Three-body resonant radiative capture reactions in astrophysics

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We develop the formalism based on the S matrix for $3 \to 3$ scattering to derive the direct three-body resonant radiative capture reaction rate. Within this formalism, the states that decay only/predominantly directly into the three-body continuum should also be included in the capture rate calculations. Basing our work on the derivation, as well as on the modern experimental data and theoretical calculations concerning ^{17}Ne nucleus, we significantly update the reaction rate for the $^{15}\mathrm{O}(2p,\gamma)^{17}\mathrm{Ne}$ process in an explosive environment. We also discuss possible implementations for the $^{18}\mathrm{Ne}(2p,\gamma)^{20}\mathrm{Mg}$, $^{38}\mathrm{Ca}(2p,\gamma)^{40}\mathrm{Ti}$, and $^{4}\mathrm{He}(n\alpha,\gamma)^{9}\mathrm{Be}$ reactions.

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I. INTRODUCTION

The reactions of three-body radiative capture may play a considerable role in the rapid nuclear processes that take place in stellar media under conditions of high temperature and density. The possibility of bridging the waiting points of the *rp* process in explosive hydrogen burning by the $2p$ radiative capture reactions was discussed in Ref. [1]. The reactions $^{15}\mathrm{O}(2p,\gamma)^{17}\mathrm{Ne}$, $^{18}\mathrm{Ne}(2p,\gamma)^{20}\mathrm{Mg}$, and $^{38}\mathrm{Ca}(2p,\gamma)^{40}\mathrm{Ti}$ could be a more efficient way to "utilize" ^{15}O, ^{18}Ne, and ^{38}Ca than to wait for their β^+ decay (corresponding lifetimes are quite large: 122, 1.67, and 0.44 s). The $^{4}\mathrm{He}(n\alpha,\gamma)^{9}\mathrm{Be}$ reaction has been found to be important for building heavy elements in the explosions of supernovae [2,3]. This reaction has been theoretically considered several times in recent years [4–9].

Three-body radiative capture is a very improbable process. It can only be important if the sequence of two-body radiative captures involving the bound states of the intermediate system is not possible. This happens if the bound intermediate system does not exist (along the driplines, the continuum ground states are not uncommon). Then the three-body radiative capture can proceed *sequentially* via the intermediate resonances or *directly* from the three-body continuum (see Fig. 1 for illustration of these modes). The latter is the inverse process to the "true" two-proton radioactivity [10], of which studies are active now [11–22]. The relations between sequential and direct mechanisms of two-proton decay are discussed in detail in Refs. [13,19].

In the modern literature, some misunderstanding exists about the role of direct three-body capture in the theoretical calculations of three-body radiative capture rates. In most cases, this misunderstanding does not lead to any significant problems. However, in some situations, the difference is sufficiently large. In our opinion, the origin of the misunderstanding is the following. The accurate formulas for the resonant three-body capture have been known for a long time (see, e.g., Ref. [23]). But nowadays they do not seem to be always interpreted completely correctly. The possible reason is that for nonresonant capture calculations the sequential capture formalism is used in Refs. [1,6,8,9,24]. At some stage, it has become considered as obtained employing more general assumptions (see, e.g., Ref. [24]) than used in the derivation of Ref. [23], which is based on complete thermal equilibrium and detailed balance.

In this paper, we apply formalism based on the S matrix for $3 \to 3$ scattering to derive the reaction rates for three-body resonant radiative capture. In this approach, the right way of using these formulas becomes evident. We found that the direct and sequential capture mechanisms complement each other. In cases when the sequential process is prohibited energetically or suppressed dynamically, the sequential formalism should underestimate the rate. Among the processes, where significant differences with previous calculations can be found, there are reactions leading to ^{17}Ne and ^{40}Ti.

The unit system $\hbar = c = 1$ is used in this article.

II. THREE-BODY RADIATIVE CAPTURE

The derivations provided below are relatively trivial. Those of Sec. II A can be found, e.g., in Refs. [1,24]. They are presented, however, in much detail to provide unified notation, simplify the reading of the paper, and avoid any possible misinterpretation.

A. Sequential capture

The abundance Y_{A+2} for the nucleus with mass number $A+2$ due to the sequential two-proton capture reaction on the nucleus with mass number A is defined via a three-body reaction rate $\langle\sigma_{pp,\gamma}v\rangle$ as (see, e.g., Ref. [23])

$$\dot{Y}_{A+2} = (1/2)\, N_A^2\, \rho^2\, \langle\sigma_{pp,\gamma}v\rangle\, Y_p^2 Y_A, \tag{1}$$

where ρ is the density and N_A is the Avogadro number. The two-proton reaction rate is defined for the sequential capture of protons (for example, in [1]) by

$$\langle\sigma_{pp,\gamma}v\rangle = 2\sum_i \frac{\langle\sigma_{p,p}v\rangle_i}{\Gamma_{p,i}}\langle\sigma_{p,\gamma}v\rangle_i.$$

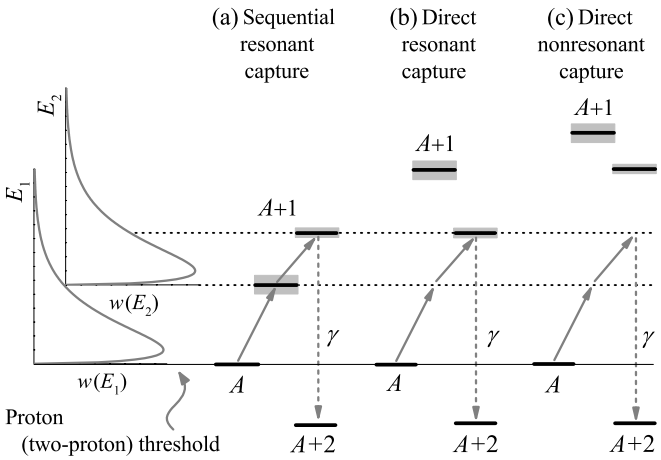

FIG. 1. Schematic illustration of different modes of two-proton radiative capture reaction. Only the ground states are shown for $A+1$ system. Protons are picked up one by one from the continuum with probabilities proportional to Boltzmann distributions $w(E_1)$ and $w(E_2)$. Difference between *sequential* (a) and *direct* (b) resonant capture is the availability of an intermediate resonance in the $A+1$ system. At very low temperature, the resonances are not populated by protons and γ deexcitation proceeds directly from the continuum (c). Note that in the case of direct captures, the ratio E_1/E_2 is not fixed, and different possible ratios should be taken into account.

This expression is a consequence of the rate equations for the resonance number i

$$\begin{aligned} \dot{Y}^{(i)}_{A+1} &= N_A\,\rho\langle\sigma_{p,p}v\rangle_i\; Y_p Y_A - \Gamma_{p,i} Y^{(i)}_{A+1}, \\ \dot{Y}_{A+2} &= \sum_i N_A\,\rho\langle\sigma_{p,\gamma}v\rangle_i\; Y_p Y^{(i)}_{A+1}, \end{aligned} \tag{2}$$

and the assumption about thermodynamic equilibrium for the intermediate resonant states $\dot{Y}^{(i)}_{A+1} = 0$.

