

Interacting quark-diquark model of baryons

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A simple quark-diquark model of baryons with direct and exchange interactions has been constructed. Spectrum and form factors have been calculated and compared with experimental data. Advantages and disadvantages of the model are discussed.

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The notion of diquark is as old as the quark model itself. Gell-Mann [1] mentioned the possibility of diquarks in his original paper on quarks. Soon afterwards, Ida and Kobayashi [2] and Lichtenberg and Tassie [3] introduced effective degrees of freedom of diquarks in order to describe baryons as composed of a constituent diquark and quark. Since its introduction, many articles have been written on this subject [4] up to the most recent ones [5].

Different phenomenological indications for diquark correlations have been collected during the years, such as some regularities in hadron spectroscopy, the $\Delta I = \frac{1}{2}$ rule in weak nonleptonic decays [6], some regularities in parton distribution functions [7] and in spin-dependent structure functions [7]. Finally, although the phenomenon of color superconductivity [8] in quark dense matter cannot be considered an argument in support of diquarks in the vacuum, it is nevertheless of interest since it stresses the important role of Cooper pairs of color superconductivity, which are color antitriplet, flavor antisymmetric, scalar diquarks.

The introduction of diquarks in hadronic physics has some similarities to that of correlated pairs in condensed matter physics (superconductivity [9]) and in nuclear physics (interacting boson model [10]), where effective bosons emerge from pairs of electrons [11] and nucleons [12], respectively. However, while the origin of correlated electron and nucleon pairs is clear (the electron-phonon interaction in condensed matter physics and the short-range pairing interaction in nuclear physics), the microscopic origin of diquarks is not completely clear and its connection with the fundamental theory (QCD) not fully understood, apart from the cold asymptotic high dense baryon case, in which the quarks form a Fermi surface and perturbative gluon interactions support the existence of a diquark BCS state known as color superconductor [8]. We can only say that in perturbative QCD the color antitriplet, flavor antisymmetric scalar channel is favored by one gluon exchange [13] and in nonperturbative regime by instanton interactions [14]. Regarding the hadron spectrum we are interested in the role of diquark correlations in nonperturbative QCD. In this respect it is interesting the discussion of nonperturbative short-range, spin and flavor dependent correlations in hadrons, given in Ref. [15] considering an instanton liquid model and the comparison of some of these results with Lattice QCD (LQCD) ones. However, the question

remains open and other LQCD calculations are needed in order to understand it fully.

Nonetheless, as in nuclear physics, one may attempt to correlate the data in terms of a phenomenological model. In this article, we address this question by formulating a quark-diquark model with explicit interactions, in particular with a direct and an exchange interaction. We then study the spectrum which emerges from this model and we start to calculate the form factors which have been measured, or will be measured at dedicated facilities (TJNAF, MAMI, ...).

We think of a diquark as two correlated quarks treated as a point-like object, though this is a rough approximation of an extended effective boson degree of freedom.

We assume that baryons are composed of a constituent quark, q , and a constituent diquark, $q^2 = Q^2$. We consider only light baryons, composed of (u, d, s) quarks, with internal group $SU_s(2) \otimes SU_f(3) \otimes SU_c(3)$. Using the conventional notation of denoting spin by its value and flavor and color by the dimension of the representation, the quark has spin $s = \frac{1}{2}$, $F = \mathbf{3}$ and $C = \mathbf{3}$. The diquark must be $\bar{\mathbf{3}}$ of $SU_c(3)$ since the total hadron must be colorless. This limits the possible $SU_{sf}(6)$ representations for the diquark [3] to be only the $\mathbf{21}$ of $SU_{sf}(6)$, which is symmetric and contains $s_{12} = 0$, $F_{12} = \bar{\mathbf{3}}$ and $s_{12} = 1$, $F_{12} = \mathbf{6}$. This is because we think of the diquark as two correlated quarks in an antisymmetric nonexcited state.

