Gauge symmetric $\Delta(1232)$ couplings and the radiative muon capture in hydrogen

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By use of the difference between the gauge symmetric and standard $\pi N\Delta$ couplings, a contact $\pi\pi NN$ term, quadratic in the $\pi N\Delta$ coupling, is explicitly constructed. In addition, contribution from the Δ excitation mechanism to the photon spectrum for the radiative muon capture in hydrogen is derived from the gauge symmetric $\pi N\Delta$ and $\gamma N\Delta$ couplings. It is shown for the photon spectrum, studied experimentally recently for photon momentum $k > 60$ MeV, that this contribution is smaller by 4–10% than the one obtained from the standardly used couplings for the on-shell Δ's.

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I. INTRODUCTION

The photon spectrum in the radiative muon capture in hydrogen,

$$\mu^- + p \longrightarrow \nu_\mu + \gamma + n, \tag{1.1}$$

has recently been calculated by several authors [1–5] in search of a process that enhances the high-energy part of the photon spectrum, as calculated earlier [6]. It was concluded [3], in the studies that were made both within the small-scale expansion scheme [7] and in the heavy baryon chiral perturbation theory [8], that a combination of various small effects could explain the experimental spectrum [9,10]. However, the actual sizes of some of these effects, such as the charge symmetry breaking, are to be considered in more detail. On the other hand, this spectrum was calculated in [5] with amplitudes derived from Lagrangians possessing the hidden local $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry [11,12]. In particular, the vertices containing the $\Delta(1232)$ isobar field were chosen as

$$\mathcal{L}_{N\Delta\pi\rho a_1} = \frac{f_{\pi N\Delta}}{m_\pi}\,\bar\Psi_\mu \vec{T}\mathcal{O}_{\mu\nu}(Z)\Psi\cdot(\partial_\nu\vec\pi + 2f_\pi g_\rho \vec a_\nu) - g_\rho\frac{G_1}{M}\bar\Psi_\mu\vec T\mathcal{O}_{\mu\eta}(Y)\gamma_5\gamma_\nu\Psi\cdot\vec\rho_{\eta\nu} + \mathrm{H.c.} \tag{1.2}$$

Here $\vec T$ is the operator of the isospin $1/2 \to 3/2$ transition. The operator $\mathcal{O}_{\mu\nu}(B)$ is taken in a form [13–15]

$$\mathcal{O}_{\mu\nu}(B) = \delta_{\mu\nu} + C(B)\gamma_\mu\gamma_\nu, \tag{1.3}$$

$$C(B) = -\left(\tfrac{1}{2} + B\right). \tag{1.4}$$

The parameters Y and Z do not influence the on-shell properties of the Δ isobar; hence they are called off-shell parameters. Lagrangian (1.2) has been used frequently [13–17] for studying the πN reactions and the pion photo-production and electroproduction on a nucleon and the parameters of the model, including Y and Z, were extracted from the data.

On the other hand, one can also find an attempt [18] to show that the off-shell parameters are redundant within the framework of effective-field theories. For this purpose, Tang and Ellis considered the Lagrangian of the $\pi N\Delta$ system with the $\pi N\Delta$ interaction of the type of Eq. (1.2). After integrating out the Δ isobar field, they obtained a nonlocal πN Lagrangian, where the Z dependence is contained in couplings. This led them to conclude that these couplings can be redefined so that the Z dependence disappears, and therefore this parameter is physically irrelevant. However, after finding that it is difficult to manage the nonlocal part of the resulting Lagrangian, Tang and Ellis returned to the starting $\pi N\Delta$ Lagrangian containing the Δ field explicitly and recommended using it with some convenient choice of the parameter Z, as it is not relevant to the physics. On the other hand, they did not consider any mechanism to compensate for the Z dependence of the observables. Indeed, if such a parameter independence should take place, one should provide a mechanism to compensate for it if it appears to be due to a particular process, which can happen more generally.

It is clear [5,15] that the Z dependence of the amplitudes appears in the form of contact terms. As has recently been discussed [19,20], the contact nature of the Δ excitation amplitudes appears if the interaction vertices contain the projection operators onto the spin 1/2 space, which leads to the contribution of this space. Indeed, the Δ propagator can be written in terms of projection operators as

$$\begin{aligned} S_F^{\mu\nu} &= \frac{1}{i\,\not p + M_\Delta}\left[\delta_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{2}{3M_\Delta^2}p_\mu p_\nu + \frac{1}{3M_\Delta}(\gamma_\mu p_\nu - \gamma_\nu p_\mu)\right] \\ &= -\frac{1}{i\,\not p + M_\Delta}(P^{3/2})_{\mu\nu} + \frac{1}{\sqrt{3}M_\Delta}\left[\left(P^{1/2}_{12}\right)_{\mu\nu} + \left(P^{1/2}_{21}\right)_{\mu\nu}\right] + \frac{2}{3M_\Delta^2}(i\,\not p - M_\Delta)\left(P^{1/2}_{22}\right)_{\mu\nu}. \end{aligned} \tag{1.5}$$