The standard expression for the cross section of the resonance reaction with the entrance channel α and exit channel β is

$$\sigma(E) = \frac{\pi}{k^2_{12}}\,\frac{\Gamma_\alpha\Gamma_\beta}{(E-E_R)^2+\Gamma^2/4}\,\frac{2J_{2R}+1}{(2J_1+1)(2J_2+1)}, \tag{3}$$

where J_1 and J_2 are the total spins of incoming particles and J_{2R} is the total spin of the resonance.

In the case of intermediate capture into the narrow proton resonance number i (which also decays practically only via proton emission), $\Gamma_\alpha = \Gamma_\beta = \Gamma_{p,i}$, and

$$\begin{aligned} \langle\sigma_{p,p}v\rangle_i &= \int v\sigma_i(E_{12})w(k_{12})d^3k_{12} = \left(\frac{A_1+A_2}{A_1A_2}\right)^{3/2} \\ &\times \frac{2J_{2R,i}+1}{2(2J_I+1)}\left(\frac{2\pi}{mkT}\right)^{3/2}\exp\left[-\frac{E_{2R,i}}{kT}\right]\Gamma_{p,i}, \end{aligned} \tag{4}$$

where J_I is the total spin of the initial (core) nucleus and $J_{2R,i}$ is the total spin of the resonance number i in the core+p system. The Boltzmann distribution normalized for integration over d^3k_{12} ($k_{12} = \sqrt{2m_{12}E_{12}}$ is the relative motion momentum) is

$$w(k_{12}) = (2\pi m_{12}kT)^{-3/2}\exp[-E_{12}/kT],$$

and we approximate the integral over the resonance profile as

$$\int_{-\infty}^{\infty}\frac{dE}{(E-E_R)^2+\Gamma^2/4} = \frac{2\pi}{\Gamma}.$$

The reaction rate for the subsequent capture of the second proton on the core+p system in the resonant state number i and the following γ emission is

$$\begin{aligned} \langle\sigma_{p,\gamma}v\rangle_i &= \left(\frac{A_1+A_2+A_3}{(A_1+A_2)A_3}\right)^{3/2}\frac{2J_F+1}{2(2J_{2R,i}+1)}\left(\frac{2\pi}{mkT}\right)^{3/2} \\ &\times \exp\left[-\frac{E_{3R}-E_{2R,i}}{kT}\right]\frac{\Gamma_\gamma\Gamma'_{p,i}}{\Gamma_{3R}}, \end{aligned} \tag{5}$$

where J_F is the total spin of the resonance E_{3R} in the core+$2p$ system. $\Gamma'_{p,i}$ is the partial width for decay of this state into the binary channel (core+p)$_i$+p, where the core+p subsystem is in the resonant state number i. It is easy to determine that the two-proton reaction rate for sequential proton capture (through narrow intermediate resonances) is

$$\begin{aligned} \langle\sigma_{pp,\gamma}v\rangle &= \left(\frac{A_1+A_2+A_3}{A_1A_2A_3}\right)^{3/2}\frac{2J_F+1}{2(2J_I+1)}\left(\frac{2\pi}{mkT}\right)^{3} \\ &\times \exp\left[-\frac{E_{3R}}{kT}\right]\frac{\Gamma_\gamma\sum_i\Gamma'_{p,i}}{\Gamma_{3R}}. \end{aligned} \tag{6}$$

The following features of this rate should be noted:

(i) The reaction rate does not depend on the number and properties of the intermediate states, but only on the sum of the proton widths for the population of these states in the decay of the $A+2$ system.

(ii) In the most expected case of the *sequential* decay mode dominance for the three-body resonance E_{3R}, we have $\Gamma_\gamma \ll \Gamma_{3R}, \Gamma_{2p} \ll \Gamma_{3R}$ (Γ_{2p} is the width for the *direct* decay into the $2p$ continuum, the process not proceeding via intermediate core+p resonances), and $\Gamma_{3R} = \sum_i \Gamma'_{p,i}$. So, the reaction rate depends *only* on the γ width of the three-body resonance in $A+2$.

(iii) Equation (6) shows that if there exist other significant decay channels for three-body resonance E_{3R}, then $\Gamma_{3R} > \sum_i \Gamma'_{p,i}$ and the production rate decreases. Such a possible decay channel is, as already mentioned, a *direct* (not via resonances) decay of the three-body resonance E_{3R} into the two-proton continuum. Typically this process is suppressed, but there are cases where this process is not suppressed. Such situations are typically connected with specific separation energy conditions [13]. The direct process can be dominating, or it can even be the only possible decay channel (no intermediate resonances). Such an opportunity is considered in the next section.

B. Direct capture

The abundance Y_{A+2} due to the direct two-proton capture reaction is defined via a three-body reaction rate as

$$\dot{Y}_{A+2} = (1/2)\; N_A^2\,\rho^2\langle\sigma_{2p,\gamma}v\rangle\; Y_p^2 Y_A. \tag{7}$$

To derive the cross section of the direct capture from the three-body continuum, we use the S-matrix formalism for the $3 \to 3$

reaction. The plane wave for three particles can be decomposed over hyperspherical harmonics $\mathcal{I}^{LM_L}_{K\gamma}$ such that

$$\begin{aligned}\Psi_3^{\rm pw} &= \exp[\,i\mathbf{k}_1\mathbf{r}_1 + i\mathbf{k}_2\mathbf{r}_2 + i\mathbf{k}_3\mathbf{r}_3]\chi_{S_1M_1}\chi_{S_2M_2}\chi_{S_3M_3}\\ &= \exp[\,i\mathbf{k}_{\rm c.m.}\mathbf{R}_{\rm c.m.} + i\mathbf{k}_y\mathbf{Y} + i\mathbf{k}_x\mathbf{X}]\chi_{S_1M_1}\chi_{S_2M_2}\chi_{S_3M_3}\\ &= \exp[\,i\mathbf{k}_{\rm c.m.}\mathbf{R}_{\rm c.m.}]\,\frac{(2\pi)^3}{(\varkappa\rho)^2}\sum_{JM}\sum_{Kl_xl_ySS_x} i^K J_{K+2}(\varkappa\rho)\\ &\quad\times \mathcal{J}^{JM}_{Kl_xl_ySS_x}(\Omega_\rho)\sum_{M_L}\mathcal{I}^{LM_L*}_{Kl_xl_y}(\Omega_\varkappa)\, g^{JLSS_x}_{MM_LM_1M_2M_3}, \qquad (8)\end{aligned}$$

where $\chi_{S_iM_i}$ are spin functions for the cluster number i. $\mathcal{J}^{JM}$ denotes the hyperspherical harmonic $\mathcal{I}^{LM_L}$ coupled with spin functions of clusters to the total momentum J, that is,

$$\begin{aligned}\mathcal{J}^{JM}_{Kl_xl_ySS_x} &= \left[\mathcal{I}^{L}_{Kl_xl_y}\otimes X_{SS_x}\right]_{JM},\\ X_{SS_xM_S} &= \left[\left[\chi_{S_1}\otimes\chi_{S_2}\right]_{S_x}\otimes\chi_{S_1}\right]_{SM_S}.\end{aligned}$$

