Isospin conservation in preequilibrium reactions

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Previous studies of the extent of isospin mixing in preequilibrium reactions in the continuum have concentrated on reactions with nucleons in both the entrance and exit channels and with incident energies of 30 MeV or less. Using an expanded database for light-particle reactions that includes complex particle channels and higher incident energies, a new investigation of preequilibrium and, at the lower incident energies, equilibrium mechanisms has been carried out. Although many of the measured inclusive energy spectra are insensitive to whether isospin is assumed to be mixed or conserved during the reaction, other spectra reveal evidence that isospin is conserved during the preequilibrium phase of a reaction when the excitation energy in the intermediate nucleus is less than four times the symmetry energy. When isospin is conserved during energy equilibration, good agreement with experiment is typically obtained when the corresponding amount of isospin conservation at equilibrium is taken to be around 40%, though this number is model dependent.

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I. INTRODUCTION

Isospin is an often-neglected quantum number in continuum nuclear reaction calculations. One reason is that calculations for such reactions with light particles ($A = 1$ to 4) in the entrance and exit channels are frequently insensitive to whether isospin is assumed to be conserved during the reaction. Yet for other reactions, isospin can be important. Another reason is ignorance about the extent of isospin conservation in a given reaction. This can depend on factors such as the strength of the mixing interaction, the time available for mixing, and the properties and state densities of the states that can mix with one another. These, in turn, can be functions of the excitation energy, mass number, symmetry energy, angular momentum distribution, and, of course, the ground-state isospin value. This article focuses particularly on isospin conservation in the preequilibrium phase of a reaction, applying phenomenological calculations to the data in an attempt to address some of these issues.

Equilibrated compound nucleus reactions have been studied over a longer period of time and have included a wider range of reaction channels and experimental evidence (see Ref. [1] and references therein) than have preequilibrium reactions. The difficulty is that most of the studies have been carried out at excitation energies around 20 MeV, and the analyses are often model dependent. In addition, different groups have defined their mixing parameters differently [2]. Still, there is consistent evidence that isospin is at least partially conserved for compound nuclei up to around mass 100. The results that are most relevant to the present preequilibrium reaction studies involve comparing the evaporation spectra from four reaction channels—(p, p'), (p, α), (α, p), and (α, α')—all proceeding through the same intermediate nucleus. They suggest [3] that isospin is a better quantum number at equilibrium for light targets (mass numbers 49 to 111 were studied) and, perhaps, at higher excitation energies, though the energy range was fairly narrow and the uncertainties on the mixing fractions are often large. The energy dependence was further elucidated in Ref. [4], and, again, a trend was inferred for isospin to be more fully conserved with increasing excitation energy, up to around 63 MeV, but the reaction channels used were quite different from those studied here. All of these results are discussed further in Sec. V A.

The preequilibrium reactions most often noted as being sensitive to isospin conservation are (p, p') reactions on fairly neutron rich targets at 18–25 MeV [5–10]. Here the analyses suggest that isospin is conserved during energy equilibration and (except in the work of Refs. [7,8], as discussed in Sec. III A 2) at least partially conserved at equilibrium. It was also pointed out [10] that for (p, n) reactions leading to $N = Z$ final nuclei, isospin conservation can increase the yield by roughly 50%. At incident energies up to 25 MeV, these are often weak channels and the data were inconclusive. At 90 MeV, (p, xn) spectra on a variety of targets could only be systematically reproduced by assuming full isospin mixing, and the largest effect was for ^{58}Ni, where the (p, n) residual nucleus has $N = Z$.

Although isospin has not been carefully investigated for neutron- and α-particle-induced preequilibrium reactions, Grimes [11] anticipates, based on the isospin coupling coefficients, that the largest effects should occur for target nuclei with ground-state isospins, $T = |T_z| = (N - Z)/2$, of zero or one half, where T_z is the z component of the isospin. His perspective is mainly that of compound nucleus reactions, but the Clebsch-Gordan coefficients apply equally in the preequilibrium phase of the reaction.

Isospin conservation with incident α particles on $T_z = 0$ targets should mainly result in an increase in α-particle emission (relative to the situation when isospin is not a conserved quantum number), particularly when the nucleon channels are dominant. When the target has $T_z = 1/2$, the coupling coefficients indicate a reduction in proton emission from the compound nucleus relative to neutrons and α particles. For incident neutrons the effect for targets with $T_z \geqslant 0$ is likewise a reduction in proton emission relative to neutron and α emission. In all these reactions, however, the effect of isospin conservation on the state densities in the residual nuclei should be taken into account, at least for the

very simple states populated during the early stages of the equilibration process.

The present investigation involves a comprehensive study of isospin conservation in preequilibrium reactions. Although most earlier studies have focused on incident and emitted nucleons, this article considers complex particle channels as well. A large database of inclusive energy spectra, including complex particles with $A = 2$ to 4 in the entrance and/or exit channels, is studied looking for evidence in both the preequilibrium and, at the lower incident energies, also the equilibrium components. The goal is to discover the physical conditions under which isospin is conserved, thus facilitating predictive model calculations for reactions that are either unmeasured or, in some cases, unmeasurable. Such calculations play an important role in both basic research, such as certain rare isotope accelerator projects, and applied work, such as accelerator driven transmutation of wastes.

II. THE METHOD

The database used in this work was taken from previous work [12–14] on the exciton model and the direct reaction models that supplement it. It consists of inclusive continuum energy spectra from the literature. Targets range from aluminum through uranium; incident energies from 14 to 63 MeV for neutrons, 14 to 90 MeV for protons, 25 to 80 MeV for deuterons, 24 to 26 MeV for ^{3}He, and 35 to 140 MeV for α particles.

All of the spectra for incident neutrons, protons, and α particles in the database were analyzed here except for those measured on natural targets with more than one prominent isotope and those where known deficiencies in the calculations and/or the wide spread in the incident energy would prevent meaningful comparisons. In all, some 299 spectra representing 169 target/projectile/incident energy combinations were investigated. Calculations were also run for 33 spectra from deuteron and ^{3}He induced reactions, but the absence of a breakup mechanism in the calculations and the resulting uncertainty in the initial exciton model particle-hole configuration (as discussed in Ref. [14]) limit the usefulness of these reactions to simply determining which spectra might be sensitive to isospin conservation.

The calculations were run with the exciton and direct reaction model code PRECO-2000 [15], with the changes discussed in recent work on reactions with complex particles in the entrance and/or exit channels [14]. Most of the calculations use two-component particle-hole state densities that include the effects of shell structure and the pairing interaction as well as the possibility of isospin conservation. This code was developed primarily for studying preequilibrium phenomena and so lacks many of the features of a full Hauser-Feshbach model code for treating equilibrium emission. The calculations include primary preequilibrium emission of all light particles from the following mechanisms: direct nucleon transfer, collective excitations to both spectroscopic and giant resonance states, inelastic scattering and knockout reactions involving cluster degrees of freedom, and the exciton equilibration model. Secondary exciton model preequilibrium emission of nucleons is calculated following primary emission of nucleons in either the exciton or nucleon transfer model. Primary equilibrium emission is likewise evaluated for all light particles, whereas secondary evaporation is considered only for nucleons following primary nucleon emission. Equilibrium emission is calculated in the simple Weisskopf-Ewing model, which, like the exciton model, does not consider angular momentum. No breakup of complex projectiles is calculated.

When isospin is assumed to be conserved in a reaction, a number of modifications are applied to the basic equations. The expressions for single and double differential cross sections always include the entrance channel Clebsch-Gordan coefficient. In addition, when more than one isospin value is possible in the intermediate nucleus (typically for p and ^{3}He induced reactions on targets with $N > Z$), a separate calculation is done for each isospin value, and the results are added. The state densities in the exciton model, nucleon transfer, and evaporation calculations are all evaluated for the appropriate isospin values of the states [16]. In calculations of the particle emission rates in the exciton and evaporation models, all possible isospin values in the residual nucleus are considered, with their state densities multiplied by the appropriate exit channel Clebsch-Gordan coefficient. The same is done in the basic formula for nucleon transfer reaction cross sections.

To study the effects of isospin conservation in the preequilibrium phase of the reaction, calculations were run with the two extreme assumptions of full isospin conservation and full isospin mixing. In both cases, isospin was assumed to be mixed at equilibrium. These results were compared with the measured spectra in the database to determine whether the data are sensitive to the isospin quantum number and, if so, whether isospin appears to be conserved or mixed during equilibration. Where isospin was indicated as being conserved, additional calculations were run with the amount of isospin conservation at equilibrium set at 50 and 100%. This allows a range of acceptable equilibrium mixing fractions to be estimated for (N, xN) reactions at the lower incident energies, where no more than two particles would be expected to be emitted in a reaction.

The equilibrium components of other (N, xN) reactions at the lower incident energies (those that were insensitive to isospin conservation in the preequilibrium phase) were also studied. A few of these are sensitive to assumptions about isospin conservation at equilibrium. Clearly, if isospin is still at least partially conserved at equilibrium, it must have been conserved during the approach to equilibrium. Once again, acceptable mixing fractions at equilibrium were estimated.

