Δ effects in pion-nucleon scattering and the strength of the two-pion-exchange three-nucleon interaction

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We consider the relationship between p-wave πN scattering and the strength of the p-wave two-pion-exchange three-nucleon interaction (TPE3NI). We explain why effective theories that do not contain the Delta resonance as an explicit degree of freedom tend to overestimate the strength of the TPE3NI. The overestimation can be remedied by higher-order terms in these "Delta-less" theories, but such terms are not yet included in state-of-the-art chiral effective field theory calculations of the nuclear force. This suggests that these calculations can predict the strength of the TPE3NI only to an accuracy of $\pm 25\%$.

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I. INTRODUCTION

A long-standing quest in hadronic physics is to relate the properties of free pions, observed in, for instance, pion-nucleon (πN) scattering, to those of the pions that play such a significant role in the nuclear force. Recently, the Nijmegen group has provided a striking demonstration that one-pion exchange indeed provides the longest range component of the two-nucleon potential. They extracted, with small error bars, the masses of the charged and neutral pions and the couplings of pions to the nucleon from fits to the *pp* and *np* scattering data [1]. A subsequent Nijmegen analysis of *NN* data then confirmed that two-pion exchange [2–4] gives a significant fraction of the intermediate-range attraction in the *NN* interaction [5]. In some models other mechanisms, for example, the very broad $f_0(600)$—or σ—meson [6], also contribute to this attraction. In systems beyond $A = 2$ the three-nucleon interaction plays a subtle, but important, role. In this article we focus on the Fujita-Miyazawa (FM) [7] term in the two-pion-exchange three-nucleon interaction (TPE3NI). It appears—at least for light nuclei—that this is the largest piece of the three-nucleon force [8].

Ideally πN scattering data should be used to directly construct the TPE3NI. However, the pions that generate nuclear forces are highly virtual. The relation between the scattering they experience from nucleons inside the nucleus and that observed in free space is nontrivial. To determine it, an extrapolation of the πN amplitude from the "physical region"—where the pion energies are greater than m_π—to the "virtual region"—where pion energies are much less than m_π—is needed.

The Delta isobar is the most prominent feature of πN dynamics. The Delta peak in the $\pi^+ p$ elastic scattering cross section is larger by an order of magnitude than any other [6]. Therefore, when constructing models of the πN interaction that will be used for the extrapolation to the virtual region it is natural to include the Delta as an explicit degree of freedom. This was the path followed many years ago, and the leading two-pion-exchange two- and three-nucleon potentials with an explicit Delta were derived by Sugawara and von Hippel [2] and Fujita and Miyazawa (FM) [7], respectively. These two-pion-exchange *NN* and *NNN* potentials were recently rederived as pieces of the more general expressions for two- and three-nucleon forces that are obtained when an effective field theory (EFT) with explicit Delta degrees of freedom is applied to the problem of nuclear forces [4,9]. Here we discuss how the FM potential arises in any theory with an explicit Delta. Our expression for this potential is connected to πN scattering data through the Delta mass and the $\pi N \Delta$ coupling constant, both of which can be determined from the πN data.

But the highly virtual pions exchanged in the TPE3NI have energies much less than the Delta-nucleon mass difference. This has encouraged the development of an approach to nuclear forces that is different from that of Sugawara and von Hippel and Fujita and Miyazawa. In this approach the Delta degree of freedom—along with all other πN resonances— is "integrated out." This yields an EFT in which pions and nucleons interact in the most general way. In this EFT πN interactions are point like and organized as an expansion in the number of space and time derivatives (for a review, see Ref. [10]). The expansion parameter is essentially $\frac{\omega}{\Delta M}$, with ω the pion energy and $\Delta M \equiv M_\Delta - M \approx 300$ MeV $\sim 2m_\pi$ the Delta-nucleon mass difference. Applying this "Delta-less" EFT to πN scattering is challenging (see, e.g., Ref. [11]) because the expansion parameter is, at best, $\frac{1}{2}$, and the expansion breaks down completely at the Delta peak. However the expansion should converge well if $\omega \ll \Delta M$, a condition which should have fair validity in nuclear-structure physics. The leading contributions to *NN* and *NNN* potentials in this EFT were found in Refs. [12] and [9], respectively.[1]

We have argued that nuclear-structure physics is within the domain of validity of both the theory with explicit Deltas and the "Delta-less" EFT. We might expect then, that the

[1]The delta contributions were of course implicit in previous dispersion-theoretical approaches [13,14] and models [15,16], although the correct chiral-symmetry properties are difficult to maintain when connecting the pion-nucleon amplitude to the potential without using field theory [17].

two theories would give similar results for the strength of the TPE3NI. But this turns out not to be the case. Effective theories without an explicit Delta predict a strength for the TPE3NI that is 1.5 to 2.5 times larger than that obtained by FM [17]. Studies of the spectrum of light nuclei with the Green's function Monte Carlo method, including three-nucleon interactions, favor a strength of the TPE3NI closer to the FM value [8,18].[2]

Here we identify the origin of this discrepancy. Parameters in the Lagrangian of the theory with pions and nucleons alone must be extracted from πN scattering data. But the poor convergence of the derivative expansion in that theory tends to contaminate parameters extracted in this way. These parameters then appear in the TPE3NI and result in overestimation of its strength. Within the Delta-less EFT this problem is only mitigated if many orders in the expansion are retained.