Variables $\mathbf{k}_{\rm c.m.}$ and $\mathbf{R}_{\rm c.m.}$ describe the center-of-mass motion. Conjugated sets of Jacobi variables for the internal motion of the three-body system are $\{\mathbf{k}_x, \mathbf{k}_y\}$ and $\{\mathbf{X}, \mathbf{Y}\}$. The corresponding equivalent sets of hyperspherical variables (in momentum and coordinate spaces) are $\{\varkappa, \Omega_\varkappa\}$ and $\{\rho, \Omega_\rho\}$. Complete definitions of the hyperspherical variables and hyperspherical harmonics can be found, e.g., in Ref. [21]. The dependence on magnetic quantum numbers is

$$g^{JLSS_x}_{MM_LM_1M_2M_3} = \sum_{M_SM_x} C^{JM}_{LM_LSM_S}C^{SM_S}_{S_xM_xS_3M_3}C^{S_xM_x}_{S_1M_1S_2M_2}.$$

Following the same steps as in the two-body case, we define scattering amplitude and decompose it over hyperspherical harmonics. The asymptotic form of the three-body wave function (WF), where center-of-mass motion is omitted, is given by

$$\Psi_3(\rho\to\infty) = \Psi_3^{\rm pw} + \frac{\exp[i\,\varkappa\rho]}{\rho^{5/2}} f_{M_1M_2M_3}(\Omega_\rho, \Omega_\varkappa), \qquad (9)$$

where

$$\begin{aligned}f_{M_1M_2M_3}(\Omega_\rho, \Omega_\varkappa) &= \sum_{JM}\sum_{LSS_x}\sum_{M_LM_S} C^{JM}_{LM_LSM_S}\\ &\quad\times f^{JLSS_x}_{MM_LM_1M_2M_3}(\Omega_\rho, \Omega_\varkappa)\, X_{SS_xM_S}.\end{aligned}$$

So, the $3\to3$ scattering amplitude can be written as

$$\begin{aligned}&f^{JLSS_x}_{MM_LM_1M_2M_3}(\Omega_\rho, \Omega_\varkappa)\\ &\quad= \exp[-i\pi/4]\,(2\pi/\varkappa)^{5/2}\sum_{Kl_xl_y,K'\gamma'}\left(\delta^{K'\gamma'}_{K\gamma} - S^{K'\gamma'}_{K\gamma}\right)\mathcal{I}^{LM_L}_{Kl_xl_y}(\Omega_\rho)\\ &\quad\times\sum_{M'_L}\mathcal{I}^{L'M'_L*}_{K'l'_xl'_y}(\Omega_\varkappa) g^{JL'S'S'_x}_{MM'_LM_1M_2M_3}, \qquad (10)\end{aligned}$$

where $\gamma = \{Ll_xl_ySS_x\}$. In these equations, angles $\Omega_\varkappa$ point to the direction on the hypersphere where the particles came from [the directions defined by momenta in the plane wave (8)]. The angles Ω_ρ in the asymptotic expression (9) define the vectors of particles and the energy distribution after collision.

The cross section of $3\to3$ scattering can be written as

$$\frac{d\sigma(\Omega_\varkappa)}{d\Omega_\rho} = \sum_{SS_xM_S}\left|\sum_{LM_L} C^{JM}_{LM_LSM_S} f^{JLSS_x}_{MM_LM_1M_2M_3}(\Omega_\rho, \Omega_\varkappa)\right|^2,$$

and after integration over the angle Ω_ρ of the outgoing particles, summation over the projections of the final total spin M, and averaging over the projections of spins of the incoming clusters M_i, we obtain

$$\begin{aligned}\sigma(\Omega_\varkappa) &= \left(\frac{2\pi}{\varkappa}\right)^5 G^J_{S_1S_2S_3}\sum_{K\gamma,K'\gamma'}\delta^{S'}_S\delta^{S'_x}_{S_x}\delta^{L'}_L\\ &\quad\times\frac{1}{2L+1}\sum_{M_L}\mathcal{I}^{LM_L}_{Kl_xl_y}(\Omega_\varkappa)\,\mathcal{I}^{L'M_L*}_{K'l'_xl'_y}(\Omega_\varkappa)\\ &\quad\times\sum_{K''\gamma''}\left(\delta^{K\gamma}_{K''\gamma''} - S^{K\gamma}_{K''\gamma''}\right)^\dagger\left(\delta^{K'\gamma'}_{K''\gamma''} - S^{K'\gamma'}_{K''\gamma''}\right). \qquad (11)\end{aligned}$$

Coefficients $G^J_{S_1S_2S_3}$ are combinatorial factors

$$G^J_{S_1S_2S_3} = \frac{2J+1}{(2S_1+1)(2S_2+1)(2S_3+1)}.$$

The astrophysical production rate is given by

$$\langle\sigma_{2p,2p}v\rangle' = 2\int\frac{\varkappa}{m}\,\sigma(\Omega_\varkappa)w(k_1k_2k_3)\,d^3k_1\,d^3k_2\,d^3k_3.$$

The prime symbol in the notation of reaction rate reminds us that it is calculated in the space of scaled hyperspherical variables. The Boltzmann distribution for three particles normalized for integration over $\prod_{i=1,3}d^3k_i$ is

$$w(k_1k_2k_3) = \frac{\exp[-(E_1+E_2+E_3)/kT]}{(2\pi mkT)^{9/2}(A_1A_2A_3)^{3/2}}.$$

After transformation to hyperspherical variables,

$$d^3k_1d^3k_2d^3k_3 \to \left(\frac{A_1A_2A_3}{A_1+A_2+A_3}\right)^{3/2} d^3k_{\rm c.m.}d\Omega_\varkappa\,\varkappa^5 d\,\varkappa,$$

and after $d^3k_{\rm c.m.}$ and $d\Omega_\varkappa$ integration (we should recall here that angles $\Omega_\varkappa$ point to the directions of the incoming particles defined by vectors $\mathbf{k}_i$),

$$\begin{aligned}\langle\sigma_{2p,2p}v\rangle' &= \frac{(2\pi)^6}{\pi(2\pi mkT)^3}G^J_{S_1S_2S_3}\int\exp\left[-\frac{\varkappa^2}{2mkT}\right]\frac{\varkappa}{m}\\ &\quad\times\sum_{K\gamma,K'\gamma'}\left(\delta^{K'\gamma'}_{K\gamma} - S^{K'\gamma'}_{K\gamma}\right)^\dagger\left(\delta^{K'\gamma'}_{K\gamma} - S^{K'\gamma'}_{K\gamma}\right)d\,\varkappa.\end{aligned} \qquad (12)$$

