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Spin-isospin stability of nuclear matter

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We calculate the density-dependent spin-isospin asymmetry energy $J(k_f)$ of nuclear matter in the three-loop approximation of chiral perturbation theory. The interaction contributions to $J(k_f)$ originate from one-pion exchange, iterated one-pion exchange, and irreducible two-pion exchange with no, single, and double virtual Δ-isobar excitation. We find that the approximation to 1π-exchange and iterated 1π-exchange terms (which leads already to a good nuclear matter equation of state by adjusting an emerging contact term) is spin-isospin stable, since $J(k_{f0}) \simeq 24\,\text{MeV} > 0$. The inclusion of the chiral $\pi N\Delta$ dynamics, necessary in order to guarantee the spin stability of nuclear matter, keeps this property intact. The corresponding spin-isospin asymmetry energy $J(k_f)$ stays positive even for extreme values of an undetermined short-distance parameter J_5 (whose possible range we estimate from realistic *NN* potentials). The largest positive contribution to $J(k_f)$ (a term linear in density) comes from a two-body contact term with its strength fitted to the empirical nuclear matter saturation point.

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In recent years a novel approach to the nuclear matter problem has emerged. Its key element is a separation of long- and short-distance dynamics and an ordering scheme in powers of small momenta. At nuclear matter saturation density $\rho_0 \simeq 0.16\,\text{fm}^{-3}$ the Fermi momentum k_{f0} and the pion mass m_π are comparable scales ($k_{f0} \simeq 2m_\pi$), and therefore pions must be included as explicit degrees of freedom in the description of the nuclear many-body dynamics. The contributions to the energy per particle $\bar{E}(k_f)$ of isospin-symmetric (spin-saturated) nuclear matter as they originate from chiral pion-nucleon dynamics have been computed up to three-loop order in Refs. [1,2]. Both calculations are able to reproduce the empirical saturation point of nuclear matter by adjusting one single parameter (either a contact coupling $g_0 + g_1 \simeq 3.23$ [1] or a cutoff scale $\Lambda \simeq 0.65\,\text{GeV}$ [2]) related to unresolved short-distance dynamics.[1] The basic mechanism for saturation in this approach is a repulsive contribution to the energy per particle $\bar{E}(k_f)$ generated by Pauli-blocking in second order (iterated) pion exchange. As outlined in Sec. 2.5 of Ref. [2] this mechanism becomes particularly transparent by taking the chiral limit $m_\pi = 0$. In that case the interaction contributions to $\bar{E}(k_f)$ are completely summarized by an attractive k_f^3 term and a repulsive k_f^4 term where the parameter-free prediction for the coefficient of the latter is very close to the one extracted from a realistic nuclear matter equation of state.

In a recent work [3] we have extended the chiral approach to nuclear matter by including systematically the effects from 2π exchange with virtual $\Delta(1232)$-isobar excitation. The physical motivation for such an extension is threefold. First, the spin-isospin-3/2 $\Delta(1232)$ resonance is the most prominent feature of low-energy πN scattering. Secondly, it is well known that 2π exchange between nucleons with excitation of virtual Δ isobars generates the needed isoscalar central *NN* attraction [4] which in phenomenological one-boson exchange models is often simulated by a fictitious scalar "σ"-meson exchange. Thirdly, the delta-nucleon mass splitting $\Delta = 293\,\text{MeV}$ is of the same size as the Fermi momentum $k_{f0} \simeq 2m_\pi$ at nuclear matter saturation density and therefore pions and Δ isobars should both be treated as explicit degrees of freedom. A large variety of nuclear matter properties has been investigated in this extended framework in Ref. [3]. It has been found that the inclusion of the chiral $\pi N\Delta$ dynamics is able to remove most of the shortcomings of previous chiral calculations of nuclear matter [2,5–7]. However, there remain open questions concerning the role of yet higher orders in the small momentum expansion and its "convergence." The relation of the fitted short-distance parameters [2] to those of few-nucleon systems is not clear at this moment. Also, a rigorous power counting that justifies the perturbative chiral expansion for nuclear matter has not yet been formulated. Recent work by Bogner *et al.* [8] based on the universal low-momentum *NN* potential $V_{\text{low-}k}$ may open interesting perspectives in this direction.

