PHYSICAL REVIEW C **71**, 068203 (2005)

Relativistic calculation of the width of the $\Theta^+(1540)$ pentaquark

Hu Li and C. M. Shakin*
Department of Physics, Brooklyn College of the City University of New York, Brooklyn, New York 11210, USA

Xiangdong Li
Department of Computer System Technology, New York City College of Technology of the City University of New York, Brooklyn, New York 11201, USA

(Received 15 March 2005; published 24 June 2005)

We calculate the width of the $\Theta^+(1540)$ pentaquark in a relativistic model in which the pentaquark is considered to be composed of a scalar diquark and a spin 1/2 triquark. We consider both positive and negative parity for the pentaquark. There is a single parameter in our model which we vary and which describes the size of the pentaquark. If the pentaquark size is somewhat smaller than that of the nucleon, we find quite small widths for the pentaquark of about 1 MeV or less. Our model of confinement plays an important role in our analysis and makes it possible to use Feynman diagrams to describe the decay of the pentaquark.

DOI: 10.1103/PhysRevC.71.068203 PACS number(s): 12.39.Ki, 13.30.Eg, 12.38.Lg

There has been a great deal of interest in the recent observation of a quite narrow resonance which decays to a nucleon and a kaon [1–9]. These states have been interpreted as pentaquarks which are likely to have a $udud\bar{s}$ structure [10]. (The extremely narrow width of the state, the $\Theta^+(1540)$, has led some to question whether the state exists in nature.) In the present work, we will describe the calculation of the width of the $\Theta^+(1540)$ in a relativistic quark model which includes a model of confinement that we introduced previously in our study of meson and nucleon structure.

Several studies have attempted to calculate pentaquark widths. The work of Ref. [11] was based upon a diquark-triquark model (as is our present work) and used a QCD sum rule approach with operator product expansion and direct instanton contributions. They find stable triquark states that support the diquark-triquark model of the pentaquark. Other studies using the QCD sum rule approach are to be found in Refs. [12] and [13]. In Ref. [13] the authors find a light $ud\bar{s}$ triquark cluster with mass of about 750 MeV and a light diquark configuration containing a u and d quark with mass of 440 MeV. They claim that they can understand the small width of the pentaquark in their model.

A number of reviews have appeared. Reference [14] reviews the experimental evidence for the pentaquark. Theoretical and experimental developments are reviewed in Refs. [15–18]. A number of theoretical papers have also appeared [10,19–24]. We are particularly interested in the work of Ref. [24] in which a diquark-triquark model is introduced to describe the pentaquark. We will make use of a variant of that model in the present work, since that model lends itself to the analysis of the pentaquark decay using Feynman diagrams. The diagram we consider is presented in Fig. 1. There the pentaquark is represented by the heavy line with momentum P. The pentaquark is composed of a diquark of momentum $-k + P_N$ and a triquark of momentum $P + k - P_N$. The triquark emits a quark (u or d) which combines with the diquark to form a nucleon of momentum P_N. The final-state kaon of momentum $P - P_N$ is emitted along with the quark at the triquark vertex.

We have studied the structure of the nucleon in a quark-diquark model in Ref. [25]. In that work we considered both scalar and axial-vector diquarks; however, in this work, we will limit our considerations to a nucleon composed of a quark and a scalar diquark. The nucleon vertex is described in Ref. [25] for the case in which the diquark is placed on mass shell. (We remark that in Fig. 1 only the quark of momentum k and the triquark of momentum $P + k - P_N$ will be off mass shell in our analysis.) The mass of the scalar diquark was taken to be 400 MeV in Ref. [25]. We use that value here and also take the triquark to have a mass of 800 MeV. Therefore, it is clear that we need to introduce a model of confinement for the pentaquark which has a mass of 1540 MeV. Our (covariant) confinement model, which we have used extensively in other works [26–28], will be useful for this work.