To get the inelastic part of the cross section for the sufficiently narrow resonance, we should replace (see Refs. [25,26])

$$\sum_{K\gamma,K'\gamma'}\left(\delta^{K'\gamma'}_{K\gamma} - S^{K'\gamma'}_{K\gamma}\right)^\dagger\left(\delta^{K\gamma}_{K\gamma} - S^{K'\gamma'}_{K\gamma}\right)$$

with

$$\frac{\Gamma_{2p}\Gamma_\gamma}{(E-E_{3R})^2 + \Gamma^2_{3R}/4}. \qquad (13)$$

For the two-proton capture on the nucleus J_I to the final state J_F (assuming the small width of the three-body resonance),

we obtain

$$\langle\sigma_{2p,\gamma}v\rangle' = \left(\frac{2\pi}{mkT}\right)^3 \frac{2J_F+1}{2(2J_I+1)} \exp\left[-\frac{E_{3R}}{kT}\right] \frac{\Gamma_{2p}\Gamma_\gamma}{\Gamma_{3R}}. \quad (14)$$

The production rate for two-proton capture should be multiplied by squared density ρ^2 to provide the abundance, see Eq. (7). Equation (14) is written in scaled Jacobi variables. The density in these variables can be expressed via density in the ordinary space as

$$\rho^2_{\text{scaled}} = \rho^2 \left(\frac{A_1+A_2+A_3}{A_1A_2A_3}\right)^{3/2}.$$

The expression for production rate which can be used with the expression for density in normal space (indicated by the absence of the prime symbol) is, therefore,

$$\langle\sigma_{2p,\gamma}v\rangle = \left(\frac{A_1+A_2+A_3}{A_1A_2A_3}\right)^{3/2} \frac{2J_F+1}{2(2J_I+1)} \left(\frac{2\pi}{mkT}\right)^3 \times \exp\left[-\frac{E_{3R}}{kT}\right] \frac{\Gamma_{2p}\Gamma_\gamma}{\Gamma_{3R}}. \quad (15)$$

Equation (15) is absolutely the same as Eq. (6) except for the dependence on the decay width of the $A+2$ system to the three-body continuum Γ_{2p} instead of decay widths to the resonant states in the $A+1$ system $\Gamma'_{p,i}$. We can draw the following conclusions here:

(i) Equation (15) is obtained with very general assumptions about the existence of the asymptotic (9) and analytical properties of the $3 \to 3$ scattering S matrix. We also use the fact that most of the states of interest are narrow. No other assumptions are made (e.g., in sequential formalism, there is the assumption about the existence of the specific decay path). So, the direct capture (unlike sequential capture) is always possible.

(ii) It is clear that Eqs. (6) and (15) supplement each other, and the total reaction rate is

$$\langle\sigma_{2p,\gamma}v\rangle + \langle\sigma_{pp,\gamma}v\rangle = \left(\frac{A_1+A_2+A_3}{A_1A_2A_3}\right)^{3/2} \frac{2J_F+1}{2(2J_I+1)} \times \left(\frac{2\pi}{mkT}\right)^3 \exp\left[-\frac{E_{3R}}{kT}\right] \frac{\Gamma_{2p}+\sum_i \Gamma'_{p,i}}{\Gamma_{3R}}\Gamma_\gamma. \quad (16)$$

(iii) In the most likely situation $\Gamma_{2p} + \sum_i \Gamma'_{p,i} \gg \Gamma_\gamma$ (and, hence, $\Gamma_{3R} = \Gamma_{2p} + \sum_i \Gamma'_{p,i}$), the total reaction rate depends *only* on the γ width of the three-body resonance in the $A+2$ system. However, it is possible that the direct two-proton emission is the only nuclear decay branch for the state. The width Γ_{2p} could be very small (smaller than the γ width) in a relatively broad range of the three-body decay energies [21]. In that case, the reaction rate depends *only* on Γ_{2p}.

(iv) Equation (16) gives the formula for the reaction rate, which is the same as the one known for a long time [see, e.g., Eq. (20) in Ref. [23]] and was obtained by much easier means (namely, a complete thermal equilibrium and a detailed balance) than in this work. The result of our derivation here is a clearer understanding of the fact that this formula already correctly and completely includes both sequential capture and direct capture reactions.

So, we see that for the resonant part of the reaction rate, the sequential formalism treatment is overcomplicated and incomplete. This is not a great issue in most cases, but there are situations where it becomes important. The impact of the formalism on the rates of reactions of astrophysical interest is discussed in Sec. III.

C. Formal questions

The derivations of the reaction rates in Secs. II A and II B are quite schematic. They basically rely on assumptions about the existence of definite asymptotics of the three-body problem. These assumptions could be not evident, and they require if not a proof then at least some discussion.

For sequential formalism, we need there to exist a long-living resonance state in the X Jacobi subsystem [at energy $E_x = k_x/(2M_x)$ and width Γ_x]. Then the asymptotic implied in the derivations of Sec. II A is

$$\Psi_3(\{X,Y\} \to \infty) = \Psi_3^{\text{pw}} + \frac{e^{ik_xX}}{X} f(\hat{k}_x) \frac{e^{ik_yY}}{Y} f(\hat{k}_y). \quad (17)$$

For the direct capture, the assumed asymptotic is

$$\Psi_3(\rho \to \infty) = \Psi_3^{\text{pw}} + \frac{e^{i\varkappa\rho}}{\rho^{5/2}} f(\Omega_\rho, \Omega_\varkappa), \quad (18)$$

where $k_x^2/(2M_x) + k_y^2/(2M_y) = \varkappa^2/(2m) = E_{3R}$.

Expressions (17) and (18) correspond to neutral particles, whereas we are speaking about nuclei $Z = 8$–20 capturing protons. The typical densities for x-ray bursts and processes in novae are 10^3–10^6 g/cm^3 [27]. For such densities, the average distances between protons are about 6×10^2 to 2×10^4 fm; and for characteristic temperatures, the Debye screening radii, are 3×10^3 to 5×10^5 fm. Beyond these radii, we have a formal right to use the asymptotics as they are given by Eqs. (17) and (18).

It should be understood that in a very formal sense, the asymptotic (17) is not valid. Equation (17) implies that subsystem X can be found in the resonance state E_x at any distance Y of the third particle separation. This is not possible because of the finite width Γ_x of this resonant state. In the limit of infinite distance, a long-living two-body state finally decays, and the asymptotic (17) should be replaced by (18). However, from the practical side, the separation of asymptotics (17) and (18) is reasonable. A nice example (also relevant to the further discussion) of the coexistence of the three-body and binary asymptotics is the decay of the ^{9}Be $5/2^-$ state at 2.429 MeV. The branchings of this state to the three-body $\alpha+\alpha+n$ channel $Br(3)$ and binary ^{8}Be(g.s.)+n channel $Br(2)$ are comparable [$Br(3) \sim 0.93$–0.95 and $Br(2) \sim 0.07$–0.05 [28]]. Separation of these decay branches is reliably experimentally observed (see, for example, Ref. [29]). In the case of binary decay, the average flight distance of the ^{8}Be g.s. resonance ($\Gamma_x = 6.8$ eV) is around 10^6 fm. Thus, for some practical purposes, the assumption of Eq. (17) is clearly justified. It is also clear that the broader the resonance in the X subsystem, the faster the transition from asymptotic Eq. (17) to asymptotic Eq. (18) happens. For example, for the width of the intermediate resonance Γ_x around 100 keV (and typical $E_{3R} = 1$ MeV),

the average flight distance of this resonance is around 100 fm. This is much smaller than the typical distance between protons in the stellar media, and then usage of Eq. (17) and chemical balance description by Eq. (2) makes little sense.