This simple argument is presented as follows. In Sec. II we write down an EFT with nucleons, pions, and explicit Deltas, and compute, to leading order, both the p-wave πN scattering amplitude and the TPE3NI. In Sec. III we use a theory without explicit deltas to compute the TPE3NI. By construction the πN amplitudes in this theory and the theory of Sec. II agree at πN threshold. We show that they differ by a factor of $\frac{4}{3}$ in their prediction for the strength of the FM NNN potential. We then discuss how this overestimation would be remedied at higher orders in the Delta-less EFT, and what the implications of this problem are for contemporary EFT computations of the TPE3NI.

II. A THEORY WITH EXPLICIT DELTAS

Although many terms contribute to πN scattering and the three-nucleon potential, here we focus on the Delta contributions. We do not claim that this is an accurate or complete model for either πN scattering or the TPE3NI, but it serves to illustrate the point we wish to make regarding the relationship between πN data and the strength of the TPE3NI in Delta-less EFTs. For discussions of this relationship in the context of hadronic models, see, e.g., Ref. [16].

We consider p-wave πN scattering in an effective theory with an explicit Delta degree of freedom. We are interested in small pion momenta, and so we need only the leading terms in the πNN and $\pi N\Delta$ interaction Lagrangians. These are as follows:

$$\mathcal{L}_{\pi NN} = \frac{g_A}{2f_\pi} N^\dagger \boldsymbol{\sigma}\boldsymbol{\tau} N \times \nabla\Phi \tag{1}$$

$$\mathcal{L}_{\pi N\Delta} = \frac{h_A}{2f_\pi}(\Delta^\dagger \mathbf{S}\mathbf{T}N + \text{H.c.}) \times \nabla\Phi \tag{2}$$

where Φ, N, and Δ are the pion, nucleon, and Delta fields, $f_\pi \simeq 93$ MeV is the pion decay constant, $g_A \simeq 1.29$ is the axial-vector constant that corresponds to the value of the (charged) πNN coupling constant reported in Ref. [1], $h_A \simeq 2.8$ is the corresponding pion-nucleon-Delta transition strength

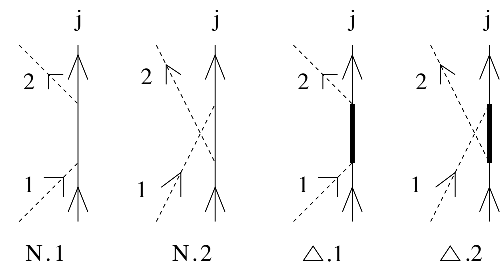

FIG. 1. Four πN scattering diagrams. Dashed lines represent pions, solid line nucleons, and thick solid lines Delta isobars.

(see, e.g. Ref. [19]), $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the Pauli spin and isospin matrices, and $\mathbf{S}$ and $\mathbf{T}$ are Rarita-Schwinger transition spin and isospin operators. Both $\mathbf{S}$ and $\mathbf{T}$ obey generalized Pauli identities of the following form:

$$\mathbf{S}^\dagger \cdot \mathbf{A}\mathbf{S} \cdot \mathbf{B} = \frac{2}{3}\mathbf{A}\cdot\mathbf{B} - \frac{1}{3} i\boldsymbol{\sigma}\cdot\mathbf{A}\times\mathbf{B}. \tag{3}$$

Alternatively, one can work with the following Hamiltonians:

$$H_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi}\boldsymbol{\sigma}\cdot\nabla(\Phi(r)\cdot\boldsymbol{\tau}), \tag{4}$$

$$H_{\pi N\Delta} = -\frac{f_{\pi N\Delta}}{m_\pi}\{\mathbf{S}\cdot\nabla[\Phi(r)\cdot\mathbf{T}] + \mathbf{S}^\dagger\cdot\nabla[\Phi(r)\cdot\mathbf{T}^\dagger]\}, \tag{5}$$

where, at this order, $f_{\pi NN} = m_\pi g_A/2f_\pi$ and $f_{\pi N\Delta} = m_\pi h_A/2f_\pi$, with m_π the pion mass.

A. πN scattering at low energies

At leading order in small momenta these Lagrangians yield four diagrams that contribute to p-wave πN scattering. They are shown in Fig. 1. Only two involve the Delta. They give the nucleon-pole-subtracted amplitude that enters the TPE3NI. Graph $\Delta.1$ is the direct (s-channel) graph, and graph $\Delta.2$ is the crossed (u-channel) graph.