III. EFFECTS OF ISOSPIN CONSERVATION

Most of the spectra studied are insensitive to whether isospin is assumed to be mixed or conserved during the reaction calculations. This is fortunate, because it allows other aspects of the reaction modeling to be studied, independent of isospin considerations. When isospin does make a difference, the observed effects are the result of (a) the isospin coupling coefficients, (b) the different number of allowed values of

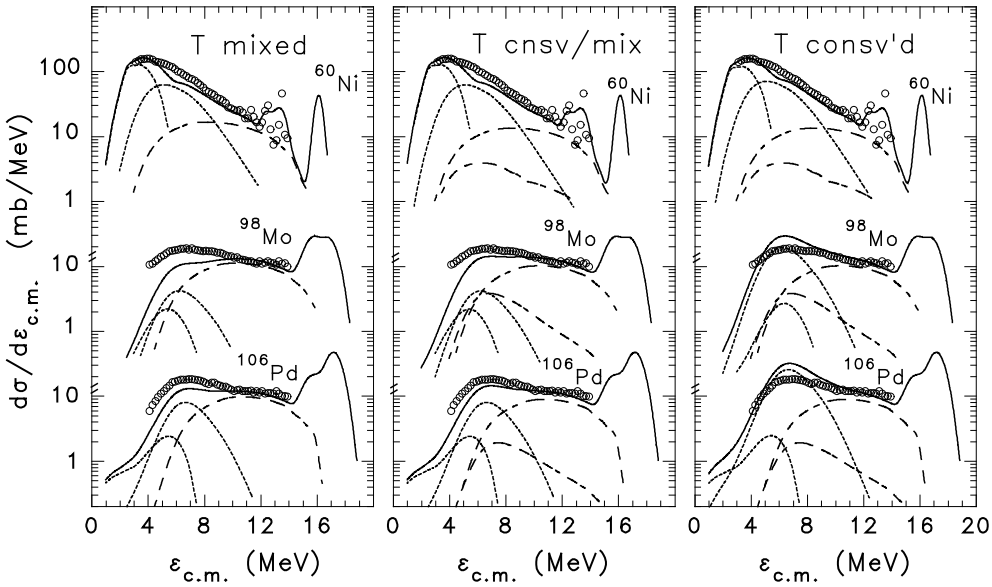

FIG. 1. The effects of isospin conservation on sample (p, xp) reactions at 18 MeV. The circles give the experimental results, the dashed curves give the preequilibrium components calculated within the exciton model, and the short-dashed curves show the primary and secondary evaporation components. The primary evaporation peak is the one extending to higher emission energies. The solid curves give the total calculated spectrum, which also includes contributions from collective excitations of both spectroscopic and giant resonance states. In the left-hand panel, isospin is assumed to be fully mixed at all stages of the reaction. In the center panel, isospin is assumed to be conserved during the equilibration stage of the reaction, but mixed at equilibrium. Here the larger and harder preequilibrium component corresponds to emission from intermediate states with the ground state isospin, while the smaller, softer component gives emission from states with one additional unit of isospin. In the right hand panel, isospin is assumed to be conserved throughout the reaction. Elastic scattering is not included in the calculations. The data are from Refs. [7,8].

the isospin quantum number in the intermediate nucleus and in the product nuclei for different reaction channels, and (c) the particle-hole state densities for the specific isospin values considered.

A. Incident protons

For target nuclei with $N > Z$, the entrance channel isospins for incident protons, unlike those for incident neutrons, deuterons, or α particles, can couple to produce two values of the intermediate isospin, corresponding to the ground-state isospin in the composite nucleus and to one unit higher or $T = |T_z| + 1$. Thus, when isospin is conserved, two separate reaction calculations are performed, one for each intermediate isospin value, and the results are added.

1. Proton emission

As stated in the introduction, the most commonly noted effect of isospin conservation in preequilibrium reactions has been for (p, xp) reactions at energies of 18 to 25 MeV on neutron rich targets. The same effect is noted here and is shown in Figs. 1 and 2 along with the corresponding data from Refs. [6–8,17]. Three calculations are shown for each reaction: (1) isospin fully mixed, (2) isospin conserved during equilibration but mixed during evaporation, and (3) isospin conserved as a quantum number throughout the reaction. Virtually all of the preequilibrium cross section is generated in the exciton equilibration model except at the highest emission energies where peaks corresponding to the excitation of strong collective states can be seen.

In each case the reaction strength in the exciton model is envisioned as beginning in states with two particle degrees of freedom and one hole degree of freedom. These are designated as three exciton (or $n = 3$) states, where the exciton number is the sum of the numbers of particle and hole degrees of freedom: $n = p + h$. The system then undergoes a series of particle-hole pair-creation interactions so that the strength passes, sequentially, through states with $n = 5, 7, 9\ldots$ until the most probable number of excitons at equilibrium is reached, until all the reaction strength has been emitted, or until a preset maximum value of n has been reached. The most probable exciton number is typically around $(gE)^{1/2}$, where E is the excitation energy of the nucleus and g is the average single particle state density, which is here taken to be $g = A/(15\ \text{MeV})$.

The preequilibrium behavior of these reactions when isospin is conserved was explained many years ago [5] in master equation calculations that studied particle emission rates as a function of time in the equilibrating nucleus. More recently it has been discussed [8] within a closed form approach. Typically most of the cross section will go into states in the intermediate nucleus which have the ground-state isospin. This follows the familiar pattern seen in calculations in which isospin is not considered: the preequilibrium emission rates decrease rapidly and the emission spectra become softer as the system progresses to states with ever greater numbers of

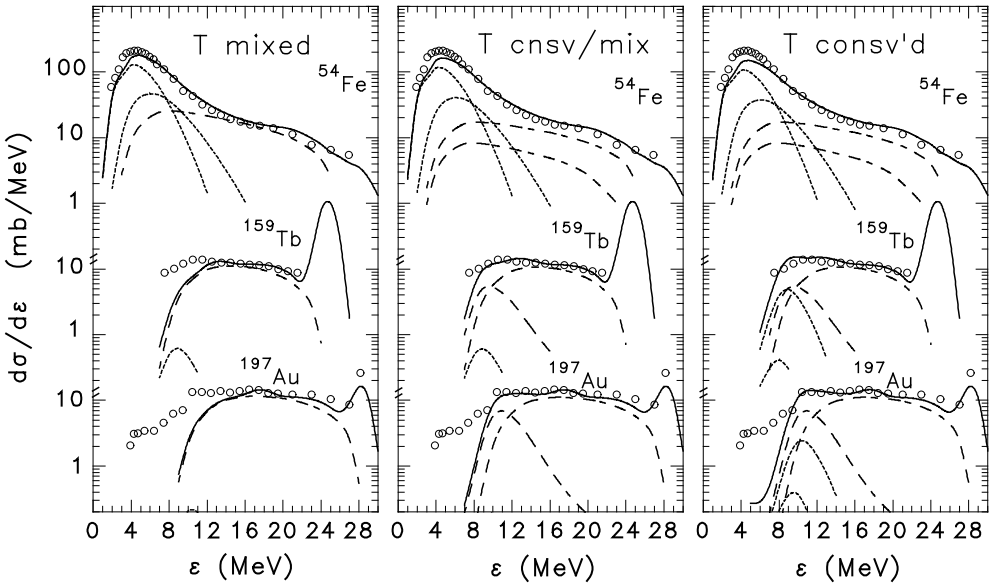

FIG. 2. The effects of isospin conservation on sample (p, xp) reactions at 25 MeV (^{159}Tb) and 28.8 MeV (^{54}Fe and ^{197}Au). The points and curves have the same significance as in Fig. 1. Here in the left two panels, the secondary evaporation components for ^{159}Tb and both evaporation components for ^{197}Au are too small to be seen. This changes in the right hand panel, where isospin conservation at equilibrium makes these components larger. The iron and gold data are from [17] and are given in the laboratory system. The terbium spectrum is obtained from [6] by taking the backward hemisphere data and extrapolating it to forward angles as discussed in [9]. It is given in terms of the channel energy. All of the calculations are likewise given in terms of the channel energy.

particle and hole degrees of freedom. Thus the preequilibrium part of the energy spectrum is dominated by emission from the initial two-particle, one-hole states in the intermediate nucleus.

The strength that goes through the intermediate states with one additional unit of isospin behaves differently. Here isospin selection rules do not allow neutron, deuteron, triton, and α-particle emission leading to states of the residual nucleus with the ground-state isospin. Emission to higher isospin states in the final nucleus occurs only if these are energetically accessible, and then such emission is typically weak because of the low final-state densities. Proton and ^{3}He emission, however, can proceed to states with the ground-state isospin. The emission rates from states with n excitons will (ignoring Pauli blocking, pairing, and shell corrections, all of which are included in the calculations), be roughly proportional to

$$\frac{(E - B_b - \varepsilon_b)^{n-2}}{(E - E_{\rm sym})^{n-1}}.$$

Here E is again the excitation energy in the intermediate nucleus relative to the ground state, $E_{\rm sym}$ is its isospin symmetry energy, B_b is the binding energy of the proton or ^{3}He in the intermediate nucleus, and ε_b is the kinetic energy in the emission channel. The energy in the numerator is the excitation energy in the residual nucleus, whereas the energy in the denominator is the effective excitation energy of the $T_>$ states in the emitting nucleus.