Irrespective of such foundational questions it is also necessary to check various stability conditions for nuclear matter in the chiral framework. In a recent paper [9] we have analyzed spin stability. It turned that the inclusion of the chiral $\pi N\Delta$ dynamics is essential in order to guarantee the spin stability of isospin-symmetric nuclear matter. The truncation to fourth order terms in the small momentum expansion with interaction contributions only from 1π and iterated 1π exchange is spin-unstable [9]. This statement holds independently of the regularization scheme if the contact terms (generating contributions linear in the nucleon density) are consistent with the empirical nuclear matter bulk properties: $\bar{E}(k_{f0}) \simeq -16\,\text{MeV}$ and $A(k_{f0}) \simeq 34\,\text{MeV}$. Now, since a nucleon possesses four internal spin and isospin degrees of freedom one can prepare nuclear matter also in a spin-isospin mixed asymmetric configuration. The stability of nuclear matter against such correlated spin-isospin deformations is the

[1]Fitting a cutoff scale, as done in Ref. [2], must be viewed as a short-term intermediate step before an eventual full effective field theory calculation. Cutoff independence of physical observables is in fact a primary goal of effective field theory.

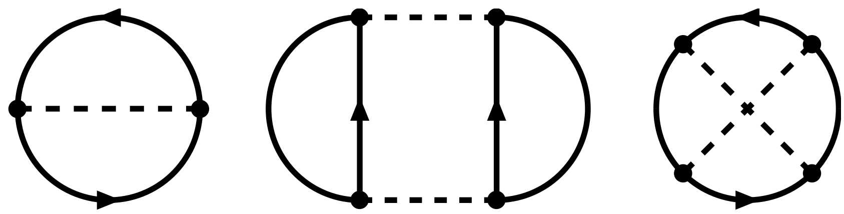

FIG. 1. The two-loop one-pion exchange Fock diagram and the three-loop iterated one-pion exchange Hartree and Fock diagrams. The combinatoric factors of these diagrams are 1/2, 1/4, and 1/4 in the order shown.

subject of the present paper. For recent work on generalized symmetry energy coefficients in the context of phenomenological Skyrme forces, also see Ref. [10]. Analogous earlier studies within Brueckner theory using the Reid soft-core *NN* potential can be found in Ref. [11].

Let us begin with defining the spin-isospin asymmetry energy $J(k_f)$ of (infinite) nuclear matter. Consider this many-nucleon system in a state where the equal densities of the spin-up protons ($p\uparrow$) and the spin-down neutrons ($n\downarrow$) have an excess over the equal densities of the spin-down protons ($p\downarrow$) and the spin-up neutrons ($n\uparrow$). With the help of the spin- and isospin-projection operators, $(1 \pm \sigma_3)/2$ and $(1 \pm \tau_3)/2$, such a spin-isospin mixed asymmetric configuration is realized by the substitution

$$\theta(k_f - |\vec{p}|) \to \frac{1+\sigma_3\tau_3}{2}\theta(k_+ - |\vec{p}|) + \frac{1-\sigma_3\tau_3}{2}\theta(k_- - |\vec{p}|), \tag{1}$$

in the medium insertion.[2] Here, $k_\pm = k_f(1 \pm \epsilon)^{1/3}$ (with ϵ a small parameter) are different Fermi momenta, chosen such that the total nucleon density $\rho = (k_+^3 + k_-^3)/3\pi^2 = 2k_f^3/3\pi^2$ stays constant. Note that Eq. (1) describes a rather peculiar asymmetric configuration of nuclear matter with equal densities of protons, neutrons, spin-up states, and spin-down states: $\rho_p = \rho_n = \rho_\uparrow = \rho_\downarrow = k_f^3/3\pi^2$. The expansion of the energy per particle of spin-isospin polarized nuclear matter,

$$\bar{E}(k_+, k_-)_{\sigma\tau-\text{pol}} = \bar{E}(k_f) + \epsilon^2 J(k_f) + \mathcal{O}(\epsilon^4), \tag{2}$$

defines the spin-isospin asymmetry energy $J(k_f)$. The obvious criterion for the spin-isospin stability of nuclear matter is then the positivity of the spin-isospin asymmetry energy: $J(k_f) > 0$. The energy per particle at fixed nucleon density ρ must take on its absolute minimum value in the spin- and isospin-saturated configuration.

The first contribution to the spin-isospin asymmetry energy $J(k_f)$ comes from the kinetic energy $\sqrt{M^2+p^2} - M$ of a noninteracting relativistic Fermi gas of nucleons:

$$J(k_f) = \frac{k_f^2}{6M} - \frac{k_f^4}{12M^3}, \tag{3}$$

with $M = 939$ MeV the (average) nucleon mass. The next term in this series, $k_f^6/16M^5$, is negligibly small at the densities of interest.

[2] Medium insertion is a technical notation for the difference between the in-medium and vacuum nucleon propagator [2]. Effectively, it sums hole propagation and the absence of particle propagation below the Fermi surface $|\vec{p}\,| < k_f$.