As one can see from the simple multiplication of the Young diagrams associated with two fundamental representations of $SU_{sf}(6)$

$$\square \otimes \square = \square \square \oplus \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \quad (1)$$

$$\mathbf{6} \otimes \mathbf{6} = \mathbf{21} \oplus \mathbf{15}, \quad (2)$$

by keeping only the representation $\mathbf{21}$ of $SU_{sf}(6)$, as in the diquark case, one is deleting in baryons those states obtained by combining the representation $\mathbf{15}$ with that of the remaining quark $\mathbf{6}$, i.e., a $\mathbf{70}$ and $\mathbf{20}$.

If one treats only nonstrange baryons, the quark is in the representation $\mathbf{4}$ of the Wigner $SU_{st}(4)$, with $s = \frac{1}{2}$ and $t = \frac{1}{2}$, and the diquark is the representations with spin $s_{12} = 0$, isospin $t_{12} = 0$, and spin $s_{12} = 1$, isospin $t_{12} = 1$, i.e., the symmetric representation $\mathbf{10}$ of $SU_{st}(4) \supset SU_s(2) \otimes SU_t(2)$. The situation for the internal degrees of freedom is summarized in Table I.

The two diquark configurations $s_{12} = 0$, $F_{12} = \bar{\mathbf{3}}$ and $s_{12} = 1$, $F_{12} = \mathbf{6}$ are split, by (among other things) color

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TABLE I. Combinations of possible irreducible representations of $SU_c(3)$, $SU_s(2)$, and $SU_f(3)$ for the quark, q , and the diquark, Q^2 . The right-hand side of the table also shows the combinations of the possible $SU_s(2)$ and $SU_t(2)$ representations if only nonstrange baryons are involved.

	$SU_c(3)$	$SU_{sf}(6)$	$SU_s(2)$	$SU_f(3)$	$SU_{st}(4)$	$SU_s(2)$	$SU_t(2)$
q	3	6	$s = \frac{1}{2}$	3	4	$s = \frac{1}{2}$	$t = \frac{1}{2}$
Q^2	$\bar{\mathbf{3}}$	21	$s_{12} = 0$	$\bar{\mathbf{3}}$	10	$s_{12} = 0$	$t_{12} = 0$
			$s_{12} = 1$	6		$s_{12} = 1$	$t_{12} = 1$

magnetic forces [16], as shown in Fig. 1. It means that we have two possible diquark configurations: a scalar diquark ($s_{12} = 0$, $F_{12} = \bar{\mathbf{3}}$, $C = \bar{\mathbf{3}}$) and a vector one ($s_{12} = 1$, $F_{12} = \mathbf{6}$, $C = \bar{\mathbf{3}}$). The scalar diquark (the ‘‘good diquark’’ in Wilczek and Jaffe’s terminology) is favored since it is at lower energy, and will be the dominant configuration in the more stable states.

We call the splitting between the two diquark configurations Δ and parametrize it as

$$\Delta = (B + C\delta_0), \quad (3)$$

i.e., as a constant B which acts equally in all states with $s_{12} = 1$ plus a contact interaction which acts only on the spatial ground state and of strength C . The contact interaction emphasizes the different role that the ground state has in a quark-diquark picture as compared with all other states.

Here, we consider a quark-diquark configuration in which the two constituents are separated by a distance r . We use a potential picture and we introduce a direct and an exchange quark-diquark interaction. For the direct term, we consider a Coulomb-like plus a linear confining interaction

$$V_{\text{dir}}(r) = -\frac{\tau}{r} + \beta r. \quad (4)$$

The importance of the Coulomb-like interaction was emphasized long ago by Lipkin [17]. A simple mechanism that generates a Coulomb-like interaction is one-gluon exchange. A natural candidate for the confinement term is a linear one, as obtained in lattice QCD calculations and other considerations [18].