If $E_{\rm sym} > B_b + \varepsilon_b$ (which happens for the lowest emission energies, if the symmetry energy is large enough), then the energy in the numerator will actually be larger than the energy in the denominator. Thus the emission rates at low outgoing energies will actually increase as the system moves to states with greater and greater numbers of excitons. Even when $E_{\rm sym}$ is not greater than $B_b + \varepsilon_b$, just the fact of having the lower effective excitation energy in the emitting nucleus means that the emission rates will not decrease nearly as rapidly for the $T_>$ states as for the isospin mixed calculations or for the $T_<$ states. Thus late-stage preequilibrium emission will be more important. This produces the preequilibrium components with the softer spectral shape seen in Figs. 1 and 2, where the softness of the $T_>$ preequilibrium component can be correlated with the difference between the proton binding energy and the symmetry energy for the intermediate nucleus. These energies are given in Table I. For targets with smaller neutron excesses, the symmetry energies are smaller, and the effect is reduced and largely hidden under a prominent evaporation peak. This is seen in the nickel and iron spectra in these figures.

The effect of isospin conservation at equilibrium is primarily to redistribute the cross section between the various exit channels and, in the case of the neutron rich targets,

TABLE I. Comparison of the symmetry and binding energies for the intermediate nuclei in the proton induced reactions shown in Figs. 1 and 2.

Reaction	$E_{\rm sym}$ (MeV)	B_p (MeV)	$E_{\rm sym} - B_p$ (MeV)
^{60}Ni+p	5.1	4.8	0.3
^{98}Mo+p	11.8	6.5	5.3
^{106}Pd+p	11.0	5.8	5.2
^{54}Fe+p	2.8	5.1	−2.3
^{159}Tb+p	15.9	7.4	8.5
^{197}Au+p	17.6	7.1	10.5

to concentrate extra strength into the weak (p, xp) channel. Because these weak equilibrium components are quite sensitive to the state density parameters, and because of experimental uncertainties, it is difficult to get a good estimate of the extent of isospin conservation at equilibrium for any given reaction. Global trends can be observed, but they are model dependent, as discussed below.

2. *Comparisons with earlier results*

The results for ^{98}Mo and ^{106}Pd in the center panel of Fig. 1 can be compared with the corresponding results of Watanbe *et al*. from Figs. 7 and 8 of Ref. [7] and from Figs. 3 and 4 of Ref. [8]. The results from the two references differ only in the criterion used for terminating the sum over exciton number in the preequilibrium calculations for the $T_>$ states in the composite nucleus. The present results and those of Watanbe *et al*. are qualitatively similar but show quantitative differences. Although the preequilibrium components from the intermediate states with the ground-state isospin (the $T_< = |T_z|$ states) are much alike, the components from the higher isospin (or $T_> = |T_z| + 1$) states in the intermediate nucleus are higher in Watanabe's work and especially in Ref. [7], whereas the evaporation components are comparable for ^{98}Mo and slightly smaller for ^{106}Pd. Overall, the total spectra in [7] are closer to the current results with isospin conserved at equilibrium than the ones with isospin mixed at equilibrium, whereas those in Ref. [8] correspond to PRECO results when isospin is roughly 40% conserved at equilibrium. There are several possible reasons for the differences.

First Watanabe *et al*. use different symmetry energies. The code PRECO takes its symmetry energies from the volume and surface symmetry energy terms in the Meyers and Swiatecki semiempirical mass equation [18]

$$E_{\rm sym}^{\rm (V+S)}(|T_z|+1, T_z) = \left[\frac{112\,\text{MeV}}{A} - \frac{133\,\text{MeV}}{A^{4/3}}\right](2|T_z|+1)$$
$$= \left[\frac{112\,\text{MeV}}{A} - \frac{133\,\text{MeV}}{A^{4/3}}\right](|N-Z|+1) \quad (1)$$

where $T_z = (N - Z)/2$ is the z component of the isospin in the nucleus being considered. [This formula is also valid for nuclei with $Z > N$.] Watanabe *et al*. use a formula for nuclei with $N \geqslant Z$ based on the Q value for the (p, n) reaction populating the nucleus of interest:

$$E_{\rm sym}^{(Q)}(|T_z|+1, T_z) = (1.44\,\text{MeV})\frac{\left(Z-\frac{1}{2}\right)}{A^{1/3}} - 1.13\,\text{MeV} + Q_{p,n}(Z-1, A). \quad (2)$$

Here the first two terms give the empirical systematics of the Coulomb displacement energy [19] [or the negative of the Q value of the (p, n) reaction to the isobaric analog state]. When this is corrected for the Q value of the (p, n) reaction to the ground state of the nucleus, the symmetry energy or energy of the lowest state with $T = |T_z| + 1$ is obtained. These symmetry energies contain the effects of pairing, and shell structure, which, in the present calculations, are accounted for in separate energy adjustments in the exciton model state densities. The two sets of symmetry energies for the intermediate nuclei are given in Table II.

TABLE II. Comparison of the symmetry energies of Eqs. (1) and (2).

Target	Composite	$E_{\rm sym}^{(V+S)}$(MeV)	$E_{\rm sym}^{(Q)}$ (MeV)
^{98}Mo	^{99}Tc	11.8	12.7
^{106}Pd	^{107}Ag	11.0	12.3

Using the larger symmetry energies of Refs. [7,8] in place of the $E_{\rm sym}^{(V+S)}$ values used here would lead to a greater enhancement for the late-stage preequilibrium emission from the higher isospin states in the intermediate nucleus, and this increase would be larger for ^{106}Pd than for ^{98}Mo, which is what is observed. It does not appear, however, that this is enough to account for the entire difference between the present results and those of Watanbe *et al*.

Second, as highlighted by the difference between the results of Refs. [7,8], the $T_>$ preequilibrium component for these reactions is sensitive to the exciton number or complexity of the states at which the closed form preequilibrium calculations are terminated. This occurs because there is not a clean separation between the preequilibrium and equilibrium parts of the reaction for the higher isospin states in the composite nucleus. Normally such a separation is achieved because the particle emission rates drop rapidly as equilibrium is approached, so that the calculated preequilibrium spectrum is quite insensitive to the exciton number of the states at which one stops the calculations. [Similar problems arise with master equation calculations, where the preequilibrium component would be unusually sensitive to the time (or number of iterations) at which the calculations are terminated. For closed form calculations, this problem is, however, compounded by the fact that pair annihilation (which is neglected) becomes comparable to pair creation for the configurations that are prominent as equilibrium is approached.]

In PRECO the sum over exciton number for the $T_>$ states typically extends to the most probable number at the effective excitation energy $E - E_{\rm sym}$, which is lower than the most probable number for the isospin mixed calculations or for states with the ground-state isospin. Although Watanabe *et al*. do not specify which excitation energy they use for the $T_>$ calculations (and it is likely $E - E_{\rm sym}$, as is used here), Ref. [8] states that they use a value for the most probable exciton number that is $\bar{n} = (2gE)^{1/2}$. This result can be derived for the one component state densities that they use by ignoring the effects of the Paul exclusion principle. The actual value in either the one-component or two-component equispacing model when the Pauli corrections are included (as they are in both the present work and the work of Watanbe *et al*.) is found empirically to be close to $\bar{n} = (gE)^{1/2}$ or about a factor of 1.4 smaller. The present work uses the empirical values of $\bar{n}$. This should make no appreciable difference in the calculations for the $T_<$ states in the intermediate nucleus, but the use of $\bar{n} = (2gE)^{1/2}$ in Ref. [7] leads to extra (p, xp) preequilibrium cross section for the $T_>$ composite nucleus

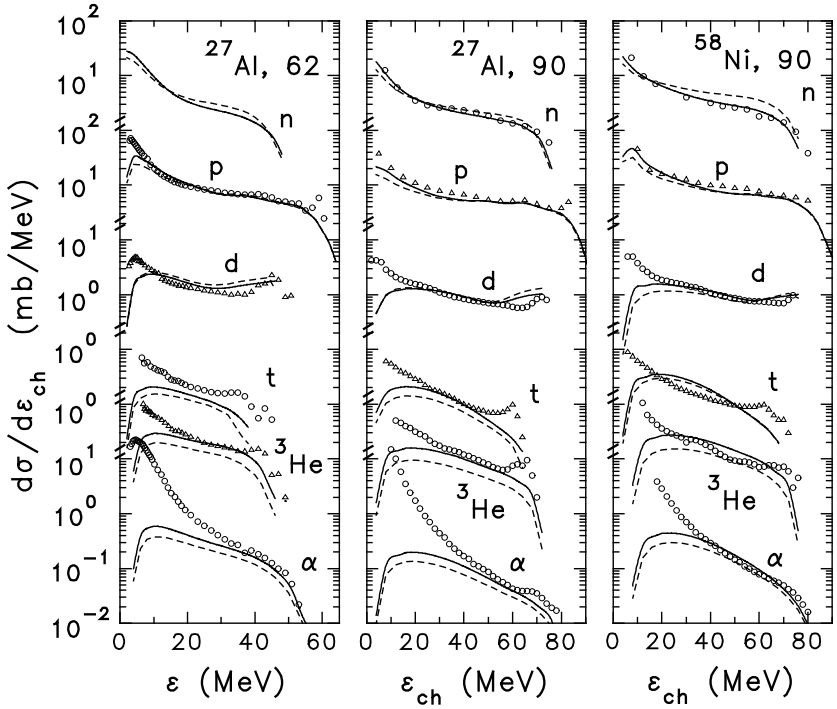

FIG. 3. The effects of isospin conservation in sample proton induced reactions on light targets. The solid curves show the results of calculations in which isospin is assumed to be a mixed (nonconserved) quantum number in all stages of the reaction; the dashed curves give the corresponding results when isospin is assumed to be a good quantum number during the equilibration phase of the reaction but to be mixed during the compound nucleus phase. The points show the experimental results. The 62 MeV data are from Ref. [17] and are given in the laboratory system, while the 90 MeV data are from Refs. [20,21] and are given in terms of the channel energy. All of the calculations are given versus the channel energy.