In its turn, operator (1.3) reads

$$\begin{aligned}\mathcal{O}_{\mu\nu}(Z) = &-(P^{3/2})_{\mu\nu} - [1+3C(Z)]\left(P^{1/2}_{11}\right)_{\mu\nu} - [1+C(Z)] \\ &\times\left(P^{1/2}_{22}\right)_{\mu\nu} - \sqrt{3}C(Z)\left[\left(P^{1/2}_{12}\right)_{\mu\nu} + \left(P^{1/2}_{21}\right)_{\mu\nu}\right].\end{aligned} \tag{1.6}$$

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Explicit form of the projection operators can be found in Ref. [16]. It is seen that if the Δ propagator $S_F^{\mu\nu}$ is sandwiched between the vertices of the type of Eq. (1.6), the nonpole contribution is present. According to [19,20], this feature of the Δ interaction is related to the change of the number of degrees of freedom in comparison with the case allowed by the kinetic-energy term of the Δ Lagrangian. As a remedy, it was proposed [19,20] that the $\pi N\Delta$ and $\gamma N\Delta$ Lagrangians would possess the same symmetry as the kinetic-energy term of the Δ Lagrangian that is invariant under a type of the gauge transformation,

$$\Psi_\mu(x) \rightarrow \Psi_\mu(x) + \partial_\mu \xi(x), \tag{1.7}$$

where $\Psi_\mu(x)$ is the Δ isobar field and $\xi(x)$ is a spinor.

Recently, new $\pi N\Delta$ and $\gamma N\Delta$ Lagrangians were proposed [19,20]. They possess the property that, with the proper choice of couplings, the new and traditional Lagrangians provide identical $\pi N\Delta$ and $\gamma N\Delta$ vertices for the on-shell particles. Further, by use of redefinition (1.7), it was shown [20] that the new and traditional $\pi N\Delta$ couplings differ by a contact term, a quadratic in the coupling constant that can be associated with the contribution of the 1/2 spin space involved because of the traditional $\pi N\Delta$ coupling.

This idea is closely related to the representation independence (equivalence) theorem that concerns the relation between different field parametrizations within the framework of a model of the perturbation theory of fields [21–27]. According to this theorem, different parametrizations of the interpolating fields can yield different Green's functions, but they should arrive at the same S-matrix elements. In other words, the theorem postulates the independence of observables for different parametrizations of fields. The equivalence transformations that are frequently used are (i) the Foldy–Dyson transformation [28,29] relating the pseudoscalar and pseudovector πNN couplings: Two chiral Lagrangians of the πN system, related by this transformation, are physically equivalent for the on-shell nucleon; (ii) Weinberg's parametrization of the nonlinear σ model [30]; (iii) the Foldy–Wouthuysen transformation [31]. In this connection, we mention also the Stückelberg transformation [32], relating the linear and nonlinear σ models, which was constructed six decades ago. It has recently been applied in the construction of chiral Lagrangians [33,34] within the approach of hidden local symmetries.

The domain of applications of the representation independence theorem is quite extensive. It is valid not only at the tree level of nonrenormalizable models, but also at the level of renormalizable models, in which case, caution is needed in the case of some regularizations [35]. It was shown [36] that the theorem is applicable also for a meson moving in the nuclear environment. In Ref. [37], the theorem was again applied to a system possessing finite density, with the result that the observables are independent of the field parametrization.

On the other hand, one should be very careful in the application of the representation independence theorem beyond the perturbation field theory. Here we have in mind the calculations based on the Bethe-Salpeter equation, the Schwinger-Dyson equation, etc. It was shown in Ref. [38] that the theorem is not valid for solutions of the Bethe-Salpeter equation. Actually, the authors concluded that the theorem is valid, but in order to satisfy the conditions of its validity in this case, the kernel of the Bethe-Salpeter equation should contain a sum of an infinite series of all loop diagrams. This result can be generalized for other techniques in which one should sum up an infinite series of connected Feynman diagrams. A similar conclusion was deduced in Ref. [39]: a redefinition of fields, relating "consistent" and "inconsistent" $\pi N\Delta$ couplings [40,41] results in the equivalence of these couplings only at the level of the perturbation field theory. This equivalence does not take place if resummation is needed. Let us note for completeness that the nonequivalence between the pseudoscalar and pseudovector πNN couplings was demonstrated for the Bethe-Salpeter equation [42,43].