In earlier work, we introduced a confining interaction that served to prevent the decay of mesons or nucleons into their constituents. Our covariant confinement model is described in a series of our papers [26–28]. In that model we solve a linear equation for a confining vertex function Γ. This function has the following property. Consider the decay $A \to B + C$, in which the hadrons A and C are on mass shell. If we include the confining vertex, we find the amplitude is zero when particle B goes on mass shell, so that the amplitude for A to decay into two on-mass-shell particles (B and C) is zero.

For the decay $A \to B + C$, we may introduce a wave function that may be expressed in terms of the momentum of the off-shell particle B. If B is a scalar, we have

$$\Psi_B(k) = \frac{1}{k^2 - m_B^2}\Gamma_B(k). \qquad (1)$$

Note that the ratio of $\Gamma_B(k)$ to $(k^2 - m_B^2)$ is an ordinary function which may often be well represented by a Gaussian function. [It is not necessary to include an $i\epsilon$ in the denominator of Eq. (1).]

*email: cshakin@brooklyn.cuny.edu

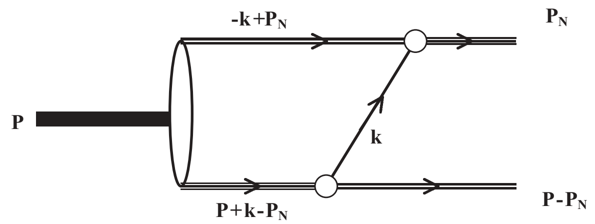

FIG. 1. Heavy line denotes the pentaquark; line of momentum $-k + P_N$ denotes an on-mass-shell diquark; line of momentum k represents the quark; and $P + k - P_N$ is the momentum of the triquark. In the final state, we have a nucleon of momentum P_N and a kaon of momentum $P - P_N$.

In the case of the nucleon, we may consider the decay into a quark and a diquark. In Ref. [25], we considered both scalar and axial-vector diquarks, but for simplicity we will limit ourselves to only the scalar diquark. The relevant wave function in this case was given in Eq. (3.3) of Ref. [25] as

$$\Psi_S(P,k,s,t) = \widetilde{\Psi}_{(1)}(P,k)\frac{2m_q\Lambda^{(+)}(\vec{k})}{\sqrt{2E(\vec{k})[E(\vec{k})+m_q]}}u_N(P,s)\chi_t, \tag{2}$$

which we will simplify for the present work to read

$$\Psi_N(P_N,k,s,t) = \widetilde{\Psi}_N(P_N,k)\Lambda^{(+)}(\vec{k})u_N(P_N,s)\chi_t. \tag{3}$$

The function $\widetilde{\Psi}_N(P,k)$ is represented in Fig. 5 of Ref. [25] by a dashed line. That function is well approximated by a Gaussian function, which we will record at a latter point in our discussion. The factor of $\Lambda^{(+)}(\vec{k}) = (\not{k}_{\text{on}} + m_q)/2m_q$ arises from an approximation made in Ref. [25]. Here $k_{\text{on}} = [E(\vec{k}), \vec{k}]$ with $E(\vec{k}) = [\vec{k}^{\,2} + m_q^2]^{1/2}$. In Ref. [25], the quark propagator was written as

$$-iS(k) = \frac{m_q}{E_q(k)}\left[\frac{\Lambda^{(+)}(\vec{k})}{k^0 - E_q(\vec{k})} - \frac{\Lambda^{(-)}(-\vec{k})}{k^0 + E_q(\vec{k})}\right]. \tag{4}$$

The second term was neglected in our formalism when we studied the nucleon. (Thus we limited our analysis to positive-energy quark spinors.) Since we wish to make use of the nucleon wave function determined in Ref. [25], we will continue to include the projection $\Lambda^{(+)}(\vec{k})$ in our formalism. Here k is the quark momentum.

