Energy loss of charm quarks in the quark-gluon plasma: Collisional vs radiative losses

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In considering the collisional energy loss rates of heavy quarks from hard light parton interactions, we computed the total energy loss of a charm quark for a static medium. For the energy range $E \sim 5$–10 GeV of charm quark, it proved to be almost the same order as that of radiative ones estimated to a first-order opacity expansion. The collisional energy loss becomes much more important for lower energy charm quarks, and this feature could be very interesting for the phenomenology of hadrons spectra. Using such collisional energy loss rates, we estimate the momentum loss distribution employing a Fokker-Planck equation and the total energy loss of a charm quark for an expanding quark-gluon plasma under conditions resembling the energies presently available at the BNL Relativistic Heavy Ion Collider. The fractional collisional energy loss is found to be suppressed by a factor of 5 as compared to the static case and does not depend linearly on the system size. We also investigate the heavy to light hadrons D/π ratio at moderately large (5–10 GeV/c) transverse momenta and comment on its enhancement.

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I. INTRODUCTION

In the initial stage of ultra-relativistic heavy-ion collisions, energetic partons are produced from hard collisions between the partons of the nuclei. Receiving a large transverse momentum, these partons will propagate through the fireball, which might consist of a quark-gluon phase for a transitional period of a few fm/c. These high-energy partons will manifest themselves as jets leaving the fireball. Owing to the interaction of the hard partons with the fireball, these partons will lose energy. Hence jet quenching will result. The amount of quenching might depend on the state of matter of the fireball, i.e., quark-gluon plasma (QGP) or a hot hadron gas. Therefore, jet quenching has been proposed as a possible signature for the QGP formation [1]. Indeed, first results from Au+Au at the BNL Relativistic Heavy Ion Collider (RHIC) have shown a suppression of high-$p_\perp$ hadron spectra [2], which could possibly indicate the quenching of light quark and gluon jets [3–8]. On the other hand, the data [9] from light ion interactions d+Au at RHIC indicate no evidence of suppression in high-$p_\perp$ hadron spectra, implying the absence of jet quenching as there is no formation of an extended dense medium in the final state in such light ion interactions. However, this information from the light ion interactions in turn gives strong circumstantial support to the idea that the observed suppression in Au+Au is due to the final state energy loss of jets in the dense QGP matter.

Hadrons containing heavy quarks are important probes of strongly interacting matter produced in heavy ion collisions and have also excited considerable interest. Heavy quark pairs are usually produced early on at a time scale of $1/2M_C \approx 0.07$ fm/c from the initial fusion of partons (mostly from $gg \rightarrow c\bar{c}$, but also from $q\bar{q} \rightarrow c\bar{c}$) and also from QGP, if the initial temperature is high enough. There is no production at later times in the QGP and none in the hadronic matter. Thus, the total number of charm quarks gets frozen very early in the history of collision, which makes them a good candidate for a probe of QGP, as one is then left with the task of determining the $p_\perp$ distribution, whose details may reflect developments in the plasma. The momenta distribution of c quarks are likely to be reflected in the corresponding quantities in D mesons as the c quarks should pick up a light quark, which are in great abundance and hadronize. The first PHENIX data [10] from RHIC in Au+Au collisions at $\sqrt{s} = 130A$ GeV on prompt single electron production are now available, which gives us an opportunity to have an experimental estimate of the $p_\perp$ distribution of heavy quarks. Within the admittedly large experimental error, the data indicate the absence of a QCD medium effect. We hope that the future experimental study will provide data with improved statistics and wider $p_\perp$ range, which could then help us understand the effect of medium modifications on the heavy quark spectra.

To see the effect of medium modifications on the final states, the energy loss of hard partons in the QGP has to be determined. There are two contributions to the energy loss of a parton in the QGP: one is caused by elastic collisions among the partons in the QGP and the other by radiation of the decelerated color charge, i.e., bremsstrahlung of gluons. The energy loss rates due to collisional scatterings among partons were estimated extensively [11–18] in the literature. Using the hard-thermal-loop (HTL) resummed perturbative QCD at finite temperature [19], the collisional energy loss of a heavy quark could be derived in a systematic way [20–24]. It was also shown [12,17,18,24] that the drag force can be related to the elastic scatterings among partons in a formulation based on the Fokker-Planck equation, which is equivalent to the treatment of HTL approximations [21]. From these results, an estimate for the collisional energy loss of energetic gluons and light quarks also could be derived [25], which was rederived later using the Leontovich relation [26,27].

Estimates of energy loss due to multiple gluon radiation (bremsstrahlung) showed it to be the dominant process. For a review on the radiative energy loss, see Ref. [28]. Recently, it has also been shown [29] that for a moderate value of the parton energy there is a net reduction in the parton energy loss induced by multiple scattering due to the partial cancellation between stimulated emission and thermal absorption.

This reduction can cause a reduction of the light hadrons quenching factor as was first anticipated in Ref. [8], though most of the earlier studies insisted that the collisional energy loss is insufficient to describe the medium modification of hadronic spectra. These studies, however, were limited to the case of massless energetic quarks and gluons.

The first estimate of heavy quark radiative energy loss found that it dominates [30] the average energy loss rate, and subsequently it was found that the charmed hadron [31] and dilepton [32] spectra have a strong dependence on the heavy quark radiative energy loss. Most recent studies of the medium modifications of the charm quark spectrum have computed energy loss by emphasizing only the energy loss of heavy quarks by gluon bremsstrahlung [33–36]. In Ref. [33], it was shown that the appearance of the kinematic dead cone effect due to the finite mass of the heavy quarks leads to a large reduction in radiative energy loss and affects significantly the estimation of the quenching of charm quarks and the D/π ratio. Also, in Refs. [34–36], a surprising degree of reduction in radiative energy loss for heavy quarks was obtained by taking into account the opacity expansion with and without the Ter-Mikayelian (TM) effect. In the framework in which all these calculations have been performed, the heavy quarks are possibly not ultra-relativistic ($\gamma v \sim 1$) [12,17,18,24] for much of the measured momentum range, and in this case it is far from clear that radiative energy loss dominates over collisional energy loss.

This paper is organized as follows. In Sec. II, the collisional energy loss for charm quarks is compared with the radiative loss computed in Refs. [34,35]. The quenching of hadron spectra in a medium is briefly reviewed in Sec. III. The charm quark in a thermally evolving plasma is modeled in Sec. IV as an expanding fireball created in relativistic heavy ion collisions. We first obtain the Fokker-Planck equation for a Brownian particle from a generic kinetic equation (Sec. IV A); we next compute analytically the momentum loss distribution for charm quarks using the collisional energy loss rates through the transport coefficients based on the elastic perturbative cross section implemented into a Fokker-Planck equation (Sec. IV B); the total energy loss for the charm quark is obtained for an expanding plasma (Sec. IV C); and then the quenching of hadron spectra and D/π are estimated for RHIC energies (Sec. IV D). We conclude in Sec. V with a brief discussion.

II. HEAVY QUARKS IN A STATIC QUARK-GLUON PLASMA

At leading order in strong coupling constant α_s, the energy loss of a heavy quark comes from elastic scattering from thermal light quarks and gluons. The energy loss rate of heavy quarks in the QGP due to elastic collisions was estimated in Ref. [21]. In the domain $E \ll M^2/T$, it reads

$$-\frac{dE}{dL} = \frac{8\pi\alpha_s T^2}{3}\left(1+\frac{n_f}{6}\right)\left[\frac{1}{v} - \frac{1-v^2}{2v^2}\ln\left(\frac{1+v}{1-v}\right)\right] \times \ln\left[2^{\frac{n_f}{6+n_f}} B(v)\frac{ET}{m_g M}\right], \quad (1)$$

whereas for $E \gg M^2/T$, it is

$$-\frac{dE}{dL} = \frac{8\pi\alpha_s T^2}{3}\left(1+\frac{n_f}{6}\right)\ln\left[2^{\frac{n_f}{2(6+n_f)}} 0.92\frac{\sqrt{ET}}{m_g}\right], \quad (2)$$

where n_f is the number of quark flavors, α_s is the strong coupling constant, $m_g = \sqrt{(1+n_f/6)gT/3}$ is the thermal gluon mass, E is the energy, and M is the mass of the heavy quarks. $B(v)$ is a smooth velocity function, which can be taken approximately as 0.7. Following (1) and (2), one can now estimate the static energy loss for heavy quarks at the energies (temperatures) of interest.

