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Attractive and repulsive contributions of medium fluctuations to nuclear superfluidity

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Oscillations of mainly surface character ($S = 0$ modes) give rise, in atomic nuclei, to an attractive (induced) pairing interaction, while spin ($S = 1$) modes of mainly volume character generate a repulsive interaction, the net effect being an attraction which accounts for a sizeable fraction of the experimental pairing gap. Suppressing the particle-vibration coupling mediated by the proton degrees of freedom, i.e., mimicking neutron matter, the total surface plus spin-induced pairing interaction becomes repulsive.

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A central issue in a quantitative description of superconductors (metals, doped fullerides, etc.) as well as superfluid Fermi systems (^{3}He, trapped gases of fermionic atoms, atomic nuclei, neutron stars, etc.) is related to the interaction between fermions that gives rise to Cooper pairs. The glue holding together the fermions of each Cooper pair is the result of the bare interaction between fermions, strongly renormalized by medium polarization effects: Coulomb interaction plus plasmon and phonon exchange in metals [1], Van der Waals interaction plus spin and density mode exchange in trapped gases of fermionic atoms [2,3], and strong force plus spin and density mode exchange in the case of atomic nuclei and neutron stars [4–6].

While broad consensus exists concerning the mechanism of electron-electron and electron-phonon interaction leading to superconductivity in metals, the situation is much less clear in the case of strongly interacting particles in the various scenarios found in nature. In particular, it has been found that medium polarization effects associated with the exchange of vibrations lead to a quenching of the bare pairing interaction in the $^1S_0$ channel in the inner crust of neutron stars [7], and at the same time, account for a sizeable fraction of the pairing gap in open shell nuclei [8–10]. In the present paper, we present, for the first time, evidence which allows us to understand these seemingly contradictory results.

We address the question at hand within the scenario provided by the paradigmatic (superfluid) open shell nucleus ^{120}Sn. The starting point corresponds to the calculation of the mean field potential and associated quasiparticle properties within the framework of Hartree-Fock (HF) plus BCS theory [11] using the SkM* force [12]. The polarization quanta were worked out within the framework of the quasiparticle random phase approximation (QRPA). The particle-hole residual interaction is derived in a self-consistent way from the Skyrme energy functional, with the exception of the spin-orbit and the Coulomb part (cf. [13] for more details). On the other hand, we neglect the momentum-dependent part of the interaction in the calculation of the particle-vibration coupling discussed below [14]. The relevant part of the particle-hole interaction can then be written

$$v_{\rm ph}(\vec{r},\vec{r}') = \delta(\vec{r}-\vec{r}')\{[F_0 + F_0'\vec{\tau}\cdot\vec{\tau}'] + [(G_0 + G_0'\vec{\tau}\cdot\vec{\tau}')\,\vec{\sigma}\cdot\vec{\sigma}']\}. \quad (1)$$

We shall only consider the $\tau_z \cdot \tau_z$ term, in keeping with the fact that here we are interested in the neutron-neutron pairing interaction. Off-diagonal terms are associated with charge-exchange modes. Thus, in lowest order, they do not contribute to the neutron-neutron interaction but are expected to be of relevance in the discussion of the proton-neutron pairing interaction.

The functions $F_0(r)$, $F_0'(r)$, $G_0(r)$, and $G_0'(r)$ (generalized Landau-Migdal [15,16] parameters) controlling the isoscalar and isovector (spin-independent and spin-dependent) channels are displayed in Fig. 1.

Strictly speaking, in the case of atomic nuclei, spin is not a good quantum number with which to identify the polarization quanta, because of the strong spin-orbit term present in these systems. We have thus adopted the criterion of distinguishing between natural ($\pi = (-1)^J$) and non-natural ($\pi = -(-1)^J$) parity modes, where J indicates the total angular momentum of the quanta. Vibrations of multipolarity and parity $J^\pi = 1^+, 2^+, 2^-, 3^+, 3^-, 4^+, 4^-, 5^+$, and 5^- were worked out. Those having energy $\leqslant$30 MeV were used in the calculation of the induced interaction [cf. Fig. 2(a)]. From the QRPA calculation, one gets [17], along with the energy of the excited states, their transition densities, which will be used as a form factor for the particle-vibration coupling vertex [cf. Fig. 2(b)]:

$$\delta\rho^i_{J^\pi}(r) = \frac{1}{\sqrt{2J+1}}\sum_{\nu_1,\nu_2}\left(X_{\nu_1,\nu_2}(i,J^\pi) + Y_{\nu_1,\nu_2}(i,J^\pi)\right) \times \left(u_{\nu_1}v_{\nu_2} + u_{\nu_2}v_{\nu_1}\right) \times \langle\nu_1||i^J Y_J||\nu_2\rangle\varphi_{\nu_1}(r)\varphi_{\nu_2}(r). \quad (2)$$

$$\delta\rho^i_{J^\pi L}(r) = \frac{1}{\sqrt{2J+1}}\sum_{\nu_1,\nu_2}\left[X_{\nu_1,\nu_2}(i,J^\pi) - Y_{\nu_1,\nu_2}(i,J^\pi)\right] \times \left(u_{\nu_1}v_{\nu_2} + u_{\nu_2}v_{\nu_1}\right) \times \langle\nu_1||i^L[Y_L\times\sigma]_J||\nu_2\rangle \times \varphi_{\nu_1}(r)\varphi_{\nu_2}(r). \quad (3)$$

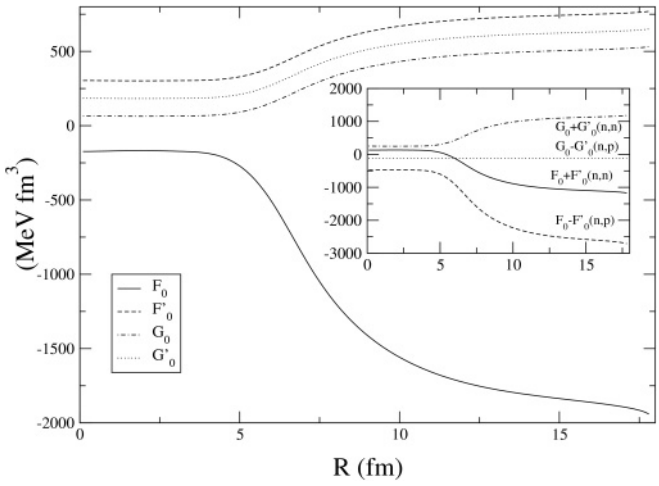

FIG. 1. Generalized Landau parameters associated with the interaction SkM* defining the strength of the particle-hole interaction in the isoscalar (F_0), isovector (F_0'), spin isoscalar (G_0), and spin isovector (G_0') channels. In the inset, the functions $F_0 + F_0'$ (n-n interaction), $F_0 - F_0'$ (n-p), $G_0 + G_0'$ (n-n), and $G_0 - G_0'$ (n-p) are also shown.

The function (2) is associated with the response to external fields which induce a density oscillation; it vanishes for phonons of non-natural parity. The function (3) is instead associated with the response to magnetic external fields and hence the coupling to excited states mediated by the part of the residual interaction which depends on the spin, and it applies to phonons of both non-natural (when $J \neq L$) and natural (when $J = L$) parity. The index i labels the different vibrational modes of a given spin and parity; X and Y are the forward and backward QRPA amplitudes of the corresponding modes; and the index ν denotes the quantum numbers n, l, j of the single-particle states.

