Combined method to extract spectroscopic information

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Spectroscopic factors (SFs) play an important role in nuclear physics and astrophysics. The traditional method of extracting SFs from direct transfer reactions suffers from serious ambiguities. We discuss a modified method that is based on including the asymptotic normalization coefficient of the overlap functions into the transfer analysis. In the modified method the contribution of the external part of the reaction amplitude, typically dominant, is fixed and the SF is determined from the fitting of the internal part. We illustrate the modified method with (d,p) reactions on ^{208}Pb, ^{12}C, and ^{84}Se targets at different energies. The modified method allows one to extract the SFs, which do not depend on the shape of the single-particle nucleon-target interaction, and has the potential of improving the reliability and accuracy of the structure information. This is specially important for nuclei on dripline, where not much is known.

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Spectroscopic factors (SFs) were introduced by the shell-model formalism and are typically related to the shell occupancy of a state n in one nucleus relative to a state m in a nearby nucleus [1]. Today, phenomenological SFs are extensively used in a variety of topics, from nuclear reactions to astrophysics or applied physics, yet the procedure for their extraction from the data has remained essentially the same for decades. For more than 40 years since the dawn of nuclear physics, direct transfer reactions, such as (d,p), (d,t), $(^3\mathrm{He},d)$, and $(^3\mathrm{He},\alpha)$, have been the central tools for determining SFs [2–4]. Extracting SFs with good precision from data is very important to test the validity of today's many-body theories. For conventional nuclei there are many experiments available that provide SFs, which are often lower than those predicted by shell model [1]. Electron-induced knockout or electron scattering is supposed to provide a better accuracy in extracting SFs than transfer [5,6]. However, for exotic nuclei near or on the driplines, transfer reactions are a unique tool and hence can have a large impact on the programs of the new-generation rare-isotope laboratories. Given the experimental difficulties faced with measurements on the driplines, it is crucial to have a reliable method for analyzing and extracting useful information from each single data set.

Usually transfer angular distributions are analyzed within the framework of the distorted-wave Born approximation (DWBA). The SF determined by normalization of the calculated DWBA differential cross section to the experimental one (e.g., Refs. [7–9]) is compared with the SF predicted by shell model. Even when error bars in the experimental cross section are low, the uncertainty of the extracted SF resulting from the normalization of the DWBA cross section is often large, regardless of whether it agrees with the shell-model prediction. The reasons for this inaccuracy are typically (i) optical potentials ambiguity, (ii) the inadequacy of the DWBA reaction theory, or (iii) the dependence on the single-particle potential parameters. The first point has been the object of a recent systematic study [10]. The second point needs to be addressed case by case, and examples of improved reaction models are the coupled-channel Born approximation (e.g., Ref. [11]) or the continuum discretized coupled-channel method (e.g., Ref. [12]). This paper critically reviews the standard procedure of extracting SFs from transfer reactions by focusing on the third point; the modified method eliminates the dependence of the extracted SFs on the single-particle potentials, the main advantage of the method.

We address a modified approach to spectroscopy from transfer reaction that includes the asymptotic normalization coefficient (ANC) in the analysis [2]. For simplicity, in the following formulation, we consider the $A(d,p)B$ reaction and disregard spins (naturally these are included in the applications). The DWBA amplitude for this reaction is given by

$$M = \left\langle \psi_f^{(-)} I_{An}^B \middle| \Delta V \middle| \varphi_{pn}\, \psi_i^{(+)} \right\rangle, \tag{1}$$

where $\Delta V = V_{pn} + V_{pA} - U_{pB}$ is the transition operator in the postform, V_{ij} is the interaction potential between i and j, and U_{pB} is the optical potential in the final state. The distorted waves in the initial and final states are $\psi_i^{(+)}$ and $\psi_f^{(-)}$, respectively, φ_{pn} is the deuteron bound-state wave function, and $I_{An}^B(\mathbf{r})$ is the overlap function of the bound states of nuclei B and A that depends on $\mathbf{r}$, the radius vector connecting the center of mass of A with n. The overlap function is not an eigenfunction of an Hermitian Hamiltonian and is not normalized to unity [13]. The square norm of the overlap function gives a model-independent definition of the SF:

$$S = N \left\langle I_{An}^B \middle| I_{An}^B \right\rangle. \tag{2}$$

Here, N is the antisymmetrization factor in the isospin formalism (N is included in the overlap function from now on).

