One-particle resonant levels in a deformed potential

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Solving the Schrödinger equation in coordinate space with the appropriate asymptotic boundary conditions, neutron one-particle resonant levels in Y_{20}-deformed Woods-Saxon potentials are studied. These resonance levels are the natural extension of one-particle bound levels to the continuum and are defined in terms of eigenphase. For one-particle bound levels with $\Omega^\pi \neq 1/2^+$ the corresponding one-particle resonant levels can be always found for small positive energies. For one-particle bound levels with $\Omega^\pi = 1/2^+$ the corresponding one-particle resonant levels are either absent or disappearing quickly as energy increases, when we use well-deformed potentials with a realistic size of diffuseness. The possible presence of $\Omega^\pi = 1/2^+$ one-particle resonant levels, in which $\ell = 0$ components in the wave functions play a crucial role, is further studied using a simplified model without spin-orbit potential.

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I. INTRODUCTION

The properties of nuclei far from the β stability line, especially close to the neutron drip line, provide a challenge to the conventional theory of nuclear structure. A characteristic feature unique in the system with some weakly bound neutrons is the importance of the coupling to the nearby continuum of unbound states, as well as the impressive role played by weakly bound neutrons with low orbital angular momenta ℓ. Weakly bound small ℓ neutrons have an appreciable probability to be outside the core nucleus and are thereby insensitive to the strength of the potential provided by the well-bound core nucleons, while the wave functions of weakly bound large ℓ neutrons stay mostly inside the nuclear potential. Since the Fermi level of drip line nuclei in the mean-field approximation is very close to the continuum, the many-body correlation in the ground state necessarily receives contributions by some of the infinite number of one-particle levels in the continuum. The role of low-lying positive-energy one-particle levels in the representative many-body correlations, deformation and pair correlation, is currently of major interest [1].

In Ref. [2] we have studied the many-body pair correlation of spherical neutron-drip-line nuclei, while in Ref. [3] the structure of weakly bound neutron orbits in deformed potentials is investigated in the absence of pair correlation. Thus, we are almost ready to proceed to studying the structure of neutron-drip-line nuclei in the presence of both deformation and pair correlation. While the effective pair gap of weakly bound neutrons in deformed nuclei has been investigated in Ref. [4], we have realized the absolute necessity for having a good knowledge of one-particle resonant levels in deformed potentials, before we can fully understand both deformation and many-body pair correlation in neutron-drip-line nuclei. In the present work we are not interested in the one-particle resonances in general, which may play a role, for example, in the neutron scattering on deformed nuclei. Instead, we concentrate on studying the one-particle resonant levels in deformed potentials, which can be regarded as a natural extension of one-particle bound levels in the same potentials. These resonant levels will play a crucial role in neutron-drip-line nuclei, when the many-body pair-correlation is taken into account on top of deformation.

When we examine a weakly bound system, it is of basic importance not to limit the system to a finite box, since we are particularly interested in the behavior of one-particle resonant levels to which $\ell = 0$ neutrons contribute. We must solve the Schrödinger equation in coordinate space with appropriate boundary conditions [5,6]. Searching for the behavior of resonant wave functions sufficiently outside the deformed potential we find it appropriate to define one-particle resonant levels in terms of eigenphase, which increases through $\frac{1}{2}\pi$ as energy increases. The eigenphase is defined as the phase shift which is obtained by diagonalizing the S matrix. See Ref. [7] for eigenphase, and references quoted therein.

In Sec. II our model and formulas are presented, while numerical results and discussions are given in Sec. III. Conclusions and perspectives are given in Sec. IV.

II. MODELS AND FORMULAS

In the present paper we study the structure of one-particle orbits in axially symmetric quadrupole-deformed Woods-Saxon potentials, solving the Schrödinger equation in coordinate space with appropriate asymptotic boundary conditions. While one-particle bound levels in deformed potentials are obtained by solving the eigenvalue problem, we look for one-particle resonant levels, which can be regarded as a natural continuation of weakly bound neutron levels in the same deformed potentials.

Except for a slight change of the expression of $k(r)$ we use the same model Hamiltonian as that used in Ref. [3], in which one-particle bound orbits are studied. Our one-body potential consists of the following three parts:

$$\begin{aligned} V(r) &= V_{\mathrm{WS}} f(r), \\ V_{\mathrm{coupl}}(\vec{r}) &= -\beta k(r) Y_{20}(\hat{r}), \\ V_{\mathrm{so}}(r) &= -V_{\mathrm{WS}} v \left(\frac{\Lambda}{2}\right)^2 \frac{1}{r}\frac{df(r)}{dr}(\vec{\sigma}\cdot\vec{\ell}), \end{aligned} \tag{1}$$