We evaluate these graphs in the center-of-mass frame in which the pion energy is ω and denote the momentum and isospin of the initial (final) pion by $\mathbf{q}_1$ and $\mathbf{t}_1$ ($\mathbf{q}_2$ and $\mathbf{t}_2$). Because we limit ourselves to pion momenta of the order of the pion mass, the nucleon kinetic energies are smaller than ω by a factor of order m_π/M and can be neglected in this leading-order calculation. For the same reason we neglect the kinetic energy of the delta.

The delta contribution to the πN amplitude is then given by the following:

$$\mathcal{A}_{\pi N} = -\frac{f^2_{\pi N\Delta}}{m^2_\pi}\langle\chi'_j|\mathbf{S}^\dagger_j\cdot\mathbf{q}_2\mathbf{S}_j\cdot\mathbf{q}_1\ \mathbf{T}^\dagger_j\cdot\mathbf{t}_2\mathbf{T}_j\cdot\mathbf{t}_1\frac{1}{\Delta M-\omega} + \mathbf{S}^\dagger_j\cdot\mathbf{q}_1\mathbf{S}_j\cdot\mathbf{q}_2\ \mathbf{T}^\dagger_j\cdot\mathbf{t}_1\mathbf{T}_j\cdot\mathbf{t}_2\frac{1}{\Delta M+\omega}|\chi_j\rangle. \tag{6}$$

The χ_j and χ'_j are spin-isospin quantum numbers of the nucleon before and after scattering, and $\Delta M \equiv M_\Delta - M$.

[2]This conclusion is somewhat dependent on the regulator used in the three-nucleon force but holds definitively if one requires that the cutoffs used in the NN and NNN system be the same.

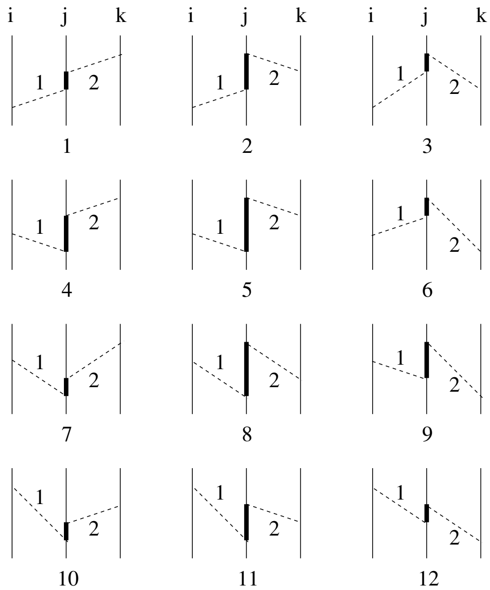

FIG. 2. The twelve "direct" *NNN* diagrams in time-ordered perturbation theory. Notation as in Fig. 1.

Using Eq. (3), we can rewrite this amplitude as follows:

$$\begin{aligned}\mathcal{A}_{\pi N} = -\frac{f^2_{\pi N\Delta}}{m^2_\pi}\langle\chi'_j|\frac{4}{9}\bigg[&\mathbf{q}_1\cdot\mathbf{q}_2\ \mathbf{t}_1\cdot\mathbf{t}_2 - \frac{1}{4}\boldsymbol{\sigma}_j\cdot\mathbf{q}_1\\ &\times\mathbf{q}_2\ \boldsymbol{\tau}_j\cdot\mathbf{t}_1\times\mathbf{t}_2\bigg]\left(\frac{2\Delta M}{(\Delta M)^2-\omega^2}\right)\\ &+ i\frac{2}{9}[\boldsymbol{\sigma}_j\cdot\mathbf{q}_1\times\mathbf{q}_2\ \mathbf{t}_1\cdot\mathbf{t}_2 + \boldsymbol{\tau}_j\cdot\mathbf{t}_1\times\mathbf{t}_2\ \mathbf{q}_1\cdot\mathbf{q}_2]\\ &\times\left(\frac{2\omega}{(\Delta M)^2-\omega^2}\right)|\chi_j\rangle.\end{aligned}\tag{7}$$

B. The three-nucleon scattering amplitude

We now turn our attention to the tree-level Delta contribution in the TPE3NI. To this end we consider the amplitude for nucleon i emitting or absorbing a pion of momentum $\pm\mathbf{q}_1$ and isospin $\mathbf{t}_1$ and nucleon k emitting or absorbing a pion of momentum $\pm\mathbf{q}_2$ and isospin $\mathbf{t}_2$. In "direct" diagrams the pion "1" converts nucleon j to a Δ and "2" reconverts it to nucleon. In the "crossed" diagrams "2" converts and "1" reconverts. There are 12 "direct" and 12 "crossed" diagrams in time-ordered perturbation theory. The 12 direct diagrams are shown in Fig. 2.

