

Single particle sum rules in the nuclear deep-inelastic region

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We have modeled the parton distribution in nuclei using a suitably modified nuclear Fermi motion. The modifications concern the nucleon rest energy which changes the Björken x in a nuclear medium. We also introduce final state interactions between the scattered nucleon and the rest of the nucleus. The energy-momentum sum rule is saturated. Good agreement with experimental data of the EMC effect for $x > 0.15$ and nuclear lepton pair production data has been obtained.

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Deep-inelastic scattering of electrons on nuclei (NDIS) proceeding with a highly virtual photon, $|Q^2| > 1$ GeV, provides us with a picture of bound nucleons with partially deconfined constituents because of the presence of the surrounding nuclear matter [1]. The resolution $1/\sqrt{|Q^2|}$ is good enough to allow for the space localization of nucleonic constituents. The influence of a nuclear medium on the nucleon structure function (SF), $F_2(x)$, which is known as the EMC effect, also depends on the longitudinal scale, $z = 1/(M_N x)$ [2], on which it is observed; the scale is determined by the mean free path and/or lifetimes of the struck partons [3]. For small x , the corresponding scale z exceeds the mean nucleon-nucleon (NN) distance in the nuclear matter. Therefore, in this region, the EMC effect should be described using some collective degrees of freedom characterized by the corresponding size of the nuclear area and by nucleonic (or partonic) clusters. On the other hand, for $x > x_B = 0.6$, one has $z < 0.3$ fm; i.e., the lifetime of the hit parton is sufficiently small for its final state not to be affected by the NN interaction because it travels a distance comparable with the nucleon hard core radius, $r_C \simeq 0.35$ fm, which is much smaller than the nucleon average radius, $r_N \simeq 0.8$ fm. It follows that in this region the single nucleon degrees of freedom play a crucial role in the description of the nuclear influence on the partonic SF of the nucleon.

In this work, we investigate the region $0.15 < x < 1$. Following [3], a usual two-step mechanism of NDIS is adopted, accounting for the fact that (a) nuclei are composed of A nucleons, which are distributed according to the nuclear distribution function, $\rho^A(y_A)$; and (b) nucleons are composed of partons, which are described by the free nucleon SF, $F_2^N(x)$. The nuclear SF is therefore the following convolution (see Fig. 1) [4]:

$$\frac{1}{x_A} F_2^A(x_A) = A \int \int dy_A \frac{dx}{x} \delta(x_A - y_A x) \rho^A(y_A) F_2^N(x). \quad (1)$$

This simple formula failed as a good description of nuclear SF. Some additional degrees of freedom present only in nuclear matter, usually attributed to additional mesonic [1], could be present. However, mesons alone cannot account for data on lepton pair production on nuclei [5]. It is suggested that the nucleon SF F_2^N itself is modified in the medium. Both effects should be included.

In this paper, we will examine the following picture. At large values of x , $x \in (x_B, 1)$, the longitudinal resolution in z

is small enough and NDIS proceeds only on a single nucleon but with its mass modified accordingly by the presence of the nuclear medium. On the other hand, approaching $x = 0.15$ limit, the longitudinal scale gradually grows, eventually exceeding nucleonic size and later also internucleonic distances in the nucleus. The action of other nucleons is now strong and should be accounted for. Because the approximation of independent nucleonic pairs is already responsible for $\sim 80\%$ of nuclear binding, we will consider in this region NDIS as proceeding on such an interacting pair which forms the quasibound state (cf. Fig. 1). This effect will be modeled by an x -dependent mass of the nucleon [6]. However, to satisfy exactly the momentum sum rule (MSR), one has to allow for a nuclear pion excess, which should be small ($\sim 1\%$) in order to simultaneously describe the lepton pair production data.

Let us start with the $x > x_B$ regime, where the partonic mean free paths z are much shorter than the average distances between nucleons. It means that we can treat nuclear nucleons as noninteracting objects remaining on the energy shell and not affected by the FSI shown in Fig. 1:

$$\sqrt{p^2} \equiv M_{B \rightarrow \text{rest}} p^+. \quad (2)$$

Let M_B , identified with the p^+ component in the nucleon rest frame, denote the rest energy of the nucleon in this case. To calculate it, we assume that the nuclear, longitudinal momentum P_A^+ component is given as a sum of all partonic momenta k_{Ai}^+ . For A nucleons, we have

$$\frac{1}{A} \sum_{i=1}^{nA} k_{Ai}^+ = \frac{M_A}{A} \equiv M_N + \epsilon = \int d^3 p \sqrt{M_B^2 + \vec{p}^2}, \quad (3)$$

where $\epsilon \simeq -8$ MeV is the usual nucleon binding energy and n is the number of partons inside the nucleon (in what follows we will work with $A > 50$ nuclei and assume uniform momentum distribution of nucleons).