Next, we come to interaction contributions to $J(k_f)$. The closed in-medium diagrams related to one-pion exchange (Fock diagram) and iterated one-pion exchange (Hartree and Fock diagrams) are shown in Fig. 1. Differences in comparison to the calculation of the energy per particle $\bar{E}(k_f)$ in Ref. [2] occur only with respect to the factors emerging from the spin and isospin traces over closed nucleon lines and the radii $k_\pm = k_f(1 \pm \epsilon)^{1/3}$ of the Fermi spheres to be integrated over. After some analytical calculation we find the following contribution to the spin-isospin asymmetry energy $J(k_f)$ from the 1π exchange Fock diagram in Fig. 1 (including its relativistic $1/M^2$-correction):

$$\begin{aligned} J(k_f) = \frac{g_A^2 m_\pi^3}{(4\pi f_\pi)^2}\bigg\{ &\frac{u^3}{9} - \frac{u}{2} + \left(\frac{2u}{9} + \frac{1}{8u}\right)\ln(1+4u^2) \\ &+ \frac{m_\pi^2}{M^2}\bigg[\frac{19u^3}{18} - \frac{4u^5}{9} - \frac{u^2}{2}\arctan 2u \\ &- \frac{u}{72}(1+18u^2)\ln(1+4u^2)\bigg]\bigg\}. \end{aligned} \tag{4}$$

Here, we have introduced the abbreviation $u = k_f/m_\pi$ where $m_\pi = 135$ MeV stands for the (neutral) pion mass. As usual $f_\pi = 92.4$ MeV denotes the weak pion decay constant and we choose the value $g_A = 1.3$ of the nucleon axial-vector coupling constant in order to have a pion-nucleon coupling constant of $g_{\pi N} = g_A M/f_\pi = 13.2$. In the second and third diagrams in Fig. 1 the 1π-exchange interaction is iterated (once) with itself. These second order diagrams carry the large scale enhancement factor M (the nucleon mass). It stems from an energy denominator that is equal to a difference of small nucleon kinetic energies. With a medium insertion at each of two equally oriented nucleon propagators we obtain from the three-loop Hartree diagram in Fig. 1 the following contribution to the spin-isospin asymmetry energy:

$$\begin{aligned} J(k_f) = \frac{\pi g_A^4 M m_\pi^4}{6(4\pi f_\pi)^4}\bigg\{ &\left(15u + \frac{7}{2u}\right) \\ &\times \ln(1+4u^2) - 14u - 16u^2\arctan 2u\bigg\}. \end{aligned} \tag{5}$$

The right Fock diagram of iterated 1π exchange (see Fig. 1) with two medium insertions on non-neighboring nucleon propagators gives rise on the other hand to a contribution to the spin-isospin asymmetry energy of the form:

$$\begin{aligned} J(k_f) = \frac{\pi g_A^4 M m_\pi^4}{9(4\pi f_\pi)^4}\bigg\{ &\frac{21u}{5} - \frac{64u^3}{15} \\ &- \left(9 + 16u^2 + \frac{64u^4}{15}\right)\arctan u + \left(\frac{33}{10u} + \frac{14u}{3}\right) \end{aligned}$$

$$\times \ln(1+u^2) - \left(\frac{3}{u}+2u\right)\ln(1+4u^2) + (9-4u^2)\arctan 2u + (9+4u^2) \times \int_0^u dx \frac{\arctan x - \arctan 2x}{u(1+2x^2)}\Bigg\}. \qquad (6)$$

This expression does not include the contribution of a linear divergence $\int_0^\infty dl\, 1$ of the momentum-space loop integral. In dimensional regularization such a linear divergence is set to zero, whereas in cut-off regularization it is equal to a momentum space cut-off Λ. The additional term specific for cut-off regularization will be given in Eq. (13). An in-medium diagram with three medium insertions represents Pauli-blocking effects in intermediate *NN* states induced by the filled Fermi sea of nucleons. The unequal filling of the $(p\uparrow, n\downarrow)$ and $(p\downarrow, n\uparrow)$ Fermi seas shows its consequences in the spin-isospin asymmetry energy. After some extensive algebraic manipulations we end up with the following double-integral representation of the contribution to the spin-isospin asymmetry energy $J(k_f)$ from the Hartree diagram in Fig. 1 with three medium insertions:

$$J(k_f) = \frac{g_A^4 M m_\pi^4}{(4\pi f_\pi)^4 u^3}\int_0^u dx\, x^2 \int_{-1}^1 dy \times\Bigg\{\left[\frac{2uxy(3u^2-5x^2y^2)}{(u^2-x^2y^2)} - (u^2+5x^2y^2)H\right] \times\left[\frac{2s^2+s^4}{1+s^2} - 2\ln(1+s^2)\right] + \frac{4u^2 H\, s^5(8s'-9s)}{9(1+s^2)^2} + [2uxy + (u^2-x^2y^2)H] \times [(5+s^2)(9s^2-16ss'+16s'^2) + 8s(1+s^2)(2s''-10s'+9s)]\frac{s^4}{9(1+s^2)^3}\Bigg\}, \qquad (7)$$

where we have introduced several auxiliary functions

$$H = \ln\frac{u+xy}{u-xy}, \qquad s = xy + \sqrt{u^2-x^2+x^2y^2},$$
$$s' = u\frac{\partial s}{\partial u}, \qquad s'' = u^2\frac{\partial^2 s}{\partial u^2}. \qquad (8)$$