An exchange interaction is also needed, as emphasized by Lichtenberg [19]. This is indeed the crucial ingredient of a quark-diquark description of baryons. We consider

$$V_{\text{ex}} = (-1)^{l+1} 2Ae^{-ar} [s_{12}^z \cdot s_3^z + t_{12}^z \cdot t_3^z + 2s_{12}^z \cdot s_3^z t_{12}^z \cdot t_3^z]. \quad (5)$$

Here s_{12}^z , s_3^z and t_{12}^z and t_3^z are the spin and the isospin operators of the diquark and the quark, respectively. The sign $(-1)^l$

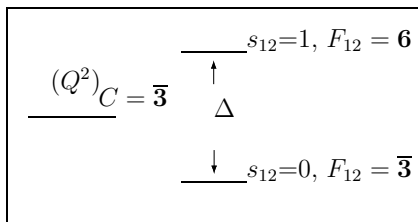


FIG. 1. Schematic picture of the split, between the two diquark configurations $s_{12} = 0$, $F_{12} = \bar{\mathbf{3}}$ and $s_{12} = 1$, $F_{12} = \mathbf{6}$.

reflects the angular momentum dependence of the exchange interaction.

A feature of the present approach, as compared with previous ones, is that we attempt a simultaneous description of all states; indeed, in previous approaches the ground state has usually been excluded. It is certainly the case that diquark correlations are particularly important for high ℓ -states [20] which have a more pronounced string-like behavior. However, if one is interested in the overall spectroscopy and also in the form factors (elastic and inelastic) that have been measured or will be measured (TJLAB, MAMI, . . .) the ground state must be included in this description.

Another important observation is that, in the quark-diquark model, $SU_{sf}(6)$ is badly broken and thus a classification of the spin-flavor wave functions in terms of $SU_{sf}(6)$ is inappropriate. The appropriate classification is that in terms of $SU_s(2) \otimes SU_f(3)$

$$|s_{12}, F_{12}; \frac{1}{2}, \mathbf{3}; S, F\rangle, \quad (6)$$

where s_{12} and F_{12} are, respectively, the spin and the flavor of the diquark, $\frac{1}{2}$, $\mathbf{3}$ are the spin and flavor of the quark, and S , F are the total spin and flavor obtained by coupling the spin and flavor of the diquark with those of the quark.

If only nonstrange baryons are considered, one can use the isospin of the diquark, t_{12} , and quark, $1/2$, and the total isospin, T . The possible states are

$$\begin{aligned} &|s_{12} = 0, t_{12} = 0; \frac{1}{2}, \frac{1}{2}; S = \frac{1}{2}, T = \frac{1}{2}\rangle, \\ &|s_{12} = 1, t_{12} = 1; \frac{1}{2}, \frac{1}{2}; S = \frac{1}{2}, T = \frac{1}{2}\rangle, \\ &|s_{12} = 1, t_{12} = 1; \frac{1}{2}, \frac{1}{2}; S = \frac{1}{2}, T = \frac{3}{2}\rangle, \\ &|s_{12} = 1, t_{12} = 1; \frac{1}{2}, \frac{1}{2}; S = \frac{3}{2}, T = \frac{1}{2}\rangle, \\ &|s_{12} = 1, t_{12} = 1; \frac{1}{2}, \frac{1}{2}; S = \frac{3}{2}, T = \frac{3}{2}\rangle. \end{aligned} \quad (7)$$