The contribution of the direct diagrams to the three-nucleon scattering amplitude is given by the following:

$$\begin{aligned}\boldsymbol{\mathcal{A}}^{\text{direct}}_{3N} = \frac{f^2_{\pi NN}}{m^2_\pi}&\langle\chi'_k|\boldsymbol{\sigma}_k\cdot\mathbf{q}_2\ \boldsymbol{\tau}_k\cdot\mathbf{t}_2|\chi_k\rangle\langle\chi'_i|\boldsymbol{\sigma}_i\cdot\mathbf{q}_1\ \boldsymbol{\tau}_i\cdot\mathbf{t}_1|\chi_i\rangle\\ &\times\left(\frac{1}{4\omega_1\omega_2}\right)\left[\sum_{\alpha=1}^{12}\frac{1}{\Pi_\alpha}\right]\frac{f^2_{\pi N\Delta}}{m^2_\pi}\\ &\times\langle\chi'_j|\mathbf{S}^\dagger_j\cdot\mathbf{q}_2\mathbf{S}_j\cdot\mathbf{q}_1\ \mathbf{T}^\dagger_j\cdot\mathbf{t}_2\mathbf{T}_j\cdot\mathbf{t}_1|\chi_j\rangle.\end{aligned}\tag{8}$$

Here $\chi_{i,j,k}$ and $\chi'_{i,j,k}$ denote the initial and final spin-isospin states of nucleons i, j, and k, and Π_α is the product of the three energy denominators in diagram α of Fig. 2. The values of Π_α can be read off the diagrams, and they are listed in Table I. Once again we have neglected nucleon and Δ kinetic energies in computing these denominators, which is valid in our leading-order calculation.

From Table I we can easily verify the following:

$$\sum_{\alpha=1}^{12}\frac{1}{\Pi_\alpha} = \frac{-4}{\omega_1\omega_2\Delta M}.\tag{9}$$

Substituting this in Eq. (8) gives the following:

$$\begin{aligned}\boldsymbol{\mathcal{A}}^{\text{direct}}_{3N} = \frac{f^2_{\pi NN}}{m^2_\pi}&\langle\chi'_k|\boldsymbol{\sigma}_k\cdot\mathbf{q}_2\ \boldsymbol{\tau}_k\cdot\mathbf{t}_2|\chi_k\rangle\langle\chi'_i|\boldsymbol{\sigma}_i\cdot\mathbf{q}_1\ \boldsymbol{\tau}_i\cdot\mathbf{t}_1|\chi_i\rangle\\ &\times\frac{f^2_{\pi N\Delta}}{m^2_\pi}\langle\chi'_j|\mathbf{S}^\dagger_j\cdot\mathbf{q}_2\mathbf{S}_j\cdot\mathbf{q}_1\ \mathbf{T}^\dagger_j\cdot\mathbf{t}_2\mathbf{T}_j\cdot\mathbf{t}_1|\chi_j\rangle\\ &\times\left(\frac{-1}{\omega_1^2\omega_2^2\Delta M}\right).\end{aligned}\tag{10}$$

The contribution of the crossed diagrams involves analogous energy denominators and can be calculated similarly. The sum of direct and crossed diagrams,

$$\begin{aligned}\mathcal{A}_{3N} = \frac{f^2_{\pi NN}}{m^2_\pi}&\langle\chi'_k|\boldsymbol{\sigma}_k\cdot\mathbf{q}_2\ \boldsymbol{\tau}_k\cdot\mathbf{t}_2|\chi_k\rangle\langle\chi'_i|\boldsymbol{\sigma}_i\cdot\mathbf{q}_1\ \boldsymbol{\tau}_i\cdot\mathbf{t}_1|\chi_i\rangle\\ &\times\left(\frac{-1}{\omega_1^2\omega_2^2\Delta M}\right)\frac{f^2_{\pi N\Delta}}{m^2_\pi}\langle\chi'_j|\mathbf{S}^\dagger_j\cdot\mathbf{q}_2\mathbf{S}_j\cdot\mathbf{q}_1\ \mathbf{T}^\dagger_j\cdot\mathbf{t}_2\mathbf{T}_j\cdot\mathbf{t}_1\\ &+\mathbf{S}^\dagger_j\cdot\mathbf{q}_1\mathbf{S}_j\cdot\mathbf{q}_2\ \mathbf{T}^\dagger_j\cdot\mathbf{t}_1\mathbf{T}_j\cdot\mathbf{t}_2|\chi_j\rangle,\end{aligned}\tag{11}$$

TABLE I. The values of $(-1/\Pi_\alpha)$ for direct diagrams.