Note that Eq. (7) stems from a nine-dimensional principal-value integral over the product of three Fermi spheres of varying radii $k_\pm = k_f(1\pm\epsilon)^{1/3}$ which has been differentiated twice with respect to ϵ at $\epsilon = 0$. Of similar structure is the contribution to $J(k_f)$ from the iterated 1π-exchange Fock diagram with three medium insertions. Because of the two different pion propagators in the Fock diagram one ends up (partially) with a triple-integral representation for its contribution to the spin-isospin asymmetry energy:

$$J(k_f) = \frac{g_A^4 M m_\pi^4}{72(4\pi f_\pi)^4 u^3}\int_0^u dx \Bigg\{ G(9G_{20}+2G_{11}+9G_{02} - 16G_{01} - 9G) + 9G_{10}^2 + 2G_{01}G_{10} - 5G_{01}^2 + 4x^2\int_{-1}^1 dy \int_{-1}^1 dz \frac{yz\,\theta(y^2+z^2-1)}{|yz|\sqrt{y^2+z^2-1}} \times\Bigg[\frac{2s^3t^3(16s't-9st-12s't')}{(1+s^2)(1+t^2)} + \frac{s^2[t^2-\ln(1+t^2)]}{(1+s^2)^2}[(3+s^2)(16ss'-9s^2-16s'^2) + 4s(1+s^2)(12s'-9s-4s'')]\Bigg]\Bigg\}. \qquad (9)$$

Here, we have split into factorizable and nonfactorizable parts. These two pieces are distinguished by whether the (remaining) nucleon propagator in the three-loop Fock diagram can be canceled or not by terms from the product of πN-interaction vertices. The factorizable terms can be expressed through the auxiliary function

$$G = u(1+u^2+x^2) - \frac{1}{4x}[1+(u+x)^2][1+(u-x)^2] \times \ln\frac{1+(u+x)^2}{1+(u-x)^2}, \qquad (10)$$

and its partial derivatives for which we have introduced a (short-hand) double-index notation:

$$G_{ij} = x^i u^j \frac{\partial^{i+j} G}{\partial x^i \partial u^j}, \quad 1 \leqslant i+j \leqslant 2. \qquad (11)$$

For the presentation of the nonfactorizable terms one needs also copies of the quantities s and s' defined in Eq. (8) which depend (instead of y) on another directional cosine z:

$$t = xz + \sqrt{u^2-x^2+x^2z^2}, \qquad t' = u\frac{\partial t}{\partial u}. \qquad (12)$$

In the chiral limit $m_\pi = 0$ the fourth order contributions in Eqs. (5)–(9) sum up to a negative k_f^4 term of the form: $J(k_f)|_{m_\pi=0} = -(g_A k_f/4\pi f_\pi)^4(M/405)(32\pi^2 + 741 + 1848\ln 2)$. Finally, we give the expression for the linear divergence specific to cut-off regularization:

$$J(k_f) = \frac{10 g_A^4 M\Lambda}{3(4\pi f_\pi)^4}k_f^3, \qquad (13)$$

to which only the iterated 1π exchange Fock diagram (with two medium insertions) has contributed. In the case of the Hartree diagram the linear divergence drops out after taking the second derivative with respect to ϵ. One observes that the term in Eq. (13) is just $-1/3$ of the corresponding contribution to the energy per particle $\bar{E}(k_f)$ (see Eq. (15) in Ref. [2]). In this context it is interesting to note that for terms linear in density ρ the relation $3J(k_f)_{\rm lin} = -\bar{E}(k_f)_{\rm lin}$ holds generally. It is a consequence of the spin-isospin structure $3 - \vec\sigma_1\cdot\vec\sigma_2\vec\tau_1\cdot\vec\tau_2$ of a Fierz-antisymmetric *NN* contact interaction (see, e.g., Eq. (35) in Ref. [12]).