The topics of this work, dealing with the Δ couplings, contain an important application of the representation independence theorem—the problem related to off-shell parameters and, more generally, to off-shell effects. These effects are related to a certain vertex, and they occur only if at least one of the particles at the vertex is off the mass shell. The problem related to these effects is that their description exhibits not only a model dependence but also a nonphysical dependence on the field parametrization.[1] The origin of such a dependence can be seen in the very essence of the off-shell effects. It holds that the effect of an off-shell vertex, if it is nonzero, cannot be distinguished from the effect of contact vertices.[2] It is true that the field transformations change off-shell terms in a Lagrangian to contact terms and vice verse.

This phenomenon has recently been studied and identified [37,45] as the reshuffling of relevant contributions between an off-shell two-body interaction and many-body forces was observed.[3] Similarly, it was shown [37] how the Born and the contact terms are related. Studying the Compton effect [46,47] belongs in this category.

Erroneous interpretation of the off-shell terms and the complementary contact terms can lead to an erroneous understanding of the equivalence of models. Neglecting complementary contact terms in passing from one field model to another one makes these models nonequivalent. Typically this error can appear in passing from the model with one type of the πNN coupling to the model with another type of the πNN coupling by use of the equation of motion for the free nucleon instead of application of the Foldy-Dyson transformation, as was done [48,49] in calculations of the photon spectrum for radiative muon capture in hydrogen. This procedure lead to the loss of the equivalence of the models, thus making the physical content of both models distinct [50].

[1]This is why it is demanded in [44] that one should strictly distinguish between the form factors and form functions, as the form factors correspond to observables, whereas this is not true for the form functions.

[2]In particular, it is shown in this work at the level of the Feynman diagrams for the process $N(\pi,\pi)N$ that the contribution of the off-shell coupling in the $\pi N\Delta$ vertex is equivalent to the contribution of a specific $NN\pi\pi$ contact vertex.

[3]It means that it makes no sense to speak about off-shell parameters without simultaneously specifying a many-body (contact) sector.

Similar problems occur in the chiral perturbation theory also. The substitution from the equation of motion into the Lagrangian is used to transform redundant structures, containing multiderivatives of chiral fields, into canonical ones. However, in this case, this substitution can be done correctly. The added terms, being proportional to the equation of motion, can be interpreted as off-shell terms. A field transformation was found [51] that transformed the Lagrangian equivalently, as did the above-mentioned substitution. It was also shown [51] that this field transformation generated higher-order contact terms, producing the same effect as that by the off-shell terms.

The understanding of the nature of off-shell effects is important for interpretation of experiments, as is the measurement of the photon spectrum (bremsstrahlung) in the nucleon-nucleon collision, Compton scattering on pions, etc. It was believed for a long time that the study of the $NN\gamma$ vertex in the reaction $N + N \to N + N + \gamma$ can provide information on the off-shell behavior of the *NN* amplitude. However, it has been shown [46,52,53] that the off-shell properties of the *NN* amplitude can be changed by use of a field redefinition. It means that it is impossible to distinguish experimentally between *NN* potentials that differ off-shell. Analogous conclusions are true also for the Compton scattering on pions [44,46,47,54].[4]

In our opinion, one can draw the following lesson from the above discussion:

(i) One should precisely distinguish between dependence on a model and on the field parametrization of a model. The model dependence can provide different physical outputs and therefore can lead to the choice of a certain model based on the data analysis whereas the dependence on the field parametrization is deprived of it.

(ii) It makes no sense to keep off-shell terms in a Lagrangian without simultaneous specification of related contact terms.

Let us apply these conclusions to our case. As we have already noted, it was shown [20] that the traditional $\mathcal{L}_{\rm int}$, and gauge symmetric $\mathcal{L}'_{\rm int}$ couplings describing the $\pi N\Delta$ system are related by field transformation (1.7) as

$$\mathcal{L}_{\rm int} = \mathcal{L}'_{\rm int} + \mathcal{L}_C, \tag{1.8}$$

where $\mathcal{L}_C$ is quadratic in the coupling constant contact interaction of the type $NN\pi\pi$, independent of the Δ isobar degrees of freedom. If both the Lagrangians and the field transformation satisfy the conditions of the representation independence theorem [24], then the description in terms of $\mathcal{L}_{\rm int}$ or $\mathcal{L}'_{\rm int} + \mathcal{L}_C$ are equivalent at the level of the *S*-matrix elements [20]. However, when the contact Lagrangian $\mathcal{L}_C$ on the right-hand side of Eq. (1.8) is neglected, the equivalence is lost, and one has two independent models of describing the $\pi N\Delta$ system that differ by the effect of the contact $NN\pi\pi$ term.

[4]A detailed discussion not only of the Compton scattering on pions but also of the off-shell effects generally in the electromagnetic structure of hadrons can be found in Ref. [55].