The total wave functions are combinations of the spin-flavor wave functions with the radial and orbital wave functions. These are very simple and straightforward since the main advantage of the quark-diquark model is to reduce the baryon problem (a three-body problem) to a two-body problem. The spatial part of the wave functions is, for the central interaction of Eq. (4),

$$\Psi_{n,l,m}(\vec{r}) = R_{n,l}(r)Y_{l,m}(\theta, \varphi), \quad (8)$$

where the radial wave function $R_{n,l}(r)$ can be obtained by solving the radial equation. Although the numerical solution poses no problem, we prefer to exploit here the special nature of the interaction. For a purely Coulomb-like interaction the

problem is analytically solvable. The solution is trivial, with eigenvalues

$$E_{n,l} = -\frac{\tau^2 m}{2n^2}, \quad n = 1, 2, \dots \quad (9)$$

Here m is the reduced mass of the diquark-quark configuration and n the principal quantum number. The eigenfunctions are the usual Coulomb functions

$$R_{n,l}(r) = \sqrt{\frac{(n-l-1)!(2g)^3}{2n[(n+l)!]^3}} (2gr)^l e^{-gr} L_{n-l-1}^{2l+1}(2gr), \quad (10)$$

where for the associated Laguerre polynomials we have used the notation of Ref. [21] and $g = \frac{\tau m}{n}$.

We treat all other interactions as perturbations. We begin with the linear term. The matrix elements of βr can be evaluated analytically, with the result

$$\Delta E_{n,l} = \int_0^\infty \beta r [R_{n,l}(r)]^2 r^2 dr = \frac{\beta}{2m\tau} [3n^2 - l(l+1)]. \quad (11)$$

This perturbative estimate is only valid for small n and l . (For large n and l , the radial equation must be solved numerically.) Combining Eq. (9) with Eq. (11), we can write the energy eigenvalues as

$$E_{n,l} = -\frac{\tau^2 m}{2n^2} + \frac{\beta}{2m\tau} [3n^2 - l(l+1)]. \quad (12)$$

Next comes the exchange interaction of Eq. (5). The spin-isospin part is obviously diagonal in the basis of Eq. (7)

$$\langle \vec{s}_{12} \cdot \vec{s}_3 \rangle = \frac{1}{2} [S(S+1) - s_{12}(s_{12}+1) - s_3(s_3+1)] \quad (13)$$

$$\langle \vec{t}_{12} \cdot \vec{t}_3 \rangle = \frac{1}{2} [T(T+1) - t_{12}(t_{12}+1) - t_3(t_3+1)].$$

To complete the evaluation, we need the matrix elements of the exponential. These can be obtained in analytic form

$$I_{n,l}(\alpha) = \int_0^\infty e^{-\alpha r} [R_{n,l}(r)]^2 r^2 dr. \quad (14)$$

The results are straightforward. Here, by way of example, we quote the result for $l = n - 1$

$$I_{n,l=n-1}(\alpha) = \left(\frac{1}{1 + \frac{n\alpha}{2\tau m}} \right)^{2n+1}. \quad (15)$$

Combining all pieces together, the Hamiltonian is

$$H = E_0 + \frac{p^2}{2m} - \frac{\tau}{r} + \beta r + (B + C\delta_0)\delta_{S_{12},1} + (-1)^{l+1} 2A e^{-\alpha r} [s_{12}^z \cdot \vec{s}_3 + t_{12}^z \cdot \vec{t}_3 + 2s_{12}^z \cdot \vec{s}_3 t_{12}^z \cdot \vec{t}_3], \quad (16)$$

where δ_0 stands in short for $\delta_{n,1}\delta_{l,0}$. This Hamiltonian is characterized by the parameters τ , β , E_0 , A , α , B , C , which are chosen by comparing with experimental data. Since all contributions are given in explicit analytic form, the determination of the parameters is straightforward. The procedure that we use is the following:

(i) the parameters $\tau^2 m$, $\frac{\beta}{m\tau}$, and E_0 are determined from the location of the lowest state for each orbital angular