α	$-1/\Pi_\alpha$	α	$-1/\Pi_\alpha$
1	$\omega_2\Delta M\omega_1$	2	$(\Delta M+\omega_2)\Delta M\omega_1$
3	$(\Delta M+\omega_2)(\omega_1+\omega_2)\omega_1$	4	$\omega_2\Delta M(\Delta M+\omega_1)$
5	$(\omega_2+\Delta M)\Delta M(\omega_1+\Delta M)$	6	$(\omega_2+\Delta M)(\omega_1+\omega_2)\omega_2$
7	$\omega_2(\omega_1+\omega_2)(\omega_1+\Delta M)$	8	$(\omega_2+\Delta M)(\omega_1+\omega_2+\Delta M)(\omega_1+\Delta M)$
9	$(\omega_2+\Delta M)(\omega_1+\omega_1+\Delta M)\omega_2$	10	$\omega_1(\omega_1+\omega_2)(\omega_1+\Delta M)$
11	$\omega_1(\omega_1+\omega_2+\Delta M)(\omega_1+\Delta M)$	12	$\omega_1(\omega_1+\omega_2+\Delta M)\omega_2$

gives the Fujita-Miyazawa potential $V_{ijk}^{2\pi,\mathrm{FM}}$ [7] as follows:

$$V_{ijk}^{2\pi,\mathrm{FM}} = \frac{f_{\pi NN}^2}{m_\pi^2}\left(\frac{1}{\omega_1^2\omega_2^2}\right)\boldsymbol{\sigma}_k\cdot\mathbf{q}_2\,\boldsymbol{\tau}_k\cdot\mathbf{t}_2\,\boldsymbol{\sigma}_i\cdot\mathbf{q}_1\,\boldsymbol{\tau}_i\cdot\mathbf{t}_1 \times\left(-\frac{f_{\pi N\Delta}^2}{m_\pi^2}\frac{4}{9}\frac{2}{\Delta M}\right)\bigg(\mathbf{q}_1\cdot\mathbf{q}_2\,\mathbf{t}_1\cdot\mathbf{t}_2 - \frac{1}{4}\boldsymbol{\sigma}_j\cdot\mathbf{q}_1 \times\mathbf{q}_2\,\boldsymbol{\tau}_j\cdot\mathbf{t}_1\times\mathbf{t}_2\bigg). \tag{12}$$

This result agrees with many previous rederivations of the FM potential (e.g. Ref. [9]). It is exact at tree level in the static limit if the only terms in the πNN and $\pi N\Delta$ Lagrangians are those in Eqs. (1) and (2).

III. RELATION TO THEORIES WITHOUT EXPLICIT DELTAS

We now attempt to find a more direct connection between πN scattering data and $V_{ijk}^{2\pi,\mathrm{FM}}$—one that does not invoke the Delta as an explicit degree of freedom. Such attempts have been reviewed in Ref. [17], whose notation we follow below.

A key aspect of this connection is that πN scattering involves pions with $\omega \sim m_\pi$, whereas in $V_{ijk}^{2\pi,\mathrm{FM}}$ we have $\omega \sim m_\pi^2/M$. (The typical nucleon momentum in the nucleus is of order the pion mass, and the pion energy is then smaller by a factor m_π/M.) Because we have already been neglecting terms suppressed by m_π/M we take $q_1^0 = q_2^0 = 0$. Given this kinematics, the three-nucleon potential can be written as follows:

$$\bar{V}_{ijk}^{2\pi} = \frac{f_{\pi NN}^2}{m_\pi^2}\frac{\boldsymbol{\sigma}_i\cdot\mathbf{q}_1\boldsymbol{\sigma}_k\cdot\mathbf{q}_2}{\omega_1^2\omega_2^2}\left[-F_j^{\alpha\beta}\tau_i^\alpha\tau_k^\beta\right], \tag{13}$$

where $\omega_i \equiv \sqrt{\mathbf{q_i^2}+\mathbf{m_\pi^2}}$ comes from the pion propagators and

$$-F_j^{\alpha\beta} = \delta^{\alpha\beta}\left[a + b\mathbf{q}_1\cdot\mathbf{q}_2 + c\left(q_1^2+q_2^2\right)\right] - d\left(\tau_j^\gamma\epsilon^{\alpha\beta\gamma}\boldsymbol{\sigma}_j\cdot\mathbf{q}_1\times\mathbf{q}_2\right) \tag{14}$$

is the Born-subtracted πN subamplitude. Because of chiral symmetry, the third term is zero at low orders in an expansion in powers of momenta and the pion mass [17]. The model considered here is consistent with this result: we have $c = 0$. Meanwhile, the first term is because of s-wave scattering. It is very small in the context of $V_{ijk}^{2\pi}$ [8] and is zero in the present model. The second and fourth terms, which we focus on in this article, are the anticommutator and commutator parts of the TPE3NI.