Earlier the contact $NN\pi\pi$ term [20] was generated by application of field transformation (1.7). In Sec. II, we show how one can construct such a contact term directly. For this purpose, we first use an identity to show that the new and traditional $\pi N\Delta$ couplings differ by a sum of the $\pi N\Delta$ terms that vanish for the on-shell Δ isobar. Next we construct, in the tree approximation, the contribution of the Δ excitation to the πN scattering amplitude, and we show that these $\pi N\Delta$ terms give rise to a contact term quadratic in the $\pi N\Delta$ coupling constant. In Sec. III, we use the new $\pi N\Delta$ and $\gamma N\Delta$ couplings to calculate the Δ excitation contribution to the photon spectrum for reaction (1.1). We show for the recently measured spectrum [9,10] that it is suppressed in comparison with the one obtained earlier by use of the traditional couplings. It is shown that the difference in the spectra is due to a contact term, in full agreement with the preceding general discussion. In Sec. IV, we discuss the results and conclusions are presented.

II. THE $\pi N\Delta$ COUPLINGS

The gauge symmetric $\pi N\Delta$ coupling [19] is

$$\mathcal{L}^{\rm g.s.}_{\pi N\Delta} = f\,\varepsilon_{\mu\nu\alpha\beta}\,(\partial_\mu \bar{\Psi}_\nu)\vec{T}\gamma_5\gamma_\alpha\,\Psi\cdot(\partial_\beta\vec{\pi}) + \text{H.c.} \tag{2.1}$$

With the choice

$$f = \frac{f_{\pi N\Delta}}{m_\pi M_\Delta} \tag{2.2}$$

and for the Δ isobar on-shell ($Z = -1/2$), this coupling is equivalent to the traditional one:

$$\mathcal{L}_{\pi N\Delta}(Z=-1/2) = \frac{f_{\pi N\Delta}}{m_\pi}\,\bar{\Psi}_\mu\vec{T}\mathcal{O}_{\mu\nu}(Z=-1/2)\Psi\cdot(\partial_\nu\vec{\pi}) + \text{H.c.} \tag{2.3}$$

Using the identity

$$\varepsilon_{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha = -\delta_{\mu\nu}\gamma_\beta + \delta_{\beta\nu}\gamma_\mu - \delta_{\beta\mu}\gamma_\nu + \gamma_\nu\gamma_\mu\gamma_\beta, \tag{2.4}$$

we get

$$\mathcal{L}^{\rm g.s.}_{\pi N\Delta} = \mathcal{L}_{\pi N\Delta}(Z) + \delta\,\mathcal{L}_{\pi N\Delta}(Z), \tag{2.5}$$

where

$$\begin{aligned}\delta\,\mathcal{L}_{\pi N\Delta}(Z) = f\{&-(\partial_\mu\bar{\Psi}_\mu)\vec{T}\gamma_\nu\,\Psi - (\partial_\nu\bar{\Psi}_\alpha\gamma_\alpha)\vec{T}\,\Psi \\ &+(\partial_\mu\bar{\Psi}_\alpha)\gamma_\alpha\vec{T}\gamma_\mu\gamma_\nu\,\Psi + \bar{\Psi}_\nu[(\gamma_\mu\overleftarrow{\partial}_\mu) \\ &- M_\Delta]\vec{T}\Psi - C(Z)\,M_\Delta\,\bar{\Psi}_\mu\gamma_\mu\vec{T}\gamma_\nu\,\Psi\}\cdot\partial_\nu\vec{\pi}.\end{aligned} \tag{2.6}$$

For the on-shell Δ isobar,

$$\partial_\mu\Psi_\mu(x) = \gamma_\mu\Psi_\mu(x) = [\not{\partial} + M_\Delta]\Psi(x) = 0. \tag{2.7}$$

It follows from these equations that for the Δ isobar on-shell $\delta\,\mathcal{L}_{\pi N\Delta}(Z) = 0$.

In the tree approximation, the πN scattering via the Δ isobar excitation is described by the Feynman graphs of Figs. 1(a) and (b).

The *S*-matrix element, corresponding to Fig. 1(a), can be calculated by use of either the left-hand side of Eq. (2.5) or, equivalently, its right-hand side. If one considers a part of the

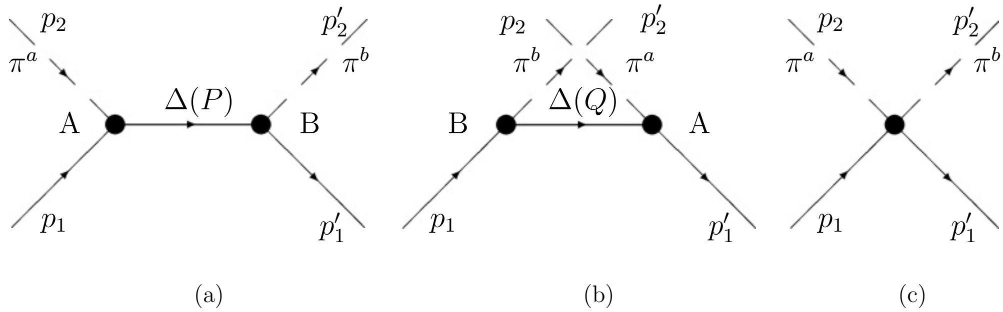

FIG. 1. The πN scattering amplitudes in the tree approximation: a, b, the Δ excitation amplitudes; c, the contact term.

