Multiplicity fluctuations in the string clustering approach

L. Cunqueiro, E. G. Ferreiro, F. del Moral, and C. Pajares
Departamento de Física de Partículas and Instituto Galego de Física de Altas Energías, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain
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We present our results on multiplicity fluctuations in the framework of the string clustering approach. We compare our results—with and without clustering formation—with CERN Super Proton Synchrotron NA49 data. We find a nonmonotonic behavior of these fluctuations as a function of the collision centrality, which has the same origin as the observed fluctuations of transverse momentum: the correlations between the produced particles because of the cluster formation.

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Nonstatistical event-by-event fluctuations have been proposed as a possible signature for quantum chromodynamics (QCD) phase transition. In a thermodynamical picture of the strongly interacting system formed in heavy-ion collisions, the fluctuations of the mean transverse momentum or mean multiplicity are related to the fundamental properties of the system, such as the specific heat, so it may reveal information about the QCD phase boundary.

Event-by-event fluctuations of transverse momentum have been measured both at CERN Super Proton Synchroton (SPS) [1–7] and BNL Relativistic Heavy Ion Collider (RHIC) [8–12]. The data show a nontrivial behavior as a function of the centrality of the collision. Concretely, the nonstatistical normalized fluctuations grow as the centrality increases, with a maximum at midcentralities, followed by a decrease at larger centralities. Different mechanisms [13–30] have been proposed to explain those data, including complete or partial equilibration [15,16,23,25], critical phenomena [17,30], production of jets [14,29] and also string clustering or string percolation [27,28].

In particular, we have proposed [27] an explanation for those fluctuations based on the creation of string clusters. In our approach, we find an increase of the mean p_T fluctuations at midcentralities, followed by a decrease at large centrality. Moreover, we obtain a similar behavior at SPS and RHIC energies. In the framework of string clustering such a behavior is naturally explained. As the centrality increases, the strings overlap in the transverse plane forming clusters. These clusters decay into particles with mean transverse momentum and multiplicities that depend on the number of strings that conform each cluster. The event-by-event fluctuations on mean p_T and mean multiplicity correspond then to fluctuations of the transverse momentum and multiplicity of those clusters and behave essentially as the number of clusters conformed by a different number of strings. If the number of different clusters—different in this context means that the clusters are made of different numbers of strings—grows, that will lead to an increase of fluctuations. And in fact this number grows with centrality up to a maximum. For higher centralities, the number of different clusters decreases.

Conversely, in a jet production scenario [9], the mean p_T fluctuations are attributed to jet production in peripheral events, combined with jet suppression at larger centralities. A possible way to discriminate between the two approaches could be the study of fluctuations at SPS energies, where jet production cannot play a fundamental role.

Recently, the NA49 Collaboration have presented their data on multiplicity fluctuations as a function of centrality [31,32] for Pb+Pb collisions at SPS ($P_{\rm lab} = 158A$ GeV/c) energies. To develop the experimental analysis, the variance of the multiplicity distribution $\mathrm{Var}(N) = \langle N^2\rangle - \langle N\rangle^2$ scaled to the mean value of the multiplicity $\langle N\rangle$ has been used. A nonmonotonic centrality—system size—dependence of the scaled variance was found. In fact, its behavior is similar to the one obtained for the $\Phi(p_T)$ measure [33] used by the NA49 Collaboration to quantify the p_T fluctuations, suggesting they are related to each other [34,35]. The Φ measure is independent of the distribution of number of particle sources if the sources are identical and independent from each other. This implies that Φ is independent of the impact parameter if the nucleus-nucleus collision is a simple superposition of nucleon-nucleon interactions.

Our aim in this note is to calculate the event-by-event multiplicity fluctuations applying the same mechanism—clustering of color strings—that we have used previously [27] for the study of the p_T fluctuations. Let us remember the main features of our model. In each nuclear collision, color strings are streached between partons from the projectile and the target, which decay into new strings by sea $q - \bar{q}$ production and finally hadronize to produce the observed particles. For the decay of the strings we apply the Schwinger mechanism of fragmentation [36], where the decay is controlled by the string tension that depends on the color charge and color field of the string. The strings have longitudinal and transverse dimensions, and the density of created strings in the first step of the collision depends on the energy and the centrality of the collision. Roughly speaking, one can consider the number of strings N_s in the central rapidity region as proportional to the number of collisions, $N_A^{4/3}$, whereas in the forward region it becomes proportional to the number of participants, N_A. We define the density of strings in the transverse space as $\eta = N_s S_1/S_A$, where N_s is the total number of strings created in the collision, each one of an area $S_1 = \pi r_0^2$ ($r_0 \simeq 0.2 \div 0.3$ fm), and S_A corresponds to the nuclear overlap area, $S_A = \pi R_A^2$ for central collisions. With the increase of energy and/or atomic number of the colliding nuclei, this density grows, so the strings begin to overlap forming clusters [37].