Therefore for our purposes the crucial point is the determination of the coefficients b and d. In a theory without explicit Delta fields, they are fitted to πN data near threshold. If we lived in a world where there were no contributions to πN scattering other than from the s- and u-channel delta and nucleon poles, comparing Eq. (14) and Eq. (7) shows that a fit to threshold πN data would result in the following:

$$b = 4d = -\frac{f_{\pi N\Delta}^2}{m_\pi^2}\frac{4}{9}\left[\frac{2\Delta M}{(\Delta M)^2 - m_\pi^2}\right]. \tag{15}$$

The TPE3NI corresponding to this amplitude is given by the following:

$$\bar{V}_{ijk}^{2\pi} = \frac{f_{\pi NN}^2}{m_\pi^2}\frac{1}{\omega_1^2\omega_2^2}\boldsymbol{\sigma}_i\cdot\mathbf{q}_1\boldsymbol{\sigma}_k\cdot\mathbf{q}_2\boldsymbol{\tau}_i\cdot\mathbf{t}_1\boldsymbol{\tau}_k\cdot\mathbf{t}_2\mathbf{O}_j^{\pi N}. \tag{16}$$

The factor $1/\omega_1^2\omega_2^2$ comes from the pion propagators, and the factors in addition to $\mathbf{O}_j^{\pi N}$ describe the coupling of the pions to the nucleons i and k. The πN interaction is described by the following:

$$\mathbf{O}_j^{\pi N} = b\bigg(\mathbf{q}_1\cdot\mathbf{q}_2\mathbf{t}_1\cdot\mathbf{t}_2 - \frac{1}{4}\boldsymbol{\sigma}_j\cdot\mathbf{q}_1\times\mathbf{q}_2\boldsymbol{\tau}_j\cdot\mathbf{t}_1\times\mathbf{t}_2\bigg), \tag{17}$$

with b given by Eq. (15). Of course, this is just the usual FM form, but with specific choices for the coefficients b and d.

A. The problem

Comparing the $\bar{V}_{ijk}^{2\pi}$ in Eq. (16) with the the "exact" result for our model [$V_{ijk}^{2\pi,\mathrm{FM}}$ of Eq. (12)] we find that they are the same apart from the crucial fact that the strength of the interaction in the "Delta-less" theory has the factor $2\Delta M/(\Delta M)^2 - m_\pi^2$, instead of the $2/\Delta M$ of the "exact" result. Because $\Delta M \simeq 2m_\pi$, these factors are $\simeq 4/3m_\pi$ and $\simeq 1/m_\pi$, respectively. One way to understand this result is to realize that the direct term for the πN scattering amplitude in Eq. (7) and Fig. 1 is evaluated at the energy of a real pion and so has the energy denominator $\Delta M - m_\pi$ for low-momentum pions. This denominator is half of the average denominator, ΔM, of the diagrams in Fig. 2 that contribute to the TPE3NI. The crossed pion term mitigates this discrepancy but not enough to cure the problem. Ultimately, the $\bar{V}_{ijk}^{2\pi}$ *that is extracted "directly" from πN scattering data is too strong by a factor of 4/3.*

The difference between $\bar{V}_{ijk}^{2\pi}$ and $V_{ijk}^{2\pi,\mathrm{FM}}$ is of order $(m_\pi/\Delta M)^2$. It will vanish in the limit $\Delta M \gg m_\pi$, which includes the chiral limit $m_\pi \to 0$. However, in the context of the nuclear many-body problem m_π is not small. The range of the one-pion-exchange potential is comparable to the mean internucleon spacing in nuclei, and the energies required to excite nucleons to isobar states such as the delta are not much larger than m_π.

Of course, in the real world there are contributions to the πN amplitude other than the two graphs we have considered here. Also b and d will probably be determined from data that are not exactly at threshold. Although we cannot say *a priori* in which direction these effects go, fitting πN data at higher energies will presumably only make the extrapolation problem worse.

Parts of this problem have been understood for a long time, but, as discussed in the introduction, the prevailing folklore has been that an EFT without explicit deltas could still work well in nuclei, because the relevant energies in nuclear-structure physics are much smaller than ΔM. However, the poor convergence of the EFT without explicit deltas for πN scattering affects the TPE3NI because b and d are not calculated from first principles; instead they are fitted to threshold πN data. This necessitates an extrapolation from pion energies $\omega \sim m_\pi$ to the energies of the highly virtual pions in the TPE3NI, which are of order m_π^2/M. This extrapolation takes place over an energy range that is sizable compared to the radius of convergence of the "Delta-less" theory—ΔM.

Here we have explicitly considered the implications of such an extrapolation for the three-nucleon potential, but other few-nucleon potentials (including the two-nucleon force) will be afflicted by the same problem. All use πN parameters that are potentially contaminated in a similar way. Such contamination will occur in all EFTs for low-energy hadronic physics that contain only pion and nucleon degrees of freedom.

B. The solution

In a theory with explicit deltas this extrapolation is under much better control, because the pion-energy dependence of the πN amplitude is better reproduced. In contrast, at leading order in the "Delta-less" theory the coefficients of the two operators in $\mathbf{O}_j^{\pi N}$ are energy independent, and so the value extracted for them at threshold, where $\omega = m_\pi$, is used in the TPE3NI, where $\omega \simeq 0$.

