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Improved (e, e') response functions at intermediate momentum transfers: The ^{3}He case

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A possibility of extending the applicability range of nonrelativistic calculations of electronuclear response functions in the quasielasic peak region is studied. We show that adopting a particular model for determining the kinematic inputs of the nonrelativistic calculations can extend this range considerably, almost eliminating the reference frame dependence of the results. We also show that there exists one reference frame where essentially the same result can be obtained with no need of adopting the particular kinematic model. The calculation is carried out with the Argonne V18 potential and the Urbana IX three-nucleon interaction. A comparison of these improved calculations with experimental data shows a very good agreement for the quasielastic peak positions at $q = 500$, 600, 700 MeV/c and for the peak heights at the two lower q values, while for the peak height at $q = 700$ MeV/c, one finds differences of about 20%.

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In Ref. [1], we studied the longitudinal response functions for electron scattering from three-nucleon systems in the momentum transfer range between 250 and 500 MeV/c. A nonrelativistic (n.r.) formulation of the nuclear three-body problem was adopted, and the full dynamics was taken into account in both the initial and final states. A related study was recently presented in Ref. [2]. To check the validity of our n.r. calculation, we checked in [1], among other issues, the reference frame dependence and found that it is not negligible for momentum transfers $q \geqslant 400$ MeV/c. A frame dependence of a similar type had already been observed in deuteron electrodisintegration [3–5]. In Ref. [1], the hadronic current was evaluated in the Breit frame and the results were compared with experimental data. In the present work, we reconsider the frame dependence and present results up to $q = 700$ MeV/c.

It is clear that as q increases, the results of purely n.r. calculations must become increasingly questionable. One manifestation of the importance of relativity is the frame dependence that occurs in such n.r. calculations at high q. Of course, use of any frame in a genuine relativistic calculation must lead to the same laboratory (LAB) frame result. We will show that certain frames in a n.r. calculation may tend to minimize the error because of the lack of a proper relativistic calculation. We also suggest a procedure to reduce the frame dependence in the quasielastic peak region.

In the one-photon exchange approximation, the inclusive electron scattering cross section in the LAB frame is given by

$$\frac{d^2\sigma}{d\Omega\, d\omega} = \sigma_M\left[\frac{(q^2-\omega^2)^2}{q^4}R_L(q,\omega) + \left(\frac{(q^2-\omega^2)}{2q^2} + \tan^2\frac{\theta}{2}\right)R_T(q,\omega)\right], \quad (1)$$

where R_L and R_T are the LAB longitudinal and transverse response functions, respectively. The LAB frame electron variables are denoted by ω (energy transfer), q (momentum transfer), and θ (scattering angle).

In addition to R_L, one may define related responses $R_L^{\rm fr}$ expressed in terms of quantities pertaining to reference frames obtained via boosting the LAB frame along $\mathbf{q}$. In general, nuclear states are products of internal and center-of-mass momentum substates. In the n.r. approximation after integrating over the center-of-mass momentum, one has

$$R_L^{\rm fr} = \sum\!\!\!\!\!\!\int df \left|\langle\psi_i|\sum_j \rho_j(\mathbf{q}^{\rm fr},\omega^{\rm fr})|\psi_f\rangle\right|^2 \delta\left(E_f^{\rm fr} - E_i^{\rm fr} - \omega^{\rm fr}\right). \quad (2)$$

Here $q^{\rm fr}$ and $\omega^{\rm fr}$ are the momentum and energy transfer in a new reference frame, the internal substates are indicated with ψ_i and ψ_f, and $\rho_j(\mathbf{q}^{\rm fr},\omega^{\rm fr})$ are the internal single-nucleon charge operators as defined in Ref. [1] (the energy dependence is due to the inclusion of the nucleon form factors).

The summation-integration symbol denotes the usual summation or integration over final state variables in addition to averaging over the initial state magnetic quantum numbers. In the relativistic case, we have the same formula, but with the substates ψ_i and ψ_f depending, respectively, on the total momenta $\mathbf{P}_i^{\rm fr}$ and $\mathbf{P}_f^{\rm fr} = \mathbf{P}_i^{\rm fr} + \mathbf{q}^{\rm fr}$ of the initial and final states in a given reference frame. Thus ψ_i and ψ_f are frame dependent in the relativistic case. We disregard this frame dependence of the states in our calculations, and we do not consider the boost corrections of the states.

