PHYSICAL REVIEW C **72**, 024319 (2005)

Energy shifts and configuration mixing in the $A = 15$ quartet

H. T. Fortune

Department of Physics & Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

R. Sherr

Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

(Received 18 May 2005; published 31 August 2005)

In a model that treats $T = 3/2$ states in $A = 15$ as combinations of one-particle, two-hole and three-particle, four-hole, states, we have computed Coulomb energies and hence are able to predict energies in ^{15}N, ^{15}O, and ^{15}F, given the energies in ^{15}C. Among our findings is that the first two levels of ^{15}F are reasonable well described in this $(0 + 2)\hbar\omega$ space, using only the three lowest configurations for each state.

DOI: 10.1103/PhysRevC.72.024319 PACS number(s): 21.10.Dr, 21.10.Jx, 21.10.Pc, 27.20.+n

In ^{15}F, the lowest $1/2^+$and $5/2^+$ energies are poorly known, and their widths are even less reliable. In ^{15}O, the $5/2^+$ state with $T = 3/2$ is known [populated in ^{17}O(p, t)], but the $1/2^+$ remains to be identified. (As we will see in the following, a tentative suggestion of $T = 3/2$ for a $1/2^+$ state at 11.56 MeV is probably not correct). In the present paper we present results of a simple calculation of expected energies (and widths, when unbound) of theses first two $T = 3/2$ states across the $A = 15$ quartet.

The earliest estimates of the $1/2^+$ energy in ^{15}F were $E_p = 1.6 \pm 0.2$ [1] and 1.37 ± 0.18 [2] MeV above threshold. The $5/2^+$ resonance was located 1.2–1.3 MeV above the $1/2^+$. Newer experiments [3,4] have provided $1/2^+$ energies from 1.29 to 1.51 MeV (see Table I) depending on reaction used and on definition of a wide resonance. For the "standard" definition of resonance energy having a phase shift of $\delta =$ 90 degrees, Goldberg *et al.* [4] found $1.45^{+0.16}_{-0.1}$ MeV, whereas for the peak in the interior wave function their energy was 1.29 MeV. This difference is discussed further in the following.

The $5/2^+$ energy is more reliably determined primarily because it is narrower and stronger. The two most recent energies for it are $E_p = 2.853 \pm 0.045$ [3] and 2.795 ± 0.045 [4]. Both these determinations come from elastic scattering of ^{14}O from a H target. We have computed these energies in ^{15}F and other members of the $A = 15$ quartet in a simple model.

In the simplest model, the ground state (gs) of ^{14}C consists of two proton holes in the 1p shell. In that same model, the lowest $1/2^+$ and $5/2^+$ states of ^{15}C are primarily a $2s_{1/2}$ and $1d_{5/2}$ neutron, respectively, coupled to ^{14}C(gs). Some coupling of the neutron to a p-shell 2^+ state is also expected as a component in those lowest states. The absence of other low-lying excited states in ^{14}C makes it unlikely that a single particle coupled to an excited core is responsible for all the missing strength. The next layer of complication in both ^{14}C and ^{15}C is a $2\hbar\omega$ excitation, that is, two (in ^{14}C) or three (in ^{15}C) sd-shell neutrons, respectively, coupled primarily to a ^{12}C ground state. The admixture of this configuration in the ^{14}C(gs) has been estimated [5] to be about 12% in intensity. Spectroscopic factors provide significant information about the structure components. For example, in the simplest model, the S factor in, for example, ^{14}C(d, p) is equal to the percentage of ^{14}C(gs) + n in the ^{15}C state. In a $(0 + 2)\hbar\omega$ picture, the situation is only slightly more complicated. In the present paper, we investigate the extent to which the level energies, widths, and/or S factors of the lowest $1/2^+$ and $5/2^+$ states in the $T = 3/2$, $A = 15$ quartet can be understood in these simple models.