where Λ is the reduced Compton wavelength of the nucleon $\hbar/m_r c$,

$$f(r) = \frac{1}{1+\exp\left(\frac{r-R}{a}\right)} \tag{2}$$

and

$$k(r) = r V_{\rm WS} \frac{df(r)}{dr}. \tag{3}$$

In the following, if it is not specifically mentioned, we employ $a = 0.67$ fm, $V_{\rm WS} = -51$ MeV, and $v = 32$, which are the standard parameters used in β stable nuclei [8]. The nuclear radius R is varied so as to vary the strength of our one-body potential. In other words, we vary the mass number A of the system with $R = r_0 A^{1/3}$ where $r_0 = 1.27$ fm is used. This manner of viewing the changing potential strength has the disadvantage of changing the height of the centrifugal barrier, which can be crucial especially to the discussion of the widths of one-particle resonances. An alternative way is to vary the values of $V_{\rm WS}$ for a given radius R. However, then, different $\Omega^\pi = 1/2^+$ levels belonging to a given major shell cannot be compared for realistic values of $V_{\rm WS}$. In any case, we have confirmed that our findings described in the present article can be obtained also by varying $V_{\rm WS}$ values. In the expression (1) we have included only the lowest-order term in deformation parameter β of the deformed Woods-Saxon potential. This is an approximation, but this simple form of the deformed potential is sufficient for the present purpose of exhibiting the possible continuation of one-particle bound levels into the one-particle resonance spectrum. Writing the single-particle wave function as

$$\Psi_\Omega(\vec{r}) = \frac{1}{r}\sum_{\ell j} R_{\ell j\Omega}(r)\mathbf{Y}_{\ell j\Omega}(\hat{r}), \tag{4}$$

which satisfies

$$H\Psi_\Omega = \varepsilon_\Omega \Psi_\Omega, \tag{5}$$

where Ω expresses the component of one-particle angular momentum $\vec{j}$ along the symmetry axis, which is a good quantum number, and

$$\mathbf{Y}_{\ell j\Omega}(\hat{r}) \equiv \sum_{m_\ell, m_s} C\left(\ell, \frac{1}{2}, j; m_\ell, m_s, \Omega\right) Y_{\ell m_\ell}(\hat{r})\chi_{m_s}. \tag{6}$$

The coupled equations for the radial wave functions are written as

$$\left\{\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2m}{\hbar^2}[\varepsilon_\Omega - V(r) - V_{\rm so}(r)]\right\} R_{\ell j\Omega}(r) = \frac{2m}{\hbar^2}\sum_{\ell' j'}\langle \mathbf{Y}_{\ell j\Omega}|V_{\rm coupl}|\mathbf{Y}_{\ell' j'\Omega}\rangle R_{\ell' j'\Omega}(r), \tag{7}$$

where

$$\begin{aligned}\langle \mathbf{Y}_{\ell j\Omega}|V_{\rm coupl}|\mathbf{Y}_{\ell' j'\Omega}\rangle &= -\beta k(r)\langle \mathbf{Y}_{\ell j\Omega}|Y_{20}(\hat{r})|\mathbf{Y}_{\ell' j'\Omega}\rangle \\ &= -\beta k(r)(-1)^{\Omega-1/2}\sqrt{\frac{(2j+1)(2j'+1)}{20\pi}} \\ &\quad \times C(j, j', 2; \Omega, -\Omega, 0)C\left(j, j', 2; \frac{1}{2}, -\frac{1}{2}, 0\right).\end{aligned} \tag{8}$$

The eigenvalues $\varepsilon_\Omega(<0)$ of the coupled equations (7) for a given value of Ω, which is equivalent to Ω appearing in the asymptotic quantum numbers $[Nn_z\Lambda\Omega]$, are obtained by solving the equations in coordinate space for a given set of potential parameters, with both the condition, $R_{\ell j\Omega}(r) = 0$ at $r = 0$, and the asymptotic behavior of $R_{\ell j\Omega}(r)$ for $r \to \infty$ as

$$R_{\ell j\Omega} \propto r h_\ell(\alpha_b r), \tag{9}$$

where $h_\ell(-iz) \equiv j_\ell(z) + i n_\ell(z)$, in which j_ℓ and n_ℓ are spherical Bessel and Neumann functions, respectively, and

$$\alpha_b^2 \equiv -\frac{2m\varepsilon_\Omega}{\hbar^2}. \tag{10}$$

The normalization condition is written as

$$\sum_{\ell, j}\int_0^\infty |R_{\ell j\Omega}(r)|^2 dr = 1. \tag{11}$$

For the levels in the continuum $\varepsilon_\Omega > 0$ we solve the coupled equations (7) in coordinate space for a given set of potential parameters, requiring