As stated earlier, the diquark of momentum $-k + P_N$ in Fig. 1 will be placed on mass shell, so that $k^0 = E_N(\vec{P}_N) - E_D(\vec{P}_N - \vec{k})$, where $E_D(\vec{P}_N - \vec{k}) = [(\vec{P}_N - \vec{k})^2 + m_D^2]^{1/2}$ and $E_N(\vec{P}_N) = [\vec{P}_N^{\,2} + m_N^2]^{1/2}$. That approximation is achieved by writing

$$\frac{1}{(P_N - k)^2 - m_D^2 + i\epsilon} \to -2\pi i\delta^{(+)}\big[(P_N - k)^2 - m_D^2\big], \tag{5}$$

as described in detail in Ref. [29]. Note that

$$\delta^{(+)}\big[(P_N - k)^2 - m_D^2\big] = \frac{1}{2E_D(\vec{P}_N - \vec{k})} \times \delta\big[P_N^0 - k^0 - E_D(\vec{P}_N - \vec{k})\big]. \tag{6}$$

The on-mass-shell specification used here arises when performing an integral in the complex k^0 plane [29].

We consider the diagram shown in Fig. 1, where the heavy line of momentum P denotes the pentaquark. The line carrying momentum $-k + P_N$ is the on-mass-shell scalar diquark and the line with momentum $P + k - P_N$ is the triquark. The momentum k is that of an up or down quark, $P - P_N$ is the momentum of the nucleon. The pentaquark, nucleon, and kaon are on mass shell. In our analysis the diquark is also on mass shell, so that only the triquark and quark propagate off mass shell, as noted earlier. (As stated earlier, the on-shell characterization of the diquark arises when we complete the k^0 integral in the complex k^0 plane.)

We now make use of the formula [30]

$$d\Gamma = |\mathfrak{M}|^2\frac{d^3k_1}{(2\pi)^3}\frac{m_N}{E_N(\vec{k}_1)}\frac{d^3k_2}{(2\pi)^3}\frac{1}{2E_K(\vec{k}_2)} \times (2\pi)^4\delta^4(P - k_1 + k_2), \tag{7}$$

where $\vec{k}_1$ and $\vec{k}_2$ are the momenta of the outgoing particles. We may put $\vec{k}_1 = \vec{P}_N$ and $\vec{k}_2 = \vec{P} - \vec{P}_N = -\vec{P}_N$ for a pentaquark at rest. (It is convenient to take $\vec{P}_N$ along the z axis when calculating the width.)

In writing our expression for Γ we will represent the product of the quark propagator and nucleon vertex function by the nucleon wave function of Eq. (3). In a similar fashion, we will represent the product of the triquark propagator and the pentaquark-triquark vertex function by a triquark wave function. (In the vertex, the diquark is on mass shell.) We then have to specify the vertex function of the triquark which describes the decay into the quark of momentum k and the kaon. That scalar part of the vertex is usefully written as

$$\Gamma_T(k) = \Psi_T(k)\big(k^2 - m_q^2\big). \tag{8}$$

Thus, the wave functions $\Psi_N(k)$, $\Psi_T(k)$, and $\Psi_\Theta(P_N - k)$ will appear in our expression for Γ_Θ. Note that

$$\vec{P}_N^2 = \left(\frac{m_\Theta^2 - m_N^2 + m_K^2}{2m_\Theta}\right)^2 - m_K^2, \tag{9}$$

which yields $\vec{P}_N^2 = 0.0722$ GeV^2, or $|\vec{P}_N| = 0.269$ GeV.