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The leading asymptotic term of the radial overlap function (for $B = A + n$) is

$$I^B_{An(lj)}(r) \overset{r>R}{\approx} C_{lj}\, i\,\kappa\, h_l(i\,\kappa\, r), \tag{3}$$

where $h_l(i\,\kappa\, r)$ is the spherical Bessel function, $\kappa = \sqrt{2\,\mu_{An}\,\varepsilon_{An}}$, ε_{An} is the binding energy for $B \to A + n$, and μ_{An} is the reduced mass of A and n. Similarly, the asymptotics of the neutron single-particle wave function is $\varphi_{An(n_r lj)}(r) \overset{r>R}{\approx} b_{n_r lj}\, i\,\kappa\, h_l(i\,\kappa\, r)$, where n_r is the principle quantum number. The asymptotic behavior is valid beyond R, the channel radius. It is clear that, in the asymptotic region, the overlap function is proportional to the single-particle wave function. The normalization C_{lj} introduced in approximation (3) is the ANC that is related to the single-particle ANC (SPANC) $b_{n_r lj}$ by $C_{lj} = K_{n_r lj}\, b_{n_r lj}$, where $K_{n_r lj}$ is an asymptotic proportionality coefficient. It is standard practice to assume that the proportionality between the overlap function and the single-particle function extends to all r values:

$$I^B_{An(lj)}(r) = K_{n_r lj}\, \varphi_{An(n_r lj)}(r). \tag{4}$$

Because $\varphi_{An(n_r lj)}(r)$ is normalized to unity, this approximation [Eq. (4)] implies that $S_{lj} = K^2_{n_r lj}$. We have to emphasize, however, that the overlap function in the interior is nontrivial and may well differ from the single-particle wave function. Approximating the radial dependence of the overlap function as already described leads to the DWBA amplitude $M = K_{n_r lj} \langle \psi^{(-)}_f \varphi_{An(n_r lj)} | \Delta V | \varphi_{pn}\, \psi^{(+)}_i \rangle$. Normalizing the calculated DWBA cross section,

$$\sigma^{\rm DW} = |\langle \varphi_{An(n_r lj)} | \Delta V | \varphi_{pn}\, \psi^{(+)}_i \rangle|^2, \tag{5}$$

to the experimental data provides the phenomenological SF $S_{lj} = K^2_{n_r lj}$. Assuming that Eq. (4) is valid for all r, we can infer from Eq. (2) that the main contribution to the norm of the overlap function comes from the nuclear interior.

To make the dependence on the SPANC more explicit, we split the reaction amplitude into an interior part and an exterior part:

$$M = K_{n_r lj}\, \tilde{M}_{\rm int}[b] + K_{n_r lj}\, b_{n_r lj}\, \tilde{M}_{\rm ext}, \tag{6}$$

where the internal part of the matrix element $\tilde{M}_{\rm int}[b_{n_r lj}] = \langle \psi^{(-)}_f \varphi_{An(n_r lj)} | \Delta V | \varphi_{pn}\, \psi^{(+)}_i \rangle_{r<R}$ depends on $b_{n_r lj}$ through the bound-state wave function $\varphi_{An(n_r lj)}$, whereas the external part $\tilde{M}_{\rm ext} = \langle \psi^{(-)}_f\, i\,\kappa\, h_l(i\,\kappa\, r) | \Delta V | \varphi_{pn}\, \psi^{(+)}_i \rangle_{r>R}$ does not depend on $b_{n_r lj}$. Here, R is the channel radius taken so that for $r > R$ the overlap function can be approximated by its asymptotic form [Eq. (3)] (R is used only to illustrate the method as in the end this separation is not required). The contribution from the nuclear exterior is fixed by the ANC, whereas the SF determines the normalization of the internal part of the radial matrix element. Since transfer reactions are dominantly peripheral, SFs can only be extracted from transfer reactions due to a small contribution from the nuclear interior. We now introduce the ANC into the DWBA cross section:

$$\frac{d\,\sigma^{\rm DW}}{d\Omega} = C^2_{lj}\, \frac{\sigma^{\rm DW}}{b^2_{n_r lj}}. \tag{7}$$