On the other hand, heavy quark medium-induced radiative energy loss [34,35] to all orders in opacity expansion, L/λ_g (L is the length of the plasma, λ_g is the mean free path of the gluon), has been derived by generalizing the massless case [37] to heavy quarks with mass in a QCD plasma with a gluon dispersion characterized by an asymptotic plasmon mass. This also provides the estimate of the influence of a plasma frequency cutoff on a gluon radiation (Ter-Mikayelian effect) and thus shields the collinear singularities ($k_\perp \to 0$) that arise due to massless quarks.

The medium-induced radiative energy loss [35] for charm quark in first-order opacity expansion has been computed with a fixed Debye screening mass, $\mu = 0.5$ GeV with $\alpha_s = 0.3$, and a static plasma length, $L = 4$ fm with $\lambda_g = 1$ fm. The scaled energy loss was found to obey a linear Bethe-Heitler-like form, $\frac{\Delta E}{E}|^{\rm rad} \propto L \sim CL$, where C is constant of proportionality per unit length. The differential energy loss follows as $\frac{d(\Delta E)}{dL}|^{\rm rad} \sim CE$. Now, C can be estimated from the right panel of Fig. 1 (also from Fig. 2 of Ref. [35]), as $C \sim \frac{\Delta E/E}{L} \sim \frac{0.15}{4}$ fm^{-1} at a plasma length $L = 4$ fm, $\mu = 0.5$ GeV, and $\alpha_s = 0.3$. For a charm quark with energy $E = 10$ GeV, the differential radiative energy loss is estimated as $\frac{d(\Delta E)}{dL}|^{\rm rad} \sim 0.375$ GeV/fm. The Debye screening mass is given as $\mu = T\sqrt{4\pi\alpha_s(1+\frac{n_f}{6})} = 2.2415T$, for two light flavors, $n_f = 2$ and $\alpha_s = 0.3$. The Debye screening mass $\mu = 0.5$ GeV corresponds to a temperature $T = 0.225$ GeV. With the plasma parameters corresponding to $\mu = 0.5$ GeV, the differential collisional energy loss for a 10 GeV charm quark in a static medium can also be estimated from (1) and (2) as $\frac{d(\Delta E)}{dL}|^{\rm coll} \sim 0.36$ GeV/fm, and it is found to be of the same order as that of radiative ones in Ref. [35].

Now the total collisional energy loss can simply be evaluated from (1) and (2). The scaled collisional (solid line) and the radiative ones with (dotted) and without (dashed line) TM effect of a charm quark as a function energy E are displayed in the left panel of Fig. 1 for a static plasma of length $L = 4$ fm, with parameters $T = 0.225$ GeV ($\mu = 0.5$ GeV) and $\alpha_s = 0.3$. As discussed earlier, the radiative energy loss is proportional to E, resulting in the scaled energy loss being almost constant in E. In the energy range $E \sim$ (5–10) GeV, the scaled collisional energy loss is found to be similar to that of radiative loss [35], but it decreases with E because the differential rates in (1) and (2) depend on a log factor involving E. Therefore, the collisional energy loss will become much more important in the lower energy range; this feature itself will be quite interesting in the phenomenology of particle spectra.

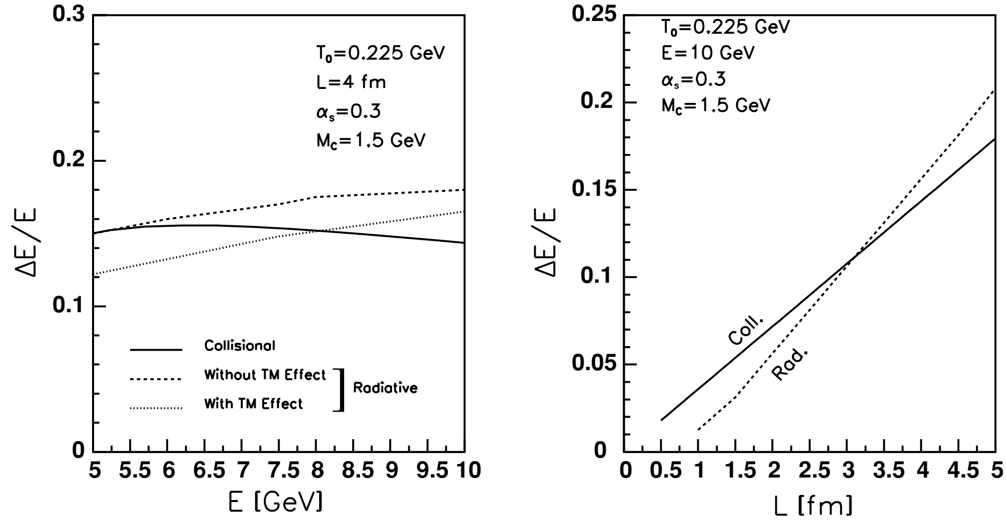

FIG. 1. Left panel: Scaled static energy loss of a charm quark $\Delta E/E$ as a function of energy E for a given length of the plasma $L = 4$ fm. Collisional loss is represented by the solid line with plasma parameters for RHIC energy. Radiative energy losses according to Ref. [35] are also plotted in first-order opacity expansion with (dotted) and without (dashed) the Ter-Mikayelian (TM) effect at a plasma length $L = 4$ fm and a fixed Debye screening mass $\mu = 0.5$ GeV (see text for details). Right panel: The effective shift of the scaled collisional (solid curve) and radiative (dashed) energy loss $\Delta E/E$ as a function of distance L for a charm quark of energy $E = 10$ GeV.

In the right panel of Fig. 1, we display the scaled effective energy loss of a charm quark due to collisional (solid curve) and radiative [35] (dashed curve) losses in a static medium as a function of its thickness L for a given charm quark energy $E = 10$ GeV. The thickness dependence of the scaled collisional energy loss for a given E is linear as in the radiative case [7,35], whereas earlier calculations [28,37,38] show a quadratic form. This scaling clearly reflects a random walk in E and L as a fast parton moves in the medium [7,35] with some interactions resulting in an energy gain and others in a loss of energy.

In the energy range 5–10 GeV, which is much of the experimentally measured range ($\gamma v \leqslant 4$), the charm quark is not very ultra-relativistic, and the collisions are found to be one of the most dominant energy loss mechanisms. In the weak coupling limit, bremsstrahlung [24] is the dominant energy loss mechanism if the charm quark is ultra-relativistic ($\gamma v \geqslant 4$). Though the collisions have a different spectrum than radiation, the collisional rather than radiative energy loss should in principle determine the medium modifications of the final state hadron spectra. In the following, we will study the suppression of heavy quark spectra.

III. QUENCHING OF HADRON SPECTRA

We will follow the investigations by Baier *et al.* [6] and Müller [7], using the collisional instead of the radiative parton energy loss. Following Ref. [6], the $p_\perp$ distribution is given by the convolution of the transverse momentum distribution in elementary hadron-hadron collisions, evaluated at a shifted value $p_\perp + \epsilon$, with the probability distribution $D(\epsilon)$, in the energy ϵ, lost by the partons to the medium by collisions, as

$$\frac{d\sigma^{\rm med}}{d^2p_\perp} = \int d\epsilon D(\epsilon)\frac{d\sigma^{\rm vac}(p_\perp+\epsilon)}{d^2p_\perp}$$
$$= \int d\epsilon D(\epsilon)\frac{d\sigma^{\rm vac}}{d^2p_\perp} + \int d\epsilon D(\epsilon)\epsilon\frac{d}{dp_\perp}\frac{d\sigma^{\rm vac}}{d^2p_\perp} + \cdots$$
$$= \frac{d\sigma^{\rm vac}}{d^2p_\perp} + \Delta E\cdot\frac{d}{dp_\perp}\frac{d\sigma^{\rm vac}}{d^2p_\perp}$$
$$= \frac{d\sigma^{\rm vac}(p_\perp+\Delta E)}{d^2p_\perp} = Q(p_\perp)\frac{d\sigma^{\rm vac}(p_\perp)}{d^2p_\perp}. \tag{3}$$

Here $Q(p_\perp)$ is the suppression factor due to the medium, and the total energy loss by partons in the medium is

$$\Delta E = \int \epsilon D(\epsilon) d\epsilon. \tag{4}$$

We need to calculate the probability distribution $D(\epsilon)$ that a parton loses the energy ϵ due to the elastic collisions in the medium. This requires the evolution of the energy distribution of a particle undergoing Brownian motion, which will be obtained in Sec. IV.