S-matrix element, S_p, given by the sum of the partial S-matrix elements, calculated with the choices

$$A = \mathcal{L}_{\pi N\Delta}(Z), \qquad B = \delta\,\mathcal{L}^{+}_{\pi N\Delta}(Z); \tag{2.8}$$

$$A = \delta\,\mathcal{L}_{\pi N\Delta}(Z), \qquad B = \mathcal{L}^{+}_{\pi N\Delta}(Z); \tag{2.9}$$

$$A = \delta\,\mathcal{L}_{\pi N\Delta}(Z), \qquad B = \delta\,\mathcal{L}^{+}_{\pi N\Delta}(Z); \tag{2.10}$$

the difference between the S-matrix elements, calculated first with the new Lagrangian $\mathcal{L}^{\text{g.s.}}_{\pi N\Delta}$ and then only with the traditional Lagrangian $\mathcal{L}_{\pi N\Delta}(Z)$, is obtained. Explicit calculations yield S_p in the form of the $\pi\pi NN$ contact graph of Fig. 1(c). Defining

$$S_p = -i(2\pi)^4\,\delta^{(4)}(P_f - P_i)(\chi^b)^+\,T_p^{ba}(s)\,\chi^a, \tag{2.11}$$

we obtain for the amplitude $T_p^{ba}(s)$ the following equation:

$$\begin{aligned} T_p^{ba}(s) &= f^2 M_\Delta\,\bar u(p_1')\Big\langle p'_{2\nu}\Big[-\delta_{\nu\mu} + \frac{1}{3}\gamma_\nu\gamma_\mu + \frac{i}{3M_\Delta}(3\,\not{P}\delta_{\nu\mu} \\ &\quad + \gamma_\nu\,\not{P}\gamma_\mu - P_\nu\gamma_\mu - P_\mu\gamma_\nu)\Big]p_{2\mu} \\ &\quad + \frac{2}{3}C(Z)\left\{\not{p}_2'\,\not{p}_2 + \frac{i}{M_\Delta}[(p_2'\cdot P)\,\not{p}_2 + \not{p}_2'(P\cdot p_2)]\right\} \\ &\quad + \frac{2}{3}C^2(Z)\,\not{p}_2'\left(2 + i\frac{\not{P}}{M_\Delta}\right)\not{p}_2\Big\rangle(T^+)^b\,T^a\,u(p_1). \end{aligned} \tag{2.12}$$

Here $P = p_1 + p_2 = p_1' + p_2'$, $(T^+)^b\,T^a = \frac{2}{3}\delta_{ba} - \frac{1}{3}i\varepsilon^{bac}\tau^c$, and τ^c are the isospin Pauli matrices. In deriving Eq. (2.12) we assume $C(Z)$ to be a real function of Z.

The amplitude $T_p^{ba}(s)$ corresponds to an effective contact Lagrangian:

$$\begin{aligned} \mathcal{L}_{\pi\pi NN}(Z) &= -f^2 M_\Delta\,(\partial_\nu\pi^b)\bar\Psi\left\{\delta_{\nu\mu} - \frac{1}{3}[1 + 2C(Z) + 4C^2(Z)]\right. \\ &\quad \times\gamma_\nu\gamma_\mu - \frac{1}{M_\Delta}\delta_{\nu\mu}\not\partial - \frac{1}{3M_\Delta}[1 + 2C^2(Z)]\gamma_\nu\gamma_\mu\not\partial \\ &\quad \left. + \frac{1}{3M_\Delta}[1 - 2C(Z)](\gamma_\nu\partial_\mu + \gamma_\mu\partial_\nu)\right\} \\ &\quad \times(T^+)^b\,T^a[\Psi(\partial_\mu\pi^a)]. \end{aligned} \tag{2.13}$$

Let us take $Z = 1/2$, which provides $C(Z) = -1$. Then the full amplitude reads

$$\begin{aligned} T_p^{ba}(s) + T_p^{ba}(u) &= f^2\,\bar u(p_1')p'_{2\nu}\left[\tfrac{1}{2}\gamma_{\nu\mu\alpha}(p_1' + p_1)_\alpha + M_\Delta\gamma_{\nu\mu}\right] \\ &\quad \times[(T^+)^a\,T^b - (T^+)^b\,T^a]p_{2\mu}u(p_1), \end{aligned} \tag{2.14}$$

where

$$\gamma_{\nu\mu\alpha} = \tfrac{1}{2}\{\gamma_{\nu\mu},\,\gamma_\alpha\}, \qquad \gamma_{\nu\mu} = \tfrac{1}{2}[\gamma_\nu,\,\gamma_\mu]. \tag{2.15}$$