We find that the width is given by

$$\begin{aligned}\Gamma_\Theta = {} & \frac{1}{2}\int\frac{d\vec{k}}{(2\pi)^3}\frac{1}{2E_D(\vec{k} - \vec{P}_N)}\int\frac{d\vec{k}'}{(2\pi)^3}\frac{1}{2E_D(\vec{k}' - \vec{P}_N)} \\ & \times \frac{1}{N_\Theta N_T N_N}\frac{1}{(2m_N)(2m_\Theta)}\Psi_N(\vec{k})\,\Psi_N(\vec{k}')\,\Psi_\Theta(\vec{P}_N - \vec{k}) \\ & \times \Psi_\Theta(\vec{P}_N - \vec{k}')\left(k^2 - m_q^2\right)\Psi_T(\vec{k})\left(k'^2 - m_q^2\right)\Psi_T(\vec{k}') \\ & \times \mathrm{Tr}[(\not{k} - \not{P}_N + \not{P} + m_T)(\not{k}_{\text{on}} + m_q)(\not{P}_N + m_N) \\ & \times \gamma^0(\not{k}'_{\text{on}} + m_q)(\not{k}' - \not{P}_N + \not{P} + m_T)\gamma^0(\not{P} + m_\Theta)]\rho,\end{aligned} \tag{10}$$

where ρ is the phase space factor. We find $\rho = \frac{1}{2\pi}\frac{m_N}{m_\Theta}|\vec{P}_N| = 0.0261$ GeV, since $|\vec{P}_N| \simeq 0.269$ GeV.

The factors $(1/N_\Theta)^{1/2}$, $(1/N_T)^{1/2}$, and $(1/N_N)^{1/2}$ serve to normalize the wave functions. We determine that $N_N = 0.316$ and, $N_T = 0.0982$ and calculate N_Θ for each choice of the

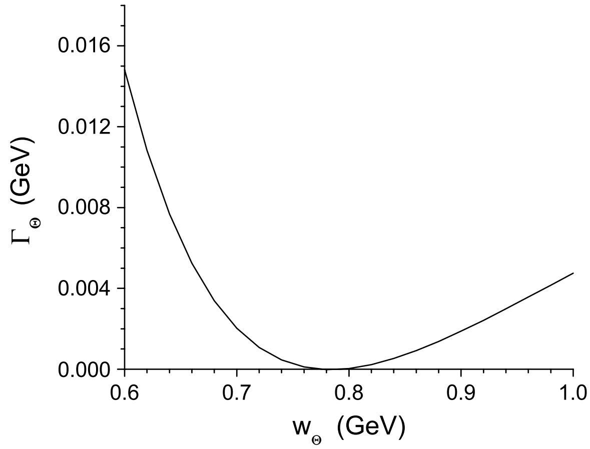

FIG. 2. Width of negative-parity pentaquark as a function of the parameter w_Θ. (The quite small values in the vicinity of the minimum have rather large uncertainties because of the limitation of the number of points used in our five-dimensional integral which determines the width.) The triquark had negative parity and mass of $m_T = 0.800$ GeV. The diquark mass was 0.400 GeV.

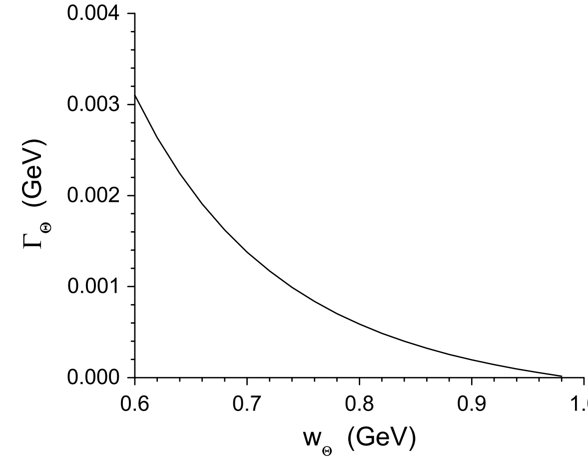

FIG. 3. Width of the positive-parity pentaquark as a function of the parameter, w_Θ, which governs the extent of the pentaquark wave function in momentum space. (In the case of the nucleon we found $w = 0.66$ GeV for the wave function calculated in Ref. [25].) In this calculation, the triquark had positive parity with mass $m_T = 0.800$ GeV. The diquark mass was 0.400 GeV.