calculations. In Ref. [8] an additional criterion for terminating the $T_>$ calculations was added: the requirement that the total particle emission rate during the calculations not exceed the total internal transition rate for a given value of n. This lowers the exciton number at which the $T_>$ calculations are terminated, whereas apparently still leaving it larger than what is used here. (For ^{106}Pd at 16 MeV, shown in Fig. 10(b) of Ref. [8], $\bar{n} = 11$, as compared to $[2g(E - E_{\rm sym})]^{1/2} = 12.4$ and the value of $\bar{n} = 9$ found in PRECO. Only odd values of n occur in the calculations.)

A third factor that, in principle, could be affecting the results is the assumed exciton number dependence of the mean-square matrix elements for the effective, residual interactions bringing about energy equilibration in the intermediate nucleus. The typical n-dependence would cause the matrix elements to increase as the excitation energy per exciton decreases, which is what happens in going to states of greater complexity. This, in turn, would make the internal interactions—pair creation, pair annihilation, and exciton scattering—compete more favorably with particle emission, thus reducing late-stage preequilibrium emission. However, neither calculation uses an exciton number dependence for the mean-square matrix element, and other differences in the calculations would be expected to have much smaller effects.

These comparisons indicate the degree of model dependence in the results. The conclusion about whether isospin needs to be conserved during equilibration is the same in all the calculations, but the conclusion about the amount of mixing at equilibrium will be different, in large part because of the lack of a clean separation between the two phases of the reaction for the higher isospin states in the composite nucleus and because of uncertainties about the appropriate values for the symmetry energies. Again, this is only a problem when the entrance channel can populate $T_>$ states and the symmetry energy is larger than the proton binding energy.

3. Neutron emission

For other channels, the effects of isospin conservation are seen mainly for light targets with small neutron excesses. Figure 3 shows results for ^{27}Al at 62 and 90 MeV and for ^{58}Ni at 90 MeV, along with the data of Refs. [17,20,21]. Similar results are seen for ^{54}Fe at 29 to 62 MeV, though they are less conclusive. Neutron emission is discussed first because, as was the case for proton emission, most of the calculated preequilibrium cross section comes from the exciton model.

It was previously mentioned that isospin conservation enhances the cross section in (p, xn) reactions on light targets with $N > Z$ at both lower (18–25 MeV) and higher (90 MeV) incident energies. Experimental results at the lower energies were sparse and the results of comparisons with the calculations were inconclusive. The 90 MeV results indicated isospin mixing. The isospin coupling coefficients (the squares of the Clebsch-Gordan coefficients) for the reactions shown in Fig. 3 are given in Table III.

For ^{27}Al, the neutron and proton coefficients are equal because the intermediate nucleus, ^{28}Si, has $N = Z$ and therefore a ground-state isospin of zero. Thus to first order, the neutron to proton yield ratio should be the same as in the calculations where isospin is assumed to be mixed. Although this is approximately true in the figure, small differences occur as a result of different isospin effects on the residual state densities, because the particle-hole configurations are different in the two cases. For the dominant preequilibrium emission that occurs from the two-particle, one-hole states, proton emission produces states with either a proton particle-hole pair or a

TABLE III. Isospin coupling coefficients for proton induced reactions on ^{27}Al and ^{58}Ni. The columns are labeled by the reaction partners and whether the entrance channel isospins couple to $T_< = |T_z|$ or $T_> = |T_z| + 1$ in the composite nucleus. The first number in each cell of the table gives the coupling coefficient for the ground-state isospin in the residual nucleus, with each succeeding number corresponding to one higher unit of isospin. The coefficients are the same for emitted particles that have the same N-Z, such as the neutron and triton.

Emitted particles	^{27}Al+p, $T_<$	^{27}Al+p, $T_>$	^{58}Ni+p, $T_<$	^{58}Ni+p, $T_>$
n, t	1/2, 0	1/2, 1/2	1, 1/3	0, 2/3, 2/5
p, ^{3}He	1/2, 0	1/2, 1/2	2/3, 0	1/3, 3/5
d, α	1, 0	0, 1	1, 0	0, 1

neutron particle-hole pair, whereas for neutron emission the residual nucleus has a proton particle and a neutron hole degree of freedom.

The results for ^{58}Ni show the more typical case where $N > Z$ in the intermediate nucleus. Here the isospin coupling coefficients and the number of allowed isospin values lead to a significant enhancement in neutron emission relative to proton emission when isospin is assumed to be conserved as an effective quantum number during energy equilibration. The experimental neutron spectrum in Fig. 3 clearly favors isospin mixing.

With regard to the 90 MeV neutron spectra, it should be noted that the experimental results shown in Fig. 3 for emission energies above 20 MeV are not the laboratory angle integrated cross sections of Ref. [20], which were obtained by taking 5-MeV energy bins and summing over the appropriate angle bins. Rather they are the result of the present author converting the original double differential cross sections into the center of mass system and fitting the resulting angular distributions with an exponential in $\cos\theta_{\rm cm}$. This form is suggested by the global angular distribution systematics [22] and generally worked well. In many cases, however, the cross section at a laboratory angle of 20° was significantly higher than the trend indicated by the remaining angles, so the 20° point was ignored in the fitting. This was the most forward angle at which measurements were made, and it was the one most subject to background problems. The published laboratory angle integrals are significantly more intense at the highest emission energies than the results shown in Fig. 3, so that they coincide more with the isospin conserved results for ^{58}Ni+p and are often above both sets of calculations for ^{27}Al+p. The same data set had two heavier targets: ^{90}Zr and ^{209}Bi. For ^{90}Zr, the two angle-integrated spectra are quite similar, whereas for ^{209}Bi, the original laboratory integral is again higher so that it cannot be satisfactorily reproduced by calculations with either isospin assumption. Thus it is only with the center-of-mass angle integrals and the assumption of isospin mixing that a consistent set of fits to the data can be achieved. The conclusion with respect to isospin is confirmed by the results for complex particle emission.

4. Complex particle emission

When the complex particle emission channels are considered, other effects emerge as the nucleon transfer mechanism becomes important. It represents roughly half of the deuteron preequilibrium cross section and is the major preequilibrium reaction mechanism for triton, ^{3}He, and α-particle emission. Once again, the effects are slightly different for ^{27}Al, where the intermediate system has $N = Z$, and ^{58}Ni, where it has $N > Z$. In nucleon transfer reactions, isospin conservation enters through the coupling coefficients in the entrance and exit channels, plus any reduction in the residual nucleus state densities. The complex particle spectra are shown in Fig. 3.

For ^{58}Ni, where neutron emission in the exciton model was enhanced, triton emission (also a $T_z = 1/2$ particle) is nearly unchanged over most of the spectrum during nucleon transfer. The enhancement in the exit channel coupling coefficients for the $T_<$ calculations balances out the reduction in the residual state densities when they are restricted to specific isospin quantum numbers. (The exciton model emission rates involve a ratio of state densities for the residual and composite nuclei, so the state density effect partly cancels out.) The other nucleon transfer channels are all reduced significantly in intensity, because of the combined effects of the coupling coefficient and reductions in the residual state densities. For ^{27}Al, all of the nucleon transfer cross sections are reduced.

The exciton model cross section contributes significantly only to the deuteron channel. For ^{58}Ni it is roughly unchanged from the isospin mixed result, whereas for ^{27}Al it is larger and is responsible for the increased deuteron emission cross section at the highest emission energies. The ^{27}Al behavior occurs because the coupling coefficients reduce the rates for nucleon emission, which competes with deuteron emission. In the ^{58}Ni system, the combined neutron and proton emission rates have the same size change because of isospin conservation as do the deuteron emission rates, so the exciton model deuteron cross section is nearly unchanged.

In each case where the data seem to prefer one isospin assumption over the other, the choice again seems to be that the isospin quantum number is mixed during equilibration.