Introducing Eqs. (6) and (7) and dividing by C^2_{lj}, we arrive at a function $R^{\rm DW}(b)$:

$$R^{\rm DW}(b_{n_r lj}) = \left| \frac{\tilde{M}_{\rm int}[b]}{b_{n_r lj}} + \tilde{M}_{\rm ext} \right|^2. \tag{8}$$

Note that the SPANC $b_{n_r lj}$ itself is a function of the geometrical parameters of the bound-state $n - A$ nuclear potential (r_0, a) that are, *a priori*, not known. If the ANC and the cross section for the (d, p) reaction have been measured, the experimental counterpart of $R^{\rm DW}$, $R^{\rm exp} = \frac{d\,\sigma^{\rm exp}}{d\Omega} / C^2_{lj}$ can be experimentally fixed. Then, imposing the equality

$$R^{\rm exp} = R^{\rm DW}\big(b_{n_r lj}\big) \tag{9}$$

will provide the correct b_{nlj} and consequently the SF $S_{lj} = C^2_{lj}/b^2_{nlj}$.

At this stage, a few points should be made clear. First, for specific optical potentials, Eq. (5) depends on two independent parameters, S_{lj} and b_{nlj}. In the standard approach, to evaluate this cross section, the second parameter is fixed by an arbitrary choice of the bound-state $n - A$ potential geometry. Thus the extracted product $S_{lj}\, b^2_{n_r lj}$ does not coincide necessarily with the correct ANC. Because the ANC determines the normalization of the external part of the DWBA amplitude, in the standard approach the SF is determined by an unrealistic variation of the external contribution. In the modified method discussed here, because the contribution of the external part is fixed through the correct ANC, the whole DWBA procedure loses this artificial degree of freedom.

Second, if the reaction is peripheral, i.e., the first term in Eq. (6) is negligible, one can determine the ANC. Therefore the modified approach makes use of two experiments: the first to fix the ANC, the second to determine the SF consistent with that ANC. In present experiments and with the new generation of rare-isotope facilities, ANCs can be determined with 5% accuracy. Because the determination of the SF comes from the internal region, the second experiment needs to be performed at a beam energy for which the contribution from the interior is significant. The higher the contribution of the internal region, the stronger the dependence on $b_{n_r lj}$ in $R^{\rm DW}(b_{n_r lj})$ and the smaller the uncertainty of the extracted SF, although a balance needs to be found because large interior contributions may not be well described by the DWBA. The DWBA differential cross section near the main peak of the angular distribution and, correspondingly, $R^{\rm DW}(b_{n_r lj})$ are the functionals of the SPANC $b_{n_r lj}$. One given $b_{n_r lj}$ can be produced by an infinite number of single-particle potentials, local and nonlocal. However, the dependence of $d\,\sigma^{\rm DW}/d\Omega$ or $R^{\rm DW}(b_{n_r lj})$ on the shape of the single-particle potential is minor. Hence the extracted SF in the modified method does not depend on the single-particle potential. We illustrate the method by presenting three different applications: (i) ^{209}Pb, (ii) ^{13}C, and (iii) ^{85}Se. We drop the subscripts on b for simplicity.

Let us consider the reaction ^{208}Pb$(d, p)^{209}$Pb from Ref. [12]. Although the ANC for $\langle {}^{209}{\rm Pb} | {}^{208}{\rm Pb} \rangle$ is not published, it can be determined from the sub-Coulomb reaction [14] ^{208}Pb(^{13}C, ^{12}C)^{209}Pb as the other vertex $\langle {}^{13}{\rm C} | {}^{12}{\rm C} \rangle$ is well known [15]. Sub-Coulomb reactions are extremely peripheral and insensitive to details of the optical potentials.