Applying the field redefinition, Eq. (1.7), with a particular choice of the field $\partial_\mu\,\xi(x) \to g\xi_\mu(x)$,

$$\xi_\mu = -\frac{1}{M_\Delta}\mathcal{O}_{\mu\rho}(Z = -5/6)\mathcal{O}_{\rho\nu}(Z)T^a\Psi\partial_\nu\phi^a, \tag{2.16}$$

in the Lagrangian (2.3), besides the gauge symmetric Lagrangian (2.1), we obtain the following effective contact Lagrangian [20]:

$$\begin{aligned} \mathcal{L}'_{\pi\pi NN}(Z) &= -f^2\,\bar\Psi\mathcal{O}_{\rho\mu}(x)(\gamma_{\mu\nu\alpha}\partial_\alpha - M_\Delta\gamma_{\mu\nu})\mathcal{O}_{\nu\sigma}(x) \\ &\quad \times(T^+)^b\,T^a\,\Psi\,[\partial_\rho(\pi^+)^b](\partial_\sigma\pi^a), \end{aligned} \tag{2.17}$$

where $x = -\frac{1}{3}[1 + C(Z)]$.

Instead of the direct comparison of the Lagrangians (2.13) and (2.17), we observe that the Lagrangian $\mathcal{L}'_{\pi\pi NN}(Z)$ was used [20] for constructing the amplitude for $Z = 1/2 (x = 0)$. This amplitude, presented in Eq. (21) of Ref. [20], coincides with our amplitude of Eq. (2.14). So we conclude that the particular choice of field transformation (2.16) provides the same contact Lagrangian as our procedure based on identity (2.4).

III. THE PHOTON SPECTRUM IN THE RADIATIVE MUON CAPTURE IN HYDROGEN

The new Lagrangians needed for the calculations of the photon spectrum that are derived from the gauge symmetric ones [19] read

$$\begin{aligned} \mathcal{L}^{\text{g.s.}}_{\pi N\Delta a_1} &= f\,\varepsilon_{\mu\nu\alpha\beta}[(\partial_\mu\bar\Psi_\nu)\vec T\gamma_5\gamma_\alpha\Psi]\cdot(\partial_\beta\vec\pi + 2f_\pi g_\rho\vec a_\beta) \\ &\quad + \text{H.c.}, \end{aligned} \tag{3.1}$$

$$\begin{aligned} \mathcal{L}^{\text{g.s.}}_{\rho N\Delta} &= f_P\,g_\rho\,\varepsilon_{\mu\nu\alpha\beta}\,[(\partial_\mu\bar\Psi_\nu)\,\vec T\gamma_\alpha\gamma_\lambda\,\Psi]\cdot\vec\rho_{\lambda\beta} \\ &\quad + f_P\,g_\rho[(\partial_\mu\bar\Psi_\nu - \partial_\nu\bar\Psi_\mu)\vec T\gamma_5\gamma_\mu\gamma_\lambda\,\Psi]\cdot\vec\rho_{\lambda\nu} + \text{H.c.} \end{aligned} \tag{3.2}$$

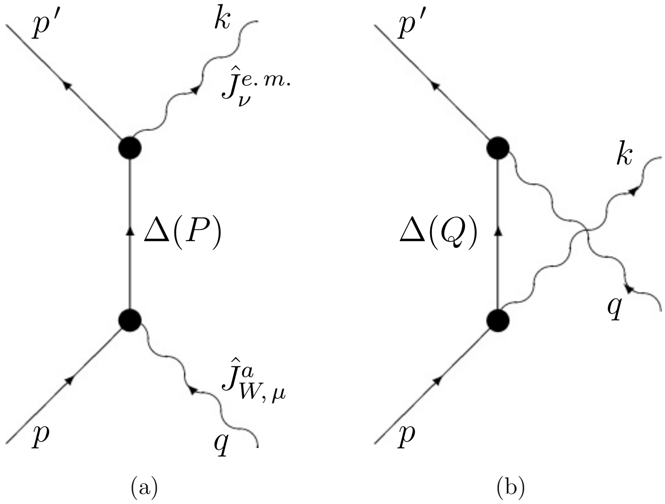

FIG. 2. The Δ excitation amplitudes contributing to the radiative muon capture in hydrogen. The weak hadron current $\hat{J}^{a}_{W,\mu}$, interacting with the nucleon, is exciting it and the Δ isobar in the intermediate state appears, which decays into the nucleon and the photon in the final state.