B. Other light projectiles

Of the remaining light projectiles, only ^{3}He has entrance channel isospins that couple to two possible intermediate values for targets with $N > Z$. Incident neutrons, deuterons, tritons, and α particles all couple only to the lowest allowed intermediate isospin, making the effects of isospin conservation

simpler to analyze. Deuteron, triton, and ^{3}He induced reactions also have a strong projectile breakup component that is not yet included in the model calculations, and so comparisons with even the limited data that exist are difficult. Nevertheless, the calculations for the reactions in the database were run without projectile breakup to see if the results were sensitive to isospin assumptions. For all of these projectiles except ^{3}He, the largest isospin effects should occur for targets with low-ground-state isospin values.

1. Incident ^{3}**He**

The behavior of ^{3}He-induced reactions is expected to be very much like that for proton-induced reactions, at least as far as the exciton model and equilibrium components are concerned. Unfortunately there are very few data available on ^{3}He-induced reactions, and calculations for the systems in the database—^{62}Ni at 24.3 MeV with light-charged-particle emission [23] plus (^{3}He, xp) spectra on ^{57}Fe, ^{62}Ni, and ^{116}Sn at 25.6 MeV [24]—were all insensitive to whether isospin was assumed to be conserved or mixed.

To look for similarities to the (p, p') sensitivity at 18–25 MeV on neutron rich targets, calculations were performed for two hypothetical ^{3}He-induced reactions. These are ^{120}Sn+^{3}He at 25.6 MeV (a more neutron rich version of the ^{116}Sn case) and ^{157}Gd+^{3}He at 18 MeV (forming the same intermediate nucleus as the ^{159}Tb+p system, which showed a large isospin effect). The ^{120}Sn(^{3}He, p) reaction shows little sensitivity to isospin conservation during the preequilibrium phase of the reaction but increased proton emission when isospin is also conserved at equilibrium. The ^{157}Gd(^{3}He, p) reaction shows sensitivity to isospin conservation at both stages of the reaction, and the behavior is quite similar to that seen in the corresponding proton-induced reaction. The direct nucleon transfer reaction components were nearly independent of the assumptions regarding isospin conservation.

In addition, calculations were run for the ^{56}Co+^{3}He reaction at the same excitation energy as was populated in the ^{58}Ni+p reaction at 90 MeV. This is an example of a reaction on a light target. Here again, behavior similar to that of the proton induced reaction is recovered with incident ^{3}He. Isospin conservation during equilibration enhances the neutron emission cross section by roughly 50% in the middle of the spectrum. The proton, deuteron, and triton spectra show much smaller changes, and these are sometimes in a different direction than for incident protons because of the varying roles of the exciton and nucleon transfer mechanisms. The α-emission spectrum is, as with incident protons, reduced, especially at the lower emission energies. The most significant difference from the corresponding proton induced reaction is that the marked decrease in ^{3}He emission seen in those calculations is not observed with incident ^{3}He. This is because in the latter case, the dominant reaction mechanism is inelastic cluster scattering. In the current formalism, cluster scattering does not have any isospin effects included because the necessary isospin-dependent state densities have not been derived. Exit channel-coupling coefficients would imply that ^{3}He emission should, again, be reduced when isospin is conserved. Assuming isospin to be also conserved at equilibrium makes relatively little difference, in part because at this high an incident energy, much of the primary emission occurs before equilibrium is reached.

TABLE IV. Isospin coupling coefficients for neutron-induced reactions on ^{27}Al and ^{28}Si. The column labels and cell contents have the same significance as in Table III.

Emitted particles	^{27}Al+n, $T_<$	^{28}Si+n, $T_<$
n, t	1, 1/4	1, 1/3
p,^{3}He	3/4, 0	2/3, 0
d, α	1, 0	1, 0

2. Incident neutrons

Of the remaining light projectiles, the mechanisms for reactions with incident neutrons are the best understood and characterized in the models used here, so neutron induced reactions are considered next.

For incident neutrons, Grimes [11] anticipates, based on the coupling coefficients, that neutron emission for targets with $T_z = 0$ or 1/2 will be increased slightly relative to proton and α-particle emission. Their isospin partners—tritons, ^{3}He, and deuterons, respectively—should show similar behavior. The case of a ^{28}Si target ($T_z = 0$) is considered, because it shows the largest effect of those systems included in the database. The coupling coefficients for ^{27}Al ($T_z = 1/2$) and ^{28}Si are given in Table IV.

The results of calculations on ^{28}Si at 50 and 62.7 MeV are shown in Fig. 4, along with the data from Refs. [25,26], and generally confirm the predictions. The effects seen are also quite similar to those for protons on ^{58}Ni, considered above, because the exit channel-coupling coefficients here match the $T_<$ values in that case. Once again, the biggest effect is observed for ^{3}He emission, where nucleon transfer is the dominant preequilibrium reaction mechanism and where the reduction in the residual state densities works together with the coupling coefficients to reduce the cross section when isospin is conserved. Unfortunately, it was not possible to cleanly separate the ^{3}He and α spectra in the data. Thus ^{3}He emission is included in the much larger α-particle spectrum and the calculated curves include both α and ^{3}He contributions. The data thus are generally inconclusive with regard to isospin conservation.

The calculations also indicate that isospin conservation increases the neutron yield, but not nearly as much as for the ^{58}Ni+p system. The reason for this difference is that the (n, xn) spectra contain a collective component for which isospin effects are not yet included.

3. Incident α particles

The situation for incident α particles is quite similar to that for incident neutrons, and the results are shown in Fig. 5, along with the experimental results of Refs. [27,28]. The isospin coupling coefficients for ^{27}Al ($T_z = 1/2$) and for ^{54}Fe and ^{58}Ni ($T_z = 1$) are given in Table V.

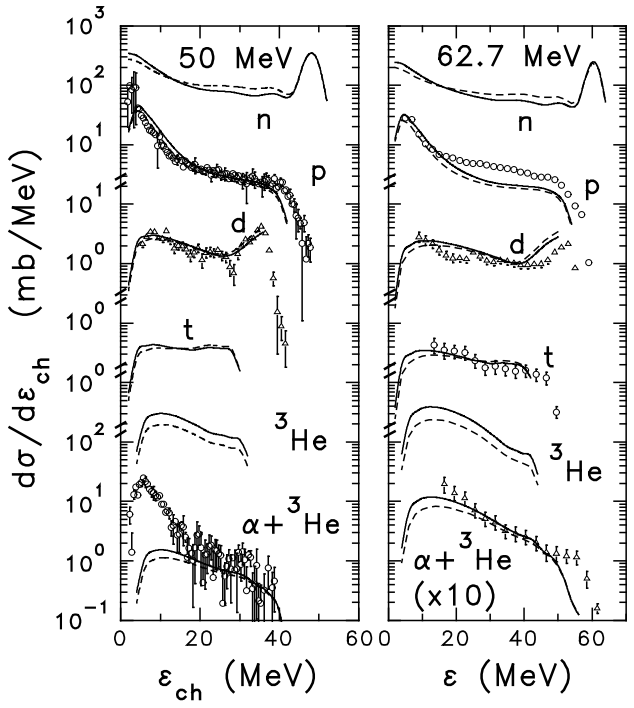

FIG. 4. The effects of isospin conservation in neutron-induced reactions on ^{28}Si at 50 and 62.7 MeV. The curves and points have the same significance as in Fig. 3. The 50-MeV data are from Ref. [25] and are given in terms of the channel energy, whereas the 62.7-MeV data are from Ref. [26] and are given in the laboratory system. All calculations are given in terms of the channel energy. Although the calculated ^{3}He spectra are shown separately, the experiments were unable to resolve ^{3}He and α. Thus the contributions from ^{3}He are included with the α spectra in both the data and the calculations.

Once again, the largest effect observed in the calculations is a reduction in the intensity of ^{3}He emission when isospin is conserved during the equilibration phase of the reaction. Proton emission is reduced by a smaller amount because the dominant preequilibrium component is from the exciton model, where a reduction in the composite nucleus state density because of isospin conservation can partially compensate for the coupling coefficients and where the effects on the pair creation interactions, which compete with particle emission, must also be considered. The emission of ^{3}He is predominantly via direct nucleon transfer, where isospin enters in a simpler, more direct way. Comparisons of the 140-MeV data with the calculated results are complicated by the fact that, at this high an incident energy, α-particle breakup may be playing a significant role in the reaction [14]. This may explain why the spectral shapes are not as well reproduced by the calculations as at lower incident energies.

Despite uncertainties in the data and the lack of α breakup in the 140-MeV calculations, whenever the data indicate a preference between isospin being mixed and conserved for these light targets, they point toward isospin mixing.

4. Incident deuterons

Because both the deuterons and the α particles are $T = 0$ projectiles, their reactions would be expected to show similar effects because of isospin conservation. The deuteron reactions investigated are for ^{63}Cu at 24.7 MeV; ^{27}Al and ^{58}Ni at 80 MeV; and ^{90}Zr, ^{208}Pb, and ^{232}Th at 70 MeV. Of these, only the aluminum and nickel targets (with $T_z = 1/2$ and 1, respectively) show significant effects. The coupling coefficients are shown in the table for incident α particles. As in α-particle-induced reactions, the biggest change resulting from assuming isospin conservation during equilibration is a reduction in the intensity of the calculated ^{3}He emission spectra. The other spectra show much smaller effects that are related to the relative influence of all the kinds of factors mentioned with regard to the other projectiles. These reactions should be studied again, once a breakup component has been included in the overall calculations.