The other difficulty is that asymptotics (17) and (18) are located in the same space and could have the same quantum numbers. However, we can speak about orthogonality of these asymptotics in a definite sense and then treat currents associated with them independently. Only this assumption makes possible the separate treatment of sequential and direct decay channels in Secs. II A and II B. The asymptotic (17) is typically localized in a very small part of the phase space. Formally, this corresponds to the fact that the hyperspherical series for binary channel (at given hyperradius) is very long (with many significant terms), while for asymptotic (18) we can expect that only the lowest hyperspherical harmonics in the decomposition are significant. Thus the sequential and direct channels are practically orthogonal on the hypersphere of a large radius. With an increase in the radius, this "orthogonality" (called asymptotic orthogonality) becomes better, until the effect of the X subsystem decay becomes important. Again, using the example of the $5/2^-$ state of ^{9}Be, the level of overlap between the direct and sequential decay channels in the momentum space can be estimated as

$$Br(3)\,Br(2)\,\frac{\Gamma_{^8\mathrm{Be}}(0^+)}{E_{3R}(5/2^-)} = 0.06\,\frac{6.8\ \mathrm{eV}}{764.1\ \mathrm{keV}} \sim 5\times 10^{-7},$$

which is clearly a very small value. When there is no longer such reliable separation between channels (17) and (18) (the intermediate resonances are too broad), we have a formal right to speak only about asymptotic (18).

To finalize this discussion, let us emphasize again that the derivations in Secs. II A and II B were done as if only one type of the asymptotic exists. From a formal point of view, only the asymptotic of Eq. (18) exists. For practical purposes, either (i) only the asymptotic of Eq. (18) exists or (ii) both asymptotics (17) and (18) are present simultaneously. Asymptotic (18) exists in the three-body problem (of the type we consider, see Fig. 1) unconditionally, while the existence of asymptotic (17) is subject to the availability of *sufficiently narrow* intermediate resonances for the decay. In case (ii), the regions of the domination of each asymptotic are separated by a complicated surface in the phase space. Some discussion of the relevant questions can be found, for example, in Ref. [26]. So, the phase space integration in Secs. II A and II B should have been done not over the whole space, but over regions of validity for each type of asymptotic. It is clear, however, that this imperfection does not influence the final result because (i) the contribution of the asymptotic of the selected kind in the phase space outside the region of its domination is typically negligible and (ii) we are interested in the contribution of both kinds of asymptotics simultaneously.

III. DISCUSSION

A. ^{15}O$(2p,\gamma)^{17}$Ne reaction

The results of resonant rate calculations for this reaction are shown in Fig. 2 and in Table I. They differ significantly from the results of Ref. [1] (shown in Fig. 2 by dashed and dotted curves). For the temperature range of astrophysical interest (∼0.3–3 GK, see [27], for example), the expected increase of the rate, compared to Ref. [1], is up to four orders of the magnitude, while the maximum possible increase is up to nine orders of magnitude.

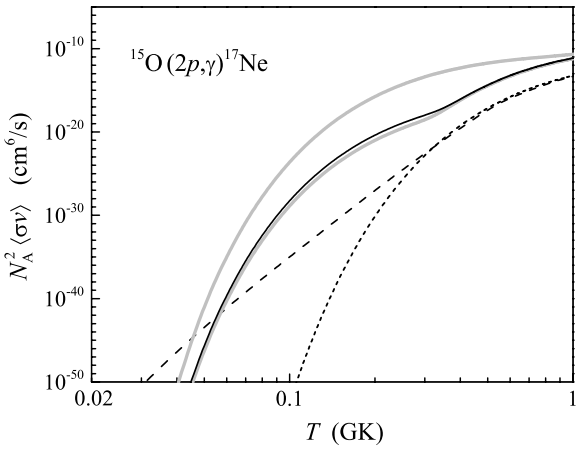

FIG. 2. Reaction rate for ^{15}O$(2p,\gamma)^{17}$Ne reaction. Solid curves show calculations of this paper. Gray curves indicate boundaries due to uncertainties in the input (they are obtained with "Lower" and "Upper" columns from Table II). Dashed and dotted curves show full result from Ref. [1] and resonance contribution to it, respectively.

The reasons for the difference are evident from Table II, which gives inputs for calculations of this work and the previous one.

(i) The level scheme of ^{17}Ne has been somewhat updated (see, e.g., Ref. [30]) since the work [1] was written.

(ii) The use of Eq. (15) includes the first $3/2^-$ excited state of ^{17}Ne into treatment (it was omitted in Ref. [1], as there is no sequential capture path to this state). The important difference in the situation with this state from the others is that the γ width of this state is known to be much larger than the $2p$ width, and the reaction rate Eq. (16) is entirely defined by the $2p$ width for the simultaneous two-proton emission. At the moment, there exist two theoretical calculations of this width: $\Gamma_{2p} = 4.1\times 10^{-16}$ MeV [18] and $\Gamma_{2p} = 3.6\times 10^{-12}$ MeV [31], and there is a quite relaxed experimental lower lifetime limit of

TABLE I. Reaction rates multiplied by N_A^2 (in cm^6/s) for ^{15}O$(2p,\gamma)^{17}$Ne reaction.

T (GK)	Ref. [1]	This work	This work upper
0.3	2.9×10^{-23}	4.9×10^{-19}	2.9×10^{-14}
0.5	6.0×10^{-18}	2.1×10^{-15}	1.3×10^{-12}
0.6	1.2×10^{-16}	2.8×10^{-14}	2.8×10^{-12}
0.8	5.6×10^{-15}	6.3×10^{-13}	6.9×10^{-12}
1.0	5.0×10^{-14}	3.5×10^{-12}	1.3×10^{-11}
1.5	1.1×10^{-12}	2.5×10^{-11}	3.8×10^{-11}
2.0	6.0×10^{-12}	5.1×10^{-11}	6.8×10^{-11}
3.0	4.3×10^{-11}	7.6×10^{-11}	1.1×10^{-10}
5.0	2.5×10^{-10}	7.3×10^{-11}	2.3×10^{-10}

TABLE II. Resonance parameters of ^{17}Ne states used in the ^{15}O$(2p,\gamma)^{17}$Ne reaction calculations. Lower and Upper columns show parameter sets used to estimate the lower and upper limits for the rate. They are defined using the experimental or theoretical uncertainties of the widths. The listed set of states is sufficient for rate calculations up to 5 GK. Widths and branchings not specifically discussed are from [28].