$$R_{\ell j\Omega}(r) = 0 \qquad \text{at} \quad r = 0 \tag{12}$$

and the asymptotic behavior of $R_{\ell j\Omega}(r)$ for $r \to \infty$ as

$$R_{\ell j\Omega}(r) \propto \cos(\delta_\Omega) r j_\ell(\alpha_c r) - \sin(\delta_\Omega) r n_\ell(\alpha_c r), \tag{13}$$

where

$$\alpha_c^2 \equiv \frac{2m}{\hbar^2}\varepsilon_\Omega. \tag{14}$$

The coupled equations (7) are integrated both outward from $r = 0$ and inward from a large r value. Then, we look for the eigenphase δ_Ω so that at $r = R_m$ we can match a combination of inward-integrated (ℓ, j) wave functions and that of their derivatives with a combination of the outward-integrated (ℓ, j) wave functions and that of their derivatives, respectively. The way of solving the coupled channel equations (7) is taken from Ref. [9]. We note that the eigenphase is common to all the open channels, (ℓ, j), for given Ω. For given ε_Ω and potential we have several solutions for δ_Ω. The number of solutions is equal to that of wave function components with different (ℓ, j) values. The value of δ_Ω determines the relative amplitudes of different (ℓ, j) components. The total normalization of the positive-energy deformed one-particle wave functions can be at present left arbitrary.

As one-particle resonant levels we look for the ε_Ω values, for which the eigenphase δ_Ω increases through $\frac{1}{2}\pi$ as ε_Ω increases. Furthermore, at present we are interested only in those one-particle resonant levels, for which the energies are smoothly connected to eigenenergies of weakly bound one-particle levels. Inside the nuclear potential the relative amplitudes of wave function components of those one-particle resonant levels with very small positive energies are very similar to those of weakly bound one-particle levels, though the normalization of bound states is totally different from that of positive-energy states. When we find one-particle resonant levels in terms of eigenphase, we calculate the width of the

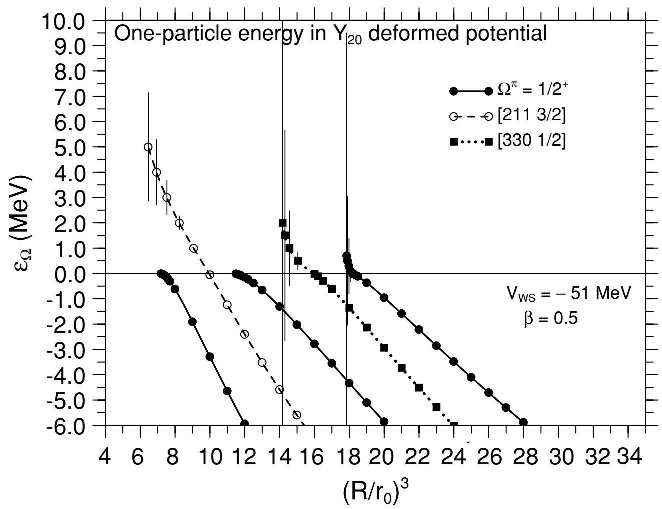

FIG. 1. Neutron one-particle levels in axially-symmetric quadrupole-deformed Woods-Saxon potentials as a function of the potential strength. The radius of the Woods-Saxon potential is expressed by R, while $r_0 = 1.27$ fm is used. The asymptotic quantum numbers $[Nn_z\Lambda\Omega]$ assigned traditionally to those levels are denoted for the [211 3/2] and [330 1/2] levels, while those of three $\Omega^\pi = 1/2^+$ levels plotted are, from left to right, [220 1/2], [211 1/2], and [200 1/2]. The width of one-particle resonant levels with $\varepsilon_\Omega > 0$ denoted by thin vertical lines is calculated in terms of eigenphase, which is defined by Eq. (15). See the text for details.

resonance using the formula

$$\Gamma \equiv \frac{2}{\frac{d\delta_\Omega}{d\varepsilon_\Omega}}, \tag{15}$$

where the denominator is calculated at the resonance energy.

III. NUMERICAL CALCULATIONS AND DISCUSSIONS

Since we have presented in Ref. [3] the structure of weakly bound one-neutron orbits in Y_{20}-deformed Woods-Saxon potentials taking examples of *sd*-shell nuclei, in the present work we keep the same mass number region showing numerical results. In Sec. III A we examine one-particle levels for realistic parameters, while in Sec. III B we simplify the deformed potential by switching off the spin-orbit potential so that the deformation couples only two channels, s and d.