The value of the coupling constant $f_P = (G_1/MM_\Delta)$ is obtained from the condition that the new $\rho N\Delta$ Lagrangian, Eq. (3.2), and the standard $\rho N\Delta$ Lagrangian, Eq. (1.2), are equivalent for the on-shell Δ isobar. The notation of this section coincides with the notation of Refs. [5,11].

The contribution from the Δ excitation processes to the photon spectrum for reaction (1.1) is given by Fig. 2.

From various form factors, calculated in Sec. II. B of Ref. [5], we need to consider

$$\Delta g_2 = -\frac{8}{9M_\Delta} f_{\pi N\Delta} G_1 \frac{f_\pi}{m_\pi}\eta k[-(1+2R)+2(1-2R) \times C(Y) + 2(1-R)C(Z) + 4(2-R)C(Y)C(Z)], \tag{3.3}$$

and

$$\Delta g_3 = -\frac{16}{9}\lambda f_{\pi N\Delta} G_1 \frac{f_\pi}{m_\pi}\eta k \frac{1}{M_\Delta - M}\{1+(1-R) \times [C(Y)+C(Z)+2(2+R)C(Y)C(Z)]\}. \tag{3.4}$$

Here λ is the photon polarization, k is the photon momentum, $\eta = (m_\mu/2M)$ and $R = M/M_\Delta$. The form factor Δg_2, Eq. (3.3), is of the contact origin, whereas inspection of Eq. (3.4) shows that the dependence of the form factor Δg_3 on the off-shell parameters Y and Z is located entirely in its contact part.

By using Lagrangians, Eqs. (3.1) and (3.2), and performing calculations identical to those presented in Sec. II. B of Ref. [5], one obtains

$$\Delta g_3 = -\frac{16}{9}\lambda f_{\pi N\Delta} G_1 \frac{f_\pi}{m_\pi}\left(\frac{M}{M_\Delta}\right)^2 \eta\, k\, \frac{1}{M_\Delta - M}. \tag{3.5}$$

In contrast to the calculations with the standard couplings, now the form factor Δg_3 contains only the Δ pole contribution.

Let us write the contribution Δg_3^0, given in Eq. (3.4), for the Δ isobar on-shell ($Y = Z = -1/2$),

$$\Delta g_3^0 = -\frac{16}{9}\lambda f_{\pi N\Delta} G_1 \frac{f_\pi}{m_\pi}\eta k \frac{1}{M_\Delta - M}, \tag{3.6}$$

and calculate the difference with the form factor Δg_3:

$$\Delta g_3 - \Delta g_3^0 = -\frac{16}{9}\lambda f_{\pi N\Delta} G_1 \frac{f_\pi}{m_\pi}\eta \frac{k}{M_\Delta}\left(1 + \frac{M}{M_\Delta}\right). \tag{3.7}$$

The difference is equal to a contact term, in agreement with the more general discussion in Sec. I.

In Fig. 3, we present the change in the photon spectrum that is due to the difference in the form factors given by Eq. (3.7). Other contributions are the same as those in Ref. [5]. The spectrum measured in the TRIUMF experiment [9,10] is given as

$$S_T = 0.061 S_s + 0.854 S_o + 0.085 S_p. \tag{3.8}$$

Here S_s, S_o, and S_p correspond to the muon-hydrogen singlet system and to the ortho-molecular and paramolecular $p\mu p$ states, respectively. As is clear from Fig. 3, the spectrum S_T, calculated with Δg_3 from Eq. (3.5) in the region $k > 60$ MeV is suppressed, in comparison with the spectrum obtained by use