State	Type	Ref. [1]		This work			
J^π		E (keV)	Γ (eV)	E (keV)	Lower Γ (eV)	Median Γ (eV)	Upper Γ (eV)
$3/2^-$	Γ_{2p}			1288	2×10^{-10}	[a]4.1×10^{-10}	[b]2.5×10^{-5}
$5/2^-$	Γ_γ	1907	6.0×10^{-5}	1764	1.5×10^{-3}	[c]1.7×10^{-3}	[c]2.0×10^{-3}
$1/2^+$	Γ_γ	1850	1.6×10^{-5}	1908	1.1×10^{-5}	1.6×10^{-5}	[d]2.1×10^{-4}
$5/2^+$	Γ_γ	2526	2.0×10^{-5}	2651	8.2×10^{-6}	[e]9.0×10^{-6}	[d]9.9×10^{-5}
$3/2^-$	Γ_γ	3204	0.022	3204	0.019	[f]0.019	[d]0.19

[a]A $2p$ width [see Eq. (16): the γ width is dominating the decay of this state], calculated theoretically in Ref. [18].
[b]A $2p$ width; this experimental limit is found in Ref. [17].
[c]Calculated from $B(E2) = 124(18)\ e^2$ fm^4 given in Ref. [17].
[d]A partial width (88% branching) into the ground and first excited states of ^{17}Ne.
[e]An assumed value (in analogy with the more than an order of magnitude increase for the $5/2^-$ state from column 3 to 5).
[f]A partial width (45% branching) into the $1/2^-$ ground and the first excited $3/2^-$ states of ^{17}Ne. γ transition to $1/2^+$ state returns the system into $2p$ continuum.

$\tau > 26$ ps [17] (which corresponds to the width $\Gamma_{2p} < 2.5 \times 10^{-11}$ MeV). Values Γ_{2p} from Refs. [18] and [17] are used to estimate, respectively, the lower and the upper boundaries for the band of expected values of the rate (see Fig. 2). The resonance contribution of this state is dominating the rate in the temperature range 0.05–0.35 GK if we take the theoretical $2p$ width from Ref. [18], and up to 1.2 GK if we consider the experimental limit.

(iii) In paper [1] the γ widths for ^{17}Ne were taken from transitions studied in the isobaric mirror partner ^{17}N. Recently, the decay of the first excited states of ^{17}Ne $(3/2^-, 5/2^-)$ has been studied via the intermediate-energy Coulomb excitation of a radioactive ^{17}Ne beam on a ^{197}Au target [11,17]. In these papers, the transition matrix elements $B(E2,1/2^- \rightarrow 3/2^-)$ and $B(E2,1/2^- \rightarrow 5/2^-)$ were deduced. We used the deduced $B(E2)$ value from Ref. [17] to calculate the γ width of the $5/2^-$ state. The result is shown in Table II. This width appears to be about 30 times larger than the corresponding width of the mirror state in ^{17}N. This is probably connected to the fact that at the proton-rich side, the number of protons contributing to gamma transitions is larger and these protons are situated at larger distances compared with the tightly bound protons in ^{17}N. The relation of this observable to the possible existence of a proton halo in ^{17}Ne is discussed in paper [32]. This situation is also expected for the other states in ^{17}Ne (compared to the states in ^{17}N), which is reflected by an order of magnitude increase in the other widths for estimates of the upper limits (column "Upper" in Table II).

The uncertainty of the obtained rate is quite large, which is connected mainly with the discrepancy between available theoretical results [18,31]. Both theoretical calculations predict almost equal structure for the ^{17}Ne $3/2^-$ state (see Table II in Ref. [31]). However, the widths differ by four orders of magnitude. Theoretical width calculations in paper [18] are based on the quantum mechanical model with approximate boundary conditions of the three-body "democratic" Coulomb problem [12,13]. The model was developed for studies of $2p$ radioactivity and was tested on a broad range of nuclear decays from the lightest clusterized systems (^{6}Be [12], ^{12}O, ^{16}Ne [14], ^{9}Be*,^{8} Li* [33]) to the heaviest known $2p$ emitters (^{45}Fe [15,21] and ^{54}Zn [22]). Width calculations in paper [31] were performed using a quasiclassical WKB formula, where penetration was considered via the lowest branch of the hyperspherical adiabatic potential. This model provides reasonable results for the decays of the ^{6}He 2^+ state and ^{12}C 0^+ states [34]. Reasons for disagreement between the models in the particular case of ^{17}Ne $3/2^-$ states are not yet clear and require further investigation. A further experimental improvement of the lifetime limit, obtained in [17], is an extremely complicated task, but could possibly resolve this problem.

B. ^{18}Ne$(2p,\gamma)^{20}$Mg reaction

For the ^{18}Ne$(2p,\gamma)^{20}$Mg reaction, there are no three-body decaying states that were not taken into account in Ref. [1], so, no significant update of the rate is expected here. However, the level scheme and γ widths are not known experimentally for this nucleus, and this should be reflected in the rate calculations.

In the cases of $2p$ capture into ^{17}Ne and ^{40}Ti, the γ widths from mirror isobaric partners were used in Ref. [1]. In contrast, for capture into ^{20}Mg, the systematics values were utilized. The theoretical $B(E2)$ values for some low-lying states in ^{20}Mg have been calculated recently in Ref. [35]. The $B(E2,4_1^+ \rightarrow 2_1^+)$ was found to be 28.2 or 11.6 e^2 fm^4 (for V2 and MN forces, respectively), and $B(E2,2_2^+ \rightarrow 0_1^+)$ was found to be 2.9 or 1.9 e^2 fm^4. These reduced probabilities give γ widths 5.6×10^{-4} or 2.3×10^{-4} eV for the 4_1^+ state (which is comparable to the value 2.1×10^{-4} eV used in [1]) and 2.0×10^{-3} or

TABLE III. Resonance parameters of ^{20}Mg states used in the ^{18}Ne$(2p, \gamma)^{20}$Mg reaction calculations.

State	This work lower		This work upper	
J^π	E (keV)	Γ_γ (eV)	E (keV)	Γ_γ(eV)
4_1^+	3570	2.1×10^{-4}	3451	5.6×10^{-4}
2_2^+	4072	1.3×10^{-3}	3857	8.9×10^{-2}
0_2^+	4456	1.4×10^{-3}	4317	1.4×10^{-3}
4_2^+	4850	2.6×10^{-3}	4699	2.6×10^{-3}
2_3^+	5234	2.9×10^{-1}	4978	2.9×10^{-1}

1.3×10^{-3} eV for the 2_2^+ state (which is significantly less than the 8.9×10^{-2} eV used in [1]). We combine the largest and the lowest γ widths from Refs. [1] and [35] to estimate the upper and the lower boundaries for the rate (see Table III). To incorporate the sensitivity to the level scheme in this estimate, we also use for the lower estimate the energies of the states from ^{20}O. The distance between levels here is expected to be somewhat larger than in ^{20}Mg [1], and the reaction rate thus should further decrease.

The results of calculations are shown in Table IV. The upper boundary in our calculations is in good agreement with results of [1] (factor of 2) at $T \geqslant 0.8$ GK. It was shown in [1] that below 0.8 GK, the nonresonant contribution to the reaction rate dominates, which explains the discrepancy in Table IV at low temperatures.