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We assume that a cluster of n strings that occupies an area S_n behaves as a single color source with a higher color field, generated by a higher color charge Q_n. This charge corresponds to the vectorial sum of the color charges of each individual string $\mathbf{Q}_1$. The resulting color field covers the area S_n of the cluster. As $Q_n^2 = (\sum_1^n \mathbf{Q}_1)^2$, and the individual string colours may be arbitrarily oriented, the average $\mathbf{Q}_{1i}\mathbf{Q}_{1j}$ is zero, so $Q_n^2 = nQ_1^2$ if the strings fully overlap. Because the strings may overlap only partially we introduce a dependence on the area of the cluster. We obtain $Q_n = \sqrt{nS_n/S_1}\,Q_1$ [38]. Now we apply the Schwinger mechanism for the fragmentation of the cluster, and one obtains a relation between the mean multiplicity $\langle\mu\rangle_n$ and the average transverse momentum $\langle p_T\rangle_n$ of the particles produced by a cluster of n strings that covers an area S_n as follows:

$$\langle\mu\rangle_n = \sqrt{\frac{nS_n}{S_1}}\langle\mu\rangle_1 \quad \text{and} \quad \langle p_T\rangle_n = \left(\frac{nS_1}{S_n}\right)^{1/4}\langle p_T\rangle_1, \tag{1}$$

where $\langle\mu\rangle_1$ and $\langle p_T\rangle_1$ correspond to the mean multiplicity and the mean transverse momentum of the particles produced by one individual string.

To obtain the mean p_T and the mean multiplicity of the collision at a given centrality, one needs to sum over all formed clusters and to average over all events:

$$\langle\mu\rangle = \frac{\sum_{i=1}^{N_{\rm events}}\sum_j\langle\mu\rangle_{n_j}}{N_{\rm events}},\ \langle p_T\rangle = \frac{\sum_{i=1}^{N_{\rm events}}\sum_j\langle\mu\rangle_{n_j}\langle p_T\rangle_{n_j}}{\sum_{i=1}^{N_{\rm events}}\sum_j\langle\mu\rangle_{n_j}}. \tag{2}$$

The sum over j goes over all individual clusters j, each one formed by n_j strings and occupying an area S_{n_j}. The quantities n_j and S_{n_j} are obtained for each event, using a Monte Carlo code [39,40], based on the quark gluon string model. With our code, once we fix the energy of the collision and the participant nuclei, we obtain for each event a number of participant nucleons and a configuration for the created strings. Each string is generated at an identified impact parameter in the transverse space. Knowing the transverse area of each string, we identify all the clusters formed in each event, the number of strings n_j that conforms each cluster j, and the area occupied by each cluster S_{n_j}. Note that for two different clusters, j and k, formed by the same number of strings $n_j = n_k$, the areas S_{n_j} and S_{n_k} can vary. Because of this we do the sum over all individual clusters. So we use a Monte Carlo for the cluster formation to compute the number of strings that come into each cluster and the area of the cluster. Conversely, we do not use a Monte Carlo code for the decay of the cluster, because we apply analytical expressions [Eqs. (1)] for the transverse momentum $\langle p_T\rangle_{n_j}$ and the multiplicity $\langle\mu\rangle_{n_j}$ of each individual cluster.

To obtain the scaled variance we calculate $\langle\mu^2\rangle$:

$$\langle\mu^2\rangle = \frac{1}{N_{\rm events}}\left[\sum_{i=1}^{N_{\rm events}}\left(\sum_j\sqrt{\frac{n_jS_{n_j}}{S_1}}\right)^2\langle\mu\rangle_1^2 + \sum_{i=1}^{N_{\rm events}}\sum_j\sqrt{\frac{n_jS_{n_j}}{S_1}}\langle\mu\rangle_1\right], \tag{3}$$

where we have supposed that the multiplicity of each cluster follows a Poissonian of mean value $\langle\mu\rangle_{n_j}$, and we have applied the property for a Poissonian:

$$\langle\mu^2\rangle_{n_j} = \langle\mu\rangle_{n_j}^2 + \langle\mu\rangle_{n_j}.$$

Finally, our formula for the scaled variance obeys the following:

$$\frac{Var(\mu)}{\langle\mu\rangle} = 1 + \langle\mu\rangle_1\frac{\left\langle\left(\sum_j\sqrt{\frac{n_jS_{n_j}}{S_1}}\right)^2\right\rangle - \left\langle\sum_j\sqrt{\frac{n_jS_{n_j}}{S_1}}\right\rangle^2}{\left\langle\sum_j\sqrt{\frac{n_jS_{n_j}}{S_1}}\right\rangle}, \tag{4}$$

where the mean value in the r.h.s. corresponds to an average over all events.