IV. CHARM QUARK IN AN EXPANDING PLASMA

A. Generic kinetic equation, Fokker-Planck equation, drag and diffusion coefficients

The operative equation for the Brownian motion of a test particle can be obtained from the Boltzmann equation, whose covariant form can be written as

$$p^\mu\partial_\mu D(\mathbf{x},\mathbf{p},t) = C\{D\}, \tag{5}$$

where $p^\mu(E_{\mathbf{p}},\mathbf{p})$ is the four-momentum of the test particle, $C\{D\}$ is the collision term, and $D(\mathbf{x},\mathbf{p},t)$ is the distribution due to the motion of the particle. If we assume a uniform plasma, the Boltzmann equation becomes

$$\frac{\partial D}{\partial t} = \frac{C\{D\}}{E} = \left(\frac{\partial D}{\partial t}\right)_{\rm coll}. \tag{6}$$

We intend to consider only the elastic collisions of the test parton with other partons in the background. The rate of collisions $w(\mathbf{p},\mathbf{k})$ is given by

$$w(\mathbf{p},\mathbf{k}) = \sum_{j=q,\bar{q},g} w^j(\mathbf{p},\mathbf{k}), \quad (7)$$

where w^j represents the collision rate of a test parton i with other partons j in the plasma. The expression for w^j can be written as

$$w^j(\mathbf{p},\mathbf{k}) = \gamma_j \int \frac{d^3\mathbf{q}}{(2\pi)^3} D_j(\mathbf{q}) v_{\rm rel}\sigma^j, \quad (8)$$

where γ_j is the degeneracy factor, $v_{\rm rel}$ is the relative velocity between the test particle and other participating partons j from the background, D_j is the phase space density for the species j, and σ^j is the associated cross section. Due to this scattering, the momentum of the test particle changes from $\mathbf{p}$ to $\mathbf{p}-\mathbf{k}$. Then the collision term on the right-hand side of (6) can be written as

$$\left(\frac{\partial D}{\partial t}\right)_{\rm coll} = \int d^3\mathbf{k}[w(\mathbf{p}+\mathbf{k},\mathbf{k})D(\mathbf{p}+\mathbf{k}) - w(\mathbf{p},\mathbf{k})D(\mathbf{p})], \quad (9)$$

where the collision term has two contributions. The first one is the gain term where the transition rate $w(\mathbf{p}+\mathbf{k},\mathbf{k})$ represents the rate that a particle with momentum $\mathbf{p}+\mathbf{k}$ loses momentum $\mathbf{k}$ due to the reaction with the medium. The second term represents the loss due to the scattering of a particle with momentum $\mathbf{p}$.

Now under the Landau approximation, in which most of the quark and gluon scattering is soft which implies that the function $w(\mathbf{p},\mathbf{k})$ is sharply peaked at $p \approx k$, one can expand the first term on the right-hand side of (9) by a Taylor series as

$$w(\mathbf{p}+\mathbf{k},\mathbf{k})D(\mathbf{p}+\mathbf{k}) \approx w(\mathbf{p},\mathbf{k})D(\mathbf{p}) + \mathbf{k}\cdot\frac{\partial}{\partial \mathbf{p}}(wD) + \frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i \partial p_j}(wD) + \cdots. \quad (10)$$

Combining (6), (9), and (10), one obtains a generic kinetic equation of the form

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial p_i}[\mathcal{T}_{1i}(\mathbf{p})D] + \frac{\partial^2}{\partial p_i \partial p_j}[\mathcal{B}_{ij}(\mathbf{p})D], \quad (11)$$

where the transport coefficients for momentum dispersion are given as

$$\mathcal{T}_{1i}(\mathbf{p}) = \int d^3\mathbf{k} w(\mathbf{p},\mathbf{k})k_i = \int d^3\mathbf{k} w(\mathbf{p},\mathbf{k})(p-p')_i, \quad (12)$$

$$\mathcal{B}_{ij}(\mathbf{p}) = \frac{1}{2}\int d^3\mathbf{k} w(\mathbf{p},\mathbf{k})k_ik_j = \frac{1}{2}\int d^3\mathbf{k} w(\mathbf{p},\mathbf{k})(p-p')_i(p-p')_j. \quad (13)$$

These transport coefficients in (12) and (13) depend on the distribution function D through the transition probability $w(\mathbf{p},\mathbf{k})$ in (8), and they can have different values depending upon the problem. The kinetic equation in (11) is the well-known Landau equation [39], a nonlinear integrodifferential equation, which describes, in general, collision processes between two particles. It should therefore depend, in a generic sense, on the states of two participating particles in the collision process and hence on the product of two distribution functions, making it nonlinear in D. Therefore, it requires to be solved in a self-consistent way, which is indeed a nontrivial task.

However, the problem can be simplified [39] if one considers a large amount of weakly coupled particles in thermal equilibrium at a temperature T constituting the heat bath in the background, and because of the fluctuation there can be some nonthermal but homogeneously distributed particles constituting the foreground. It is assumed that the overall equilibrium of the bath will not be disturbed by the presence of such a few nonthermal particles. Because of their scarcity, one can also assume that these nonthermal particles will not interact among themselves but only with particles of the thermal bath in the background. This requires one to replace the phase space distribution functions of the collision partners from the heat bath appearing in (8) by time-independent or thermal distribution $f_j(q)$. This will reduce the generic Landau kinetic equation (11), a nonlinear integrodifferential equation, to the Fokker-Planck (FP) equation, a linear differential equation for the Brownian motion of the nonthermal particles in the foreground.

Now, one can write the transport coefficients in (12) and (13) for such a FP equation in terms of the two body matrix elements $\mathcal{M}$ between a foreground and a background particle [12,17]:

$$\mathcal{T}_{1i}^{\rm FP}(\mathbf{p}) = \frac{1}{2E_\mathbf{p}}\int \frac{d^3\mathbf{q}}{(2\pi)^3 2E_\mathbf{q}}\int \frac{d^3\mathbf{q}'}{(2\pi)^3 2E_{\mathbf{q}'}}\int \frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{\mathbf{p}'}}\frac{1}{\gamma_c} \times \sum |\mathcal{M}|^2(2\pi)^4\delta^4(p+q-p'-q') \times [p_i - p_i']f(\mathbf{q})\tilde{f}(\mathbf{q}) \equiv \langle\langle(p-p')_i\rangle\rangle, \quad (14)$$

$$\mathcal{B}_{ij}^{\rm FP}(\mathbf{p}) = \tfrac{1}{2}\langle\langle(p-p')_i(p-p')_j\rangle\rangle. \quad (15)$$

In our case, the incoming particle is a heavy quark which is different from the background. So, $\mathbf{p}(\mathbf{p}')$ and $\mathbf{q}(\mathbf{q}')$ represent the momenta of the incoming (outgoing) charm and background light-quark/gluon, respectively. For each background species, there is a similar additive contribution to the collisional integral in (14). γ_c is the spin and color degeneracy factor of the foreground particle arising due to the initial reaction channels. $f(\mathbf{q})$ is the particle distribution of the thermal background, and $\tilde{f}(\mathbf{q}) = [1 \pm f(\mathbf{q})]$ corresponds to a Bose enhancement/Pauli suppression factor for scattered background particles, as appropriate. Because of this thermal $f(\mathbf{q})$ the contents of (14) and (15) are different from (12) and (13), and the charm quark in the foreground of a weakly coupled system is driven by a Brownian motion mechanism [12,17,18,24,40–42].