Energy conservation is explicit in the argument of the δ function where $E_f^{\rm fr}$ and $E_i^{\rm fr}$ denote the total initial and final energies and can be expressed with relativistic or n.r. kinematics (both cases will be considered in the following). In the n.r. case, the center of mass and internal energies can be

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separated such that

$$\delta\left(E_f^{\rm fr} - E_i^{\rm fr} - \omega^{\rm fr}\right) \approx \delta\big[e_f^{\rm fr} + \left(P_f^{\rm fr}\right)^2/(2M_T) - e_i^{\rm fr} - \left(P_i^{\rm fr}\right)^2/(2M_T) - \omega^{\rm fr}\big] \quad (3)$$

$$\equiv \delta\left[e_f^{\rm fr} - e_f^{\rm nr}(q^{\rm fr}, \omega^{\rm fr})\right], \quad (4)$$

where $e_f^{\rm fr}$, $e_i^{\rm fr}$ are intrinsic energies of the final and initial states.

The response R_L can be expressed in terms of $R_L^{\rm fr}$ with the help of the relationship

$$R_L(q, \omega) = \frac{q^2}{(q^{\rm fr})^2} \frac{E_i^{\rm fr}}{M_T} R_L^{\rm fr}(q_{\rm fr}, \omega_{\rm fr}) \,. \quad (5)$$

The origin of the factor $q^2/(q^{\rm fr})^2$ is shown in Ref. [3], Eqs. (2.13), (2.14) (see also, e.g., Refs. [4,6,7]). The factor $E_i^{\rm fr}/M_T$ arises because we adopt the usual normalization of the target state to unity instead of its covariant normalization. (In [1] this factor was not included).

We will use relation (5) to get the LAB response from calculations referring to frames different from the LAB frame and study the frame dependence of n.r. calculations.

In addition to the LAB frame we consider three other frames. One is the so-called antilab (AL) frame, where the total momentum in the final state is zero. Thus the target nucleus has a momentum $-\mathbf{q}_{\rm AL}$. If one neglects the internal motion of the nucleons inside the nucleus, then one could say that the nucleon momenta in the inital state are about $-\mathbf{q}_{\rm AL}/A$ in this reference frame. Absorption of a virtual photon of momentum $\mathbf{q}_{\rm AL}$ by a ground state nucleon (the quasielastic process) would result in a final state, where one nucleon has a momentum of about $\mathbf{q}_{\rm AL}(A-1)/A$ and $A-1$ slower nucleons each have a momentum of about $-\mathbf{q}_{\rm AL}/A$.

If one chooses to minimize the sum of the center-of-mass kinetic energies of initial and final states, one is led to the Breit (B) frame. In the Breit frame, the target nucleus moves with $-\mathbf{q}_B/2$ and the nucleon momenta are thus about $-\mathbf{q}_B/(2A)$. According to the above picture, the final state in the vicinity of the quasielastic peak corresponds roughly to one nucleon with momentum $\mathbf{q}_B(2A-1)/(2A)$ and $A-1$ nucleons with momenta of about $-\mathbf{q}_B/(2A)$. At fixed q and ω values, the antilab and Breit responses tend to the LAB response with increasing A.

As a fourth reference frame, we introduce what we call the active nucleon Breit frame (ANB). In this frame, the target nucleus consisting of A nucleons has a momentum of $-A\mathbf{q}_{\rm ANB}/2$ so that the nucleons have momenta about $-\mathbf{q}_{\rm ANB}/2$ in the initial state. The final state in the vicinity of the q.e. peak would correspond to an active nucleon with momentum of about $\mathbf{q}_{\rm ANB}/2$, while the other nucleons continue moving with the momenta about $-\mathbf{q}_{\rm ANB}/2$. Thus within these approximations, the maximum nucleon momentum is limited by $q_{\rm ANB}/2 \simeq q/2$ in the ANB frame, whereas in other reference frames nucleons with momenta up to q are present. The momentum of the active nucleon is largest in the LAB frame, so one may expect that this reference frame is the least suitable within a n.r. approach. In particular, the relativistic correction related to the kinetic energy is 4 times larger in the LAB frame than in the ANB frame. In the following, we will calculate R_L^B, $R_L^{\rm AL}$, and $R_L^{\rm ANB}$ and then use (5) to give the predicted R_L from each of these.