Now we can turn to numerical results. In Fig. 2 we show the spin-isospin asymmetry energy $J(k_f)$ of nuclear matter as a function the nucleon density $\rho = 2k_f^3/3\pi^2$. The solid line corresponds to a calculation up to fourth order in small momenta. It includes besides the kinetic energy term Eq. (3) the contributions from static 1π exchange and iterated 1π exchange. For reasons of consistency we have dropped the small relativistic $1/M^2$ correction in Eq. (4) since it is of fifth order in the small momenta k_f and m_π.

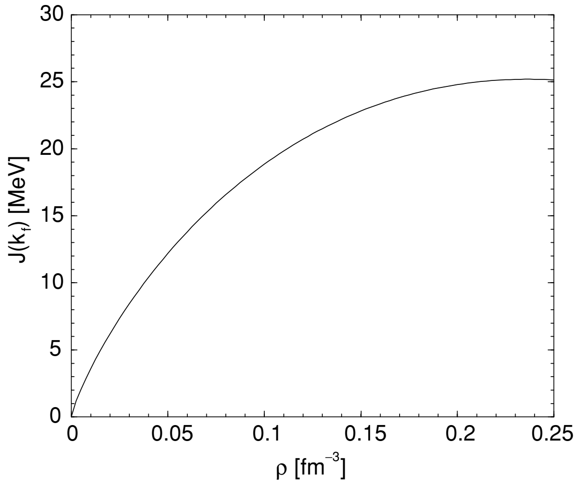

FIG. 2. The spin-isospin asymmetry energy $J(k_f)$ of nuclear matter versus the nucleon density $\rho = 2k_f^3/3\pi^2$. The solid line shows the result of a calculation up to fourth order in small momenta including 1π exchange and iterated 1π exchange. The cut-off scale $\Lambda = 0.61$ GeV has been adjusted to the saturation point: $\rho_0 = 0.173\,\text{fm}^{-3}$, $\bar{E}(k_{f0}) = -15.3$ MeV. The positive values of $J(k_f)$ indicate the spin-isospin stability of nuclear matter in this approximation.

The cut-off scale $\Lambda = 0.61$ GeV has been adjusted to the nuclear matter saturation point $\rho_0 = 0.173\,\text{fm}^{-3}$ and $\bar{E}(k_{f0}) = -15.3$ MeV. The resulting value of the nuclear matter compressibility $K = k_{f0}^2 \bar{E}''(k_{f0}) = 252$ MeV is consistent with a recent extrapolation from giant monopole resonances of heavy nuclei [13], which gave $K = (260 \pm 10)$ MeV. One can read off from Fig. 2 a positive value of the spin isospin asymmetry energy at saturation density: $J(k_{f0}) = J(2m_\pi) = 23.9$ MeV. It indicates the spin-isospin stability of nuclear matter in this approximation. The largest positive contribution to $J(2m_\pi) = 23.9$ MeV comes from the term, Eq. (13), linear in density and amounts to 59.2 MeV at saturation density $k_{f0} = 2m_\pi$. Compared to that the largest negative contribution is -32.8 MeV and it stems from the iterated 1π-exchange Fock diagram with two medium insertions, Eq. (6). The remaining numerically smaller contributions cancel each other to a large extent. It must however be stressed that at this level of approximation, with interaction terms only from 1π exchange and iterated 1π exchange, nuclear matter is spin unstable [9]. The inclusion of higher order terms (in particular 2π exchange with virtual Δ-isobar excitation) is mandatory in order to achieve spin-stability of nuclear matter.

Therefore, we turn now to contributions to $J(k_f)$ of fifth order in the small momentum expansion. At three-loop order these terms are generated by (irreducible) two-pion exchange between nucleons. The corresponding one-loop diagrams for elastic NN scattering are shown in Fig. 3. Since we are counting the delta-nucleon mass splitting $\Delta = 293$ MeV (together with k_f and m_π) as a small momentum scale the diagrams with single and double virtual $\Delta(1232)$-isobar excitation shown in Fig. 4 belong to the same order. By closing the two open nucleon lines of the one-loop diagrams in Figs. 3 and 4 to either two or one ring one gets (in diagrammatic representation) the Hartree or Fock contribution to the energy density. The Hartree contribution to the spin-isospin asymmetry energy $J(k_f)$ vanishes identically because the relevant 2π-exchange NN T matrix in forward direction is spin-independent [4,12]. The Fock contribution on the other hand is obtained by integrating the spin- and isospin-contracted T matrix over the product of two Fermi spheres of radii $k_\pm = k_f(1 \pm \epsilon)^{1/3}$. We separate regularization dependent short-range contributions to the T matrix (originating from the ultraviolet divergences of the one-loop diagrams in Figs. 3 and 4) from the unique long-range terms with the help of a twice-subtracted dispersion relation. The occurring subtraction constants give rise to a contribution to the spin-isospin asymmetry energy of the form