A. One-particle levels with realistic parameters

In Fig. 1 we show calculated neutron energy levels in axially-symmetric quadrupole-deformed Woods-Saxon potentials as a function of the potential strength, as described by the radius of the potential. The chosen deformation parameter $\beta = 0.5$ may appear a bit large; however, we consider it realistic, since the example involves a light deformed nucleus and, moreover, we have included only a linear term of deformation in Eq. (1). In order to have an easily readable figure, we have included only the [211 3/2] and [330 1/2] levels in addition to all three $\Omega^\pi = 1/2^+$ levels in the *sd* shell. In the calculation of positive-parity levels we have included $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$ orbits, while $p_{1/2}$, $p_{3/2}$, $f_{5/2}$, and $f_{7/2}$ are coupled for negative-parity levels. All bound energies $\varepsilon_\Omega < 0$ are calculated in the same way as those shown in Fig. 1 of Ref. [3]. The only difference from Ref. [3] is the expression used for $k(r)$ in Eq. (3). Though the resulting numerical difference is small, the presently adopted expression (3) appears more reasonable when a stretching of length scale rather than a displacement of the surface is considered.

The major component of the well-bound [330 1/2] level is $f_{7/2}$, while the p component, especially $p_{3/2}$, becomes larger as the binding energy approaches zero as shown in Fig. 4 of Ref. [3]. As the resonant energy for $\varepsilon_\Omega > 0$ increases, the width rapidly increases. Consequently, the resonance is not identified above 2 MeV, which is approximately the energy characteristic of the disappearance of $\ell = 1$ one-particle resonances having the centrifugal barrier height for $\ell = 1$ orbits. The [211 3/2] level has no $\ell = 0$ component and the possible lowest ℓ component is $\ell = 2$. Therefore, as seen in Fig. 1, the continuation as a resonant level into the region of $\varepsilon_\Omega > 0$ can be seen up to about 10 MeV. Among the three $\Omega^\pi = 1/2^+$ levels, which can be identified as [220 1/2], [211 1/2] and [200 1/2] from the left to the right in Fig. 1, we find no one-particle resonant states as a continuation of the bound [220 1/2] and [211 1/2] levels. The vanishing slope of one-particle eigenvalues for $|\varepsilon| \to 0$ is a characteristic feature of $\ell = 0$ orbits in spherical finite-well potentials. The tendency of the vanishing slope is clearly seen in the [220 1/2] and [211 1/2] eigenvalues of Fig. 1. The absence of the resonant states can be easily confirmed by examining the eigenphase for the potentials, of which the strength is slightly weaker than that of the potential producing almost zero-binding energy. In those weaker potentials in which no weakly bound state exists any longer, the eigenphase steeply increases from zero (or *mod* $n\pi$) as energy increases from zero; however, the phase starts to decrease before reaching $\pi/2$. For illustration, in Fig. 2(a) the eigenphase as a function of ε_Ω is exhibited for the potential producing a weakly bound level, while in Fig. 2(b) the one for the slightly weaker potential.

For the [200 1/2] level a continuation by one-particle resonant levels is found, as plotted in Fig. 1. However, the large width becomes increasingly larger and the resonant states disappear before reaching $\varepsilon_\Omega = 0.8$ MeV. The existence of resonant states is realized by the admixed $\ell = 2$ components, while the large width is due to the admixed $\ell = 0$ component. See Fig. 5 and discussions in Sec. III B. For comparison between the wave functions of one-particle bound and resonant states inside the potential range, in Fig. 3(a) we plot the components of the radial wave function with $\varepsilon_\Omega = -0.1$ keV, while in Fig. 3(b) those with $\varepsilon_\Omega = +100$ keV are shown. In Fig. 3(a) we have artificially normalized the bound-state wave function by integrating only up to $r_{\max} = 60$ fm, in order to keep a finite size of the amplitudes inside the potential. We should compare only the relative amplitudes of various components in Fig. 3(a) with those in Fig. 3(b), since the normalization of resonant states is arbitrary and furthermore continuum wave functions have a dimension different from that of bound states. It is seen that the relative amplitudes of various components