For the uniform nucleon Fermi distribution, the average nucleon Fermi energy is

$$E_{\text{Fermi}} \simeq 0.6 [\bar{p}_F^2 / (M_N + \epsilon)], \quad (4)$$

where $p_F = (3\pi^2 \bar{n}_A / 2)^{1/3}$ is the average Fermi momentum given by average nuclear density \bar{n}_A . One gets from (3) that

$$M_B \cong M_N + \epsilon - E_{\text{Fermi}}. \quad (5)$$

It means that in a nuclear medium characterized by ϵ and E_{Fermi} , the rest energy of the nucleon, $M_B = \sum_i k_{Ni}^+$, takes

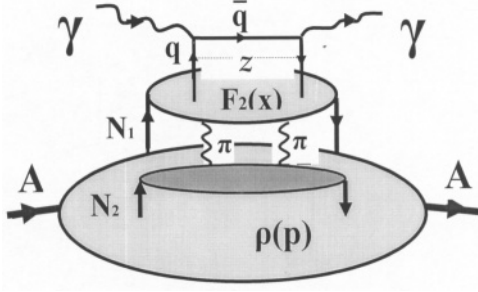


FIG. 1. Graphical representation of the convolution model for the deep-inelastic electron nucleus scattering with 2π final state interaction (FSI) between the active nucleon N_1 (with hit quark q) and the spectator N_2 .

in the large x limit, $x > x_B$, a value different from the sum of the corresponding partonic energies k_{Ni}^0 expressed in the rest frame of the nucleon [notice that they differ from k_{Ai}^0 in (3)]; here and in what follows we put $x_B = 0.6$ because, as was stated before, this value corresponds to longitudinal scale z starting to be smaller than the nucleon hard core radius). $M_B < M_N$ represents therefore the additional effect of Fermi motion emerging from the partonic (x) level of description.

Following the line of reasoning proposed by us in [8], we are not changing the expression for the nuclear spectral function $\rho^A(y_A)$ and take it from the relativistic mean field (RMF) model of the nucleus [9]. In the relativistic Fermi gas model [10], it is given by

$$\rho^A(y_A) = \frac{4}{\rho} \int \frac{d^4 p}{(2\pi)^4} S_N(p^0, \mathbf{p}) \times \left[1 + \frac{p_3}{E(p)} \right] \delta \left[y - \frac{(p_0 + p_3)}{\mu} \right], \quad (6)$$

where the factor $[1 + p_3/E(p)]$ representing relativistic correction [11] has been obtained for the RMF form of the nucleon spectral function: $S_N = n(p)\delta[p^0 - (E(p) + U_V)]$ with $E(p) = \sqrt{(M_N + U_S)^2 + \mathbf{p}^2}$.

Let us now proceed to lower values of x , $x < x_B$. This is the region in which an increase in the partonic mean free paths makes them eventually comparable with internucleonic distances, and one faces the problem of how to properly treat forces binding nucleons in nuclei. To solve it, notice that for sufficiently small values of x , in the region of $x < x_N \simeq 0.3$, the uncertainties in the lifetime of an intermediate parton state are so big that one should include exchanges of nuclear mesons (such as two π , but also σ , ω , and ρ) between nucleons, cf. Fig. 1. In standard low-energy nuclear physics, this is usually done by adding to Eqs. (3) and (5) the nuclear potential energy term resetting the effective nucleon mass to its free nucleon value M_N . To be more specific, for $x > x_B = 0.6$ we have $z(x) = 1/(xM_N) < r_C = 0.35$ fm. In this region, the nearby second nucleon, which is separated by the average distance $r_h \simeq 1.7$ fm (obtained from independent pair approximation), will not affect collisions proceeding on the active nucleon. The situation changes when x is getting smaller and the uncertainty z exceeds the nucleon radius $r_N = 0.85$ fm $\simeq r_h/2$. For such an

x , the single nucleon approximation is no longer applicable. We will model this process by introducing the probability $f(x)$ that the struck quark originates from the nucleon which interacts with a nearby nucleon spectator and assume for its simple linear form in the $z = 1/(M_N x)$ variable (i.e., in the longitudinal scale)

$$f(x) = \begin{cases} 0 & \text{if } x \geq x_B \quad (\simeq 0.6) \\ \left[\frac{1}{M_N x} - r_c \right] / (r_N - r_c) & \text{if } x \leq \frac{1}{r_N M_N} \quad (\simeq 0.25) \\ 1 & \end{cases} \quad (7)$$