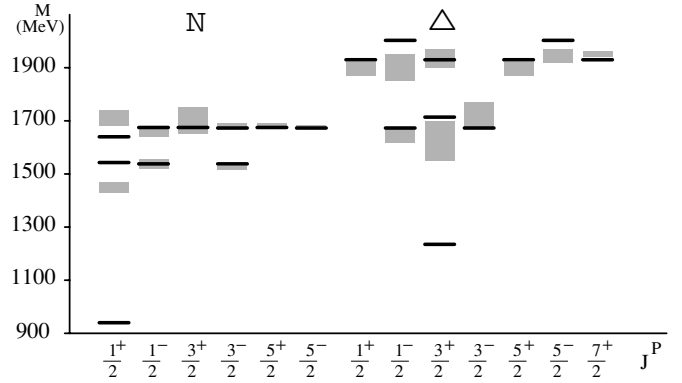


FIG. 2. Comparison between the calculated masses (black lines) of nonstrange baryon resonances up to 2 GeV and the experimental masses (grey boxes) from PDG [22] for the 4^* and 3^* resonances.

momentum; we obtain $\tau^2 m = 1546$ MeV, $\frac{\beta}{m\tau} = 5$ MeV, and $E_0 = 1706$ MeV;

(ii) the parameters B and C are determined by the splitting between the state $s_{12} = 0, t_{12} = 0$ and the average of the states $s_{12} = 1, t_{12} = 1$. We find $B = 300$ MeV and $C = 400$ MeV. The value $B = 300$ MeV is consistent with earlier estimates, $B = 250$ MeV, arising from an evaluation of the color magnetic interaction [16];

(iii) the parameters A and α are determined from the splitting of the multiplet within $s_{12} = 1, t_{12} = 1$. We find $A = 205$ MeV, $\frac{\alpha}{\tau m} = 0.30$.

In Fig. 2 and in Table II the results of the model are compared with the experimental data [22]. It can be seen that the quark-diquark model with the specific interaction, Eq. (16), provides a good description of the masses of 4^* and 3^* resonances. The quality of this description is similar to that of the usual three-quark model in its various forms [23,24], and it can be observed that the predicted value for the mass of the Roper resonance is higher than the experimental data. However, in the quark-diquark model, some degrees of freedom are frozen. There are therefore far fewer missing resonances, and in particular no missing resonances in the lower part of the spectrum. On the contrary, the problem of missing resonances plagues all models with three constituent quarks.

It will also be noted that the clustering of states expected by the quark-diquark model is particularly evident for $l^P = 1^-$ and $l^P = 2^+$, at both the qualitative and quantitative levels. This clustering appears both in the nucleon, N , and in the Δ . The clustering of states is another feature that is difficult to obtain in a three quark description. In the quark-diquark model, however, it is obtained automatically.

For the model described here, in which all interactions in addition to the Coulomb-like interaction are treated in perturbation theory, elastic and transition form factors can all be calculated analytically. The scalar matrix elements are of the type

$$\int d^3r R_{n',l'}(r) Y_{l',m'}(\Omega) e^{-i\vec{k}\cdot\vec{r}} R_{n,l}(r) Y_{l,m}(\Omega). \quad (17)$$

TABLE II. Comparison between the calculated masses of non-strange baryon resonances up to 2 GeV, M_{calc} , and the experimental masses [22], M_{exp} , for the 4^* and 3^* resonances. The masses are given in MeV.

Baryon	Status	M_{exp} (MeV)	J^P	l^P	S	M_{calc} (MeV)
N(939) P_{11}	****	938	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	940
N(1440) P_{11}	****	1430–1470	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1543
N(1520) D_{13}	****	1515–1530	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1538
N(1535) S_{11}	****	1520–1555	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1538
N(1650) S_{11}	****	1640–1680	$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1675
N(1675) D_{15}	****	1670–1685	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1673
N(1680) F_{15}	****	1675–1690	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	1675
N(1700) D_{13}	***	1650–1750	$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1673
N(1710) P_{11}	***	1680–1740	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1640
N(1720) P_{13}	****	1650–1750	$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	1675
$\Delta(1232)P_{33}$	****	1230–1234	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1235
$\Delta(1600)P_{33}$	***	1550–1700	$\frac{3}{2}^+$	0^+	$\frac{1}{2}$	1714
$\Delta(1620)S_{31}$	****	1615–1675	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1673
$\Delta(1700)D_{33}$	****	1670–1770	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1673
$\Delta(1900)S_{31}$	***	1850–1950	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	2003
$\Delta(1905)F_{35}$	****	1870–1920	$\frac{5}{2}^+$	2^+	$\frac{3}{2}$	1930
$\Delta(1910)P_{31}$	****	1870–1920	$\frac{1}{2}^+$	2^+	$\frac{3}{2}$	1930
$\Delta(1920)P_{33}$	***	1900–1970	$\frac{3}{2}^+$	2^+	$\frac{3}{2}$	1930
$\Delta(1930)D_{35}$	***	1920–1970	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	2003
$\Delta(1950)F_{37}$	****	1940–1960	$\frac{7}{2}^+$	2^+	$\frac{3}{2}$	1930