C. ^{38}Ca$(2p, \gamma)^{40}$Ti reaction

For the ^{38}Ca$(2p, \gamma)^{40}$Ti reaction, one three-body decaying state has been omitted in Ref. [1]. According to isobaric symmetry, there should be a 0_2^+ state located at about 2.121 MeV excitation energy. The two-proton separation energy used in Ref. [1] is $S_{2p} = 1.582$ MeV. Another estimate (e.g. [36]) is $S_{2p} = 1.370$ MeV. In the first case, the $2p$ emission energy for the 0_2^+ state is 539 keV, and (following Refs. [19,21]) the two-proton width can be estimated as about 10^{-21} MeV. In the second case, the $2p$ energy is 751 keV and the estimated two-proton width is around 10^{-18} MeV. For other states, we use parameters from Ref. [1] (see Table V), which mainly come from the isobaric mirror partner ^{40}Ar. To estimate the upper boundary for the reaction rate we increase the γ widths of 4_1^+ and 2_3^+ states by an order of magnitude. As already discussed, one could expect a significant increase in the γ widths when we come to the proton-rich mirror partner. To estimate the sensitivity to the level scheme (which is not known for ^{40}Ti), we use the smaller $2p$ separation energy $S_{2p} = 1.370$ MeV for the estimate of the lower boundary and $S_{2p} = 1.582$ MeV for the upper boundary. Again, as in the case of ^{20}Mg, the increase of the state energy above the $2p$ threshold leads to a decrease of the corresponding reaction rate. For that reason, we use the larger $2p$ width as the estimate of the lower boundary (Table V, line 1): the larger two-proton width corresponds to the case of larger energy of states above the $2p$ separation threshold.

TABLE IV. Reaction rates multiplied by N_A^2 (in cm^6/s) for ^{18}Ne$(2p, \gamma)^{20}$Mg reaction.

T (GK)	Ref. [1]	This work lower	This work upper
0.3	4.4×10^{-21}	9.3×10^{-28}	2.5×10^{-25}
0.5	3.3×10^{-17}	4.3×10^{-20}	1.8×10^{-18}
0.6	4.8×10^{-16}	3.0×10^{-18}	8.3×10^{-17}
0.8	1.8×10^{-14}	5.1×10^{-16}	9.5×10^{-15}
1.0	2.4×10^{-13}	9.6×10^{-15}	1.8×10^{-13}
1.5	6.8×10^{-12}	3.7×10^{-13}	1.1×10^{-11}
2.0	9.7×10^{-11}	2.0×10^{-12}	8.0×10^{-11}
3.0	4.8×10^{-10}	1.5×10^{-11}	4.4×10^{-10}
5.0	2.4×10^{-9}	1.4×10^{-10}	1.2×10^{-9}

TABLE V. Resonance parameters of ^{40}Ti states used in the ^{38}Ca$(2p, \gamma)^{40}$Ti reaction calculations. In the Type column, the type of width is specified, which defines the contribution of the state to reaction rate. The Lower set of widths is used in the calculations with $S_{2p} = 1.370$ MeV; the Upper set, with $S_{2p} = 1.582$ MeV.

J^π	E (keV)	Type	Lower Γ (eV)	Upper Γ (eV)
0_2^+	2121	Γ_{2p}	10^{-12}	10^{-15}
2_2^+	2524	Γ_p	1.0×10^{-5}	1.0×10^{-5}
4_1^+	2892	Γ_γ	2.0×10^{-4}	2.0×10^{-3}
2_3^+	3208	Γ_γ	1.0×10^{-2}	1.0×10^{-1}

The results of calculations for ^{38}Ca$(2p, \gamma)^{40}$Ti are given in Fig. 3 and Table VI. Our results are somewhat larger (one to two orders of magnitude) than results of [1] for temperatures $T > 1$ GK. They more or less overlap at lower temperatures. The effect of inclusion of the 0_2^+ 2.121 MeV state can be seen in Fig. 3: the range between upper and lower boundaries shrinks at $T < 0.35$ GK because the contribution of the 0_2^+

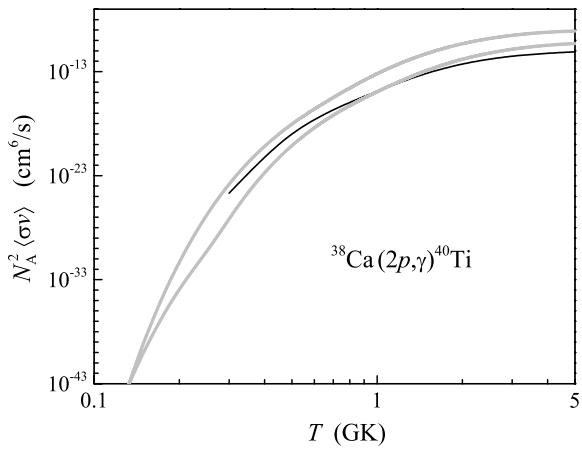

FIG. 3. Reaction rate for ^{38}Ca$(2p, \gamma)^{40}$Ti reaction. Solid curve shows the result from [1]. Gray curves indicate upper and lower boundaries for our results (see Table VI) due to existing uncertainties in the input.

TABLE VI. Reaction rates multiplied by N_A^2 (in cm^6/s) for ^{38}Ca$(2p,\gamma)^{40}$Ti reaction.

T (GK)	Ref. [1]	This work lower	This work upper
0.3	2.1×10^{-25}	7.0×10^{-28}	2.3×10^{-24}
0.5	1.0×10^{-19}	7.8×10^{-21}	1.1×10^{-18}
0.6	2.4×10^{-18}	4.0×10^{-19}	3.1×10^{-17}
0.8	1.1×10^{-16}	5.3×10^{-17}	3.0×10^{-15}
1.0	1.3×10^{-15}	1.2×10^{-15}	7.1×10^{-14}
1.5	7.0×10^{-14}	1.3×10^{-13}	6.0×10^{-12}
2.0	5.2×10^{-13}	1.5×10^{-12}	5.1×10^{-11}
3.0	3.0×10^{-12}	1.4×10^{-11}	3.2×10^{-10}
5.0	8.0×10^{-12}	5.0×10^{-11}	8.2×10^{-10}

state is much larger in the lower parameter set, which otherwise provides a smaller reaction rate.

D. ^{4}He$(n\alpha,\gamma)^9$Be reaction

The stellar reaction rate for ^{4}He$(n\alpha,\gamma)^9$Be process has been studied several times in recent years [4–9]. The results are in overall agreement, except for the latest paper [9]. In this work, the rate obtained is significantly higher (for temperatures $T > 3$ GK) than the rates in the previous studies.

In our studies here, we have found that the sequential formalism underestimates the reaction rate only if the width of the state for direct decay into the continuum is dominating (see Sec. II A). The low-lying ($E \leqslant 3$ MeV) ^{9}Be states typically have strong ^{8}Be $+ n$ decay branchings. Only the $5/2^-$ 2.429 MeV state is an exception: the branching to the three-body channel is 93–95% Refs. [28,29]. The γ width of this state is 0.091 eV [28]. The results of our calculations are shown in Fig. 4 and Table VII. In these calculations, we use a version of Eq. (16) without an assumption about narrow widths of the resonances, and the capture cross section is parametrized as in Ref. [8] (with the exception that the $5/2^-$ state is included).