pentaquark wave function. We write, with $k = |\vec{k}|$,

$$\Psi_N(\vec{k}) = \frac{1}{\sqrt{N_N}}\left(y_0 + \frac{A}{w\sqrt{\frac{\pi}{2}}}\, e^{-\frac{2(k-k_c)^2}{w^2}}\right) \tag{11}$$

and a similar equation for $\Psi_T(\vec{k})$. We have determined y_0, A, k_c, and w from a fit to the wave function given in Fig. 5 of Ref. [25]. We find $y_0 = -3.66$, $k_c = -0.013$ GeV, $w = 0.660$ GeV, and $A = 27.73$ GeV. For the pentaquark we write

$$\Psi_\Theta(\vec{k} - \vec{P}_N) = \frac{1}{\sqrt{N_\Theta}}\frac{A}{w_\Theta\sqrt{\frac{\pi}{2}}}\, e^{-\frac{2(\vec{k}-\vec{P}_N)^2}{w_\Theta^2}}, \tag{12}$$

where w_Θ is a variable in our analysis. (Note that N_Θ depends upon the choice of w_Θ.) Here $\vec{k} - \vec{P}_N$ is the relative momentum of the diquark and triquark when $\vec{P} = 0$.

In Fig. 2, we present the results of our calculation of Γ_Θ as a function of w_Θ. The minimum values are very small, and it is quite difficult to make an accurate calculation of widths significantly less than 1 MeV. [We calculated five-dimensional integrals with 40 points for each variable. Thus the number of points calculated was $(40)^5 \sim 10^8$.]

In the case of a positive-parity pentaquark, we assume that the pentaquark decays to a positive-parity diquark and a positive-parity triquark. (We note that the triquark and diquark have zero relative angular momenta.) In this case, we need to insert factors of $i\gamma_5$ at the triquark-kaon vertex where the quark of momentum k is emitted. In addition, the calculation of the normalization factor N_T is modified. In Eq. (10), the trace becomes

$$\begin{aligned}\text{Trace} = \text{Tr}[&(\not{k} - \not{P}_N + \not{P} + m_T)(-\not{k}_{\text{on}} + m_q)(-\not{P}_N + m_N)\\ &\times(-\widetilde{\not{k}}'_{\text{on}} + m_q)(\widetilde{\not{k}}' - \widetilde{\not{P}}_N + \widetilde{\not{P}} + m_T)(\not{P} + m_\Theta)],\end{aligned} \tag{13}$$

where $\widetilde{\not{k}} = \gamma^0\not{k}\gamma^0$, etc.

In this case, we find $N_N = 0.316$ and $N_T = 0.0201$, where only the second value has changed relative to the values given above. Figure 3 shows the values obtained for the width of the positive-parity pentaquark.

Some researchers question the existence of the pentaquark because of its very small width [17]. We have shown in the present work that very small widths are obtained in our model if the pentaquark wave function parameter w_Θ is somewhat larger than that found for the nucleon ($w_N = 0.66$ GeV) in Ref. [25].

There have been recent lattice QCD studies of the pentaquark [31–34]. A pentaquark was not found in the work described in Refs. [31,32]; however, pentaquarks were found in Refs. [33,34]. In Ref. [33], a positive-parity pentaquark was found with a mass of $m_\Theta = 1562(121)$ MeV. In Ref. [34], the negative-parity pentaquark had the smaller mass.

It is clear that further experimental studies are needed to determine whether various pentaquarks are present in nature. Further theoretical studies which improve upon the work described here are desirable.

[1] T. Nakano *et al.* (LEPS Collaboration), Phys. Rev. Lett. **91**, 012002 (2003).
[2] V. V. Barmin *et al.* (DIANA Collaboration), Phys. At. Nucl. **66**, 1715 (2003).
[3] C. Alt *et al.* (NA49 Collaboration), Phys. Rev. Lett. **92**, 042003 (2004).
[4] A. Aktas *et al.* (H1 Collaboration), Phys. Lett. **B588**, 17 (2004).