TABLE V. Isospin coupling coefficients for neutron-induced reactions on ^{27}Al, ^{54}Fe, and ^{58}Ni. The column labels and cell contents have the same significance as in Table III.

Emitted particles	^{27}Al+α, $T_<$	^{54}Fe+α or ^{58}Ni+α, $T_<$
n, t	1, 1/3	1, 1/4
$p, ^3$He	2/3, 0	3/4, 0
d, α	1, 0	1, 0

IV. EXPERIMENTAL EVIDENCE ON ISOSPIN CONSERVATION

A. During equilibration

Comparisons between calculation and experiment such as those shown in Figs. 1–5 have been carried out for all the reactions in the database, and the results were assigned to one of five categories: (i) good evidence that isospin is conserved (i.e., a good quantum number), (ii) weak evidence that isospin is conserved, (iii) calculations are insensitive or data comparisons are inconclusive, (iv) weak evidence that isospin is mixed (i.e., not a good quantum number), and (v) good evidence that isospin is mixed.

The results of these comparisons are shown in the upper panels of Fig. 6. They suggest that the criterion for isospin conservation during the preequilibrium phase of a reaction may be related to the relative sizes of the excitation energy of the intermediate nucleus and its symmetry energy; i.e., isospin is conserved when $E < K\,E_{\rm sym}$, where K is between 2.5 and five.

B. At equilibrium

For reactions at incident energies low enough so that no more than two nucleons are likely to be emitted, it was possible to interrogate the equilibrium portion of the spectrum. The purpose here is twofold. First, when isospin is conserved during the preequilibrium phase, there is the obvious question of the extent to which it is also conserved at equilibrium, where particle emission occurs over a much longer time span. Second, the data for some reactions may be sensitive to isospin conservation in the equilibrium phase of the reaction when they are not sensitive in the preequilibrium phase.

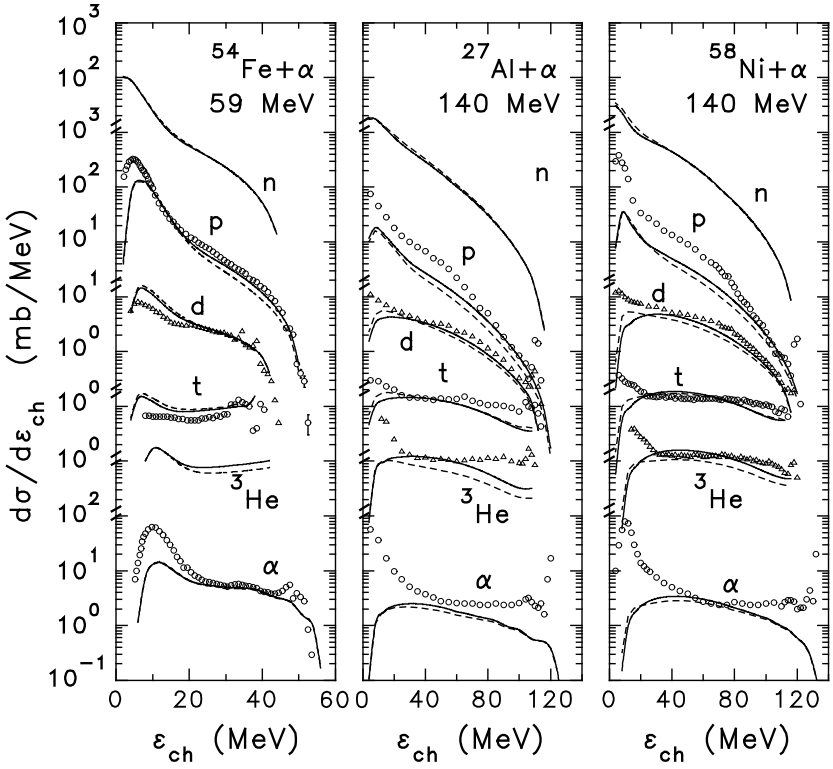

FIG. 5. The effects of isospin conservation in sample α-particle-induced reactions on light targets at 59 and 140 MeV. The points and curves have the same significance as in Fig. 3. The 59- and 140-MeV results are from Refs. [27] and [28], respectively. All of the results are given in terms of the channel energy.

To investigate these issues, additional calculations were run for nucleon-induced reactions at incident energies of 14 to 25 MeV. The additional calculations had isospin conserved during the preequilibrium phase of the reaction and either fully conserved or only 50% conserved at equilibrium. These results, along with the earlier ones with isospin conserved during equilibration but mixed at equilibrium, were compared with experiment. These comparisons indicate whether the data are sensitive to isospin conservation at equilibrium and, if so, what range of values for the extent of isospin conservation yield acceptable fits to the data.

Two points need to be stressed here. First, as discussed in Sec. III A 2, the conclusions on equilibrium isospin mixing in proton- and ^{3}He-induced reactions are model dependent. They can depend on the values chosen for the symmetry energies and on the form of the mean square matrix elements for the effective residual interactions used in the exciton model. Larger symmetry energies result in more late-stage preequilibrium emission from the $T_>$ states in proton- or ^{3}He-induced reactions on neutron-rich targets. This would reduce or possibly eliminate the need for isospin conservation at equilibrium, as was the case in the work of Watanabe *et al.* [7]. Conversely, mean-square matrix elements that increase with increasing exciton number would likely lead to less late stage preequilibrium emission and would increase the amount of isospin conservation needed at equilibrium.

Second, for the $T_>$ states in the intermediate nucleus in proton- and ^{3}He-induced reactions on very neutron-rich targets, there is no longer a sharp natural division between the preequilibrium and equilibrium contributions to the emission spectra. Thus the conclusions about isospin mixing at equilibrium can depend on the criterion used for the division between the two reaction phases.

With these qualifiers in mind, it is found that the (p, xp) reactions at 18 and 25 MeV on ^{103}Rh, ^{105}Pd, ^{106}Pd, and $^{\rm nat}$Ag show sensitivity to isospin conservation at equilibrium but were insensitive for preequilibrium emission. In addition, the (n, xp) reactions at 14.9 MeV on ^{46}Ti and ^{50}Cr weakly indicate isospin mixing at equilibrium. Both have excitation energies in the intermediate nucleus that are close to $4E_{\rm sym}$. The ^{56}Fe(n, xp) calculations also show a weak sensitivity, and again the data would suggest mixed isospin, even though $E/E_{\rm sym} = 2.8$. The difficulty here is that the calculated proton spectra all fall 35–40% below the data in the preequilibrium region of the spectrum, where isospin effects are negligible, and assuming isospin conservation at equilibrium simply gives the best agreement in spectral shape. This disagreement in intensity implies that some of the state density parameters may need adjustment, so the results here may be misleading. All of these points except that for ^{56}Fe are indicated in the lower panel in Fig. 6, where the solid triangles indicate evidence from the data for significant, though typically not full, isospin conservation. The indicated ranges for the fraction of isospin conservation at equilibrium are shown as a function of the ratio $E/E_{\rm sym}$ in Fig. 7.

C. A working prescription

Taken together, the results of Fig. 6 suggest that isospin is conserved during equilibration when

$$E < 4E_{\rm sym}. \tag{3}$$

This criterion is indicated by the dashed lines in the figure, and it accommodates all the results except for three reactions

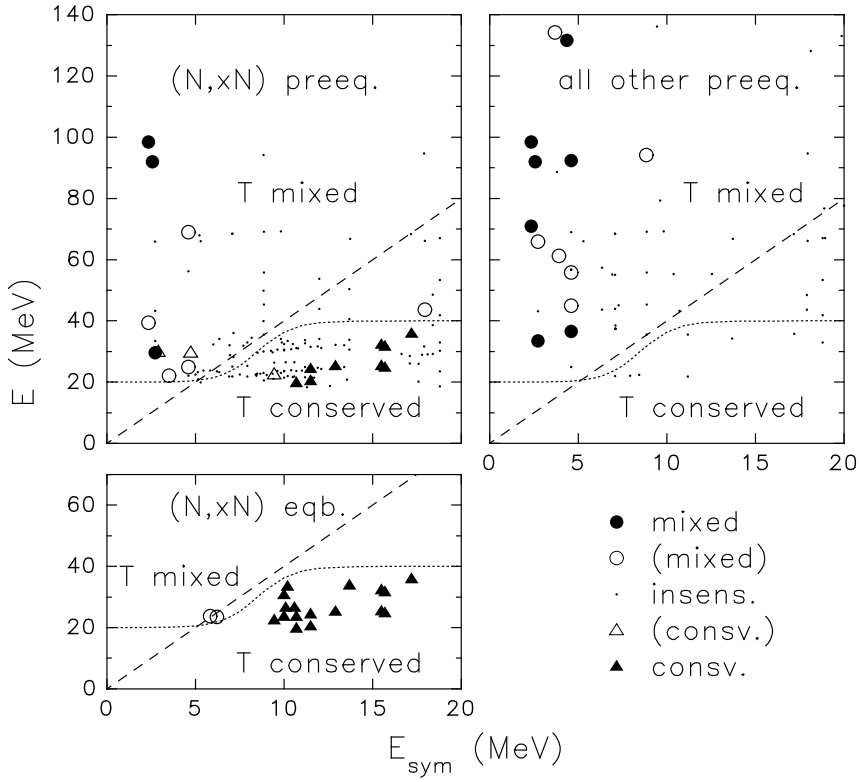

FIG. 6. Summary of experimental evidence on when isospin is conserved. The results are displayed as a function of the excitation energy and the symmetry energy in the intermediate nucleus. Solid points in the legend indicate stronger evidence for isospin mixing or conservation, whereas open symbols indicate weaker evidence. The dots represent systems for which the calculations are insensitive to isospin assumptions or where comparisons with the experimental results are inconclusive either because of the size of the error bars or because of discrepancies in spectral shape between calculation and experiment. The dots have been omitted from the panel displaying the equilibrium results. The dashed lines correspond to $E = 4E_{\rm sym}$ and show the adopted criterion in this work for separating the isospin mixed and isospin conserved regimes. The dotted lines give an alternative boundary between these regimes, given by Eq. (4).