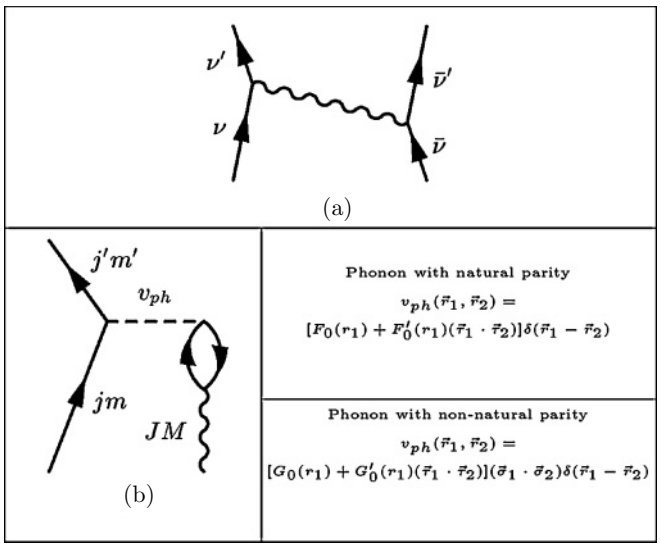

FIG. 2. (a) Diagram depicting the pairing interaction induced by the exchange of phonons; (b) particle-vibration coupling vertex associated with the particle-hole interaction $v_{\rm ph}$ which gives rise to the QRPA phonons (wavy lines). We also indicate the Landau parameters giving the dominant contributions to $v_{\rm ph}$, for phonons of natural and non-natural parity.

By using Eq. (1), one can find the expression of the vertices produced by the spin-independent part of the residual interaction:

$$f^{\nu'm'}_{\nu m;J^\pi Mi} = \langle \nu' m' | [F_0(r) + F_0'(r)\vec{\tau} \cdot \vec{\tau}']\delta(\vec{r} - \vec{r}')|\nu m; J^\pi Mi\rangle. \tag{4}$$

It can be rewritten by using the multipole expansion for the δ function as

$$f^{\nu'm'}_{\nu m;J^\pi Mi} = i^{l-l'}\langle j'm'|(i)^J Y_{JM}|jm\rangle \times \int dr \varphi_{\nu'}\left[(F_0 + F_0')\delta\rho^i_{J^\pi n} + (F_0 - F_0')\delta\rho^i_{J^\pi p}\right]\varphi_\nu, \tag{5}$$

$\delta\rho^i_{J^\pi n}$ and $\delta\rho^i_{J^\pi p}$ being the neutron and proton contributions to the transition densities defined in Eq. (2). In a similar way, the vertices produced by the spin-dependent part of the residual interaction are

$$g^{\nu'm'}_{\nu m;J^\pi Mi} = \langle \nu' m' | [G_0(r) + G_0'(r)\vec{\tau} \cdot \vec{\tau}'] \times \vec{\sigma} \cdot \vec{\sigma}'\delta(\vec{r} - \vec{r}')|\nu m; J^\pi Mi\rangle, \tag{6}$$

and as before, they can be expanded in the form

$$g^{\nu'm'}_{\nu m J^\pi Mi} = \sum_{L=J-1}^{J+1} i^{l-l'}\langle j'm'|(i)^L[Y_L \times \sigma]_{JM}|jm\rangle \times \int dr \varphi_{\nu'}\left[(G_0 + G_0')\delta\rho^i_{J^\pi Ln} + (G_0 - G_0')\delta\rho^i_{J^\pi Lp}\right]\varphi_\nu, \tag{7}$$

where $\delta\rho^i_{J^\pi Ln}$ and $\delta\rho^i_{J^\pi Lp}$ are, respectively, the neutron and proton contributions to the transition densities defined in Eq. (3).

These particle-vibration coupling matrix elements, together with the energies of the QRPA modes and the HF single-particle energies, are the basic ingredients needed to calculate the pairing induced interaction $v_{\rm ind}$ [cf. Fig. 2(a)] within the framework presented in Ref. [10]. The matrix elements between two pairs of neutrons in time-reversal states are given by

$$\langle \nu' m' \nu' \bar{m}'|v_{\rm ind}|\nu m \nu \bar{m}\rangle = \sum_{J^\pi Mi} \frac{2(f+g)^{\nu'm'}_{\nu m;J^\pi Mi}(f-g)^{\nu'm'}_{\nu m;J^\pi Mi}}{E_0 - E_{\rm int}}, \tag{8}$$

where the sum is over the phonons of multipolarity J^π, M obtained in the QRPA calculation. In the denominator, $E_{\rm int} = (|\epsilon_\nu - \epsilon_F| + |\epsilon_{\nu'} - \epsilon_F| + h\omega_i)$ denotes the energy of the intermediate state given by two particles (whose energy is calculated with respect to the Fermi energy ϵ_F) and one vibration, while E_0 is the energy of the correlated two-particle state, which must be obtained self-consistently: the denominator is therefore always negative. The important conclusion is that the sign of the matrix element (8) then depends on the relative magnitudes of the attractive contribution f^2, arising from the spin-independent term, and the repulsive contribution $-g^2$, arising from the spin-dependent term.