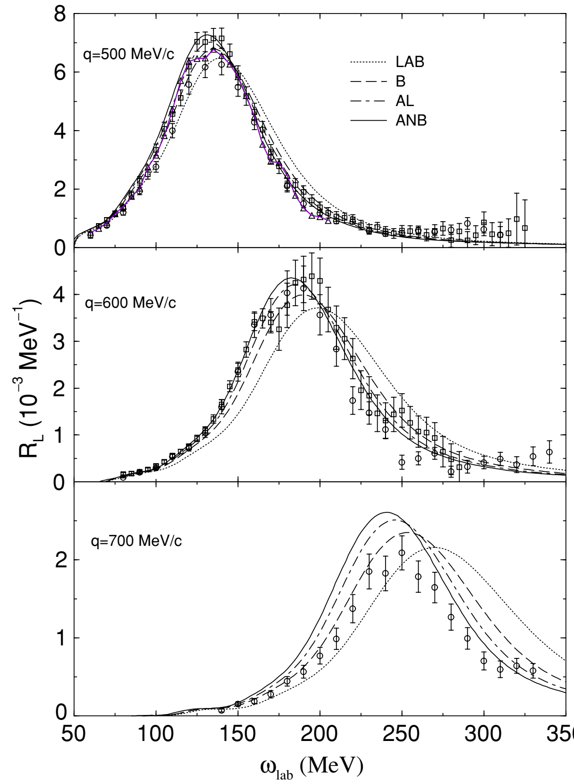

FIG. 1. (Color online) Frame dependence of the ^{3}He longitudinal response function at three different momentum transfers q (notation of curves in upper panel); experimental data are from Refs. [12] (squares), [13] (triangles), [14] (circles).

These indirectly calculated R_L are then compared with R_L as computed directly in the LAB frame at $q = 500$, 600, and 700 MeV/c. By comparing our results to experimental data, it should become apparent if the ANB frame, for example, is superior to the LAB frame.

The present calculation proceeds in the manner described in [1]. There we found only a weak potential model dependence, so in the present calculation we choose the Argonne V18 (AV18) *NN* [8] plus Urbana IX (UrbIX) *NNN* [9] potentials. As in [1], the n.r. charge operator is supplemented with the first-order relativistic corrections (Darwin-Foldy and spin-orbit terms). However, while in [1] we considered q values up to 500 MeV/c, in the present work we calculate the responses at $q = 500$, 600, and 700 MeV/c. The inclusion of these higher q values requires a larger set of basis states for convergence. For example, whereas the total angular momentum of the final states was limited to $J = 21/2$ in [1], here we include states up to $J = 31/2$. As in [1], we use the simple dipole fit for the proton electric form factor, but consider also the proton form factor fit from [10]. For the neutron electric form factor, we take the fit from [11].

Figure 1 shows the R_L results for the various frames together with experimental data at $q_{\rm LAB} = 500$, 600, and 700 MeV/c. It is readily seen that one obtains rather frame-dependent results. One finds the following differences in peak positions and peak heights between the two extreme cases (ANB and LAB frame results): 6 MeV and 13%

(500 MeV/c), 11 MeV and 19% (600 MeV/c), 20 MeV and 24% (700 MeV/c). As anticipated of all four frames, the LAB frame calculation leads to the worst result in comparison with experimental data. Let us recall that these LAB results represent just the conventional n.r. calculation. On the other hand, the ANB frame leads to a good description of the data at $q = 500$ and 600 MeV/c. This may be related to the fact that nucleons with only moderate momenta are present in this reference frame. Description of the data with the ANB frame is even better if a contemporary proton form factor in place of the dipole form factor is used. This will be demonstrated below. The above considerations demonstrate the frame dependence inherent in a n.r. calculation of the longitudinal response at high q. Clearly a proper relativistic calculation would remove this frame dependence, but one can still ask whether there is a way to modify the n.r. calculation such that the degree of frame dependence would be reduced.