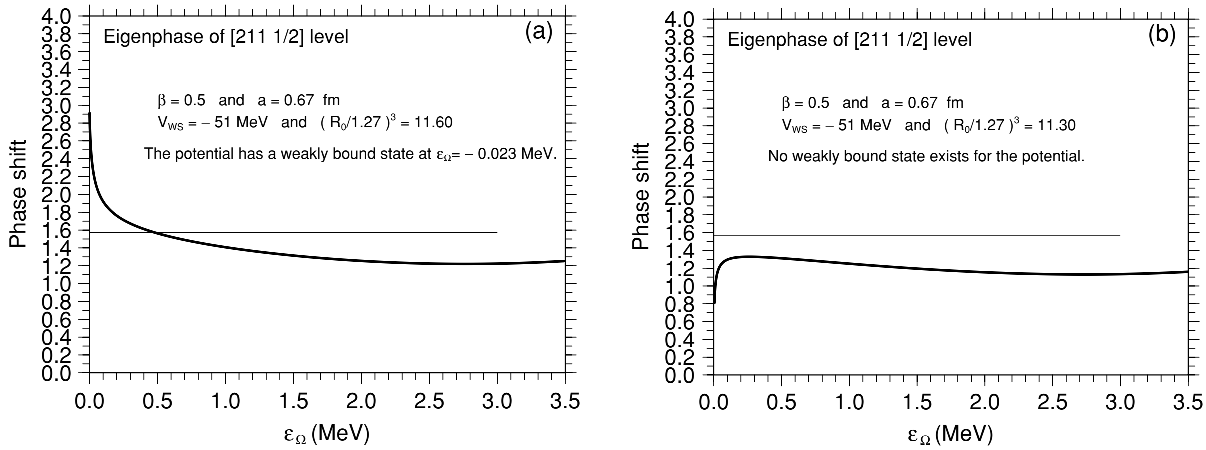

FIG. 2. (a) Eigenphase as a function of energy, for the potential producing a weakly bound state at $\varepsilon_\Omega = -0.023$ MeV. The value of $\pi/2$ is denoted by the thin horizontal line. (b) The same as in (a), but for the potential, which is slightly weaker and consequently produces no weakly bound state. Note that the eigenphase starts to decrease before reaching $\pi/2$.

inside the potential range in Fig. 3(b) are very similar to those in Fig. 3(a). A prominent difference between Figs. 3(a) and 3(b) is the behavior of the $s_{1/2}$ radial wave function, since the absence of the centrifugal barrier for $\ell = 0$ orbits affects the $s_{1/2}$ wave function already for very small values of $\varepsilon_\Omega > 0$.

The statement that the major component of $\Omega^\pi = 1/2^+$ bound levels in a Y_{20}-deformed finite-well potential becomes $s_{1/2}$ as the binding energy approaches zero [3,10] comes from the huge $s_{1/2}$ tail outside the potential seen in Fig. 3(a). Namely, it comes from the normalization unique in bound states and, in this sense, the statement is valid for all $\Omega^\pi = 1/2^+$ levels without exception. Nevertheless, this predominance of $s_{1/2}$ component in the weakly-bound wave functions does not control the structure of possible one-particle resonant levels just above zero energy. A critical element to provide the characters of resonant states is the relative amplitudes of various components inside the potential range, which are the characterization of respective one-particle levels and are the smoothly-varying quantities as energy increases from negative to positive values. Since the presence of three components, $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$, produces an interesting but complicated structure, in the following section we study a simplified model switching off the spin-orbit potential.

B. A simple model without spin-orbit potential

For a typical deformation the one-particle levels usually have several appreciable components with different ℓ values. In other words, the component of a given $\ell = 0$ orbit in one major shell is mixed into several one-particle levels with $\Omega^\pi = 1/2^+$.

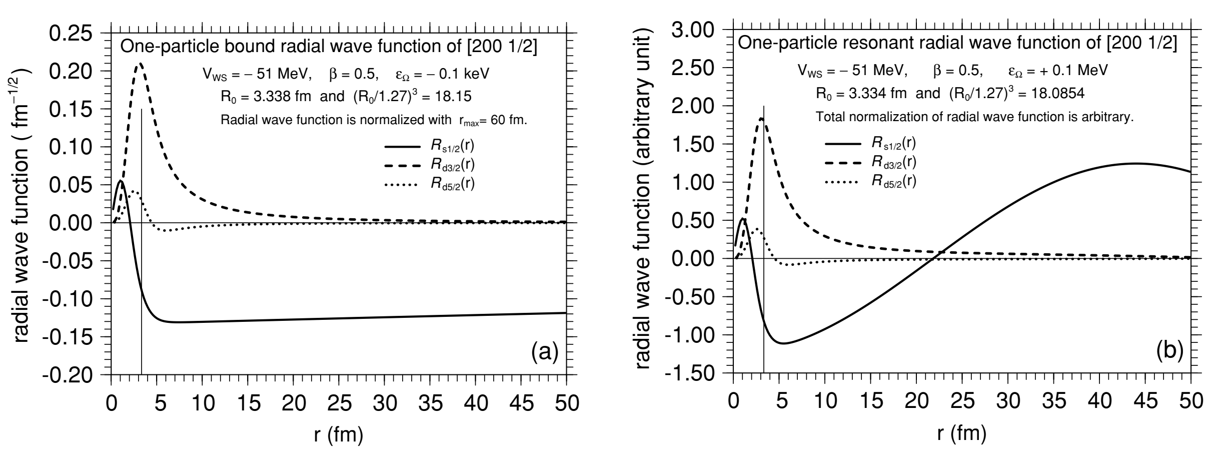

FIG. 3. (a) Components of the bound radial wave function of the [200 1/2] level as a function of radial coordinate. The eigenenergy is -0.1 keV. In order not to have almost vanishing amplitudes for finite values of r, the components are shown, which are obtained by integrating the total wave function only up to $r_{\rm max} = 60$ fm and normalizing it to unity. (b) Components of the resonant radial wave function of the [200 1/2] level as a function of radial coordinate. The resonance energy is $+100$ keV, in which the resonance is defined using the eigenphase. Only the relative amplitudes of various components are meaningful, since the total normalization of the resonant wave function is arbitrary.