$$J(k_f) = -B_3 \frac{k_f^3}{3M^2} + J_5 \frac{k_f^5}{M^4}. \tag{14}$$

The dimensionless parameters $B_3 = -7.99$ has been adjusted in Ref. [3] to the saturation minimum $\bar{E}(k_{f0}) = -16$ MeV. Again, we recognize in the first part of Eq. (14) the relation $3J(k_f)_{\text{lin}} = -\bar{E}(k_f)_{\text{lin}}$ for terms linear in the density $\rho = 2k_f^3/3\pi^2$. The other subtraction constant J_5 in front of the k_f^5/M^4-term is (*a priori*) not constrained by any empirical (ground-state) property of nuclear matter. The long-range parts of the 2π-exchange (two-body) Fock diagrams can be expressed as a dispersion-integral:

$$\begin{aligned} J(k_f) = \frac{1}{6\pi^3} \int_{2m_\pi}^\infty d\mu \Big\{ & \text{Im}(3W_C + 2\mu^2 V_T + 4\mu^2 W_T) \\ & \times \frac{k_f}{3}\left[\frac{4k_f^2}{\mu} - \frac{8k_f^4}{\mu^3} - \mu \ln\left(1 + \frac{4k_f^2}{\mu^2}\right)\right] \\ & + \text{Im}(V_C + 3W_C + 2\mu^2 V_T + 6\mu^2 W_T) \\ & \times \left[\frac{\mu k_f}{2} - \frac{k_f^3}{\mu} + \frac{8k_f^5}{3\mu^3} - \frac{\mu^3}{8k_f} \ln\left(1 + \frac{4k_f^2}{\mu^2}\right)\right]\Big\}, \end{aligned} \tag{15}$$

where $\text{Im}V_C$, $\text{Im}W_C$, $\text{Im}V_T$, and $\text{Im}W_T$ are the spectral functions of the isoscalar and isovector central and tensor NN amplitudes, respectively. Explicit expressions of these imaginary parts for the contributions of the triangle diagram

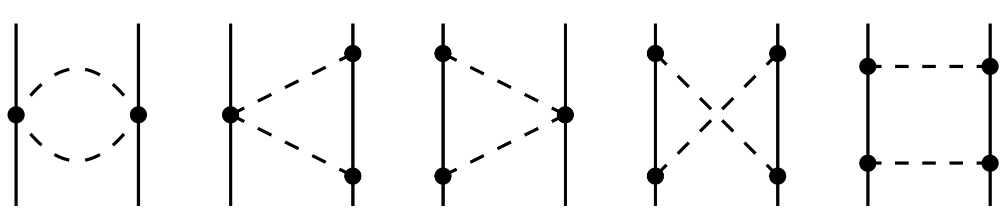

FIG. 3. One-loop diagrams of irreducible two-pion exchange between nucleons.

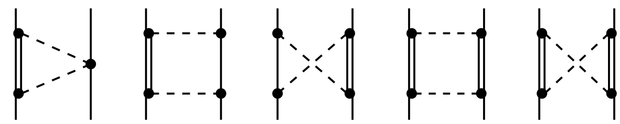

FIG. 4. One-loop two-pion exchange diagrams with single and double $\Delta(1232)$-isobar excitation. Diagrams for which the role of both nucleons is interchanged are not shown.

with single Δ excitation and the box diagrams with single and double Δ excitation can be easily constructed from the analytical formulas given in Sec. III of Ref. [4]. The μ- and k_f-dependent weighting functions in Eq. (15) take care that at low and moderate densities this spectral integral is dominated by low invariant $\pi\pi$ masses $2m_\pi < \mu < 1$ GeV. The contributions to the spin-isospin asymmetry energy $J(k_f)$ from irreducible 2π exchange (with only nucleon intermediate states, see Fig. 3) can also be cast into the form Eq. (15). The corresponding non vanishing spectral functions read [12]

$$\mathrm{Im}W_C(i\mu) = \frac{\sqrt{\mu^2-4m_\pi^2}}{3\pi\mu(4f_\pi)^4}\left[4m_\pi^2\left(1+4g_A^2-5g_A^4\right) + \mu^2\left(23g_A^4-10g_A^2-1\right) + \frac{48g_A^4m_\pi^4}{\mu^2-4m_\pi^2}\right], \tag{16}$$

$$\mathrm{Im}V_T(i\mu) = -\frac{6g_A^4\sqrt{\mu^2-4m_\pi^2}}{\pi\mu(4f_\pi)^4}. \tag{17}$$