We look first at the n spectroscopic factors in ^{14}C(d, p), of which there are only a few values in the literature. For definiteness, we take those of [6], which are listed in Table II. If we write (in an obvious notation)

$$^{15}\mathrm{C}(1/2_1^+) = \alpha(0^+ \otimes 2\mathrm{s}_{1/2}) + \beta(2^+ \otimes 1\mathrm{d}_{5/2}),$$

then, $\mathrm{S}(1/2_1^+) = \alpha^2$(with $\alpha^2 + \beta^2 = 1$). Similarly,

$$^{15}\mathrm{C}(5/2_1^+) = A(0^+ \otimes 1\mathrm{d}_{5/2}) + B(2^+ \otimes 2\mathrm{s}_{1/2}) + C(2^+ \otimes 1\mathrm{d}_{5/2}).$$

We ignore $1d_{3/2}$ throughout so that $S(5/2_1^+) = A^2$, with $A^2 + B^2 + C^2 = 1$. It is not surprising that α^2 is larger that A^2, because $s_{1/2}$ lies below $d_{5/2}$ and $2^+ \otimes$ d lies above $2^+ \otimes$ s, so that the unperturbed $5/2^+$ states will be closer together than the $1/2^+$ ones—increasing the mixing for $5/2^+$. Throughout this paper, for the energy of the 2^+ core state, we use the average of the lowest two known 2^+ levels [7], because they are nearly equal mixtures of 0 and $2\hbar\omega$, and it is the $0\hbar\omega$ 2^+ state to which we are coupling a nucleon.

We use a potential model with a Woods-Saxon nuclear potential whose geometrical parameters are $r_o = 1.25$ fm, $a =$ 0.65 fm, and whose strength is adjusted to reproduce the ^{15}C energies [7]. With addition of the Coulomb potential arising from a uniform sphere we use this potential to compute levels in ^{15}N, ^{15}O, and ^{15}F, properly weighting for z component of isospin in ^{15}N and ^{15}O. Results for the pure configurations and the mixture suggested by the spectroscopic factors are listed in Table III, along with the experimental levels to which they correspond. Two factors are apparent: (1) The appropriate state

TABLE I. Experimental energies and widths (both in MeV) of the lowest two states in ^{15}F.

	E_p	Γ	Ref.
Ground state $1/2^+$	1.6 ± 0.2	≈0.9	1
	1.37 ± 0.18	0.8 ± 0.3	2
	1.51 ± 0.15	1.2	3
	$1.45^{0.16}_{0.10}$	0.7	4 ($\delta = \pi/2$)
	$1.29^{0.08}_{0.06}$	0.7	4 (Ψ_{max})
First excited $5/2^+$	2.8 ± 0.2	0.24 ± 0.03	1
	2.67 ± 0.10	0.5 ± 0.2	2
	2.853 ± 0.045	0.34	3
	2.795 ± 0.045	0.32 ± 0.06	4

is not known in ^{15}O, and (2) the agreement with this extremely simple model is reasonably good.

If we add the next layer of complexity, we must consider $2\hbar\omega$ excitations in both $A = 14$ and $A = 15$. In ^{15}C, for example, the $2\hbar\omega$ $5/2^+$ state is taken to be the weak-coupling three-particle, four-hole (3p-4h) state whose 3p are the ^{19}O(gs) and whose 4h are the ^{12}C(gs). Similarly, the $2\hbar\omega$ $1/2^+$ in ^{15}C is 3p-4h, with the 3p corresponding to the $1/2^+$ state at an excitation energy of 1.472 MeV in ^{19}O [8]. Computation of the expected energies of these 3p-4h states, in the manner of Bansal-French Zamick [9], puts them between the $0^+ \otimes n$ and $2^+ \otimes n$ previously considered. It thus appears they should be included even in a simple calculation. Energy levels of the unmixed states are depicted in Fig. 1.

In ^{14}C then, the gs is $(1 - \epsilon^2)^{1/2}(2h) + \varepsilon(2p\text{-}4h)$, where an estimate of $\epsilon^2 = 0.12$ has been deduced from ^{12}C(t, p) results [5]. Then with

$$^{15}\mathrm{C}(1/2_1^+) = \alpha(0^+ \otimes 2s) + \beta(3p - 4h) + \gamma(2^+ \otimes d),$$

the reaction ^{14}C(d, p) can populate both the first two components, and the spectroscopic amplitude is

$$\mathcal{A}(^{15}\mathrm{C}(1/2_1^+) = \alpha(1 - \epsilon^2)^{1/2}(1.0) + \beta\epsilon\mathcal{A}(^{18}O(d, p)^{19}(1/2_1^+).$$