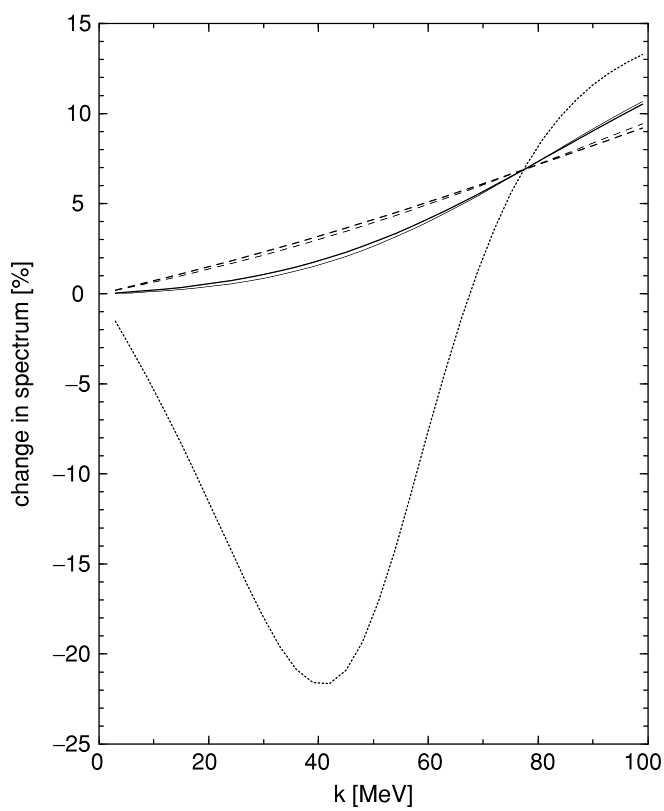

FIG. 3. The change in the photon spectrum calculated as $(S_{\text{s.c.}} - S_{\text{g.s.}})/S_{\text{s.c.}}$, where $S_{\text{s.c.}}$ ($S_{\text{g.s.}}$) is the photon spectrum obtained with the standard (gauge symmetric) couplings. Solid curve, the spectrum measured in the TRIUMF experiment; dashed curve, the spectrum for the muon-hydrogen triplet state; dotted curve, the spectrum for the muon-hydrogen singlet state.

of the traditional couplings. It means that the new couplings cannot resolve the "g_P puzzle" either. A minor difference between the curves of the same sort arises from omitting the form factor Δg_2 of Eq. (3.3) in the calculations.

It is clear from Eqs. (3.5) and (3.6) that the form factors Δg_3 and Δg_3^0 differ by a factor $(M/M_\Delta)^2 \approx 0.58$. Such a factor will appear also in the meson exchange current operators with the Δ excitation. On the other hand, a suppression factor of 0.8 is needed [56] to reduce the effect of the weak axial-meson exchange currents with the Δ excitation in order to explain the experimental value of the Gamow-Teller matrix element for the triton β decay if the value of the constant $f_{\pi N\Delta}$ is taken from the constituent quark model. In other words, it means that effectively the value of the constant $f_{\pi N\Delta}$ turns out to be unrealistically small or one should speculate about other processes that suppress the meson exchange current effect [57]. If these weak axial exchange currents are constructed from the new Lagrangians, the factor $(M/M_\Delta)^2$ appears naturally and the value of the constant $f_{\pi N\Delta}$ can be taken as larger and therefore more realistic. Simultaneously, such a factor will appear also in the vector-meson exchange currents with the Δ excitation. However, the precise data on the radiative capture of neutrons by protons do not demand any damping of the vector-meson exchange currents effect [58] and the capture rate for the reaction $\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + {}^3\mathrm{H}$, which has been measured in a precise experiment [59,60], is underestimated [57] by the suppressed weak axial exchange currents. Precise data, expected from the experiments on the ordinary muon capture in hydrogen and deuterium [61], will be very helpful for the axial sector of the weak nuclear interaction.

IV. RESULTS AND CONCLUSION

We studied some aspects of new $\pi N\Delta$ and $\gamma N\Delta$ couplings that have recently been proposed [19]. In comparison with the traditional couplings, the new ones possess an additional gauge symmetry (1.7) that is present in the kinetic-energy term of the Δ Lagrangian. This symmetry guarantees that the couplings have the same Δ degrees of freedom as the kinetic-energy term. As a consequence, the amplitudes of the processes with the Δ excitation in the intermediate state do not contain the contact terms arising from the spin 1/2 space.

In Sec. II we studied the difference between traditional and gauge symmetric $\pi N\Delta$ couplings. Using an algebraic identity between the γ matrices, we first expressed the new coupling as a sum of the traditional coupling and of terms that are zero for the on-shell Δ isobar. The πN scattering amplitude, constructed from these terms, is a contact term, quadratic in the coupling constant. Such a term was obtained [20] when symmetry condition (1.7) was imposed on the traditional coupling.

In Sec. III we used the gauge symmetric $\pi N\Delta$ and $\gamma N\Delta$ couplings [19] to calculate the photon spectra for the radiative muon capture in hydrogen. As a result, the new form factor Δg_3 contains only the Δ isobar pole contribution. This form factor differs from the old one, calculated for the on-shell Δ isobar by the damping factor $(M/M_\Delta)^2 \approx 0.58$. Consequently the new photon spectrum, corresponding to the spectrum measured in the TRIUMF experiment, is suppressed in the region $k > 60$ MeV, in comparison with the photon spectrum, which is calculated from the traditional couplings. Therefore the problem of extraction of the induced pseudoscalar form factor g_P from the photon spectrum in the radiative muon capture in hydrogen cannot be solved by use of the gauge symmetric $\pi N\Delta$ and $\gamma N\Delta$ couplings.

We note that the damping factor $(M/M_\Delta)^2$ will be also present in the meson exchange current operators with the Δ isobar excitation if the gauge symmetric couplings are used for the construction. However, a comparison of the existing calculations with the present data on the weak and electromagnetic reactions in few–nucleon systems does not allow us to decide if this factor is needed.

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