A clue is evident in the work of Arenhövel and collaborators (see, e.g. [15]) in deuteron electrodisintegration, where the relative momentum of outgoing nucleons is determined in a relativistically correct way, and the energy that is used as input to the n.r. calculation is obtained from that momentum by the usual n.r. relation. In general, in a two-body problem, one may either determine the kinetic energy in a relativistically correct way and solve the n.r. Schrödinger equation with it or determine the relative momentum p_{12} in a relativistically correct way and solve the Schrödinger equation for the "fake" kinetic energy $E_{12} = p_{12}^2/2\mu_{12}$, where μ_{12} is the reduced mass of the two particles. The reason why the latter procedure is chosen in the case of deuteron electrodisintegration is because the construction of *NN* potential models proceeds that way.

If one is mainly interested in the region of the quasielastic peak, then one can adopt an analogous procedure based on a two-body model for the quasielastic process. That is, the final state is assumed to consist of a knocked-out nucleon and an $(A - 1)$ particle residual system remaining in its lower energy state. We stress here that the two-body model is adopted only for determining the kinematic input of a calculation where the full three-body dynamics is properly taken into account.

The momenta of the knocked-out nucleon and that of the residual nucleus are denoted by $\mathbf{p}_N^{\rm fr}$ and $\mathbf{p}_X^{\rm fr}$, respectively. Then the relative and center-of-mass momenta will be given by $\mathbf{p}^{\rm fr} = \mu(\mathbf{p}_N^{\rm fr}/M - \mathbf{p}_X^{\rm fr}/M_X)$ and $\mathbf{P}_f^{\rm fr} = \mathbf{p}_N^{\rm fr} + \mathbf{p}_X^{\rm fr}$, where M_X is the mass of the residual nucleus and μ is the $N - X$ reduced mass. (Note that $\mathbf{p}^{\rm fr}$ depends on the reference frame in the relativistic case). The value of $\mathbf{p}^{\rm fr}$ can be obtained from the following relativistically correct kinematical relation

$$\omega^{\rm fr} = E_f^{\rm fr} - E_i^{\rm fr}, \qquad (6)$$

where

$$E_f^{\rm fr} = \sqrt{M^2 + \left[\mathbf{p}^{\rm fr} + (\mu/M_X)\mathbf{P}_f^{\rm fr}\right]^2} + \sqrt{M_X^2 + \left[\mathbf{p}^{\rm fr} - (\mu/M)\mathbf{P}_f^{\rm fr}\right]^2}. \qquad (7)$$

Then, in accordance with the preceding discussion on the two-body system, the final state relative energy to use in the n.r. calculation is taken to be

$$e_f^{\rm fr} = (p^{\rm fr})^2/(2\mu)\,. \qquad (8)$$

Here one has to notice that in order to solve Eq. (6) for $p^{\rm fr}$ one needs to know its direction. For the class of reference frames we consider the momentum $\mathbf{P}_f^{\rm fr}$ is directed along $\mathbf{q}$. Again, since we are mainly interested in the region of the quasielastic peak we can safely assume that $\mathbf{p}^{\rm fr}$ is also directed along $\mathbf{q}$. [Indeed, e.g., $\mathbf{p}^{\rm LAB} \simeq (\mu/M)\mathbf{q}$.]

Proceeding in the way described above is formally equivalent to replacing $(E_f^{\rm fr} - E_i^{\rm fr})$ in the δ function of Eq. (2) by a function $F(e_f^{\rm fr}) = [E_f^{\rm fr}(e_f^{\rm fr}) - E_i^{\rm fr}]$. Therefore,

$$\delta\left(E_f^{\rm fr} - E_i^{\rm fr} - \omega^{\rm fr}\right) = \left(\frac{\partial F^{\rm fr}}{\partial e_f^{\rm fr}}\right)^{-1} \delta\left[e_f^{\rm fr} - e_f^{rel}(q^{\rm fr}, \omega^{\rm fr})\right], \qquad (9)$$

with

$$\left(\frac{\partial F^{\rm fr}}{\partial e_f^{\rm fr}}\right)^{-1} = \frac{p^{\rm fr}}{\mu}\left(\frac{\partial E_f}{\partial p^{\rm fr}}\right)^{-1}. \qquad (10)$$

This leads to

$$R_L^{\rm fr}(q^{\rm fr}, \omega^{\rm fr}) = \frac{p}{\mu}\left(\frac{\partial E_f}{\partial p}\right)^{-1} \times \sum\!\!\!\!\!\!\int df \left|\langle\psi_i|\sum_j \rho_j(q^{\rm fr}, \omega^{\rm fr})|\psi_f\rangle\right|^2 \times \delta\left[e_f^{\rm fr} - e_f^{\rm rel}(q^{\rm fr}, \omega^{\rm fr})\right]. \qquad (11)$$

To calculate this quantity, a new calculation is not required. We have obtained it via interpolation with respect to the momentum transfer of the n.r. response.