We are now set to study the momentum distribution of a charm quark undergoing Brownian motion and its relation with the transport coefficients. In absence of vectors other than $\mathbf{p}$, the values of $\mathcal{T}_{1i}^{\rm FP}$ and $\mathcal{B}_{ij}^{\rm FP}$, which depend functionally on $\mathbf{p}$, and the background temperature T, must be of the form in

Langevin theory [12,17,39]

$$\mathcal{T}_{1i}^{\rm FP}(\mathbf{p},T) = p_i \mathcal{A}(p^2,T), \tag{16}$$

$$\mathcal{B}_{ij}^{\rm FP}(\mathbf{p},T) = \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right)\mathcal{B}_0(p^2,T) + \frac{p_i p_j}{p^2}\mathcal{B}_1(p^2,T), \tag{17}$$

where $p^2 = p_i^2$, summation convention is always implied. $\mathcal{A}$ is the drag, $\mathcal{B}_0$ is the transverse diffusion, and $\mathcal{B}_1$ is the longitudinal diffusion coefficients. In terms of microscopic reaction amplitudes, these functions are obtained [12,17] as

$$\mathcal{A}(p^2,T) = \langle\langle 1\rangle\rangle - \frac{\langle\langle \mathbf{p}\cdot\mathbf{p}'\rangle\rangle}{p^2}, \tag{18}$$

$$\mathcal{B}_0(p^2,T) = \frac{1}{4}\left[\langle\langle p'^2\rangle\rangle - \frac{\langle\langle(\mathbf{p}\cdot\mathbf{p}')^2\rangle\rangle}{p^2}\right], \tag{19}$$

$$\mathcal{B}_1(p^2,T) = \frac{1}{2}\left[\frac{\langle\langle(\mathbf{p}\cdot\mathbf{p}')^2\rangle\rangle}{p^2} - 2\langle\langle \mathbf{p}\cdot\mathbf{p}'\rangle\rangle + p^2\langle\langle 1\rangle\rangle\right]. \tag{20}$$

The averaging, $\langle\langle\cdots\rangle\rangle$, defined in (14) can further be simplified [17] by solving the kinematics in the center-of-mass frame of the colliding particles, as

$$\begin{aligned}\langle\langle F(\mathbf{p}')\rangle\rangle = {} & \frac{1}{512\pi^4\gamma_c}\frac{1}{E_\mathbf{p}}\int_0^\infty \frac{q^2}{E_\mathbf{q}}dq\int_{-1}^{1} d\,(\cos\chi) \\ & \times \frac{\sqrt{\left(s+M_C^2-m_{g(q)}^2\right)^2 - 4sM_C^2}}{s} f(E_\mathbf{q}) \\ & \times \int_{-1}^{1} d\cos\theta_{\rm c.m.}\sum|\mathcal{M}|^2\int_0^{2\pi} d\phi_{\rm c.m.} e^{\beta E_{\mathbf{q}'}} \\ & \times f(E_{\mathbf{q}'})F(\mathbf{p}'),\end{aligned} \tag{21}$$

where M_C is the mass of a charm quark, $s = (E_\mathbf{p} + E_\mathbf{q})^2 - (\mathbf{p}+\mathbf{q})^2$, $E_{\mathbf{q}'} = E_\mathbf{p} + E_\mathbf{q} - E_{\mathbf{p}'}$, and $\mathbf{p}'$ is a function of $\mathbf{p}$, $\mathbf{q}$ and $\theta_{\rm c.m.}$. $\mathcal{M}^2$ is the matrix elements [12] for scattering processes Qg, Qq, and$Q\bar{q}$, where Q is a heavy quark and $g(q)$ is gluon (light quarks with 2-flavors). The expression in (21) is larger by a factor of 2 than the ones originally derived in (3.6) of Ref. [12]. Apart from this, we have also introduced the thermal masses of quarks (m_q), and gluons (m_g) and the quantum statistics, as appropriate.

The momentum and temperature dependence of the $\mathcal{A}$, $\mathcal{B}_0$, and $\mathcal{B}_1$ are summarized in Figs. 1–3 in Ref. [17]; we do not repeat them here and refer the reader to this work for details. The main finding is that these coefficients are momentum independent up to $p = 5$ GeV/c, and beyond this there is a weak momentum dependence. Note that the detailed studies of the dynamics of charm quark, as discussed in Refs. [12], may only depend on $\mathcal{A}$ and $\mathcal{B}_0$, but perhaps not on $\mathcal{B}_1$ in the phenomenological relevant momentum range. In the present calculation, we are interested in the kinematic domain of $p = (5-10)$ GeV/c, for which one needs to solve the FP equation considering the momentum dependence of drag and diffusion coefficients. This will require us to solve the FP equation numerically.

Instead, we assume the momentum independence of these coefficients in (18)–(20), which will correspond to a scenario where a particle travels through an ideal heat bath and undergoes linear damping (Rayleigh's particle). So, these transport coefficients are expected to be largely determined by the properties of the heat bath and not so much by the nature of the test particle [39]. This is also a fairly good approximation, which we will justify in the next section. Under this approximation, the transport coefficients in (16) and (17) become

$$\mathcal{T}_{1i}^{\rm FP} = p_i\mathcal{A}, \tag{22}$$

$$\mathcal{B}_{ij}^{\rm FP} = \delta_{ij}\mathcal{B}_0 \equiv \delta_{ij}\mathcal{T}_2^{\rm FP}, \tag{23}$$

where $\mathcal{B}_0(\mathbf{p}\to 0,T) = \mathcal{B}_1(\mathbf{p}\to 0,T) \equiv \mathcal{T}_2^{\rm FP}$. This could be viewed as a course-grained picture in which the finer details of the collisions have been averaged out over a large number of macroscopic situations (or over an ensemble). Then, combining (11), (22), and (23), one can write the FP equation as

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial p_i}\left[\mathcal{T}_{1i}^{\rm FP}D\right] + \mathcal{T}_2^{\rm FP}\left(\frac{\partial}{\partial \mathbf{p}}\right)^2 D. \tag{24}$$

In Sec. IV B, we obtain the time evolution of the FP equation in a thermally evolving QGP.

B. Time evolution of Fokker-Planck equation, drag and diffusion coefficients in an expanding plasma

We assume that the background partonic system has achieved thermal equilibrium when the momenta of the background partons become locally isotropic. At the collider energies, it has been estimated that $t_0 = 0.2$–0.3 fm/c. Beyond this point, further expansion is assumed to be described by the Bjorken scaling law [43]

$$T(t) = t_0^{1/3}T_0/t^{1/3}, \tag{25}$$

where T_0 is the initial temperature at which the background has attained local thermal equilibrium.

We consider, for simplicity, the one-dimensional problem, for which the FP equation in (24) reduces to

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial p}\left[\mathcal{T}_1^{\rm FP}D\right] + \mathcal{T}_2^{\rm FP}\frac{\partial^2}{\partial p^2}D, \tag{26}$$

and as discussed in Sec. IV A, the coupling between the Brownian particle and the bath is weak; the quantities $\mathcal{T}_1^{\rm FP}$ and $\mathcal{T}_2^{\rm FP}$ can also be written using the Langevin formalism [39] as

$$\mathcal{T}_1^{\rm FP}(p) = \int dk\,w(p,k)k = \frac{\langle\delta p\rangle}{\delta t} = \langle F\rangle = p\mathcal{A}, \tag{27}$$

$$\mathcal{T}_2^{\rm FP} = \frac{1}{2}\frac{\langle(\delta p)^2\rangle}{\delta t} \approx T\mathcal{T}_1^{\rm FP}. \tag{28}$$

Now the work done by the drag force $\mathcal{T}_1^{\rm FP}$ acting on a test particle is

$$-dE = \langle F\rangle\cdot dL = \mathcal{T}_1^{\rm FP}\cdot dL, \tag{29}$$

which can be related to the energy loss [21,25] of a particle as

$$-\frac{dE}{dL} = \mathcal{T}_1^{\rm FP} = p\mathcal{A}. \tag{30}$$

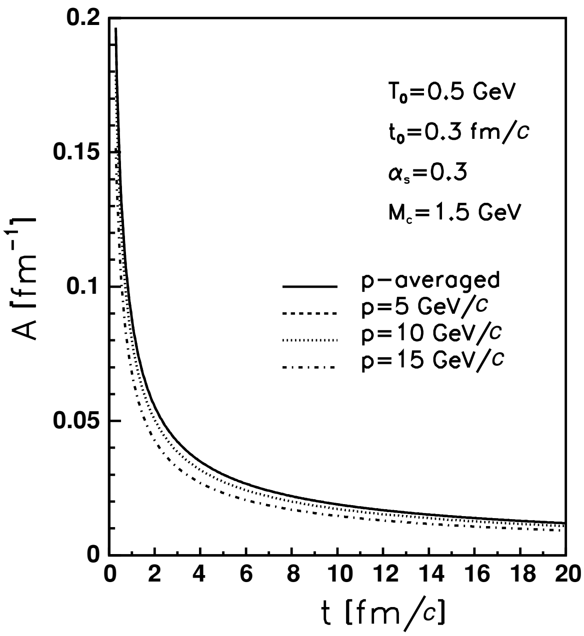

FIG. 2. Momentum averaged $\langle\mathcal{A}(t)\rangle$ in (31) and momentum dependence $\mathcal{A}(p,t)$ in (30) of the drag coefficient of a charm quark in an expanding QGP with plasma parameters (see text) suitable for RHIC energy.