Next, we come to the additional 2π-exchange three-body terms which arise from Pauli blocking of intermediate nucleon states (i.e., from the $(1\pm\sigma_3\tau_3)\theta(k_\pm-|\vec{p}\,|)$ terms in the in-medium nucleon propagators [2]). The corresponding closed Hartree and Fock diagrams with single virtual Δ excitation are shown in Fig. 5. The contribution of the left three-body Hartree diagram to the spin-isospin asymmetry energy $J(k_f)$ has the following analytical form:

$$J(k_f) = \frac{g_A^4m_\pi^6u^2}{27\Delta(2\pi f_\pi)^4}\left[\left(\frac{27}{4}+8u^2\right)\ln(1+4u^2) + 2u^4(1-9\zeta) - 22u^2 - \frac{5u^2}{1+4u^2}\right]. \tag{18}$$

The delta propagator shows up in this expression merely via the (reciprocal) mass splitting $\Delta = 293$ MeV. Furthermore, we have already inserted in Eq. (18) the empirically well-satisfied relation $g_{\pi N\Delta} = 3g_{\pi N}/\sqrt{2}$ for the $\pi N\Delta$-coupling constant. The parameter $\zeta = -3/4$ has been introduced in Sec. II of Ref. [3] in order to reduce a too strongly repulsive ρ^2 term in the energy particle $\bar{E}(k_f)$. It controls the strength of a three-nucleon contact interaction $\sim(\zeta g_A^4/\Delta f_\pi^4)(\bar{N}N)^3$ which has the property that it contributes equally but with opposite sign to the energy per particle $\bar{E}(k_f)$ and the spin-isospin asymmetry energy $J(k_f)$. The contribution of both three-body Fock diagrams in Fig. 5 to the spin-isospin asymmetry energy $J(k_f)$ can be represented as

$$J(k_f) = \frac{g_A^4m_\pi^6}{108\Delta(4\pi f_\pi)^4u^3}\int_0^u dx\big\{-4G_{S01}G_{S10} - 10G_{S01}^2 - 18G_{S10}^2 + 2G_S(9G_S + 16G_{S01} - 9G_{S02} - 2G_{S11} - 9G_{S20}) - 2G_{T01}G_{T10} - 17G_{T01}^2 - 9G_{T10}^2 + G_T(9G_T + 16G_{T01} - 9G_{T02} - 2G_{T11} - 9G_{T20})\big\}, \tag{19}$$

with the two auxiliary functions:

$$G_S = \frac{4ux}{3}(2u^2-3) + 4x[\arctan(u+x) + \arctan(u-x)] + (x^2-u^2-1)\ln\frac{1+(u+x)^2}{1+(u-x)^2}, \tag{20}$$

$$G_T = \frac{ux}{6}(8u^2+3x^2) - \frac{u}{2x}(1+u^2)^2 + \frac{1}{8}\left[\frac{(1+u^2)^3}{x^2} - x^4 + (1-3u^2)(1+u^2-x^2)\right] \times \ln\frac{1+(u+x)^2}{1+(u-x)^2}. \tag{21}$$

The double indices on G_S and G_T have the same meaning as explained in Eq. (11) for the function G.

In Fig. 6 we show again the spin-isospin asymmetry energy $J(k_f)$ of nuclear matter as a function of the nucleon density $\rho = 2k_f^3/3\pi^2$. The solid line includes all the contributions from chiral 1π and 2π exchange written down in Eqs. (3)–(9), (14)–(19). The (yet undetermined) short-range parameter J_5 has been set to zero, $J_5 = 0$. We note as an aside that the term linear in the density and the cut-off Λ, Eq. (13), is now not counted extra since the parameter $B_3 = -7.99$ [3] collects all such possible terms. Numerically, these two terms linear in density are anyhow almost identical. One observes in Fig. 6 a positive spin-isospin asymmetry energy $J(k_f)$ which rises monotonically with the density ρ.

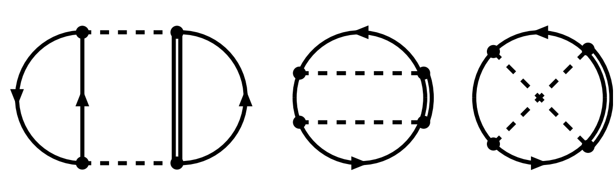

FIG. 5. Hartree and Fock three-body diagrams related to 2π exchange with single virtual Δ-isobar excitation. They represent interactions between three nucleons in the Fermi sea. The combinatoric factor is 1 for each diagram.