For this reason they present an excellent probe for extracting the ANC accurately. From Ref. [14] we obtain an ANC of $C^2_{g9/2} = 2.15(0.16)$ fm^{-1} for ^{209}Pb. Then, by using ^{208}Pb$(d,p)^{209}$Pb data at $E_d = 22$ MeV [12], we obtain $R^{\rm exp} = 2.46(0.31)$ fm mb/sr, where we calculate the error bar based on both the ANC and the cross-sections errors, taken as independent. The experimental data in Ref. [12] have 1% accuracy but are taken down to only $\theta_{\rm cm} = 35^\circ$ whereas the peak of the DWBA distribution is at $\theta_{\rm cm} = 25^\circ$. We extrapolate the data based on the shape predicted by DWBA and include a 10% error in the cross section to account for this difference. Measurements at 25° could improve the error bar in $R^{\rm exp}$ considerably. We next perform a series of finite-range DWBA calculations for ^{208}Pb$(d,p)^{209}$Pb ($E_d = 22$ MeV), using the optical potentials from Ref. [16]. The adiabatic prescription [17] was used to take into account deuteron breakup, which is important for this reaction. The Reid-soft-core potential was used for the deuteron wave function, as well as in all other examples. For illustration purposes, we use a Woods-Saxon well to generate the ^{208}Pb$+n$ single-particle wave functions and obtain a range of SPANCs b by varying the single-particle parameters (r_0, a) and adjusting the depth to reproduce the correct binding for the $2g_{9/2}$ in each case. We use the same spin-orbit strength as that in Ref. [14], although the spin-orbit strength does not affect the final result.

The results of our calculations $R^{\rm DW}$ (dot-dashed curve) and the experimental value $R^{\rm exp}$ (solid line and shaded area, respectively) are presented, as functions of b, in Fig. 1. From $R^{\rm exp}$ we find $b = 1.82$ fm$^{-1/2}$ and $S = 0.74$. It is worth noting that, in the standard approach, typical parameters $(r_0, a) = (1.2, 0.6)$ fm produce $b = 1.34$ fm$^{-1/2}$. Direct comparison of the DWBA cross section by use of $(r_0, a) = (1.2, 0.6)$ fm with the data gives $S = 0.866$ and consequently $C^2 = 1.56$ fm^{-1}, beyond the experimental range. As pointed out before, in the standard approach the SF is determined at the cost of an artificial ANC.

The beam energy of 22 MeV is above the Coulomb barrier; thus the reaction is not peripheral. This can be seen in Fig. 1 through the slope of the dot-dashed curve. In fact, for this particular energy, the interior contribution is around 10%. The uncertainty in $b \in [1.1, 3.1]$ fm$^{-1/2}$ propagates into a large uncertainty in $S \in [0.3, 2.2]$. This is due to the fact that the contribution from the interior at this energy is still small. The scaling factor relating the uncertainty of S with that of b is $(|M_>|/|M_<|)^2$. The smaller the contribution from the interior, the smaller the accuracy with which the SF can be determined.

Also in Fig. 1 we show the results for $R^{\rm DW}$ corresponding to the calculation at $E_d = 5$ MeV (dashed line). This is to illustrate that, at sub-Coulomb energies, the reaction becomes completely peripheral and the dependence on b disappears. Measurements at these energies could provide $C^2_{g_{9/2}}$ with an accuracy of <5%. In addition, measurements at higher energies (>30 MeV) would increase the slope of $R^{\rm DW}(b)$ and decrease further the error on the extracted SF.

Another standard case is the ^{12}C$(d,p)^{13}$C reaction, for which many data sets are conveniently compiled in a recent publication [10]. We studied three cases (8.9, 30, and 51 MeV), using the same JLM optical potentials as were used in Ref. [10]. We perform a series of finite-range DWBA calculations, varying the $1p_{1/2}$ ^{12}C $- n$ single-particle parameters to obtain $R^{\rm DW}(b)$ as described before. Results for the less peripheral case (51 MeV) are plotted in Fig. 2 (dotted-dashed curve). We take the data from Ref. [10], and the ANC from Ref. [15] to obtain $R^{\rm exp} = \frac{\sigma(2.5^\circ)}{C^2_{1,1/2}} = 2.92(0.35)$ fm mb/sr. An $S = 0.66$ (shell model) would require $b = 1.89$, which is contained in our results. However, such a conclusion is misleading. Figure 2 shows that, even for this relatively large energy, the dependence of $R^{\rm DW}$ on b is weak. Consequently, it is not possible to extract a SF.

It was pointed out in Ref. [10] that the deuteron breakup is important for this reaction and should be taken into account. To emphasize this fact, we compare our results by using the adiabatic deuteron potential [17] from Ref. [10] (dot-dashed curve in Fig. 2) with those obtained with an optical potential fitted to the deuteron elastic scattering (dotted line in Fig. 2). The disagreement is very large. Interestingly, the method here described is also able to detect inadequate optical potential parametrizations.