The behavior of this quantity is as follows: in the limit of low density (isolated strings that do not interact),

$$\frac{Var(\mu)}{\langle\mu\rangle} = 1 + \langle\mu\rangle_1\frac{\langle N_s^2\rangle - \langle N_s\rangle^2}{\langle N_s\rangle}, \tag{5}$$

where N_s corresponds to the number of strings. Considering that, for a fixed number of participants, the number of strings behaves as a Poissonian distribution we obtain the following:

$$\frac{\langle N_s^2\rangle - \langle N_s\rangle^2}{\langle N_s\rangle} \simeq 1, \tag{6}$$

so

$$\frac{Var(\mu)}{\langle\mu\rangle} = 1 + \langle\mu\rangle_1. \tag{7}$$

In the large-density regime—all the strings fuse into a single cluster that occupies the whole interaction area—we have:

$$\frac{Var(\mu)}{\langle\mu\rangle} = 1 + \langle\mu\rangle_1\frac{\left\langle\left(\sqrt{\frac{N_sS_A}{S_1}}\right)^2\right\rangle - \left\langle\sqrt{\frac{N_sS_A}{S_1}}\right\rangle^2}{\left\langle\sqrt{\frac{N_sS_A}{S_1}}\right\rangle}, \tag{8}$$

where S_A is the nuclear overlap area. The second element of the r.h.s. of this equation tends to zero, and the scaled variance becomes equal to 1.

Our results for the scaled variance for negative particles $Var(n^-)/\langle n^-\rangle$ compared to experimental data on Pb+Pb collisions at $P_{\rm lab} = 158A$ GeV/c [31,32] are presented in Fig. 1. Note that to obtain these results we need to fix the value of the parameter $\langle\mu\rangle_1$. It is defined as $\langle\mu\rangle_1 = \langle\mu\rangle_0\Delta y$, where $\langle\mu\rangle_0$ is the number of particles produced by one individual string and Δy corresponds to the rapidity interval considered. We do not introduce any dependence of $\langle\mu\rangle_0$ with the energy or the centrality of the collision. The value of $\langle\mu\rangle_0$ has been previously fixed from a comparison of the model to SPS and RHIC data [38,41] on multiplicities. In the first reference of Ref. [38], the total multiplicity per unit rapidity produced by one string has been taken as $\langle\mu\rangle_{0\,\rm tot} \simeq 1$. If we assume that one-third of the created particles are negative, that would lead to a negative particle multiplicity per unit rapidity for each individual string of $\langle\mu\rangle_{0\,\rm neg} = 0.33$. The rapidity interval considered, to compare with NA49 experimental data, is $4.0 < y < 5.5$. The data are obtained in a restricted p_T

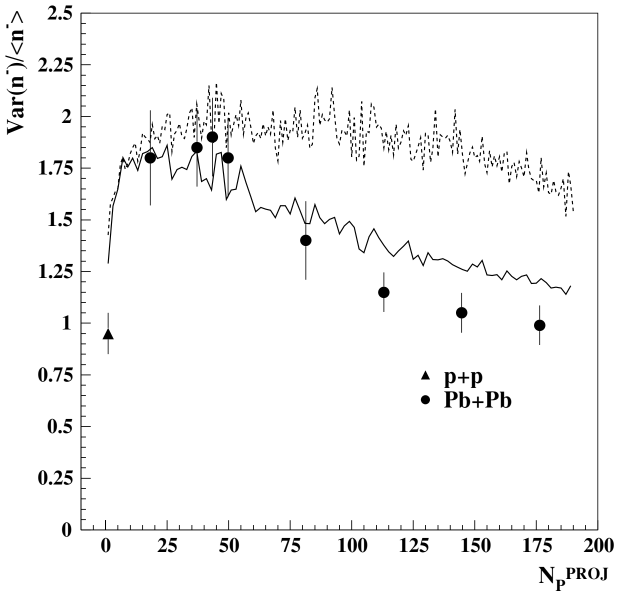

FIG. 1. Our results for the scaled variance of negatively charged particles in Pb+Pb collisions at $P_{\rm lab} = 158A$ GeV/c compared to NA49 experimental data. The dashed line corresponds to our result when clustering formation is not included, the continuous line takes into account clustering.