where the evidence was fairly weak. However, many of the experimental systems are insensitive to the assumptions made about isospin conservation in the calculations. The data for which the calculated results are signficantly different depending on whether isospin is assumed to be conserved or mixed during equilibration tend to have $E_{\rm sym} \leqslant 5$ MeV or $E \leqslant 40$ MeV (or both). Thus the solid and open points in the figure lie fairly close to one of the axes in the plot, and, as a result, other functional forms for the boundary between the isospin mixed and isospin conserved domains are clearly possible. The conclusion of Eq. (3) is not inescapable. For example, because there are no reaction systems and no $E_{\rm sym}$ values for which there is strong evidence for isospin conservation at lower excitation energies and similarly strong evidence for mixing at higher excitation energies, the results in Fig. 6 could be accounted for almost as well by making a simple cut around $E_{\rm sym} = 8$ MeV, with isospin being conserved for larger values of the symmetry energy and mixed for smaller values. The only additional result that would not be accommodated is the weak evidence from ^{90}Zr(p,^{3}He) at 90 MeV ($E_{\rm sym} = 8.84$ MeV). Other more nearly horizontal boundary lines are also possible. One example is indicated by the dotted curves in Fig. 6, which have the form

$$E = 20\ \text{MeV} + \frac{20\ \text{MeV}}{1 + \exp(8.5\ \text{MeV} - E_{\rm sym})}. \qquad (4)$$

This has the advantage of accommodating the weak evidence for isospin mixing from the ^{209}Bi(p, xp) reaction ($E_{\rm sym} =$ 17.9 MeV) at 39 MeV. Finally, the results do not look very different if plotted versus the mass number of the system rather than the symmetry energy. In this case criteria such as a cut at $E = A_{\rm CN}/2$ or $A_{\rm CN} \cong 70$ could be used to divide the isospin mixed and conserved regions. The choice for a relationship such as Eq. (3) was made because of its mathematical simplicity and because the ratio between the excitation energy and the symmetry energy seemed

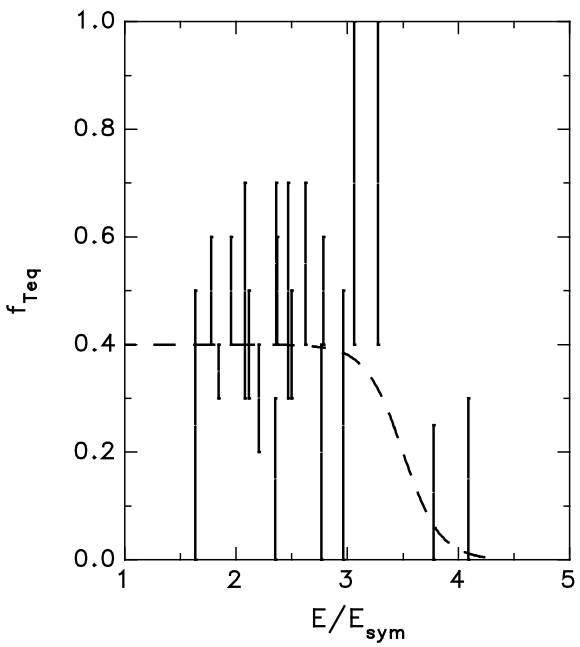

FIG. 7. Summary of experimental evidence on the amount of isospin conservation at equilibrium shown as a function of the ratio $E/E_{\rm sym}$. The vertical lines give the ranges of reasonable values extracted from the data, whereas the curve represents the prescription given by Eq. (5).

like a possibly relevant physical parameter, as discussed under Sec. V B.

The model-dependent estimates for the fraction of isospin conservation at equilibrium when isospin is conserved prior to the attainment of equilibrium show significant scatter and uncertainty. Nevertheless, they are generally consistent with a value of around 40%. The two (n, xp) reactions weakly indicating isospin mixing and shown in the lower panel of Fig. 6 can still be reasonably reproduced with some conservation, and these are the right-most two bars in Fig. 7. The results in this figure have been described by a mathematical form that is fairly constant at 0.4 for $E < 3E_{\rm sym}$ and then goes rapidly to zero as E approaches $4E_{\rm sym}$. This is represented by the dashed curve in Fig. 7, which is given by the relation

$$f_{\rm Teq} = \frac{0.4}{1 + \exp[6(E/E_{\rm sym} - 3.5)]} \quad \text{for } E/E_{\rm sym} < 4. \quad (5)$$

If some other boundary were chosen between the isospin mixed and isospin conserved domains, then clearly the form of this equation would have to change and might become significantly more complicated, even though the value of $f_{\rm Teq}$ would still be around 0.4 except for systems close to the boundary. In the specific case where the variable was changed from $E_{\rm sym}$ to $A_{\rm CN}$, and the boundary is described by $E = A_{\rm CN}/2$, then Eq. (5) would be replaced by

$$f_{\rm Teq} = \frac{0.4}{1 + \exp[36(E/A_{\rm CN} - 0.44)]} \quad \text{for } E/A_{\rm CN} < 0.5. \quad (6)$$

Together, Figs. 6 and 7 along with Eqs. (3) and (5) represent important results of this work.

V. PERSPECTIVES

A. Comparisons with evidence from compound nucleus reactions

Although the present conclusions with regard to when isospin is conserved during equilibration do not appear to conflict with other evidence from preequilibrium reaction data, they do seem contrary to some of the evidence from compound nucleus reactions. As mentioned in the introduction, the review article by Harney, Richter, and Weidenmüller [1] examined results from various types of compound nucleus reactions at excitation energies mainly around 20 MeV but extending up to near 40 MeV. All of them indicated significant isospin conservation at equilibrium and seemed to suggest that isospin is a better quantum number for light targets, where the symmetry energies are smallest. There were also indications that it was better conserved at higher excitation energies than at lower ones. The excitation energy dependence was supported by the work of Ref. [4]. These trends are the reverse of what might be inferred from the preequilibrium criterion of Eq. (3). Thus, it is worth looking at the compound nucleus results in more detail.

In fact, all of the apparent conflicts come from light nuclei with $T_z = 0$. The compound nuclei with $A \geqslant 49$ summarized in Ref. [1] have $T_z > 0$, meet the criterion of Eq. (3), and show results of partial isospin conservation at equilibrium that are quite consistent with the results of the present work. Many of these were studied by comparing the yields in (p, p'), (p, α), (α, p), and (α, α') reactions that all go through the same compound nucleus. Thus the entrance and exit channels are comparable to those studied in the present work.

All of the lighter systems involve reactions that populate $T_z = 0$ compound nuclei, ^{16}O through ^{34}Cl, so that the symmetry energies are quite low and $E > 4E_{\rm sym}$. (In all but one case E is also greater than $A/2$.) Thus the criterion of Eq. (3) would imply isospin mixing at equilibrium, whereas the results reported in Ref. [1] and the original references show a high degree of isospin conservation. The results of Ref. [4] likewise involve $T_z = 0$ nuclei (^{26}Al and ^{28}Si), though at somewhat higher excitation energies, and show partial isospin conservation at equilibrium, not full mixing. Significantly, however the kinds of reactions studied were quite different from the ones considered here.

In Ref. [4] and some cases in Ref. [1], the compound nuclei were formed by heavy-ion fusion, which would produce initial intermediate states with much larger numbers of particle and hole degrees of freedom and more angular momentum than the simple configurations prominent in the early phases of light-particle-induced reactions. In particular, the high angular-momentum values would tend to reduce the applicable state densities relative to light-particle-induced reactions and might significantly restrict the number of configurations available for mixing. In Ref. [4] and other cases in Ref. [1], it was assumed in the analysis that isospin mixing (the ratio between the mixing and escape widths of the $T_> = |T_z| + 1$ states) would be the same for these $T_z = 0$ nuclei as for the $T_z = 1$ nuclei with two extra neutrons, even though their symmetry energies differ in the current prescription by about a factor of 3. The results in Ref. [1] that are most like the light-particle reactions studied here involve comparing the relative intensities for populating discrete, low-lying final states that are isospin forbidden in the given reaction with those for states that are isospin allowed when T is a good quantum number.