For the latter value we have used S factors from [10], also listed in Table II. It is clear, here, that S for $5/2^+$ in ^{19}O should be smaller than for $1/2^+$ because ^{18}O(gs) contains more d than s neutrons. In the very simplest models of ^{18}O and ^{19}O, the shell-model upper limits on these two quantities are 2/3 and 1 for $5/2^+$ and $1/2^+$, respectively. There is sufficient evidence that the ^{19}O($1/2^+$) S factor must be somewhat less than unity, but because of the small multiplication factor of $\beta\epsilon$, that difference is almost immaterial here. We return to this point later. With the added complexity of these $2\hbar\omega$ components, available information does not allow a unique solution for the coefficients α, β, and γ, even if the S factor is reproduced. However, there does arise a constraint on the range of possible values: $0.77 \leqslant \alpha^2 \leqslant 1.0$. This limitation arises from consideration of the relevant orthonormality conditions, plus the requirement that the amplitudes should be real. At the two limits, unique values of β and γ can be computed to reproduce S. In Table IV we present predictions of the $1/2^+$ energy in the other three nuclei for these two limits and one arbitrary value in between. These all have the same value of S but will have different values of computed particle-decay width only because the energies are different, and the width depends on energy. If we had used $S = 0.80$ rather than 1.0 for ^{19}O($1/2^+$), the limiting values of α^2, β^2, and γ^2 would have

TABLE II. Experimental spectroscopic factors for the lowest $1/2^+$ and $5/2^+$ states in ^{15}C and ^{19}O.

Reaction	E_x (MeV)	J^π	S	Ref.
^{14}C$(d, p)^{15}$O	0.0	$1/2^+$	0.88	6
	0.74	$5/2^+$	0.69	6
^{18}O$(d, p)^{19}$O	0.0	$5/2^+$	0.57	10
	1.47	$1/2^+$	1.00	10

TABLE III. Computed energies (MeV) in $A = 15$, $T = 3/2$ nuclei, compared with experimental ones (Excitation energies in ^{15}C,^{15}N, and ^{15}O; E_p in ^{15}F).

		^{15}C	^{15}N	^{15}O	^{15}F
$1/2^+$	Exp[a]	0	11.615(4)	—	1.29–1.6[b]
	$0^+ \otimes 2s$	0	11.558	11.011	1.20
	$\langle 2^+\rangle \otimes 1d$	0	11.811	11.604	2.22
	0.88, 0.12 mixture	0	11.588	11.082	1.32
$5/2^+$	Exp[a]	0.74	12.522	12.225	2.7–2.8[b]
	$0^+ \otimes 1d$	0.74	12.499	12.222	2.76
	$\langle 2^+\rangle \otimes 2s$	0.74	12.491	12.202	2.72
	0.7, 0.3 mixture	0.74	12.496	12.216	2.75

[a]From ref 7.
[b]See Table I.

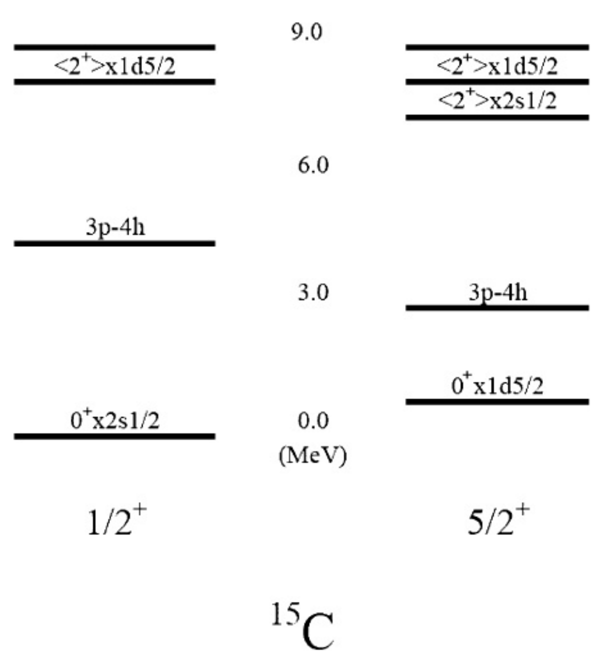

FIG. 1. Energy levels of the expected components of the ground and first-excited states of ^{15}C.