These integrals, denoted by $U_{nl,n'l'}(k)\delta_{mm'}$ are straightforward and Table III shows the corresponding results for transitions from the ground state with quantum numbers, $n = 1, l^P = 0^+$ to a state with n, l . It is observed that the elastic form factor is

$$F(k) = \frac{1}{(1 + k^2 a^2)^2}, \quad (18)$$

with $a = \frac{1}{2\tau m}$. In addition to having a power-law behavior with momentum transfer, k , typical of Coulomb-like interactions, this form factor has precisely the power dependence observed experimentally. Thus the quark-diquark model presented here has the further advantage of producing in first approximation an elastic form factor in agreement with experimental data, Fig. 3. All form factors depend on the scale a . To determine the scale a the rms radius can be calculated by using the ground state wave functions, $\langle r^2 \rangle = \frac{3}{\tau^2 m^2} = 12a^2$. This calculated value is then fitted to the experimental value $\langle r^2 \rangle_{\text{exp}} = 0.74(1) \text{ fm}^2$ [22]. The resulting value is $a = 0.25 \text{ fm}$. Since the parameter τ is determined from the spectrum, one obtains the reduced mass $m = 102 \text{ MeV}$. This value is somewhat lower than the naive expectation $m = 200 \text{ MeV}$, obtained by assuming the quark mass to be 300 MeV and the diquark mass to be 600 MeV . The results shown in Table III

TABLE III. The scalar form factors of Eq. (17) for transitions to final states labeled by the quantum numbers n, l^P , where P is the parity. The initial state is $n = 1, l^P = 0^+$ and $a = \frac{1}{2\tau m}$.

n	l^P	$\langle nl^P U 10^+ \rangle$
1	0^+	$\frac{1}{(1+k^2 a^2)^2}$
2	1^-	$\frac{i}{\sqrt{2}} \left(\frac{4}{9}\right)^3 \frac{24ka}{(1+\frac{16}{9}k^2 a^2)^3}$
2	0^+	$16\sqrt{2} \left(\frac{4}{9}\right)^3 \frac{(ka)^2}{(1+\frac{16}{9}k^2 a^2)^3}$
3	2^+	$-\frac{4}{\sqrt{6}} \left(\frac{9}{16}\right)^2 \frac{(ka)^2}{(1+\frac{9}{4}k^2 a^2)^4}$
3	1^-	$\frac{i\sqrt{2}64ka}{27} \left(\frac{9}{16}\right)^3 \frac{(1+\frac{27}{4}(ka)^2)}{(1+\frac{9}{4}k^2 a^2)^4}$
3	0^+	$\frac{4}{\sqrt{3}} \left(\frac{9}{16}\right)^2 \frac{(1+\frac{27}{4}(ka)^2)(ka)^2}{(1+\frac{9}{4}k^2 a^2)^4}$

should be compared with the analogous results in the three quark model, as for example reported in Table IX of Ref. [24].