This function determines whether interaction proceeds on the noninteracting nucleon with the rest energy equal to M_B or on the correlated nucleon with a different rest energy. One can introduce the effective nucleon SF, \tilde{F}_2^N , and express it via one- and two-nucleonic SFs:

$$\tilde{F}_2^N(x) = f(x)F_2^{2N}(x) + [1 - f(x)]F_2^N(x). \quad (8)$$

Now we can take into account the effect of two-body FSI in $F_2^{2N}(x)$ by changing the nucleon rest energy M_B and adding the NN interaction term to Eq. (5). Assuming the dominant role of two-body short-range NN correlation, it will reset the nucleon rest energy to the standard low-energy value M_N . It gives the dominant change of the nucleon SF for smaller $x < x_B$ coming from FSI. We are not including other effects of the NN interaction.

Now, the above interpolation procedure applies also approximately to the rest energy of the struck nucleon. Using the relation $\langle V_N \rangle = 2(\epsilon - E_{\text{Fermi}})$, which connects the average nucleon interacting energy $\langle V_N \rangle$ with the Fermi momentum in the nucleus, the interesting nucleon rest energy now becomes x dependent and is given by [12]

$$M_x = M_N + \frac{[1 - f(x)]}{2} \langle V_N \rangle. \quad (9)$$

Such an x -dependent nucleonic mass affects the MSR, which is now given by

$$\begin{aligned} \frac{\frac{1}{A} \int F_2^A(x_A) dx}{\int F_2^N(x) dx} &= \left(1 - C_x \frac{M_N - M_B}{M_N} \right) \frac{M_A}{AM_N} \\ &= \left(1 - C_x \frac{E_{\text{Fermi}} - \epsilon}{M_N} \right) (1 + \epsilon), \end{aligned} \quad (10)$$

where the factor C_x comes from the integration of the function $f(x)$ and indicates the amount of momentum violation (no violation for $C_x = 0$ and maximum violation for $C_x = 1$). For constant mass $M_x = M_B$, one has $C_x = 1$ in the whole range of x and consequently this sum rule is violated by $\sim 3\%$ [13]. The increase of nucleon rest energy connected with the inclusion of nuclear FSI in NDIS decreases C_x and will reduce this violation. Calculations performed for ^{56}Fe using $\bar{n}_A = 0.12$ fm $^{-3}$, $\epsilon = -8.8$ MeV and with x -dependent nucleon mass (9) reduces this violation further to the level of only 1%. This amount can be safely (i.e., in agreement with

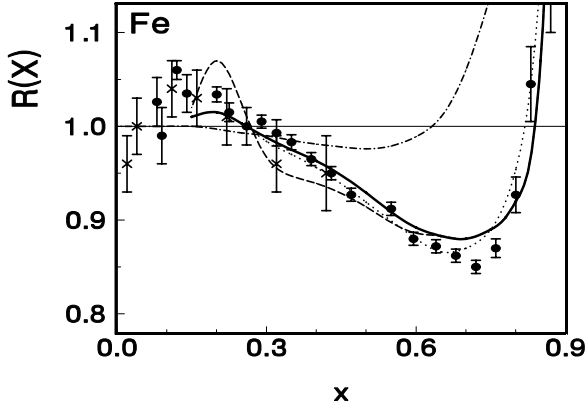


FIG. 2. Results and data [18] for the ratio $R(x) = F_2^{56}(x)/F_2^D(x)$. Fe data (solid circles) are from SLAC. See also Cu data from NA2' (x marks). Curves are described in the text.

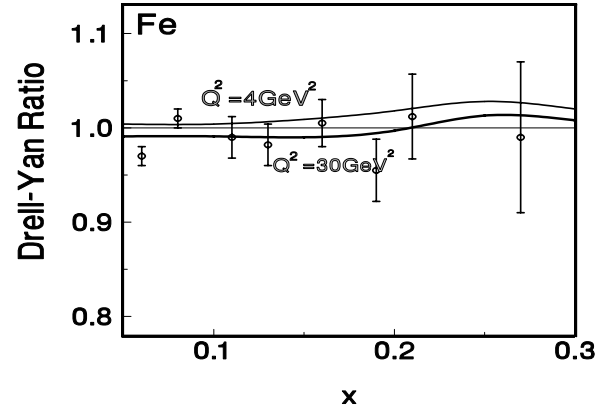


FIG. 3. Results for $Q^2 = 30 \text{ GeV}^2$ and data [5] for the ratio of Drell-Yan lepton pair production. Results for $Q^2 = 4 \text{ GeV}^2$ (as in the EMC effect) are shown for reference.

nuclear lepton pair production data) accounted for [14] by increasing the sea quarks momentum fraction in the nuclear medium. It can be done by scaling the x variable by 3% in the sea quark part of SF, $F_2^N(x)$, and it corresponds in model [8,15] to the similar increase of the mass of virtual pion for $Q^2 = 4 \text{ GeV}$ (for $Q^2 = 30 \text{ GeV}$ it would be $<1\%$) [16].