[5] V. Kubarovsky *et al.* (CLAS Collaboration), Phys. Rev. Lett. **92**, 032001 (2004).
[6] S. Stepanyan *et al.* (CLAS Collaboration), Phys. Rev. Lett. **91**, 252001 (2003).
[7] J. Barth *et al.* (SAPHIR Collaboration), Phys. Lett. **B572**, 127 (2003).
[8] M. Abdel-Bary *et al.* (COSY-TOF Collaboration), Phys. Lett. **B595**, 127 (2004).
[9] S. Chekanov *et al.* (ZEUS Collaboration), Phys. Lett. **B591**, 7 (2004).
[10] R. Jaffe and F. Wilczek, Phys. Rev. Lett. **91**, 232003 (2003); Eur. Phys. J. C. **33**, S38 (2004).
[11] H.-J. Lee, N. I. Kochelev, and V. Vento, Phys. Lett. **B610**, 50 (2005).
[12] Z.-G. Wang, W.-M. Yang, and S.-L. Wan, hep-ph/0501015; hep-ph/0504151.
[13] N. I. Kochelev, H.-J. Lee, and V. Vento, Phys. Lett. **B594**, 87 (2004).
[14] D. S. Carman, Eur. Phys. J. A **24**S1, 15 (2005).
[15] S.-L. Zhu, submitted to Int. J. Mod. Phys. A., hep-ph/0410002.
[16] Fl. Stancu, Int. J. Mod. Phys. A **20**, 209 (2005).
[17] F. E. Close, AIP Conf. Proc. **711**, 919 (2004).
[18] T. Nakano and K. Hicks, Mod. Phys. Lett. **19**, 645 (2004).
[19] E. Shuryak and I. Zahed, Phys. Lett. **B589**, 21 (2004).
[20] C. Semay, F. Brau, and B. Silvestre-Brac, Phys. Rev. Lett. **94**, 062001 (2005); C. E. Carlson, C. D. Carone, H. J. Kwee, and V. Nazaryan, Phys. Rev. D. **70**, 037501 (2004).
[21] I. M. Narodetskii, Yu. A. Simonov, M. A. Trusov, and A. I. Veselov, Phys. Lett. **B578**, 318 (2004); I. M. Narodetskii, C. Semay, B. Silvestre-Brac, and Yu. A. Simonov, Nucl. Phys. Proc. Suppl. **142**, 383 (2005).
[22] Y. Maezawa, T. Maruyama, N. Itagaki, and T. Hatsuda, Acta Phys. Hung. A **22**, 61 (2005).
[23] M. Karliner and H. J. Lipkin, Phys. Lett. **B586**, 303 (2004).
[24] M. Karliner and H. J. Lipkin, Phys. Lett. **B575**, 249 (2003).
[25] J. Szweda, L. S. Celenza, C. M. Shakin, and W.-D. Sun, Few-Body Syst. **20**, 93 (1996).
[26] L. S. Celenza, Bo Huang, H. Wang, and C. M. Shakin, Phys. Rev. C **60**, 025202 (1999).
[27] L. S. Celenza, Bo Huang, and C. M. Shakin, Phys. Rev. C **59**, 1041 (1999).
[28] L. S. Celenza, Bo Huang, H. Wang, and C. M. Shakin, Phys. Rev. C **60**, 065209 (1999).
[29] L. S. Celenza and C. M. Shakin, *Relativistic Nuclear Physics: Theories of Structure and Scattering* (World Scientific, Singapore, 1986).
[30] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
[31] B. G. Lasscock, J. Hedditch, D. B. Leinweber, W. Melnitchouk, A. W. Thomas, A. G. Williams, R. D. Young, and J. M. Zanotti, hep-lat/0503008.
[32] F. Csikor, Z. Fodor, S. D. Katz, T. G. Kovács, and B. C. Tóth, hep-lat/0503012.
[33] T.-W. Chiu and T.-H. Hsieh, Prog. Theor. Phys. Suppl. **153**, 220 (2004); hep-lat/0501021.
[34] C. Alexandron and A. Tsapalis, hep-lat/0503013.