In order to calculate the magnetic elastic form factors and the helicity amplitudes, other matrix elements are also needed to be calculated; this program will be completed in another article (not least because a relativistic version of the model is required for a good calculation of form factors), since here we are interested only to explore the qualitative features of the model and of its results. In particular the quark-diquark model presented here produces the phenomenon of stretching, which is at the basis of the Regge behavior of hadrons. The transition radii increase with n and l , as one can see from Table III, or by evaluating $\langle r^2 \rangle_{n,l=n-1} = (2n+2)(2n+1)\frac{n^2}{4\tau^2 m^2}$. In other words, hadrons swell as the angular momentum increases.

In this article, we present a simple quark-diquark model with a specific direct plus exchange interaction. This model reproduces the spectrum just as well as conventional three-quark models. However, it has far fewer missing resonances than the usual models. Most importantly, the model produces form factors with power-law behavior as a function of momentum transfer, in agreement with experimental data. Finally, it shows the phenomenon of stretching which is at the basis of the Regge behavior of hadrons.

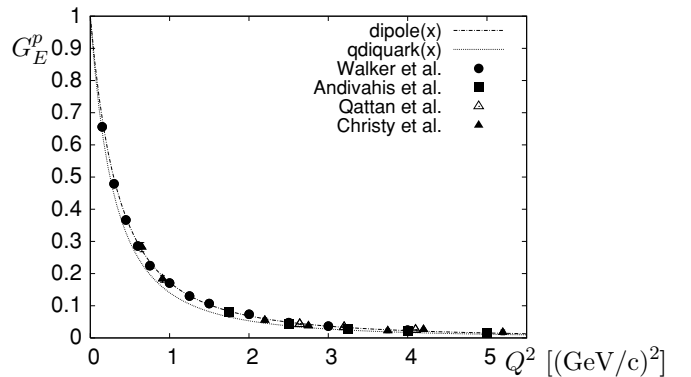


FIG. 3. The electric form factor of the proton. The dotted line is the result of the model [Eq. (18)] using $a = 0.25 \text{ fm}$, the dot-dashed line corresponds to the dipole fit, $a = 0.23 \text{ fm}$. The experimental data are from Ref. [25].

One may wonder whether there is a unique spectral signature for quark-diquark models. One of these signatures is the detection of 1^+ states which are antisymmetric in all three quarks. These states, originating from the omitted diquark representation **15** of $SU_{sf}(6)$ are not present in the quark-diquark model and occur (at different masses) in all models with three quarks. These missing states may, however, be very difficult to detect since they are decoupled and cannot be excited with electrons or photons. To excite these states, strongly interacting particles are needed, for example (\vec{p}, \vec{p}') with spin transfer. Another possibility is to study whether or not the mixed symmetry states (which are doubly degenerate in the q^3 model) are in fact simply degenerate (as in the quark-diquark model).

The work presented here can be expanded in several directions: (1) to include strange baryons; this expansion is straightforward and requires no further assumption; (2) to study multi-quark states; (3) to include relativistic corrections.

For transparency, here we have presented results in which all interactions, in addition to the Coulomb-like force, are treated in perturbation theory. Numerical diagonalization

shows that with the parameters of this article, the perturbation treatment is valid (at least for the low-lying states). The complete numerical results will be the subject of a subsequent paper.

An aspect that has not been discussed here is how to derive the quark-diquark model from microscopy. This aspect has been the subject of many investigations in the similar problems of condensed matter and nuclear physics. The usual argument in hadronic physics is that diquark correlations arise from the spin-spin interaction originating from one gluon exchange that lowers the scalar diquark relative to the vector diquark. Some interesting results are now available and show that instanton interactions favor diquark clustering [14], and that by using an instanton liquid model these generate a deeply bound scalar antitriplet diquark not point-like [15]. In this respect instantons can provide a microscopic dynamical mechanism in terms of nonperturbative QCD interactions. In this article, we have merely tried to describe many data by means of a phenomenological approach; the objective of a subsequent paper will be to understand what kind of microscopic QCD mechanisms we are trying to mimic in this oversimplified form.

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