With regard to the excitation energy dependence of the mixing fraction, the results of Ref. [4] indicate that for ^{26}Al and ^{28}Si, there is more isospin mixing in earlier measurements at excitation energies around 20 MeV than what they observe at 33 to 65 MeV, implying that isospin becomes a better quantum number at higher energies. However, closer inspection shows that, though the error bars are quite large, the specific results of that work for ^{26}Al, where the energy range studied was larger than for ^{28}Si, hint at more, not less, mixing as the excitation energy increases. A similar conclusion could be drawn by looking just at the comparison of measured and calculated γ-ray spectra for ^{28}Si at $E = 47$ and 63 MeV (shown in Fig. 2 of that article), without reference to the ^{30}Si results. Thus, although there is clearly a conflict with the present results in the *amount* of isospin mixing (they saw little mixing at equilibrium, whereas the present results from ^{27}Al+p show weak evidence for preequilibrium mixing at $E = 39$ MeV and good evidence at 71 and 98 MeV), the overall trend with excitation energy may not be in conflict with the implications of Eq. (3). And the difference in the amount of isospin mixing may be, at least in part, because of, the fact, that [4] was studying heavy-ion, not light-particle, reactions. However, the

trend between the previous 20-MeV results and the 33-MeV results of Ref. [4] remains unresolved, though the types of reactions studied were often different.

In the end, we simply need to recognize that there is some apparent disagreement between the results on isospin mixing from compound nucleus reactions with $T_z = 0$ intermediate nuclei and the results of the present preequilibrium studies, whereas the results on other systems seem to be in general agreement. This will require further investigation.

B. Possible physical significance

Although the form of the criterion for isospin conservation given in terms of E and $E_{\rm sym}$ was suggested by the results in Fig. 6, the general trend can be qualitatively supported by physical arguments. First, configuration mixing between states of different isospins will depend on their respective state densities. A higher excitation energy should favor isospin mixing because the state densities and the level widths both increase, so that there are more overlapping states that could potentially mix. The importance of the increasing state densities within an equilibrium mixing context was pointed out by Grimes [29]. The countervailing argument often cited in compound nucleus studies is that at higher excitation energies there is less time available for mixing to occur. How those two considerations should balance out is unknown, but the present results suggest that level density considerations may be dominant. The use of the symmetry energy as a crucial parameter and the state densities as significant physical quantities is supported by their important role in the strong isospin effects seen here in (p, p') reactions on neutron-rich targets at the lower incident energies.

In addition, an increasing symmetry energy (essentially the energy of the lowest lying state with $T = |T_z| + 1$) means that analog configurations will be further displaced in energy, so that states of different isospins at the same excitation energy are less likely to have similar wave functions, making mixing less likely. The countervailing argument here is that higher mass numbers (and the symmetry energy tends to increase with target mass as $N - Z$ values increase) typically result in higher single particle state densities and thus in higher total state densities at comparable excitation energies. This should favor isospin mixing. Again, both effects are likely present, with the current results suggesting that the symmetry energy effect may be dominant.

Because most mixing is expected to occur "downward," from the $T_>$ states into the typically more numerous $T_<$ states in the intermediate nucleus [30], the mixing rate should depend on the average number of $T_<$ states accessible from a given $T_>$ state and thus on their relative state densities. For states with a specific number of excitons, n, populated during equilibration, the ratio of the state densities for states with $T = T_>$ and $T = T_<$ is, to first approximation,

$$\frac{\omega(n, E, T = |T_z| + 1)}{\omega(n, E, T = |T_z|)} = \left(\frac{E - E_{\rm sym}}{E}\right)^{n-1}.$$

This can be expanded into a series in the ratio $E_{\rm sym}/E$ whenever $E \gg E_{\rm sym}$:

$$\frac{\omega(n, E, T = |T_z| + 1)}{\omega(n, E, T = |T_z|)} \approx 1 - (n-1)\frac{E_{\rm sym}}{E} + \frac{(n-1)(n-2)}{2!}\left(\frac{E_{\rm sym}}{E}\right)^2 - \cdots,$$

thus suggesting that this energy ratio is a reasonable parameter to describe mixing probabilities.

C. Calculations with gradual mixing

Although the proposed criterion for isospin conservation represents an empirical trend, there is no obvious reason to believe that there should be a sudden transition between preequilibrium isospin mixing and isospin conservation at $E/E_{\rm sym} = 4$ or at any other discrete boundary. A fully realistic calculation would take account of a gradual mixing of states of different isospin values. In principle, the mixing can be handled in very much the same way as the transitions between states of different particle-hole configurations in the exciton model, provided that the isospin nonconserving interaction is a small part of the total interaction. This was done in Ref. [31], where the time evolution of the equilibrating system was studied in the unified exciton model. There are, however, significant difficulties that have prevented this approach from being generally implemented.

First, it complicates the calculations, particularly in a two-component exciton model, where the particle-hole numbers of isobaric analog states are not always the same [16]. A two-component approach is necessary for the correct handling of shell structure and pairing effects, and the added complexity would carry over into the many secondary preequilibrium calculations required for reactions at higher excitation energies. One such equilibration calculation must be performed for each excitation energy in the residual nuclei formed by primary neutron and proton emission.

A second and more serious problem is determining the transition rates for isospin mixing. The present work is the first preequilibrium reaction study to look at a broad database, and the preceding discussion indicates that evidence from compound nucleus reactions is limited and not necessarily applicable to equilibration in light-particle-induced reactions. Rykbosch *et al.* [31] used rates estimated from isospin forbidden neutron decay of giant dipole resonance states populated by photon absorption. Assumptions also need to be made about what kinds of configurations will mix. Will they always have the same exciton number? Are they exclusively isobaric analog configurations or is the mixing broader? Much is simply unknown.

Given these difficulties and the frequent lack of sensitivity to isospin conservation observed in this and other work, the direct consideration of gradual isospin mixing seems unwarranted or, at least, premature within the scope of the exciton model. Conversely, the prescription given here (or one of its variants) represents a significant advance over simply ignoring isospin as a quantum number.

VI. SUMMARY AND CONCLUSIONS

This work has confirmed previous evidence that although calculated results for many statistical reactions are nearly independent of the assumptions made about isospin conservation, there are a few reactions for which it is important to make the right assumptions. The examination of a large database, including reactions with complex particles in the entrance and/or exit channel, have shown two types of evidence regarding isospin conservation.

For neutron-rich targets, the proton inelastic scattering reactions at incident energies up to around 30 MeV sometimes indicate that isospin is conserved during the energy equilibration process. When the symmetry energies of Eq. (1) are used and when the mean-square matrix elements for the residual two-body interactions in the exciton model are assumed to be independent of the exciton number of the configurations involved, the data also imply that isospin should be partially conserved in the compound nucleus or equilibrium part of the reaction. The sensitivity in the preequilibrium phase is largely determined by the relative sizes of the proton binding energy and the symmetry energy in the composite nucleus and is related to the way they enter into the exciton model particle emission rates.

For light targets, where N and Z are nearly equal, it is the isospin coupling coefficients—the squares of the isospin Clebsch-Gordan coefficients—that play a dominant role. This general effect was pointed out by Grimes [11] with regard to neutron and α-particle-induced reactions, and its manifestation in both exciton model and direct nucleon transfer calculations has been observed. The (n, p) and $(n,{}^3\mathrm{He})$ channels show the greatest sensitivity, and the data generally tend to indicate isospin mixing. Similarly, at incident energies of 60 to 90 MeV, there is fairly clear evidence for isospin mixing from (p, xn) reactions and from the yields in the corresponding complex particle channels.

The combined experimental evidence points to the working criterion that isospin should be assumed to be conserved during the preequilibrium phase of a reaction whenever $E < 4E_{\mathrm{sym}}$, though other criterion are possible because of the lack of data with both $E > 40$ and $E_{\mathrm{sym}} > 5$ MeV that are sensitive to isospin conservation. The suggested criterion seems to be qualitatively reasonable from simple physical arguments. Conversely, there is evidence from compound nucleus reactions at excitation energies up 65 MeV that point toward significant isospin conservation in $T_z = 0$ intermediate nuclei, where the present results suggest mixing. Many of the compound nucleus results, however, involve reactions that are quite different from the light particle reactions studied here.

When isospin is conserved during equilibration, the amount of isospin conservation at equilibrium is typically indicated to be around 40% and is given by Eq. (5), though this result is clearly model dependent.

There is still much that remains to be understood about the role of the isospin quantum number in continuum reactions, and it would clearly be useful to have data analyses that could fill in the open areas of the graphs in Fig. 6, if sensitive reactions can be identified. Unfortunately, calculations for a number of the reactions that give clear evidence for isospin conservation at incident energies of 18 to 25 MeV show that the effect of isospin conservation decreases as the incident energy (and thus the excitation energy in the intermediate nucleus) increases. The reason is that both the symmetry energy and the proton binding energy represent smaller and smaller fractions of the excitation energy. It is hoped that the comprehensive results of this investigation will be a useful guide for reaction calculations and will also stimulate further work in the field.

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