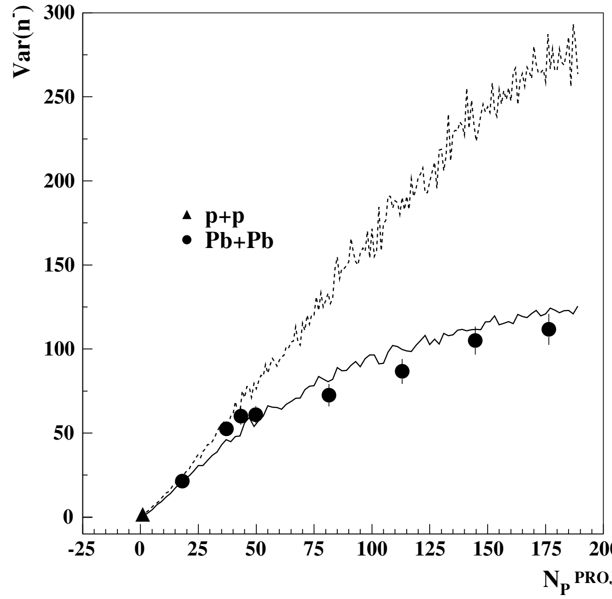

FIG. 2. Our results for the variance of negatively charged particles in Pb+Pb collisions at $P_{\rm lab} = 158A$ GeV/c compared to NA49 experimental data. The dashed line corresponds to our result when clustering formation is not included, the continuous line takes into account clustering.

range, $0.005 < p_T < 1.5$ GeV/c, whereas our results take into account all possible transverse momenta. Nevertheless, the experimental acceptance covers the small p_T region, which gives the largest contribution at SPS energies. Because of this, we obtain a good agreement for the centrality dependence of $\langle p_T \rangle$ (see Table 1 of Ref. [27] for more details). Concerning the centrality selection, it is important to remember the procedure used in the NA49 data analysis: for Pb+Pb collisions, eight narrow centrality bins, defined by the energy measured in the Veto Calorimeter, were chosen. In our model, because we know the exact number of participant nucleons for each event, we do not need to define centrality bins, it is enough for us to increase statistics to have a sufficient number of events for each number of participant nucleons. In this way, we avoid the effect of fluctuations of the number of participant nucleons on the multiplicity fluctuations.

In Figs. 2 and 3 we present separately our results for the variance $V(n^-)$ and the mean multiplicity $\langle n^- \rangle$ of negatively charged particles. We have included our results without clustering formation. One can observe that, when clustering is included, we find a perfect agreement with experimental data for the mean multiplicity. Concerning the variance and the scaled variance, the agreement is less good, but still one can see that the clustering works in the right direction: it produces a decrease of the variance in the central region—where the density of strings increases so the clustering has a bigger effect. Instead of that, without clustering, the scaled variance tends to a monotonic behavior with centrality. Note that, if no clustering is taken into account, our result for the variance is qualitatively similar to the HIJING simulation. From Eqs. (4) to (8) one can also deduce what will be the behavior of the scaled variance if both positively and negatively particles are taken into account: there will be an increase of the scaled variance in the fragmentation region—low number of participants and low density of strings—according to Eq. (7), because of the increase of $\langle \mu \rangle_1$, which now becomes proportional to 2/3 of $\langle \mu \rangle_0$. In the most central region our result for the scaled

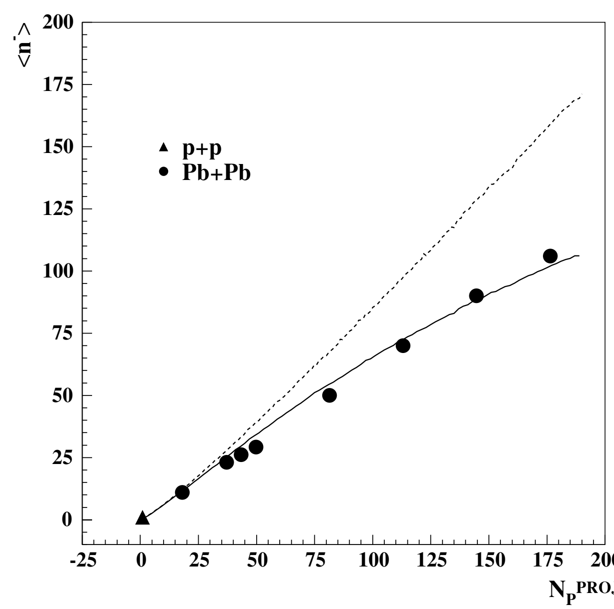

FIG. 3. Our results for the mean multiplicity of negatively charged particles in Pb+Pb collisions at $P_{\rm lab} = 158A$ GeV/c compared to NA49 experimental data. The dashed line corresponds to our result when clustering formation is not included, the continuous line takes into account clustering.