The matrix elements (8) are then used to obtain the $^1S_0$ neutron pairing gap, solving the BCS gap equation [18]

$$\Delta_\nu = -\frac{1}{2j+1}\sum_{\nu'}\frac{\Delta_{\nu'}}{2E_{\nu'}}v_{\nu,\nu'}, \tag{9}$$

with the corresponding number equation. The matrix elements $v_{\nu,\nu'}$ are given by

$$\begin{aligned} v_{\nu,\nu'} &= \sum_{mm'}\langle \nu m \nu \bar{m}|v_{\rm ind}|\nu' m' \nu' \bar{m}'\rangle \\ &= \sqrt{(2j+1)(2j'+1)}\langle jj;0|v_{\rm ind}|j'j';0\rangle. \end{aligned} \tag{10}$$

The two-particle wavefunction $|j'j';0\rangle$, coupled to zero angular momentum, contains an admixture of singlet and triplet components, with about equal weight. It is well known (see, e.g. [19–21]) that in infinite matter, when only a species is present, as in neutron matter, the matrix elements associated with the singlet component have a simple character, depending on the spin S of the exchanged fluctuation: for density modes, characterized by $S = 0$, the matrix elements are negative, while for spin modes ($S = 1$), they are positive. The suppression of the pairing gap produced by the bare neutron-neutron force through medium polarization effects in neutron matter is associated with the dominance of the repulsive contribution from the spin modes over the attractive contribution from the density modes.

In the case of ^{120}Sn, we observe that the diagonal matrix elements $\langle jj;0|v_{\rm ind}|jj;0\rangle$, shown in Fig. 3, are systematically attractive or repulsive, depending on whether the exchanged fluctuations are, respectively, of natural or non-natural parity, in direct correspondence with the case of infinite matter. On one hand, this is because for unnatural parity modes, only the spin-dependent vertices (7), which have a $S = 1$ character, contribute, while for natural parity modes, the spin-independent matrix elements (5) are the dominant ones, and one can show that they are the only ones contributing to the diagonal matrix elements. On the other hand, the contributions of the matrix elements associated with the triplet component of the two-particle $|jj0\rangle$ wavefunction are small and of variable sign [22].

However, in contrast with neutron matter, the resulting total matrix elements are predominantly attractive (in any case around the Fermi energy) [23].

The resulting state-dependent pairing gap obtained by solving the BCS gap and number equations, making use of the (total) induced pairing matrix elements, is depicted in Fig. 4(a). For states close to the Fermi energy, the gap accounts for a consistent fraction of the experimental value (1.4 MeV). If one solves the BCS equations considering only the exchange of density modes [i.e., neglecting the contributions from Eq. (7)], one obtains values which are, on average, larger [cf. Fig. 4(b)]. In fact, the exchange of $S = 1$ modes quenches the pairing gap arising from the exchange of only $S = 0$ modes by roughly 30% [24].

To gain insight into the peculiar features of finite nuclei, as compared to the case of infinite systems, it is useful to study the radial dependence of the particle-vibration coupling