The drag coefficient is a very important quantity containing the dynamics of elastic collisions, and it has a weak momentum dependence. Then one can average out the drag coefficient as

$$\langle\mathcal{A}[p,T(t)]\rangle \equiv \mathcal{A}[T(t)] = \left\langle -\frac{1}{p}\frac{dE}{dL}\right\rangle, \tag{31}$$

implying that the dynamics is solely determined by the collisions in the heat bath and independent of the initial momentum of the Brownian particle.

For averaging over the momentum, the Boltzmann distribution and the differential energy loss rates (1) and (2) were used. The time dependence of the drag coefficient comes from assuming a temperature, $T(t)$ decreasing with time as the system expands, according to the Bjorken scaling law [43] given in (25). We consider the initial temperature $T_0 = 0.5$ GeV, initial time $t_0 = 0.3$ fm/c and $\alpha_s = 0.3$ of the plasma for RHIC energy. In Fig. 2, the momentum averaged as well the momentum dependence of the drag coefficient of a charm quark in the QGP phase of the expanding fireball is shown as a function of time. As can be seen, the behavior of the momentum averaged drag coefficient (solid curve) is dominated by $T^2/p \sim t^{-1/3}$ according to the scaling law. It can also be seen that up to $p = 10$ GeV/c, there is no significant difference between momentum averaged $\langle\mathcal{A}[T(t)]\rangle$ and momentum dependence $\mathcal{A}[p,T(t)]$ of the drag coefficient, and it has only a weak p dependence beyond $p = 10$ GeV/c. Since it decreases with moderately high values of $p \geqslant$ (15 GeV/c), the momentum averaged approximation of drag coefficient $\langle\mathcal{A}[T(t)]\rangle$ would overestimate the actual $\mathcal{A}[p,T(t)]$ in this high momentum range. In our phenomenological approach, the momentum independence of the drag coefficient in (31) is a good approximation up to a moderate value of momentum $p \leqslant 15$ GeV/c.

Now, combining (27) and (28), we can write the diffusion coefficient as

$$\mathcal{T}_2^{\rm FP} = T\mathcal{T}_1^{\rm FP} = T\mathcal{A}p. \tag{32}$$

Once the drag coefficient is averaged out using the properties of heat bath, one can approximate p by the temperature T of the bath (as discussed earlier, it is independent of the initial momentum of the Brownian particle) and $\mathcal{A}$ by its average value given in (31). This leads to

$$\mathcal{T}_2^{\rm FP} = \mathcal{A}[T(t)]T^2(t), \tag{33}$$

which is also known as the Einstein relation [39] between drag and diffusion coefficients. In the left panel of Fig. 3, the diffusion coefficient obtained in (33) is represented by the solid line. It is found to have agreed quite well with the momentum-independent diffusion coefficient $\mathcal{B}_0(p \to 0)$ (dashed line) in (19) with a factor of $1/3$ multiplied with it, because we consider the one-dimensional scenario. As evident, the momentum independence of the diffusion coefficient is also a fairly good approximation.

Alternatively, one can also calculate the $\mathcal{T}_2^{\rm FP}$ in (32) by substituting $\mathcal{A}$ from (30) and averaging out the momentum dependence as

$$\left\langle\mathcal{T}_2^{\rm FP}\right\rangle = T\left\langle -\frac{dE}{dL}\right\rangle. \tag{34}$$

In the right panel of Fig. 3, the average diffusion coefficient computed in (34) (filled triangle) is displayed. The momentum-dependent transverse diffusion coefficient $\mathcal{B}_0[p^2, T(t)]$ in (19) is also plotted for different momenta. It can be seen that there is weak momentum dependence in $\mathcal{B}_0$ in the momentum range $p = 5$–20 GeV/c. The momentum averaged values are in agreement with $\mathcal{B}_0$ for higher momenta $p \geqslant 10$ GeV/c, whereas it overestimates lower momenta $p < 10$ GeV/c. We will use both the approximations for diffusion coefficient in (33) and (34) to obtain the momentum distribution below.

Combining (26) and (30), we find

$$\frac{\partial D}{\partial t} = \mathcal{A}\frac{\partial}{\partial p}(pD) + \mathcal{D}_F\frac{\partial^2 D}{\partial p^2}, \tag{35}$$

where $\mathcal{D}_F$, in general, has been used as the diffusion coefficient corresponding to (33) and (34), and $\mathcal{A}$ is the averaged drag coefficient in (31).

Next we solve the above equation with the boundary condition

$$D(p,t) \xrightarrow{t\to t_0} \delta(p - p_0). \tag{36}$$

The solution of (35) can be found by making a Fourier transform of $D(p,t)$,

$$D(p,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{D}(x,t)e^{ipx}dx, \tag{37}$$

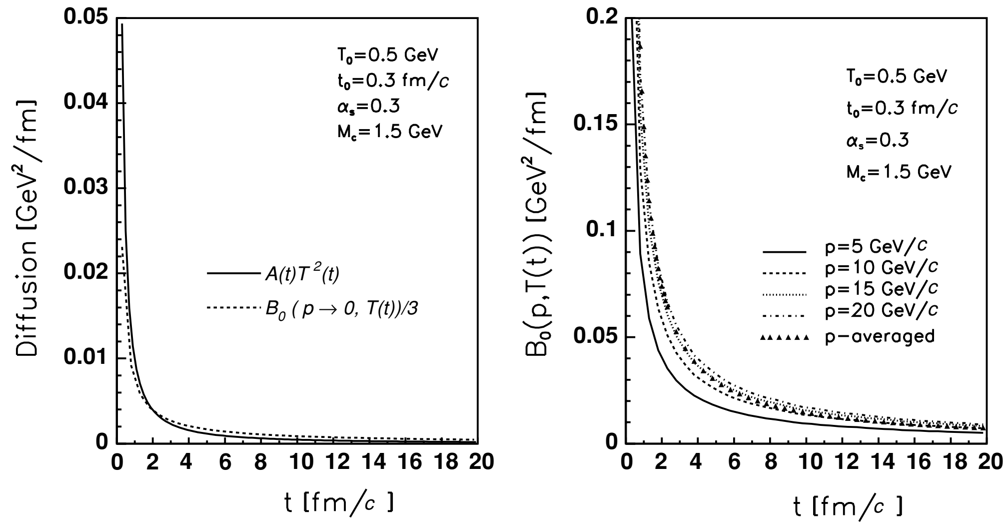

FIG. 3. Left panel: Comparison of diffusion coefficients given in (33) (solid line) and zero momentum limit of $\mathcal{B}_0$ (dashed line) given in (19). Right panel: Comparison of momentum averaged $\langle \mathcal{T}_2^{\mathrm{FP}} \rangle$ in (34) and momentum dependence of $\mathcal{B}_0$ in (19).

where the inverse transform is

$$\tilde{D}(x,t) = \int_{-\infty}^{+\infty} D(p,t) e^{-ipx} dp. \tag{38}$$

Under the Fourier transform, the corresponding initial condition follows from (36) and (38) as

$$\tilde{D}(x_0, t = t_0) = e^{-ip_0 x_0}, \tag{39}$$

where $x = x_0$ at $t = t_0$ is assumed. Replacing $p \to i\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial p} \to ix$, the Fourier transform of (35) becomes

$$\frac{\partial \tilde{D}}{\partial t} + \mathcal{A}x\frac{\partial \tilde{D}}{\partial x} = -\mathcal{D}_F x^2 \tilde{D}. \tag{40}$$