Since the admixed $\ell = 0$ component leads to either absence of one-particle resonant levels or rapidly growing width of possible resonant levels, we wish to examine the role of the $\ell = 0$ component simplifying the model as much as possible. In the present subsection we study a model consisting of the two orbits, $\ell = 0$ and 2, without spin-orbit potential. The absence of spin-orbit potential implies that in this subsection Ω expresses the component of one-particle orbital angular momentum along the symmetry axis. Furthermore, in order to control the mixture of the two components, we examine a very small deformation, $\beta = 0.1$.

In Fig. 4(a) we plot $\Omega^\pi = 0^+$ neutron one-particle levels for $\beta = 0.1$ keeping all other parameters the same as before. For the spherical Woods-Saxon potential with standard parameters [8] the $1d$ level lies lower for deeply-bound orbits, while the $2s$ level lies lower for weakly-bound orbits. Therefore, the solid curves in Fig. 4(a) exhibit a slightly complicated behavior. For reference, switching off the deformation coupling between the $\ell = 0$ and 2 channels, we plot the eigenvalues with s and d by the dotted and dashed curves, respectively, where the diagonal term of the deformed Hamiltonian with $\beta = 0.1$ is already included. First of all, from the comparison between the solid, dotted, and dashed curves around the crossing point of the latter two, $(R/r_0)^3 \approx 17.2$, we see that the sd-coupling matrix element is slightly less than 1 MeV for the present $\beta = 0.1$. For the lower-lying $\Omega^\pi = 0^+$ level, which for convenience we may call [220], we find no continuation of resonant levels. The absence of resonant levels may be expected, since the wave function at small binding energy consists predominantly of $\ell = 0$ component. In contrast, the higher-lying $\Omega^\pi = 0^+$ level, which may for convenience be called by [200] though the wave function is very different from that given by the asymptotic quantum numbers [200], has a very short continuation of resonant levels with rapidly growing widths.

In Fig. 5 the calculated eigenphase as a function of energy $\varepsilon_\Omega > 0$ is shown taking the potential, in which the [200] one-particle resonant level is obtained at $\varepsilon_\Omega^{\text{res}} = 0.6$ MeV with $\Gamma = 1.35$ MeV. If the potential strength is weaker, namely if a slightly smaller value of $(R/r_0)^3$ is chosen, the eigenphase starts to decrease before reaching $\pi/2$ and thus the one-particle resonant state is not obtained. The energy dependence of the eigenphase obtained at very low energy is different from that of the $\ell = 0$ phase shift which should increase proportional to $\sqrt{\varepsilon_\Omega}$. Instead, it comes clearly from the $\ell = 2$ component. It is also well known that the $\ell = 0$ phase shift alone never increases through $\pi/2$ as energy increases. On the other hand, if the $\ell = 0$ component is responsible for the width of the one-particle resonance, we may expect for very small energy,

$$\Gamma_{\ell=0} \propto \sqrt{\varepsilon} P(\ell = 0), \tag{16}$$

where $P(\ell = 0)$ is an appropriate measure of the relative probability of the s component in the interior region of the potential. We can, for example, obtain $P(\ell = 0)$ using the s and d components of the calculated resonant state and the expression

$$P(\ell = 0) = \frac{\langle s|V(r)|s\rangle}{\langle d|V(r)|d\rangle + \langle s|V(r)|s\rangle}. \tag{17}$$