But at higher orders in this EFT additional corrections to the πN amplitude, and in particular to the two operators in $\mathbf{O}_j^{\pi N}$, enter. To see what form this higher- order energy dependence would take, we expand the result (7) in powers of $(\omega/\Delta M)^2$. The first correction to the leading-order results for b and d (15) occurs at $O[(\omega/\Delta M)^2]$. The form of $\mathbf{O}_j^{\pi N}$ is now as follows:

$$\mathbf{O}_j^{\pi N} = (b + \tilde{b}\omega^2)\left(\mathbf{q}_1 \cdot \mathbf{q}_2 \mathbf{t}_1 \cdot \mathbf{t}_2 - \frac{1}{4}\boldsymbol{\sigma}_j \cdot \mathbf{q}_1 \times \mathbf{q}_2 \boldsymbol{\tau}_j \cdot \mathbf{t}_1 \times \mathbf{t}_2\right). \tag{18}$$

In the EFT, terms such as $\tilde{b}\omega^2$ and $\tilde{d}\omega^2$ appear in the Lagrangian as pion-nucleon interactions with time derivatives. We must fit πN data over a range of pion energies to determine both b and $\tilde{b}$. If, once again, we imagine living in a world where the true answer was given by Eq. (7), then fitting the Eq. (18) to reproduce Eq. (7) in the region around $\omega = m_\pi$ yields the following:

$$b = -\frac{4}{9}\frac{f_{\pi N\Delta}^2}{m_\pi^2}\left(\frac{2\Delta M}{(\Delta M)^2 - m_\pi^2}\right)\left(1 - \frac{m_\pi^2}{(\Delta M)^2 - m_\pi^2}\right); \tag{19}$$

$$\tilde{b} = -\frac{4}{9}\frac{f_{\pi N\Delta}^2}{m_\pi^2}\frac{2\Delta M}{\left[(\Delta M)^2 - m_\pi^2\right]^2}. \tag{20}$$

Note that at πN threshold this gives exactly the same result for $\mathbf{O}_j^{\pi N}$ as in Eq. (17). However, extrapolating to $\omega = 0$ now yields a TPE3NI that has an additional factor of $[1 - \frac{m_\pi^2}{(\Delta M)^2 - m_\pi^2}]$ in its strength. If we set $\Delta M = 2m_\pi$, this gives an overall factor of $8/9m_\pi$, instead of the $1/m_\pi$ found in the "exact" calculation with explicit deltas. This means that in the theory *without* explicit deltas the "exact" factor $1/m_\pi$ is being built up as follows:

$$\frac{1}{m_\pi} = \left(1 - \frac{1}{3} + \ldots\right)\frac{4}{3m_\pi}, \tag{21}$$

a series that converges moderately quickly.

To summarize, in the theory without explicit Deltas it is important to realize that the factor $4/3m_\pi$ obtained by fitting πN "data" with the leading-order form [Eq. (17)] is not the final answer. This result will change when higher order terms are incorporated in the theory and used to improve the extrapolation from $\omega \simeq m_\pi$ to $\omega \simeq 0$. We can estimate the size of such terms based on our knowledge that the convergence will be governed by the parameter $m_\pi/\Delta M$ and that—because of crossing—only even terms in this expansion can appear in b and d. The leading-order result should therefore be quoted as follows:

$$b = -\frac{4}{9}\frac{f_{\pi N\Delta}^2}{m_\pi^2}\frac{4}{3m_\pi}\left[1 \pm \left(\frac{m_\pi}{\Delta M}\right)^2\right]. \tag{22}$$

More conservative error bars are certainly acceptable, but the $\approx$25% we have chosen is the minimum permissible theoretical error that can be assigned to b when it is extracted in the theory without explicit deltas. Such an error bar turns out to be consistent with the "exact" answer for b in the simple model considered here.

IV. CONCLUSION

We have shown that theories without an explicit Delta tend to overestimate the delta contribution to the TPE3NI. This is because there is an error in the leading-order computation of the three-nucleon potential in the "Delta-less" theory. The error is $\sim$25%, and it is necessary to include terms suppressed by $(\omega/\Delta M)^2$ in the EFT to reduce it. The inclusion of other higher order effects, such as nucleon recoil and dispersive effects for intermediate-state Deltas, may make the extrapolation error smaller than we found, but it seems unlikely that it will completely remove the difficulty.

Unfortunately this problem is present in the state-of-the-art N^{3}LO chiral EFT computation of *NN* potentials [20] and their possible extension to the *NNN* case. The terms that ameliorate the overestimation appear in $\mathcal{L}_{\pi N}^{(4)}$, and so will not enter the chiral EFT nuclear force until N^{4}LO. Computing the two- and three-nucleon potentials to this (or higher) order will take considerable effort. It may well be that an EFT with explicit Deltas is simply a more efficient tool than one without. In fact, the first studies in nuclear EFT [4,9] included diagrams with intermediate Deltas in their calculation of the nuclear force. The drawback of such a treatment is that to fix parameters one must analyze data around the Delta resonance, which necessitates a resummation of the Delta self-energy. Only recently has a power counting been devised that allows a systematic EFT treatment of effects in this kinematic region [19].