This is a first-order partial differential equation which may be solved by the method of characteristics [44]. The characteristic equation corresponding to (40) reads

$$\frac{\partial t}{1} = \frac{\partial x}{\mathcal{A}x} = -\frac{\partial \tilde{D}}{\mathcal{D}_F x^2 \tilde{D}}. \tag{41}$$

Using the boundary condition in (39), the solution of (40) can be obtained as

$$D(p,L) = \frac{1}{\sqrt{\pi \mathcal{W}(L)}} \exp\left[-\frac{\left(p - p_0 e^{-\int_0^L \mathcal{A}(t')dt'}\right)^2}{\mathcal{W}(L)}\right], \tag{42}$$

where

$$\mathcal{W}(L) = \left(4\int_0^L \mathcal{D}_F(t') \exp\left[2\int^{t'} \mathcal{A}(t'')dt''\right] dt'\right) \times \left[\exp\left(-2\int_0^L \mathcal{A}(t')dt'\right)\right], \tag{43}$$

which is the probability distribution in momentum space. Since the plasma expands with the passage of time, we used the length of the plasma L as the maximum time limit for the relativistic case ($\gamma v \sim 1$).

In Fig. 4, we show the momentum loss probability distribution $D(p, L)$ given in (42) of a charm quark with initial momentum $p_0 = 5$ GeV/c as a function of momentum p. The solid lines represent the distribution with the diffusion coefficient $\mathcal{D}_F = \mathcal{A}T^2$ in (33), whereas the dashed lines with $\langle \mathcal{D}_F \rangle$ in (34). Both sets of curves are for two different expanded plasma lengths, $L = 1$ and 5 fm as indicated in Fig. 4. In general, the physical mechanism reflected in Fig. 4 can be understood at initial time t_0 or length of the plasma, where a momentum distribution is sharply peaked at $p = p_0$, according to (36). With passage of time (or distance traveled) the peak of the probability distribution is shifted toward smaller momentum, as a result of drag force acting on the momentum

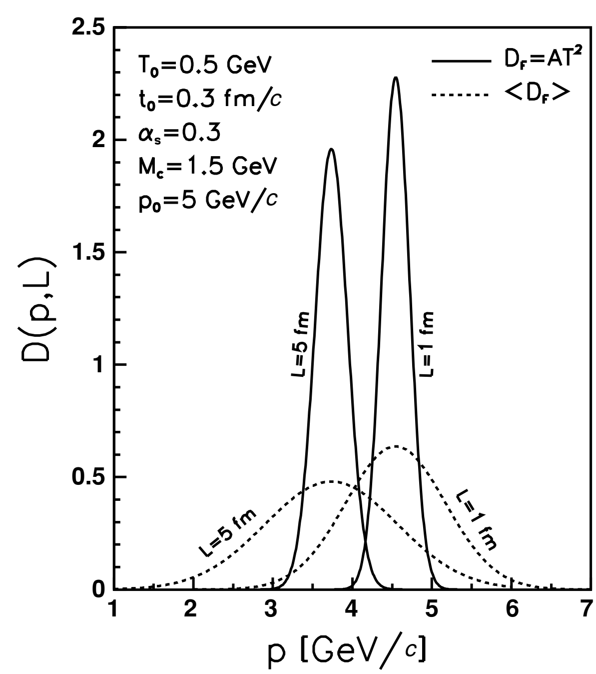

FIG. 4. Momentum loss probability distribution $D(p, L)$ of a charm quark as a function of momentum p after plasma has expanded a distance L.

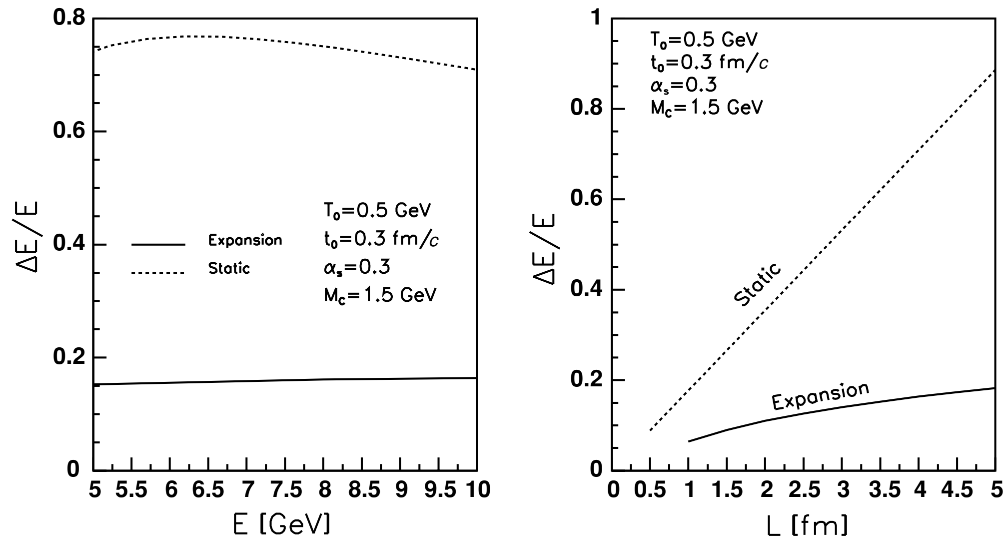

FIG. 5. Left panel: Fractional collisional energy loss of a charm quark $\Delta E/E$ as a function of energy E when plasma has expanded to $L = 4$ fm (solid line) and for static plasma of length $L = 4$ fm (dashed line). Right panel: Collisional $\Delta E/E$ as a function of length L for a charm quark of $E = 10$ GeV for expanding (solid line) and static (dashed line) plasma.

of the charm quark, indicating its most probable momentum loss due to elastic collisions in the medium. Moreover, the peak broadens slowly as a result of diffusion in momentum space, implying that a finite momentum dispersion sets in. As evident, with both the approximations in diffusion coefficient, only the momentum dispersion is affected while the peak positions remain unaltered, indicating that a drag force acting on the mean momentum of a charm quark is the same. After plasma has expanded to a length of $L = 1$ fm, the charm quark loses 10% of its momentum, whereas momentum loss is 25% at an expanded length of $L = 5$ fm. In Sec. IV C, we will use this distribution to compute the total energy loss of a charm quark for an expanding plasma.

C. Energy loss of a charm quark in an expanding plasma

In Sec. IV B, we obtained a momentum loss distribution by solving the time evolution of the FP equation in a thermally evolving plasma, which is modeled by an expanding fireball under conditions resembling central Au-Au collisions at RHIC. The mean energy of a charm quark due to the elastic collisions in a expanding medium can be estimated as

$$\langle E \rangle = \int_0^\infty E D(p, L) dp. \tag{44}$$

The average energy loss due to elastic collisions in the medium is given by

$$\Delta E = \langle \epsilon \rangle = E_0 - \langle E \rangle, \tag{45}$$

where $E = m_\perp = \sqrt{p_\perp^2 + m^2}$ at the central rapidity region $y = 0$.

The total energy loss of a charm quark has been computed in (45) by using the momentum loss distribution in (42). The numerical results for scaled collisional energy loss of a charm quark as a function of energy E in an expanding plasma (solid line) is shown in the left panel of Fig. 5 for the plasma parameters $T_0 = 0.5$ GeV, $t_0 = 0.3$ fm/c, and $\alpha_s = 0.3$ and expanded plasma length to $L = 4$ fm. In the energy range $E \sim 5$–10 GeV, the fractional collisional energy loss remains almost constant around a value 0.15, the reason for which can be traced back to the momentum independence of the drag coefficient [12,17] as discussed earlier. The corresponding scaled energy loss for a static plasma is shown by the dashed line. Taking into account the expansion, the scaled energy loss is suppressed by a factor of 5 as compared to the static case. In the right panel of Fig. 5, the scaled energy loss is plotted as a function of L for $E = 10$ GeV and found that it does not depend linearly on the system size for the expanding case (solid line) as compared to the static case (dashed line). Similar suppression should also occur in the radiative case as result of expansion [45].