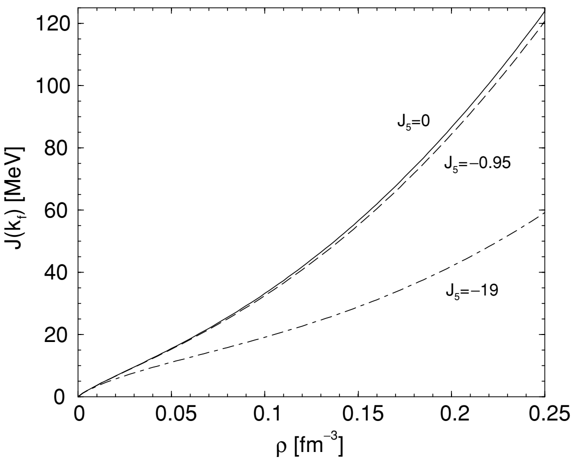

FIG. 6. The spin-isospin asymmetry energy $J(k_f)$ of nuclear matter versus the nucleon density $\rho = 2k_f^3/3\pi^2$. In comparison to Fig. 2 the effects from 2π exchange with single and double virtual Δ-isobar excitation are now included. The solid, dashed, and dashed-dotted curves correspond to the choices $J_5 = 0, -0.95$, and -19 of the short-range parameter J_5 introduced in Eq. (14). The positive values of $J(k_f)$ ensure the spin-isospin stability of nuclear matter.

The inclusion of the chiral $\pi N\Delta$ dynamics does therefore not disturb the spin-isospin stability of nuclear matter. It is also interesting to look at numerical values of $J(k_f)$ and their decomposition. At a Fermi momentum of $k_f = 2m_\pi$ (corresponding to $\rho = 0.173\,\text{fm}^{-3}$) the spin-isospin asymmetry energy is now $J(2m_\pi) = 69.5\,\text{MeV}$ (setting $J_5 = 0$). The most significant changes in comparison to the previous fourth order calculation come from the two-body Fock and three-body Hartree contributions Eqs. (15), (18) which amount together to $30.6\,\text{MeV} + 20.7\,\text{MeV} = 51.3\,\text{MeV}$. About one-third thereof (namely 16.6 MeV) stems from the three-body contact interaction proportional to $\zeta = -3/4$.

The size of the short-distance parameter J_5 in Eq. (14) is still open and large negative values could endanger the spin-isospin stability. In order to get an estimate of J_5 we bring into play the complete set of four-nucleon contact couplings written down in Eqs. (3) and (4) of Ref. [14]. This set represents the most general short-range NN interaction quadratic in momenta and it involves seven low-energy constants $C_1, \ldots, C_7$. After computing the spin-isospin asymmetry energy $J(k_f)$ from the corresponding contact-potential in Hartree-Fock approximation we find

$$J_5 = \frac{M^4}{18\pi^2}(C_2 - 4C_1) = \frac{M^4}{144\pi^3}\Big[3C\big({}^1P_1\big) + C\big({}^3P_0\big) + 3C\big({}^3P_1\big) + 5C\big({}^3P_2\big)\Big]. \tag{22}$$

In the second line of Eq. (22) we have reexpressed the relevant linear combination of $C_{1,2}$ through the so-called spectroscopic low-energy constants which characterize the short-range part of the NN potential in the spin-singlet and spin-triplet S- and P-wave states. In that representation we obtain from the entries of Table IV in Ref. [14] for the three NN potentials[3] CD-Bonn, Nijm-II, and AV-18 the numbers: $J_5 = -1.34, -0.57$, and -0.94. The dashed line in Fig. 6 shows the spin-isospin asymmetry energy $J(k_f)$ which results from taking their average value $J_5 = -0.95$. The corresponding reduction of the spin-isospin asymmetry energy is negligible. The dashed-dotted curve in Fig. 6 corresponds to the extreme choice $J_5 = -19$. One can see that even with such a large negative J_5-value the spin-isospin stability of nuclear matter remains still preserved. We can therefore conclude that spin-isospin stability is a robust property of the chiral approach to nuclear matter (at least in the three-loop approximation). This is a important finding.

In summary we have investigated in this work the spin-isospin stability of nuclear matter in the framework of chiral perturbation theory. For that purpose we have calculated the density-dependent spin-isospin asymmetry energy $J(k_f)$ of nuclear matter to three-loop order. The interaction contributions to $J(k_f)$ originate from 1π exchange, iterated 1π exchange, and (irreducible) 2π exchange with no, single, and double virtual Δ-isobar excitation. We have found that the approximation to 1π- and iterated 1π-exchange terms is spin-isospin stable, since $J(k_{f0}) > 0$. The inclusion of the chiral $\pi N\Delta$-dynamics (necessary to ensure the spin stability [9] of nuclear matter) keeps this property intact. The largest positive contribution to $J(k_f)$ comes from a two-body contact interaction with its strength fitted to the empirical nuclear matter saturation point.

[3]The short-distance structure of realistic NN potentials and effective field theory could be very different. The idea here is simply to explore the extreme possible range of J_5.

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