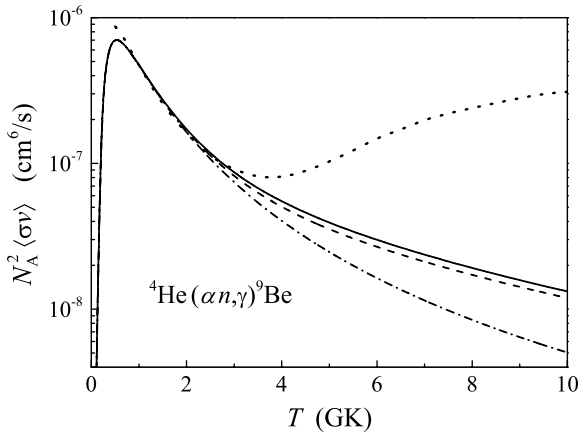

FIG. 4. Reaction rate for ^{4}He$(n\alpha,\gamma)^9$Be reaction. Solid curve shows our result (see also Table VII). Dashed curve is the same, but without $5/2^-$ 2.429 MeV state contribution (this coincides with result of Ref. [8]). Dash-dotted curve is contribution from the near threshold $1/2^+$ state. Dotted curve shows calculations of Ref. [9].

TABLE VII. Reaction rates multiplied by N_A^2 (in cm^6/s) for ^{4}He$(n\alpha,\gamma)^9$Be reaction.

T (GK)	Ref. [8]	Ref. [8] upper	This work
2	1.80×10^{-7}	2.20×10^{-7}	1.83×10^{-7}
4	5.48×10^{-8}	6.99×10^{-8}	5.94×10^{-8}
6	2.88×10^{-8}	3.83×10^{-8}	3.20×10^{-8}
8	1.81×10^{-8}	2.47×10^{-8}	2.02×10^{-8}
10	1.23×10^{-8}	1.70×10^{-8}	1.36×10^{-8}

The results obtained are in very good agreement with [8]. The rate increase due to addition of the $5/2^-$ state is 11% at most in the temperature range up to 10 GK. This small change is connected with a comparatively small γ width of this state: the γ widths of the other states in the capture cross section parametrization used in [8] are around 0.45–0.9 eV. So, the uncertainty in the reaction rate due to uncertainties in the experimental data found in [8] is significantly larger than the correction connected with the $5/2^-$ state (see Table VII).

This experimental uncertainty could be even larger than indicated in Ref. [8]. The analysis provided in Ref. [7] in the framework of the semimicroscopic model demonstrated that the older photodisintegration data for ^{9}Be [37–39] could be more preferable than the more up-to-date results [40] (on which, e.g., the parametrization of the cross section used in Ref. [8] is based). The reaction rate found in [7] (as well as in the early work [4]) is around 35% larger than the rate in Ref. [8].

Paper [9] is generally dedicated to the R-matrix analysis of the β-delayed particle decay of ^{9}C via the excited states in ^{9}B. The authors utilize the R-matrix parameters obtained in the decay studies of ^{9}B for the capture calculations in ^{9}Be. The reaction rate calculated in this work is consistent with the other results at low temperatures, but is qualitatively different at $T > 3$ GK (see Fig. 4, dotted curve). The rise of the reaction rate at higher temperatures is connected, according to [9], with the contribution of the sequential capture of an α particle on the broad ground state of ^{5}He. Such a capture path has never been considered elsewhere.

It should be noted that in the framework of sequential formalism, this is a valid question: How narrow should the intermediate state be to be considered within this formalism? Really, the sequential formalism is evidently correct in the limit of an infinitely narrow intermediate state. However, in the other limit (an infinitely broad state), we have just the nonresonant continuum, and the sequential formalism should fail at some point. This issue is qualitatively discussed in Sec. II C. Our work resolves this question in a very natural way: we state that contributions of different sequential and three-body channels should add up in a way that makes their relative contributions unimportant. So, inclusion of capture via ^{5}He into formalism should not lead to any significant changes (compared to conventional sequential capture via ^{8}Be g.s.) until there exist states with a dominating three-body decay branch (which are not accounted for in sequential formalism) and large γ widths. No such states are known in the energy range of interest. The reaction rate from Ref. [9] can be reproduced within our formalism only if we assume that the γ

width for the 3 MeV state in ^{9}Be is about 15 eV and that one more state at about 5 MeV has a γ width above 1 keV. Such assumptions are quite unrealistic.

Unfortunately, there is evidence of problems in Ref. [9] which probably have led to the discussed strange result. In Eq. (32) of that work, the penetrability is present in the first power, while it should be in the second (as we speak about elastic cross section). Possibly this is the reason for the qualitatively incorrect behavior of the intermediate population values (see Fig. 8 in Ref. [9]). For example, the population $\langle \sigma(E)v/\Gamma(E) \rangle$ for ^{5}He g.s. should decrease as T at low temperature. In Fig. 8 of Ref. [9], this value has a rapid rise at low temperature. Using Eq. (32) from Ref. [9] "as is" one gets behavior $T^{-1/2}$ at low T in agreement with this figure.

So, the difference found in our approach due to inclusion of three-body decaying states is not significant in the case of ^{4}He$(n\alpha, \gamma)^9$Be reaction. It is much smaller than the other uncertainties (see Refs. [7] and [8]). However, our formalism excludes such a possibility as a contribution of a broad intermediate ^{5}He state (as in Ref. [9]) to the capture rate.

IV. CONCLUSION

We use the formalism based on the S matrix for $3 \to 3$ scattering to derive the reaction rate for the three-body resonant radiative capture. This derivation makes especially evident that (i) all the three-body states should be included in the treatment (even if there is no opportunity of a sequential capture to the state), (ii) the detailed knowledge of the intermediate states is unnecessary to calculate the resonant rates, and (iii) only the knowledge of particle and γ widths for the three-body states is needed to calculate the resonant rates (not the relative contribution of direct and sequential mechanisms).

This formalism, together with the modern results on $2p$ and γ widths of ^{17}Ne states, allows us to update significantly the capture rate for the ^{15}O$(2p, \gamma)^{17}$Ne reaction. The updated rate is four to nine orders of magnitude larger (in the temperature range of astrophysical interest). The experimental derivation of the $2p$ width of the first excited state in ^{17}Ne is found to be very important for refining this rate. The ^{38}Ca$(2p, \gamma)^{40}$Ti reaction rate has also been a increased considerably. Thus, the conclusions about importance of the $2p$ capture reactions could possibly be more optimistic than in Ref. [1]. We also discuss the impact of our approach on the ^{18}Ne$(2p, \gamma)^{20}$Mg and ^{4}He$(n\alpha, \gamma)^9$Be reaction rates. Our studies emphasize the importance of better γ width information for $2p$ capture rates (experimental or theoretical, if the first is not available).

The studies of this work are restricted to resonant reactions (and correspondingly to relatively high temperatures). We are planning to perform accurate three-body studies of the nonresonant contributions in a forthcoming paper.

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