D. Quenching of hadron spectra in an expanding plasma

We assume that the geometry is described by a cylinder of radius R, as in the boost invariant Bjorken model [43] of nuclear collisions, and the parton moves in the transverse plane in the local rest frame. Then a parton created at point $\vec{\mathbf{r}}$ with angle ϕ in the transverse direction will travel a distance [7]

$$L(\phi) = (R^2 - r^2 \sin^2 \phi)^{1/2} - r \cos \phi, \tag{46}$$

where $\cos \phi = \hat{\vec{\mathbf{v}}} \cdot \hat{\vec{\mathbf{r}}}$; $\vec{\mathbf{v}}$ is the velocity of the parton, and $r = |\vec{\mathbf{r}}|$. The value of the transverse dimension is taken as $R \sim 7$ fm.

The quenched spectrum convoluted with the transverse geometry of the partonic system can be written from (3) as

$$\begin{aligned}\frac{dN^{\rm med}}{d^2p_\perp} &= Q(p_\perp)\frac{dN^{\rm vac}}{d^2p_\perp} \\ &= \frac{1}{2\pi^2R^2}\int_0^{2\pi} d\phi \int_0^R d^2r \frac{dN(p_\perp + \Delta E)}{d^2p_\perp}. \end{aligned} \tag{47}$$

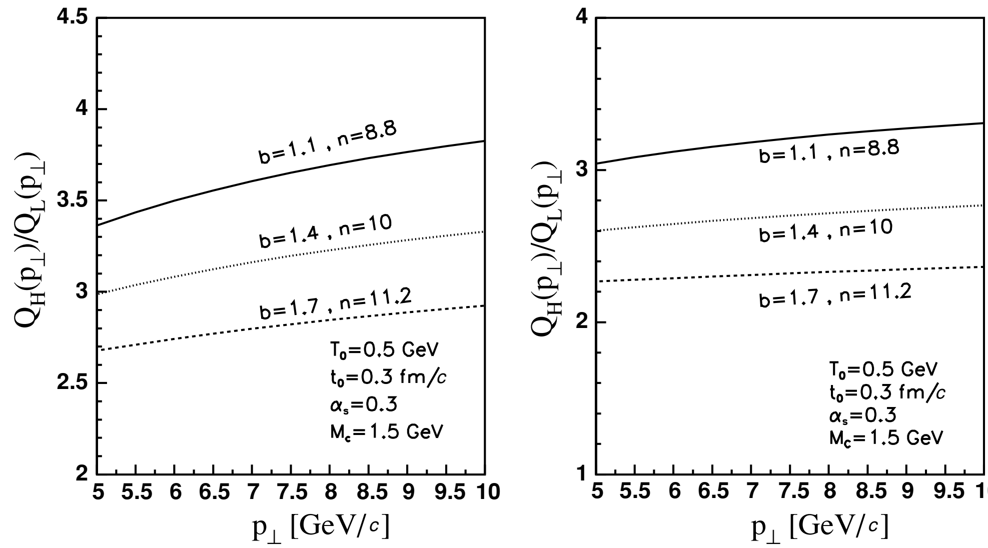

FIG. 6. Left panel: Ratio of charm to light quark quenching factors $Q_H(p_\perp)/Q_L(p_\perp)$ as a function of transverse momentum $p_\perp$ with collisional energy loss. Right panel: Ratio of charm to light quark quenching factors $Q_H(p_\perp)/Q_L(p_\perp)$ as a function of transverse momentum $p_\perp$ using both collisional and radiative energy losses.

The $p_\perp$ distribution of charmed hadrons, D mesons, produced in hadron collisions were experimentally found [46] to be well described by the following simple parametrization as

$$\frac{dN_H^{\rm vac}}{d^2p_\perp} = C\left(\frac{1}{bM_C^2+p_\perp^2}\right)^{n/2}, \tag{48}$$

where $b = 1.4 \pm 0.3$, $n = 10.0 \pm 1.2$, and $M_C = 1.5$ GeV.

The parametrization of the $p_\perp$ distribution exists in the literature [7,33,47], which describes the first RHIC light hadroproduction data for moderately large values of $p_\perp$. In this case, we consider the form given in Ref. [33], which reads as

$$\frac{dN_L^{\rm vac}}{d^2p_\perp} = A\left(\frac{1}{p_0+p_\perp}\right)^{m}, \tag{49}$$

where $m = 12.42$ and $p_0 = 1.71$ GeV/c.

The light hadron quenching obtained by using the collisional energy loss rate [25] was first anticipated [8] to be of the same order as that reached of with the radiative energy loss rate [7], though most of the previous studies insisted that the collisional energy loss is insufficient to describe the medium modifications of hadron spectra. We here estimate light hadron quenching (for details see Ref. [8]) with the energy loss due to elastic collisions [25], and the energy loss rate averaged over parton species reads

$$-\frac{dE}{dL} = \frac{4}{3}\left(1+\frac{9}{4}\right)\pi\alpha_s^2T^2\left(1+\frac{n_f}{6}\right) \times \log\left[2^{n_f/2(6+n_f)}0.92\frac{\sqrt{ET}}{m_g}\right]. \tag{50}$$

We now illustrate in the left panel of Fig. 6 the ratio of heavy to light quark quenching factors $Q_H(p_\perp)/Q_L(p_\perp)$ as a function of transverse momentum $p_\perp$, using the collisional energy loss for plasma parameters $T_0 = 0.5$ GeV, $t_0 = 0.3$ fm/c, and $\alpha_s = 0.3$ and the charm quark mass $M_C = 1.5$ GeV. As discussed earlier, this ratio may reflect the heavy to light hadrons D/π ratio originating from the fragmentation of heavy and light quarks in heavy ion collisions. As shown, the D/π ratio is enhanced significantly as compared to p-p collisions. The enhancement factor varies from 2.5 to 4 in the $p_\perp$ range (5–10) GeV/c because of the uncertainties of different choices of parameter set to parametrize the heavy hadron spectra as depicted in (48). However, the ratio also strongly depends on the quenching of light quark jets. The numerical estimate shows that the quenching of charm quarks is about half that of light quarks. The light quarks, for a given $p_\perp$, lose 10% of their energy after traversing a distance 1 fm and around 40% after 5 fm [8], whereas the charm quark loses 5% at 1 fm and around 20% at 5 fm (Fig. 5). Because of the large mass of the charm quark, the D meson will be formed in a shorter distance and hence the charm quark will have less time to propagate in the medium before transforming into the D meson. On the other hand, the light quarks will travel in the medium over a longer period and suffer more loss in energy than heavy quarks. The ratio is also found to be little more than that obtained earlier [33] by considering only the radiative energy loss with the appearance of the kinematic dead cone effect due to the finite heavy quark mass. This implies that the collision is one of the most dominant mechanisms of energy loss in the medium.

Having shown that collisional and radiative energy losses are of the same the order in magnitude, it would now be interesting to predict the D/π ratio within our model considering radiative energy loss in addition to collisional one. Since neither the drag nor diffusion coefficients have been calculated for other than collisional processes, it is not possible to infer the impact of radiative processes directly within our model. To circumvent our lack of knowledge of radiative processes in terms of the transport coefficients, we consider an alternative set [48] within our model, obtained by multiplying the transport coefficients by a factor $K = 2$. It is our hope that the experimental data will allow us to fix an approximate value of K, if at all required. In the right panel of Fig. 6, we plot the D/π ratio for such a case with the same parameter set as before. As shown, the ratio is little reduced compared to the collisional case and it varies from 2 to 3 within the $p_\perp$ range of 5–10 GeV/c. There is no striking change in the ratio mostly because of the cancellation of the introduced K factor with reference to the collisional one for respective species. However, the small change can be attributed to the charm quark which with the inclusion of radiative energy loss is relatively more quenched than that of the light quarks.

One may also add the interesting scenario that after the quark jet has hadronized to leading particles, they would scatter with hadronic matter before decoupling. Considering for example, $\sigma_{D\pi} \ll \sigma_{\pi\pi}$, it is most likely that the heavy mesons would decouple quickly from the hadronic phase. Pions would interact with each other via resonance formation and with other light hadrons. This might lead to further enhancement of the D/π ratio.