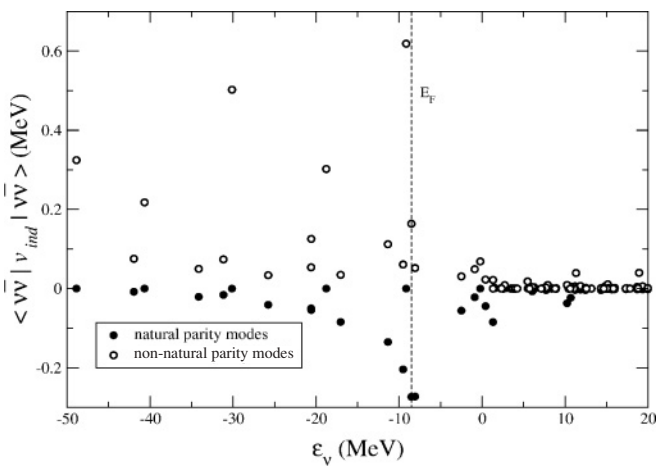

FIG. 3. Diagonal induced pairing matrix elements resulting from the exchange of phonons with natural parity (filled circles) and those resulting from the exchange of phonons with non-natural parity vibrations (empty circles), displayed as a function of the energy of the single-particle state ϵ_ν.

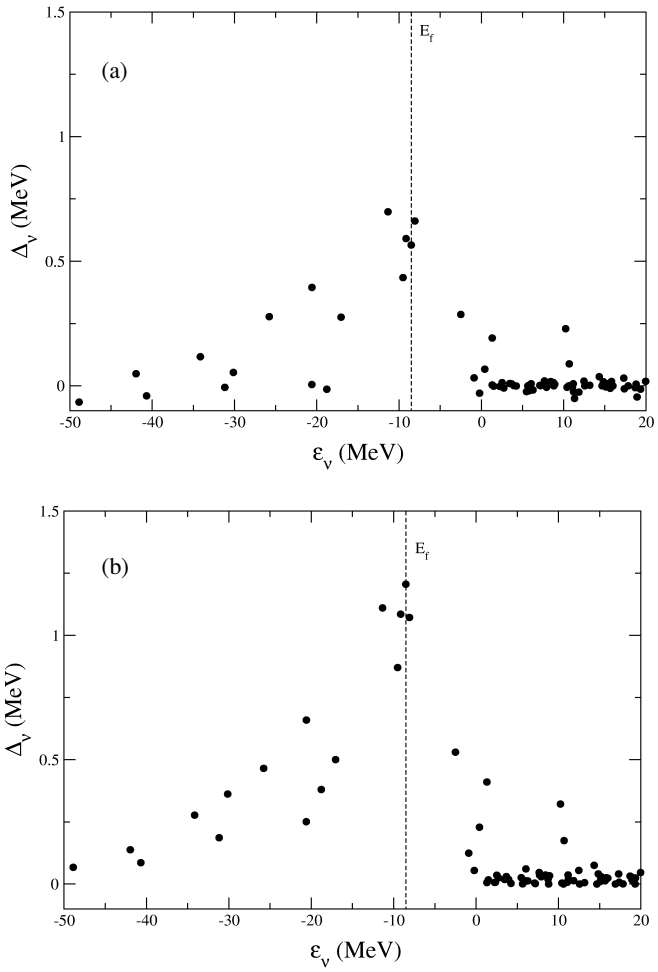

FIG. 4. (a) The state-dependent pairing gap as a function of the single-particle energies obtained by solving the BCS equations associated with the total induced interaction matrix elements; (b) same as (a) but for the matrix elements associated with the spin-independent part of the particle-hole interaction.