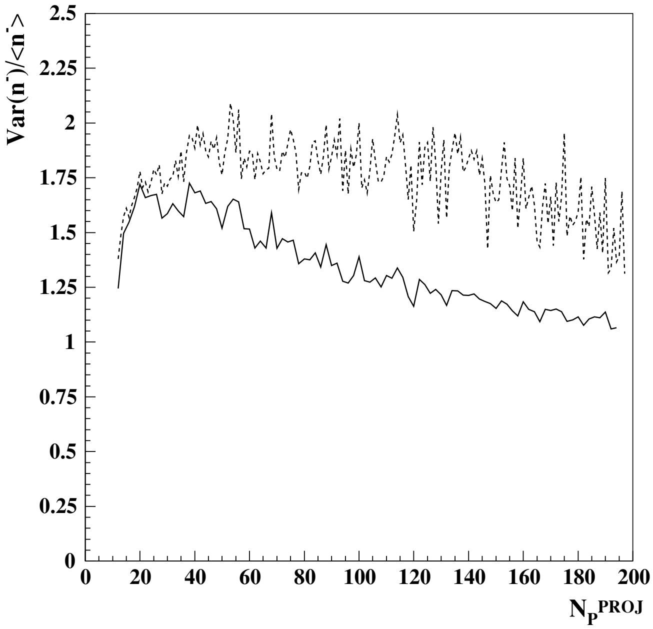

FIG. 4. Our results for the scaled variance of negatively charged particles in Au+Au collisions at $\sqrt{s} = 200$ GeV. The dashed line corresponds to our result when clustering formation is not included, the continuous line takes into account clustering.

variance essentially does not change, because the dependence on $\langle\mu\rangle_1$ is in this region much smaller, according to Eq. (8). In our approach, the scaled variance for the positive particles is equal to the one for the negatives particles, because both depend on $\langle\mu\rangle_1$ in the same way. This is in agreement with experimental data [5].

In Fig. 4 we present our prediction for the scaled variance on Au+Au collisions at RHIC ($\sqrt{s} = 200A$ GeV) energies. The behavior is similar to the one obtained at SPS energies. This is in accordance with our results for the mean p_T fluctuations. Note that now $\langle\mu\rangle_1$ is going to be smaller that in the SPS case, because we take $\Delta y = 0.7$, according with the experimental acceptance of PHENIX experiment. This in principal implicates smaller correlations. Conversely, at RHIC energies we have a higher value for the mean number of strings at fixed N_{part}. Both effects tend to compensate each other, especially in the small and midcentrality region—where $\langle\mu\rangle_1$ plays a fundamental role, according to Eq. (7). In the large centrality region we can observe that the effect of clustering leads to a scaled variance very close to one.

In conclusion, we have found a nonmonotonic dependence of the multiplicity fluctuations with the number of participants. The centrality behavior of these fluctuations is very similar to the one previously found for the mean p_T fluctuations. In our approach, the physical mechanism responsible for multiplicity and mean p_T fluctuations is the same [27]: the formation of clusters of strings that introduces correlations between the produced particles. Conversely, the mean p_T fluctuations have been also attributed [9] to jet production in peripheral events, combined with jet suppression in more central events. However, this hard-scattering interpretation, based on jet production and jet suppression, cannot be applied to SPS energies, so it does not explain the nonmonotonic behavior of the mean p_T fluctuations neither the relation between mean p_T and multiplicity fluctuations at SPS energy. Other possible mechanisms, extensively discussed in Refs. [34,35] are as follows: The combination of strong and electromagnetic interaction, dipole-dipole interactions and nonextensive thermodynamics. However, it is not clear if these fluctuations have a kinematic or dynamic origin, but clustering of color sources remains a good possibilty, because

(i) It can reproduce the qualitative behavior of the even-by-event fluctuations with centrality.
(ii) In this approach, mean p_T fluctuations and multiplicity fluctuations are naturally related.
(iii) It applies at SPS and RHIC energies.

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