This procedure should reduce the frame dependence of $R_L(q, \omega)$ considerably. This is evident in the free case, i.e., when there is no interaction between the fast nucleon and the residual system. In this case, the n.r. and relativistic final states would contain the relative motion plane wave with the same momentum $\mathbf{p}$, resulting in no frame dependence of the matrix elements due to a difference in relative motion.

In Fig. 2, we show the various R_L results in comparison with experimental data, and in fact we find an enormous reduction of the frame dependence. For the peak positions, we even have an essentially frame-independent result and also the differences of the peak heights are much reduced, namely to maximally 4, 6, and 9% at $q = 500$, 600, and 700 MeV/c, respectively. It is evident that there is good agreement between theory and experiment for the position of the quasielastic peak at all three-momentum transfers. Concerning the peak heights, one finds a relatively good agreement at $q = 500$ and 600 MeV/c, while at 700 MeV/c the theoretical peak height overestimates the experimental one between about 20% and 30%.

It is interesting to check which of the frame-dependent results of Fig. 1 reproduces best the frame-independent peak positions of Fig. 2. It turns out that this is the ANB frame. Also the peak heights of the ANB curves in Figs. 1 and 2 are not much different: 4% ($q = 500$ MeV/c), 5% ($q = 600$), 6% ($q = 700$). This is not surprising since the ANB frame is the only frame where the nucleon has equal initial and final energies (in fact, the initial nucleon momentum is about $-\mathbf{q}_{\rm ANB}/2$ and its final momentum is $\mathbf{q}_{\rm ANB}/2$). Thus the quasielastic peak

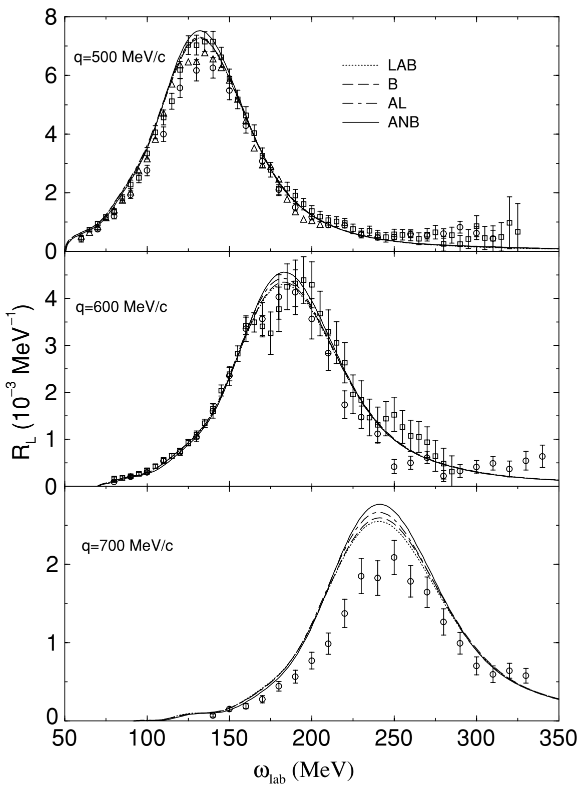

FIG. 2. As in Fig. 1, but considering two-body relativistic kinematics for the final state energy as discussed in the text.