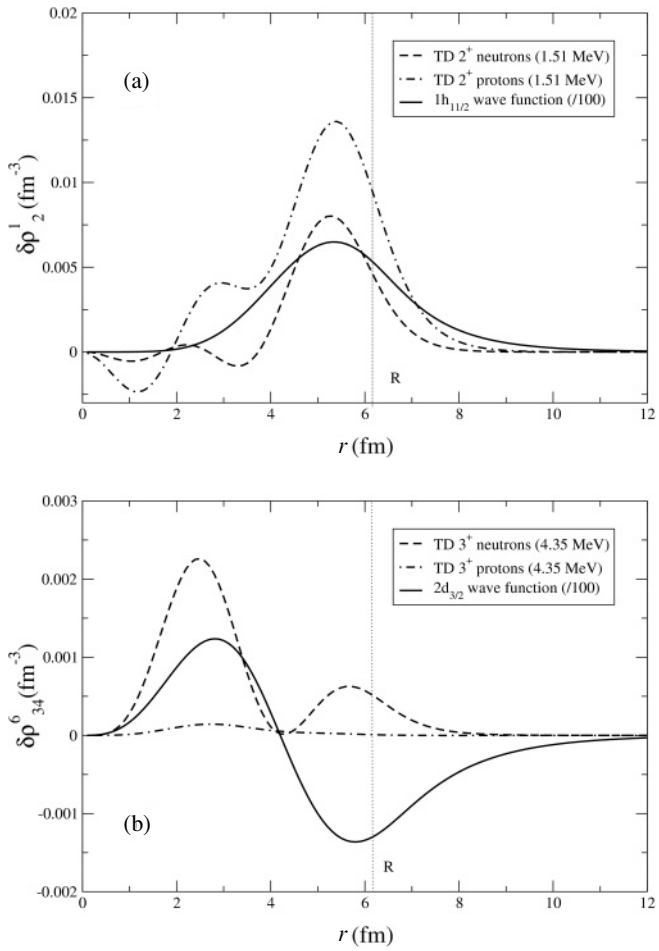

FIG. 5. (a) The dashed and dot-dashed curves are, respectively, the neutron and proton transition densities associated with the 2^+ phonon with energy 1.5 MeV; the solid curve is the wave function of the $1h_{11/2}$ state (in arbitrary units). (b) Same as (a) but for the 3^+ phonon with energy 4.35 MeV and the $2d_{3/2}$ state.

vertices shown in Fig. 2(b). The induced pairing matrix elements associated with natural parity modes have a clear surface character (cf. Ref. [25]). In particular, this is the case for the most attractive pairing matrix element which is associated with the $1h^2_{11/2}(0)$ ($\epsilon_{1h_{11/2}} = -8.027$ MeV, $\epsilon_F = -8.50$ MeV) configuration (cf. Fig. 3). Because of its large centrifugal barrier, the wave function of this single-particle state is mainly concentrated at the nuclear surface. The main contribution to the corresponding induced pairing matrix element arises from the exchange of a 2^+ phonon (of energy 1.5 MeV) between the two nucleons moving in time-reversal states in the $h_{11/2}$ orbital. The associated proton and neutron transition densities depicted in Fig. 5(a) testify to the fact that this phonon has the character of a surface vibration. Concerning the most repulsive matrix elements, we have found that the corresponding unnatural parity phonons are volume modes. In particular, one of the largest (positive) matrix elements is associated with the $2d^2_{3/2}(0)$ configuration ($\epsilon_{2d_{3/2}} = -8.52$ MeV). Because of the low angular momentum, one finds that a consistent fraction of the corresponding wave function is concentrated in the interior of the nucleus. This state can thus couple efficiently with phonons of volume character. In fact, the major contribution to the corresponding matrix element is due to the exchange of the 3^+ vibration (with energy at 4.35 MeV), which is a mode with a large volume component as demonstrated by the corresponding proton and neutron transition densities shown in Fig. 5(b). One can conclude that states lying close to the Fermi energy with high j and thus localized at the surface mainly feel the (attractive) coupling arising from the exchange of surface vibrations. The situation is expected to be quite different in the case of infinite neutron matter. In fact, in going from the finite to the infinite system, the collectivity of the natural parity modes, mostly surface modes, will be strongly reduced, while not much is expected to happen to the volume modes.

Furthermore, in going from nuclear to neutron matter, many attractive contributions vanish. In fact, if we turn off the neutron-proton particle-hole interaction contributing to the basic vertices displayed in Fig. 1, a strongly net repulsive induced interaction is obtained (cf. Fig. 6), a situation which much resembles the neutron star case. This result can be understood by realizing that, quite generally, the dominant contribution to the spin-independent (and therefore attractive) induced matrix elements arises precisely from the neutron-proton part of the particle-hole interaction, which is proportional to $(F_0 - F'_0)\delta\rho_p$ [cf. Eq. (5) and Fig. 1]. The remaining part of the spin-independent interaction depends on the function $F_0 + F'_0$ (corresponding to the particle-phonon coupling mediated by $\delta\rho^i_{J^\pi n}$), which is rather weak, and for the SkM* interaction adopted here, even displays a node at the nuclear surface. The induced interaction is then dominated by the (repulsive) spin-dependent matrix elements proportional to $G_0 + G'_0$ (corresponding to the neutron-neutron particle-phonon coupling mediated by $\delta\rho^i_{J^\pi Ln}$ [cf. Eq. (7)], which are large and without nodes.