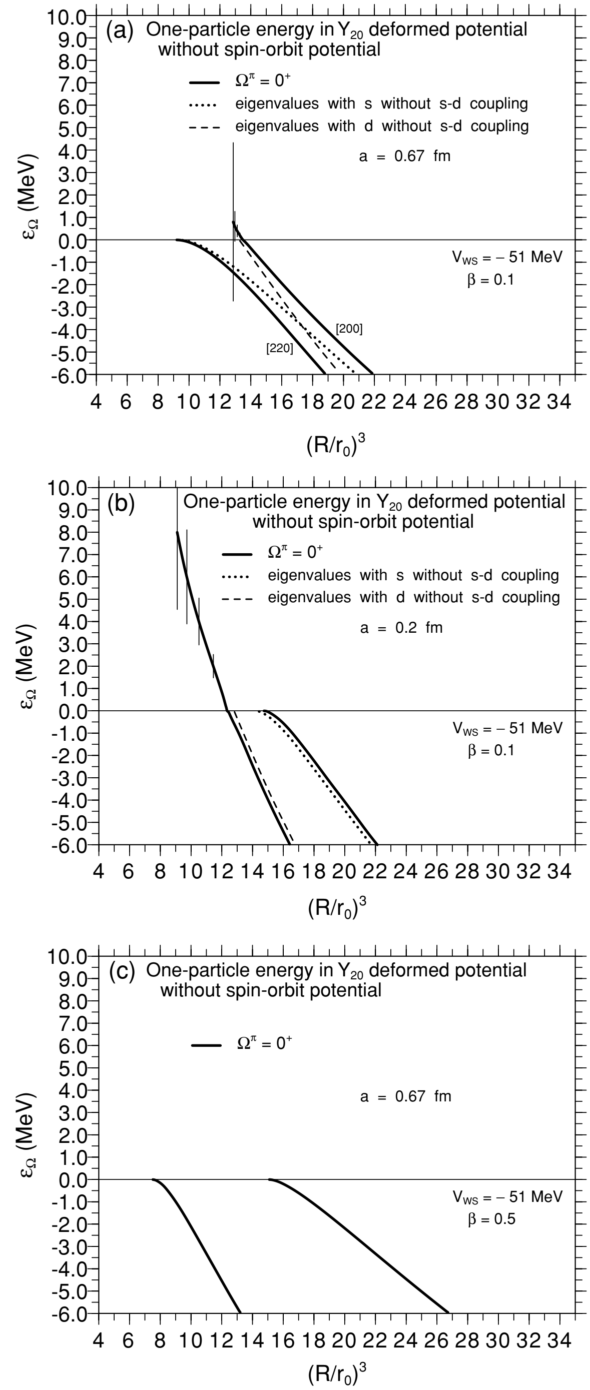

FIG. 4. (a) $\Omega^\pi = 0^+$ neutron one-particle levels in the absence of spin-orbit potential, as a function of the potential strength. For convenience, we call the higher-lying level [200], while the lower-lying one [220]. The radius of the Woods-Saxon potential is expressed by R, while $r_0 = 1.27$ fm is used. A very small deformation $\beta = 0.1$ is used as a reference example. The width of one-particle resonant levels with $\varepsilon_\Omega > 0$ denoted by thin vertical lines is calculated in terms of eigenphase, which is defined by Eq. (15). The dotted and dashed curves express the eigenvalues of $2s$ and $1d$ orbits, respectively, which are obtained when the deformation (namely, s-d) coupling is switched off. (b) The same as in (a), but a very small diffuseness $a = 0.2$ fm is used in the Woods-Saxon potential. (c) The same as in (a), except for a larger deformation $\beta = 0.5$.

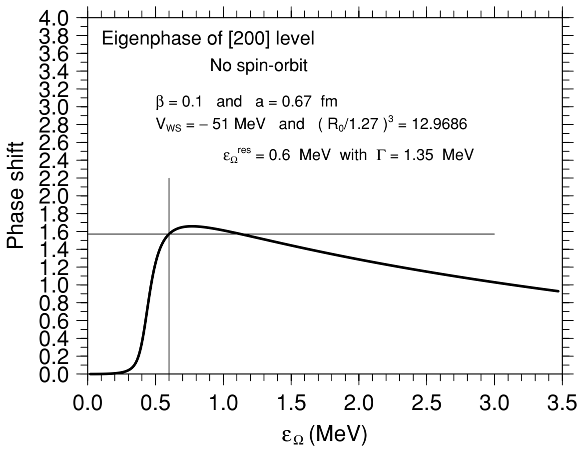

FIG. 5. Eigenphase as a function of one-particle energy is plotted for the potential, which produces the [200] one-particle resonance at $\varepsilon_\Omega = 0.6$ MeV in Fig. 4(a). The value of $\pi/2$ is denoted by the thin horizontal line.

Indeed, we have found that the energy dependence of the width of the [200] one-particle resonance shown in Fig. 4(a) is well expressed by Eq. (16) together with Eq. (17), up to the energy of a few hundred keV. From the above considerations we may conclude that the resonance width comes essentially from the admixed $\ell = 0$ component, while the resonant states are realized thanks to the presence of the $\ell = 2$ component.