TABLE IV. Computed energies (in MeV) for lowest $T = 3/2$, $1/2^+$ state in ^{15}N, ^{15}O(E_x), and ^{15}F(E_p) for three allowed wave-function admixtures. All values have $S = 0.88$.

$0^+ \otimes 2s$	3p-4h	$\langle 2^+ \rangle \otimes 1d$	^{15}N	^{15}O	^{15}F[a]
1.0	0	0	11.558	11.011	1.19 (1.39)
0.93	0.01	0.06	11.581	11.058	1.26 (1.45)
0.77	0.11	0.12	11.682	11.218	1.36 (1.51)
Exp [4,7]			11.615	—	1.29 (1.45)

[a]Numbers in parentheses correspond to $\delta = \pi/2$.

been 0.81, 0.09, and 0.10, respectively, rather than 0.77, 0.11, and 0.12, as given in Table IV. Similar results for $5/2^+$ are listed in Table V.

Throughout the results presented here, we have defined the position of an unbound state as the energy at which the energy derivative of the phase shift ($d\delta/dE$) peaks. For broad states this definition is known to lead to an energy that is lower than that estimated using other definitions (e.g., $\delta = \pi/2$). This difference is small for the $5/2^+$ state, but it amounts to about 200 keV for the $1/2^+$ level. Goldberg *et al.* [4] presented experimental results for both definitions of energy for the $1/2^+$ level of ^{15}F, so we can directly compare, as we do in Table IV. The values for $\delta = \pi/2$ are in parentheses in the last column.

TABLE V. Same as Table IV, but for the $5/2^+$ state. These all have $S = 0.69$.

$0^+ \otimes 1d$	3p-4h	$\langle 2^+ \rangle \otimes 2s$	^{15}N	^{15}O	^{15}F
0.79	0.00	0.213	12.497	12.218	2.75
0.68	0.05	0.27	12.516	12.243	2.77
0.64	0.09	0.27	12.509	12.240	2.78
Exp [4,7]			12.522	12.225	2.7–2.8

For the $5/2^+$ state, the sp $l = 2$ decay width is close to 250 keV for all the allowed wave functions. With $S = 0.69$, the computed decay width for $5/2^+$ would thus be 172 keV. For the $1/2^+$ state, the $d\delta/dE$ results for Γ_{sp} vary from 440 keV at $E_p = 1.19$ MeV to 640 keV at $E_p = 1.36$ MeV, giving (with $S = 0.88$) $\Gamma_{calc} = 0.39$–0.56 MeV. For $\delta = \pi/2$ the width is larger even for the same proton energy, but the $\delta = \pi/2$ results also produce higher resonance energies, giving $\Gamma_{calc} = 800$–910 keV for $E_p = 1.39$–1.51 MeV.

We thus conclude that inclusion of the two simplest additions to an extreme sp picture—namely, coupling of a sp to a 2^+ core state, and including the lowest 3p-4h configuration—provide an adequate description of the energies and widths of the two lowest $T = 3/2$ states in the $A = 15$ quartet.

[1] W. Benenson *et al.*, Phys. Rev. C **17**, 1939 (1978).
[2] G. J. KeKelis *et al.*, Phys. Rev. C **17**, 1929 (1978).
[3] W. Peters *et al.*, Phys. Rev. C **68**, 034607 (2003).
[4] V. Z. Goldberg *et al.*, Phys. Rev. C **69**, 031302(R) (2004).
[5] H. T. Fortune and G. S. Stephans, Phys. Rev. C **25**, 1 (1982).
[6] J. D. Goss, P. L. Jolivette, C. P. Browne, S. E. Darden, H. R. Weller, and R. A. Blue, Phys. Rev. C **12**, 1730 (1975).
[7] F. Ajzenberg-Selove, Nucl. Phys. **A523**, 1 (1991).
[8] D. R. Tilley, H. R. Weller, C. M. Cheves, and R. M. Chasteler, Nucl. Phys. **A595**, 1 (1995).
[9] R. Bansal and J. B. French, Phys. Rev. Lett. **11**, 145 (1964); L. Zamick, Phys. Lett. **19**, 580 (1965); R. Sherr and G. Bertech, Phys. Rev. C **12**, 1671 (1975).
[10] S. Sen, S. E. Darden, H. R. Hiddleston, and W. A. Yoh, Nucl. Phys. **A219**, 429 (1974).