We have chosen to adopt a force such as SkM*, which has been tested by various groups over the years. We are well aware that our results could change if we adopted a different

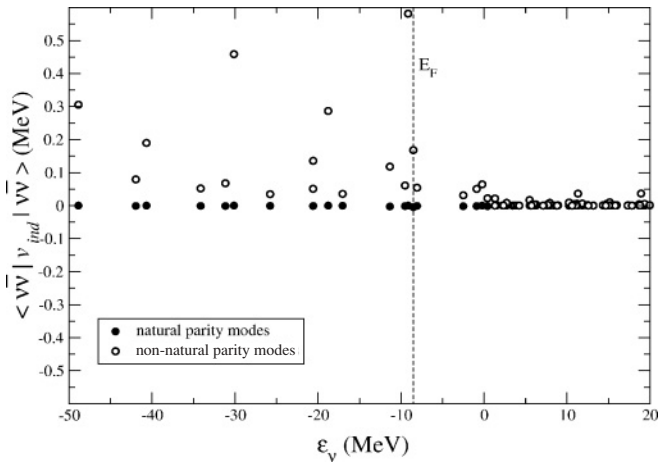

FIG. 6. The diagonal matrix elements associated with the exchange of phonons of natural (filled circles) and non-natural (open circles) parity as a function of the energy of the single-particle states. The proton part of the phonon wave functions was not included in the calculation.

effective force. It is difficult to fix the spin-dependent part of the interaction, partly because of the scarcity of experimental constraints. In fact, different approaches have been adopted in the literature. In particular, 1^+ modes have been calculated in the framework of the extended theory of finite Fermi systems, using constant values of the Landau parameters G_0 and G_0' [26,27]. Recently, a new specific parametrization of the Skyrme type has been introduced, SkO′, which takes into account the effects of time-odd spin-isospin couplings, and adopted for the description of the spin-flip transitions such as the Gamow-Teller resonance [28,29]. This force has not yet been extensively checked in the non-charge-exchange channel, and the large and negative value of G_0 associated with this force can lead to too strongly collective or unstable solutions in the spin isoscalar channel. Even using different effective forces, however, the main qualitative aspects of our results are likely to remain true, as they are based on quite general features—such as the dominance of neutron-proton over neutron-neutron interaction, and the stronger collectivity and surface localization of the low-lying density modes with respect to the spin modes.

We conclude that the exchange of low-lying surface vibrations (in which neutrons and protons participate on equal footing) between pairs of nucleons moving in time-reversal states close to the Fermi energy leads to a sizeable attractive pairing interaction which accounts for about 70% of the pairing gap. The inclusion of spin (volume) modes reduces this contribution by 30% in the case of finite nuclei, bringing the induced pairing contribution to the pairing gap to a value of the order of ≈50%, the other half coming from the bare nucleon-nucleon force. The attractive character of the neutron-neutron pairing effective interaction in finite nuclei is found to be associated with the efficient coupling of the single-particle states lying close to the Fermi energy to collective surface vibrations, as well as with the contribution of the proton part of the particle-vibration coupling. Without these two elements, spin modes dominate, and the effective interaction becomes repulsive, as in neutron matter.

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[23] Note that the induced matrix elements in this figure can be directly compared with the empirical constant $G = 25/A$ (≈0.2 MeV, $A = 120$) used to describe pairing correlations in nuclei within the framework of the BCS theory and a (pairing) force of constant matrix elements.
[24] The neutron pairing gap obtained by solving the BCS equations making use of matrix elements $v_{v,v'}$ sum of the matrix element of the induced interaction and of the bare neutron-neutron potential, turns out to be substantially larger than that calculated with the bare potential alone. This is in contrast with the neutron matter case and in keeping with previous results for finite nuclei [8,9], in which, however, the spin-dependent particle-hole interaction had not been considered.
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