to the latter there is only one branching ratio in Ref. [21] to which we can compare our results, although most of the decays are regarded as "seen." Our results are generally of the order of $\approx$20 to 90 MeV. The known branching ratio is that of the $b_1(1235)$ with $\Gamma = 142 \pm 9$ MeV, which decays dominantly into channels contained in our model; our result is 33 MeV with and 42 MeV without the form factor, underestimating experimental data by about a factor of 4.

We have applied the Poincaré invariant coupled-channel (CC) formalism of Ref. [11] to the sector of axial-vector mesons. The axial-vector quantum numbers $J^{PC} = 1^{++}/1^{+-}$ both imply orbital angular momentum $l = 1$ for the $\bar{q}q$ pair. The effects of the dynamical treatment of the one-boson exchange (OBE) as compared to the instantaneous approximation (IA) reveals different characteristics as compared to the case of vector mesons (mainly $l = 0$). There are four main observations: (i) As in the case of vector mesons, the effects are strongest in the isoscalar channels, since there one-pion exchange dominates and it is very sensitive to the dynamical setup because of the light pion mass. (ii) In the h_1 spectrum, the use of a vertex form factor (as compared to a form factor = 1) changes the sign of the mass shift from the meson exchange in the IA, while the CC results have the same sign and magnitude regardless of the details of the form factor. (iii) In the f_1 spectrum, the CC mass shifts are larger than the IA ones—opposite to the general trend (including the vector mesons). (iv) In the a_1 spectrum, the CC shifts have the opposite sign as compared to the IA ones. The main conclusion from this collection of observations must be that results from a dynamical CC treatment of one-particle exchange can differ significantly, both in magnitude and sign, in channels where this exchange is important. The results for the decay widths are sizable, but comparison with experimental data is inconclusive: the only data point with definite value [for the $b_1(1235)$] is underestimated by a factor of 4. This supports the conclusion drawn in Ref. [11] from an analogous situation in the $\omega(1420)$ case: the calculations could be improved by explicitly including the loop contributions from the interaction in Eq. (2) and/or taking into account final-state interactions.

These conclusions strongly suggest analogous investigations of qqq systems in this context, since such a dynamical, Poincaré invariant treatment of OBE in the baryon sector is still missing. In Ref. [11] and the present work, the path is laid out and also a possible intermediate step has been identified [26]. We note here that a treatment along the lines of the stochastic variational method [27] used to date in the GBE CQM [8] seems impossible because of the high dimensions and numerical nature of the integrations involved in the solution of the CC problem. A more promising approach is of the Faddeev type along the lines of Ref. [28]. It is important to proceed in this direction because a dynamical treatment of the OBE in baryons will yield hadronic baryon decay widths in a nonperturbative way, which could remedy their unsatisfactory description at the present stage of the model. Furthermore, given the limited comparison to experimental data of hadronic meson decays predicted by the present work and in Ref. [11] as well as the importance of GBE in the baryon sector, an analogous investigation of baryons will clarify the full impact of a dynamical treatment of OBE-type interactions.

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