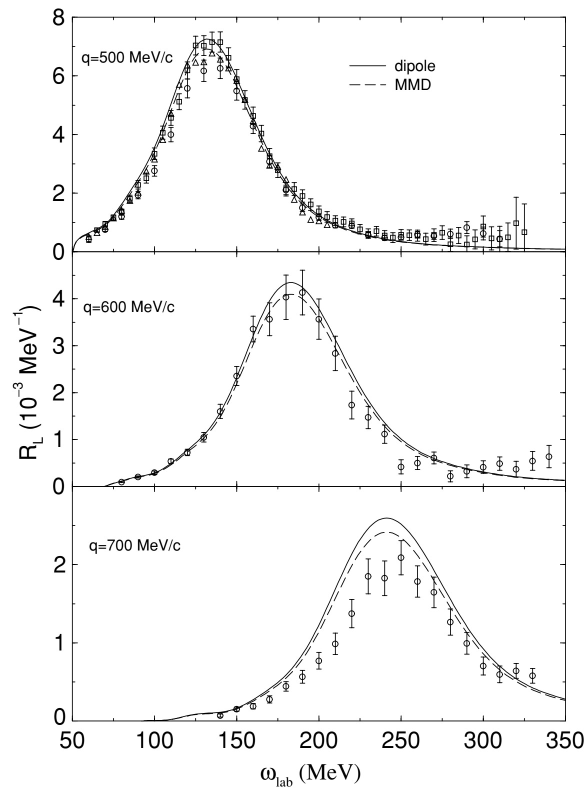

FIG. 4. R_L of B frame calculations with consideration of two-body relativistic kinematics using different proton electric form factors: dipole fit (full curves), fit from [10] (long dashed curves). Notation of experimental data as in Fig. 1.

occurs at $\omega_{\rm ANB} = 0$, independent of whether relativistic or n.r. kinematics are employed. Note that in the $A = 2$ case, the ANB frame coincides with the antilab frame, which is often chosen for the deuteron electrodisintegration.

A comparison of Figs. 1 and 2 shows that the n.r. ANB frame calculations agree with the relativistic two-body kinematics calculations not only at the peak but also in the tails. This is illustrated in another way in Fig. 3 where the n.r. ANB frame

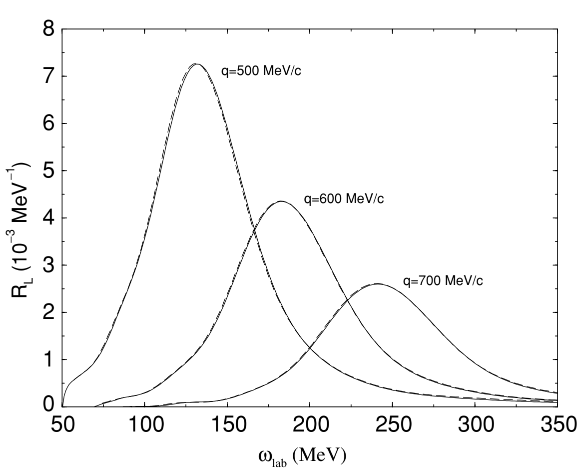

FIG. 3. R_L of ANB frame calculations without consideration of two-body relativistic kinematics (long dashed curves) in comparison to R_L of B frame calculations with consideration of two-body kinematics (full curves).

results are shown together with the relativistic kinematics Breit frame results. The choice of the Breit frame was motivated by the deuteron electrodisintegration work of [5], where it was shown that boost corrections are minimal for this frame.

Apart from theoretical uncertainties of the quasielastic R_L response due to frame dependence, probably the greatest remaining theoretical uncertainty is due to the proton electric form factor. As an illustration, we show in Fig. 4 the Breit frame results with relativistic two-body kinematics using the two above-mentioned different proton electric form factors (dipole fit, and fit from [10]). In comparison to the dipole fit the fit of [10] reduces the peak height by about 4, 6, and 7% at $q = 500$, 600, and 700 MeV/c leading to an improved agreement with experiment at the lower two q values and reducing the discrepancy at $q = 700$ MeV/c to about 15%. On the other hand, the rather large experimental uncertainties preclude making definitive conclusions.

We summarize our results as follows. We have shown that the usual n.r. calculation of the longitudinal inclusive (e, e') response leads to rather frame-dependent results at intermediate momentum transfers of $q = 500-700$ MeV/c. The frame dependence is drastically reduced if one assumes a two-body breakup model with relativistic kinematics to determine the input to the n.r. dynamics calculation. One obtains a nearly frame-independent peak position and much smaller deviations for the peak heights. Within n.r. kinematics, of the considered reference frames, the ANB frame turns out to be the best, leading to results almost identical to those obtained with the suggested two-body breakup model. In comparison with experimental data, we find good agreement for the positions of the quasielastic peak and also good agreement of the peak heights at $q = 500$ and 600 MeV/c, while at $q = 700$ MeV there is a discrepancy between about 15% and 25%.

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