The results of the calculations of the total nuclear SF, $R(x) = F_2^A(x)/F_2^D(x)$, are presented in Fig. 2 [17]. Notice that the case of pure “Fermi smearing,” where $M_x = M_N$ for the all x (no binding), represented by the dash-dotted line, fails completely. The situation improves dramatically (cf. dashed line) when one uses a modified constant nucleon rest energy, $M_x = M_B$ (i.e., $C_x = 1$). In this case, to satisfy the nuclear MSR, we have to enlarge the nuclear sea contribution (pion excess) by allowing for 3% of the momentum to be carried by the nuclear sea quarks. However, this makes agreement with lepton pair production data [5] impossible. Therefore, in the next step (solid line) we have included the x -dependent modification of the nucleon rest energy by using M_x as given by Eq. (9). In this case, the nuclear MSR is satisfied by increasing the momentum of sea quarks in bound nucleon by only 1%. Agreement with the data is now very good [19]. The dotted line shows results obtained by replacing F_2^D by F_2^N , i.e., it shows the effect caused by deuteron. As seen in Fig. 3, the 1% increase of the nuclear sea at $Q^2 = 30$ is now fully compatible with the lepton pair production data [5].

Consequently, although both results (represented by dashed and solid lines) satisfy the MSR, one can see that they differ for $x < 0.6$, with the solid line presenting better fit to data in the region of $0.3 < x < 0.6$ (because of our x -dependent effective nucleon mass). The sea quark contributions, which are located at low x , are relatively small for the solid line and agree with the data. Agreement with data for $x > 0.6$ is determined only by the values of ϵ and p_F used in (5). Finally, the overall fit represented by the solid curve is very good [19], better than the dashed one, and it shows that the physical mechanism of the x -dependent nucleon rest energy in NDIS works properly, although the results depend slightly on the actual value of x_B . The presented model makes it possible to incrementally switch

on (depending on the scale z) the effect of nucleon-nucleon interaction, giving the desired big reduction of C_x in the MSR (10).

To summarize, we propose to account for the EMC effect in NDIS by using the single particle approach with effective Fermi motion caused by nuclear interactions, which is also responsible for medium changes of the parton distributions inside nucleons. The parton distribution is the function of the nucleon rest energy (via Björken x) which depends on the interacting scale in a medium. Therefore, the nucleon rest energy is modeled by introducing x -dependent effective nucleon rest energy (nucleon mass) M_x such that for large $x > 0.6$, $M_x = M_B \simeq M_N + \epsilon - 1.8\bar{n}_A^{2/3}/M_B \approx M_N - (20 \div 30) \text{ MeV}$ (5) and for $x < 0.25$, it is equal to the free nucleon mass M_N . In this way we have obtained a very good fit to the experimental data [18] for $x > 0.15$ without additional free parameters (contrary to claims made in [21], where a similar approach but with only nucleon degrees of freedom and constant mass was used). In our model, only $\sim 1\%$ of nucleon momenta is carried by additional partons from the sea region, which is not in contradiction with lepton pair production data [5]. The medium modifications of mass proposed here can be compared with the approach using the chiral soliton model and direct quark-meson coupling mechanism [22]. In our approach, we took effectively into account the nucleon-nucleon interaction in NDIS, and the final results are expressed by the global nuclear parameters. The medium modifications depend on the value of the average Fermi energy E_{Fermi} and on the mass defect ϵ . We investigate the structure of the interacting object, and therefore its energy is not well defined. In our calculations of nucleon SF, we try to take into account the nucleon-nucleon interaction while we investigate its parton structure. Proposed changes of the nucleon rest energy should be present in the nucleon structure obtained from the heavy ion collision; however, this effect is of the same origin as the FSI between hadrons (partons) and therefore it is rather subtle and difficult to separate and detect. Nevertheless, some recent observations of the decay spectrum of delta particles in high-energy heavy ion collision seem to suggest that a similar reduction ($\sim 20 \text{ MeV}$) of its

invariant mass exists [23]. The MSR is satisfied within the conventional picture of interacting nucleons, mainly because of the x -dependent effective nucleon mass M_x that was introduced.

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