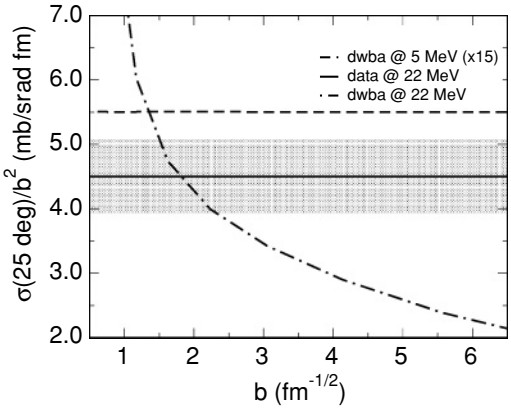

FIG. 1. Cross section for ^{208}Pb$(d,p)^{209}$Pb(ground state) at 22 MeV and the dependence on the single-particle parameters: experimental value (solid line), experimental error bar (shaded area), and the DWBA prediction (dot-dashed curve).

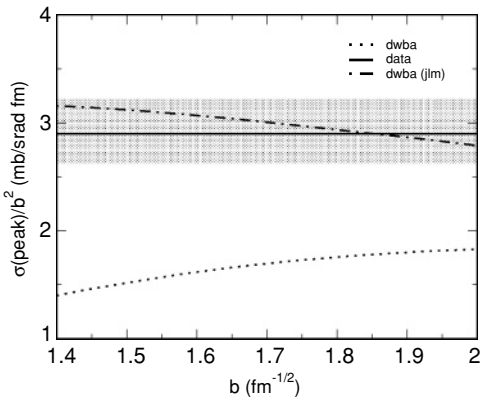

FIG. 2. Cross section for ^{12}C$(d,p)^{13}$C(ground state) at 51 MeV and the dependence on the single-particle parameters: experimental value (solid line), experimental error bar (shaded area), and the DWBA prediction (dot-dashed curve).

Oak Ridge has developed a program to measure a series of inverse kinematics (d,p) reactions for nuclei on the neutron dripline [18]. As one of the nuclei in the program is ^{85}Se, we have performed exploratory calculations for $^{84}\mathrm{Se}(d,p)^{85}\mathrm{Se}$. We take global parametrizations for the optical potentials [16] and perform a series of calculations, varying the single-particle parameters. We compare the dependence of R^{DW} on b for a range of energies $E_d = 4$–100 MeV. We verify that, expectedly, the dependence on b increases with beam energy. We find that Oak Ridge energies (10 MeV/A) are adequate for determining ANCs but not SFs. However, a facility that allows for the production of ^{84}Se at $E > 25$ MeV/A (such as the National Superconducting Cyclotron Laboratory at Michigan State University, GANIL, or RIKEN) could provide accurate spectroscopic information.

In conclusion, we have presented an alternative method for extracting SFs, taking into account the sensitivity of the transfer data to the interior part of the overlap function and combining that information with the ANC. Transfer data can become useful within this method only if it has a significant contribution from the interior and is well described through a one-step DWBA formalism. The balance between these two conditions is not a trivial one. By reducing the error bars in both the measured transfer cross section and the ANC, this prescription determines the single-particle asymptotics and from it a SF with reduced uncertainty. The ANC needs to be determined independently; it can be pinned down accurately with the same transfer reaction at sub-Coulomb energies or by use of heavy-ion-induced reactions, both safely peripheral. Note that uncertainties that are due to optical potentials and higher-order effects need to be assessed independently, as this work focuses on the single-particle parameter uncertainties only.

The method here presented has the potential of reducing the uncertainty in the overlap function considerably. However, it still assumes that the interior part has a Woods-Saxon single-particle wave-function shape. This has been corroborated by recent Green's function Monte Carlo calculations on light nuclei [19]. Even if there were nonlocalities of the single-particle potential, this would affect mostly the deep interior and thus would not be visible in the transfer reactions.

Results for (d,p) on ^{208}Pb were used to illustrate the method. We discussed previous analyses of (d,p) reactions on ^{12}C and showed the limitations. We have also demonstrated that this method can rule out inadequate choices of optical potentials. Considering specific future experiments, we have performed exploratory calculations for (d,p) on ^{84}Se. This method will become useful for a broad variety of transfer experiments in the field of rare isotopes. The same method can equally be used for transfer to excited states. These same ideas can be extended to other reactions, in particular breakup reactions, which also have an impact on astrophysics. Finally, it would be helpful if the state-of-the-art reaction codes would incorporate the formalism discussed.

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