V. CONCLUSION

Apart from uncertainties in the various parameters describing the plasma and hadron spectra, let us look at some of the assumptions made in this work which may affect our findings. First, as discussed above, the momentum dependence of the drag and diffusion coefficients, containing the dynamics of the elastic collisions, has been averaged out. A major advantage of this is the simplicity of the resulting differential equation. Of course, this simplification can lead to some uncertainty in the quenching factor. Second, the entire discussion is based on the one-dimensional Fokker-Planck equation and the Bjorken model of the nuclear collision, which may not be a very realistic description here but can provide useful information on the problem. However, extension to three dimensions is indeed an ambitious goal, which may cause many of the considerations of the present work to be revised.

In the present calculation, the hadron spectra for both light and heavy quarks have directly been used to calculate the quenching factor $Q(p_\perp)$. This is equivalent to assuming that a quark forms a hadron without much change in its energy. To calculate the effects of the parton energy loss on the quenching pattern of high-$p_\perp$ partons in nuclear collisions, one should take into account the modification of the fragmentation function [4,5] of a leading quark resulting from many soft interactions of the hard partons in the medium. This also causes a significant energy loss of parton prior to hadronization and changes the kinematic variables of the fragmentation function [5]. This can also modify the quenching factor and thus D/π ratio.

We show that the total collisional energy loss is almost the same order as that of radiative energy loss for a static plasma. Considering the collisional energy loss rates, we obtain a momentum loss distribution for charm quarks by solving the time evolution of the Fokker-Plank equation for a thermally evolving plasma. The total energy loss for an expanding plasma is found to be reduced by a factor of 5 as compared to the static case and does not depend linearly on the system size. The ratio of heavy to light hadrons D/π is also estimated. However, the collisions have a different spectrum than radiation and therefore contribute in a similar way to the suppression factor than anticipated in earlier works.

We now eagerly wait for the experimental data, and a detailed calculation has to be carried out before a realistic value of the D/π ratio can be presented. Nevertheless, the total collisional energy loss of a charm quark computed within our simplified model may imply that collision is one of the important energy loss mechanisms in the medium for the energy range 5–10 GeV, and this feature may be phenomenologically important. Also, the results for D/π presented within this model are not definitive but can provide a very intuitive picture of the medium energy loss for partons in moderately large $p_\perp$.

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[1] M. Gyulassy and M. Plümer, Phys. Lett. **B243**, 432 (1990).
[2] K. Adcox *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **88**, 022301 (2002); C. Adler *et al.* (STAR Collaboration), *ibid.* **89**, 202301 (2002); **90**, 032301 (2003); **90**, 082302 (2003); S. S. Adler *et al.* (PHENIX Collaboration), *ibid.* **91**, 072301 (2003); B. B. Back *et al.* (PHOBOS Collaboration), Phys. Lett. **B578**, 297 (2004).
[3] M. Gyulassy, P. Levai, and I. Vitev, Phys. Lett. **B538**, 282 (2002); M. Gyulassy, I. Vitev, X. N. Wang, Phys. Rev. Lett. **86**, 2537 (2001).
[4] C. A. Salgado and U. A. Wiedemann, Phys. Rev. Lett. **89**, 092303 (2002).
[5] E. Wang and X. N. Wang, Phys. Rev. Lett. **89**, 162301 (2002).
[6] R. Baier, Y. L. Dokshitzer, A. H. Mueller, and D. Schiff, J. High Energy Phys. 0109 (2001) 033.
[7] B. Müller, Phys. Rev. C **67**, 061901(R) (2003).
[8] M. G. Mustafa and M. H. Thoma, Acta Phys. Hung. A **22**, 93 (2005).
[9] B. B. Back *et al.* (PHOBOS Collaboration), Phys. Rev. Lett. **91**, 072302 (2003); S. S. Adler *et al.* (PHENIX Collaboration), *ibid.* **91**, 072303 (2003); J. Adams *et al.* (STAR Collaboration), *ibid.* **91**, 072304 (2003); I. Arsene *et al.* (BRAHMS Collaboration), *ibid.* **91**, 072305 (2003).
[10] K. Adcox *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **88**, 192303 (2002).
[11] J. D. Bjorken, Fermilab preprint 82/59-THY (1982, unpublished).
[12] B. Svetitsky, Phys. Rev. D **37**, 2484 (1988).
[13] M. H. Thoma and M. Gyulassy, Nucl. Phys. **B351**, 491 (1991).
[14] S. Mrówczyński, Phys. Lett. **B269**, 383 (1991).

[15] Y. Koike and T. Matsui, Phys. Rev. D **45**, 3237 (1992).
[16] A. K. Dutta-Mazumder, J. Alam, P. Roy, and B. Sinha, Phys. Rev. D **71**, 094016 (2005).
[17] M. G. Mustafa, D. Pal, and D. K. Srivastava, Phys. Rev. C **57**, 889 (1998).
[18] H. van Hees and R. Rapp, Phys. Rev. C **71**, 034907 (2005).
[19] E. Braaten and R. D. Pisarski, Nucl. Phys. **B337**, 569 (1990).
[20] E. Braaten and M. H. Thoma, Phys. Rev. D **44**, 1298 (1991).
[21] E. Braaten and M. H. Thoma, Phys. Rev. D **44**, R2625 (1991).
[22] H. Vija and M. H. Thoma, Phys. Lett. **B342**, 212 (1995).
[23] P. Romatschke and M. Strickland, Phys. Rev. D **69**, 065005 (2004); P. Romatschke and M. Strickland, Nucl. Instrum. A **510**, 281 (2003).
[24] G. D. Moore and D. Teaney, Phys. Rev. C 71, 064904 (2005).
[25] M. H. Thoma, Phys. Lett. **B273**, 128 (1991).
[26] M. H. Thoma, J. Phys. G **26**, 1507 (2000).
[27] M. H. Thoma, Eur. Phys. J. C **16**, 513 (2000).
[28] R. Baier, D. Schiff, and B. G. Zakharov, Annu. Rev. Nucl. Part. Sci. **50**, 37 (2000).
[29] E. Wang and X.-N. Wang, Phys. Rev. Lett. **87**, 142301 (2001).
[30] M. G. Mustafa, D. Pal, D. K. Srivastava, and M. H. Thoma, Phys. Lett. **B428**, 234 (1998).
[31] Z. Lin, R. Vogt, and X. N. Wang, Phys. Rev. C **57**, 899 (1998); Z. Lin and R. Vogt, Nucl. Phys. **B544**, 339 (1999).
[32] E. Shuryak, Phys. Rev. C **55**, 961 (1997).
[33] Y. L. Dokshitzer and D. E. Kharzeev, Phys. Lett. **B519**, 199 (2001).
[34] M. Djordjevic and M. Gyulassy, Phys. Lett. **B560**, 37 (2003).
[35] M. Djordjevic and M. Gyulassy, Nucl. Phys. **A733**, 265 (2004).
[36] N. Armesto, C. A. Salgado, and U. A. Wiedemann, Phys. Rev. D **69**, 114003 (2004).
[37] M. Gyulassy, P. Levai, and I. Vitev, Nucl. Phys. **B594**, 371 (2001).
[38] U. A. Wiedemann, Nucl. Phys. **B558**, 303 (2000); **B582**, 409 (2000).
[39] R. Balescu, *Equilibrium and Non-Equilibrium Statistical Mechanics* (Wiley, New York, 1975).
[40] S. Chakrabarty and D. Syam, Lett. Nuovo Cimento **41**, 381 (1984).
[41] D. B. Walton and J. Rafelski, Phys. Rev. Lett. **84**, 31 (2000).
[42] Jan-e. Alam, S. Raha, and B. Sinha, Phys. Rev. Lett. **73**, 1895 (1994); P. Roy, J. Alam, S. Sarkar, B. Sinha, and S. Raha, Nucl. Phys. **A624**, 687 (1997).
[43] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).
[44] W. E. Williams, *Partial Differential Equations* (Clarendon Press, Oxford, 1980).
[45] X. N. Wang, Phys. Rev. Lett. **81**, 2655 (1998).
[46] C. A. Alves *et al.* (E796 Collaboration), Phys. Rev. Lett. **77**, 2392 (1996).
[47] R. J. Fries, B. Müller, and D. K. Srivastava, Phys. Rev. Lett. **90**, 132301 (2003).
[48] P. B. Gossiaux, V. Guiho, and J. Aichelin, J. Phys. G **31**, S1079 (2005).