It is known that for a finite square-well potential the $1d$ level lies lower than the $2s$ level all the way up to zero binding. Therefore, the potentials with diffuseness much smaller than the realistic one would give us a good opportunity for studying the weakly bound levels in deformed potentials, where the $1d$ level lies lower than the $2s$ level in the spherical limit. In Fig. 4(b) we show neutron one-particle levels for potentials with the same parameters as those used in Fig. 4(a) except for the small diffuseness, $a = 0.2$ fm. Indeed, due to the smaller diffuseness the $1d$ level denoted by the dashed curve lies lower than the $2s$ level expressed by the dotted curve, when the coupling between the s and d channels is switched off. The solid curves show the result of including the s-d deformation coupling. The higher-lying solid curve, of which the predominant component has $\ell = 0$, has no continuation at resonant levels. The lower-lying solid curve continues into the positive-energy region by resonant levels. The analysis of the resonance widths from zero to the energy of a few hundred keV shows that the energy dependence of the widths is well reproduced by formulas (16) and (17). In contrast to the one-particle resonances smoothly connected to the [200] level in Fig. 4(a) where the unperturbed bound $\ell = 0$ level lies 1–2 MeV below the resonant levels, no $\ell = 0$ levels to be admixed with lie energetically nearby. The small probability of the $\ell = 0$ component defined by Eq. (17) is found to decrease rapidly in one-particle resonant states at higher energies. Consequently, the resonant levels become purer $\ell = 2$ states continuing up to 10 MeV.

For reference, in Fig. 4(c) we show $\Omega^\pi = 0^+$ neutron one-particle eigenvalues for potentials with the same parameters as those used in Fig. 4(a) except for a larger deformation $\beta = 0.5$. At this deformation no continuation by one-particle resonant levels is found in either of those two levels denoted by solid curves.

IV. CONCLUSIONS AND PERSPECTIVES

We have shown that one-particle resonant levels defined in terms of eigenphase are the natural extension of one-particle bound levels to the continuum in deformed potentials. These resonant levels will be especially useful and play an important role when the many-body pair correlation is taken into account in deformed neutron-drip-line nuclei. For $\Omega^\pi \neq 1/2^+$ one-particle levels in which $\ell = 0$ component is absent a smooth extension to the continuum by resonant levels can always be found for small positive energies. The energy, at which the resonance disappears, depends primarily on the minimum value of ℓ of possible components, but for a given Ω^π value it may appreciably depend on the structure of respective wave functions. In contrast, for $\Omega^\pi = 1/2^+$ one-particle bound levels whether or not the relevant resonant levels are present depends on deformation, the magnitude of the admixed $\ell = 0$ component, the location of nearby $\ell = 0$ bound levels, and the diffuseness of the potential. For well-deformed Woods-Saxon potentials with a realistic size of diffuseness either no one-particle resonant levels are found or resonant levels disappear quickly obtaining an increasingly large width as energy increases.

Considering that one-particle resonant levels with energies less than a few MeV may have important contributions to the many-body pair-correlation in neutron-drip-line nuclei, the properties of the $\Omega^\pi = 1/2^+$ one-particle resonance are of particular interest. Thus, we have studied a simplified model consisting of s and d orbits by switching off the spin-orbit potential, in order to learn more about the role of the $\ell = 0$ component in one-particle resonant levels with $\Omega^\pi = 1/2^+$. We have confirmed that the width of those resonant levels at low energy comes from the admixed $\ell = 0$ component, while the possible presence of the resonance is made by the $\ell = 2$ component. However, it is a future task to find a proper parameter which determines the presence or absence of one-particle resonant levels as a continuation of one-particle bound levels.

For some parameter sets of potentials we have indeed found certain one-particle resonant levels, which are obtained using the present definition in terms of eigenphase and are not directly connected with one-particle bound levels. The role and the meaning of those one-particle resonant levels will be studied in the future.

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[1] I. Hamamoto and B. R. Mottelson, C. R. Acad. Sci., Ser IV: Phys., Astrophys. **4**, 433 (2003).
[2] I. Hamamoto and B. R. Mottelson, Phys. Rev. C **68**, 034312 (2003); **69**, 064302 (2004).
[3] I. Hamamoto, Phys. Rev. C **69**, 041306(R) (2004).
[4] I. Hamamoto, Phys. Rev. C **71**, 037302 (2005).
[5] A. Bulgac, Hartree-Fock-Bogoliubov Approximation for Finite Systems, preprint No. FT-194-1980, Institute of Atomic Physics, Bucharest, 1980, nucl-th/9907088.
[6] M. Grasso, N. Sandulescu, Nguyen Van Giai, and R. J. Liotta, Phys. Rev. C **64**, 064321 (2001).
[7] K. Hagino and Nguyen Van Giai, Nucl. Phys. **A735**, 55 (2004).
[8] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, MA, 1969), Vol. I.
[9] T. Tamura, Oak Ridge National Laboratory, Report ORNL-4152, 1967 (unpublished).
[10] T. Misu, W. Nazarewicz, and S. Aberg, Nucl. Phys. **A614**, 44 (1997).