The Delta-less EFT has also found difficulties with certain πN parameters that are large because the effects of the integrated-out Delta are encoded there. In Ref. [21] Epelbaum *et al.* argued that there is a cancellation of Delta-excitation and $\pi\rho$-exchange contributions in nuclear forces. This motivated their use of *NN* and *NNN* potentials containing πN-interaction parameters smaller than those extracted from chiral analyses of πN scattering data. We stress that the reduction in strength of the *NNN* force we have discussed here is not based on such an argument. It is independent of details of nuclear dynamics at the distance scale $1/m_\rho$.

So, until the theory with explicit Delta degrees of freedom is further developed, or Delta-less theories can be extended to higher order, the πN parameters used in the *NNN* potential should be viewed as only loosely constrained by πN data. Furthermore, EFT extractions of πN parameters from *NN* data (see, e.g., Ref. [5]) and from πN data (see, e.g., Ref. [11]) can be expected to give results that differ by amounts of order $(m_\pi/\Delta M)^2$.

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[1] R. A. M. Klomp, V. G. J. Stoks, and J. J. de Swart, Phys. Rev. C **44**, 1258(R) (1991).
[2] H. Sugawara and F. von Hippel, Phys. Rev. **172**, 1764 (1968).
[3] R. A. Smith and V. R. Pandharipande, Nucl. Phys. **A256**, 327 (1976); R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C **29**, 1207 (1984); R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[4] C. Ordóñez, L. Ray, and U. van Kolck, Phys. Rev. Lett. **72**, 1982 (1994); Phys. Rev. C **53**, 2086 (1996).
[5] M. C. M. Rentmeester, R. G. E. Timmermans, J. L. Friar, and J. J. de Swart, Phys. Rev. Lett. **82**, 4992 (1999); M. C. M. Rentmeester, R. G. E. Timmermans, and J. J. de Swart, Phys. Rev. C. **67**, 044001 (2003).
[6] S. Eidelman *et al.*, Review of Particle Physics, Particle Data Group, Phys. Lett. **B592**, 1 (2004).
[7] J.-I. Fujita and H. Miyazawa, Prog. Theor. Phys. **17**, 360 (1957).
[8] S. C. Pieper, V. R. Pandharipande, R. B. Wiringa, and J. Carlson, Phys. Rev. C **64**, 014001 (2001).
[9] U. van Kolck, Phys. Rev. C **49**, 2932 (1994).
[10] V. Bernard, N. Kaiser, and U.-G. Meissner, Int. J. Mod. Phys. E **4**, 193 (1995).
[11] V. Bernard, N. Kaiser, and U.-G. Meissner, Nucl. Phys. **A615**, 483 (1997; N. Fettes and U.-G. Meissner, *ibid.* **A693**, 693 (2001).
[12] C. Ordóñez and U. van Kolck, Phys. Lett. **B291**, 459 (1992); J. L. Friar, Phys. Rev. C **60**, 034002 (1999)
[13] W. N. Cottingham, M. Lacombe, B. Loiseau, J. M. Richard, and R. Vinh Mau, Phys. Rev. D **8**, 800 (1973); A. D. Jackson, D. O. Riska, and B. Verwest, Nucl. Phys. **A249**, 397 (1975).
[14] S. A. Coon, M. D. Scadron, P. C. McNamee, B. R. Barrett, D. W. E. Blatt, and B. H. J. McKellar, Nucl. Phys. **A317**, 242 (1979).
[15] L. S. Celenza, A. Pantziris, and C. M. Shakin, Phys. Rev. C **46**, 2213 (1992); C. A. da Rocha and M. R. Robilotta, *ibid.* **49**, 1818 (1994); **52**, 531 (1995); Nucl. Phys. **A615**, 391 (1997); J.-L. Ballot, M. R. Robilotta, and C. A. da Rocha, Int. J. Mod. Phys. E **6**, 83 (1997); Phys. Rev. C **57**, 1574 (1998).
[16] H. T. Coelho, T. K. Das, and M. R. Robilotta, Phys. Rev. C **28**, 1812 (1983); M. R. Robilotta and H. T. Coelho, Nucl. Phys. **A460**, 645 (1986); T. Y. Saito and I. R. Afnan, Few Body Syst. **18**, 101 (1995); T. Y. Saito and J. Haidenbauer, Eur. Phys. J. A **7**, 559 (2000); D. P. Murphy and S. A. Coon, Few Body Syst. **18**, 73 (1995).
[17] J. L. Friar, D. Hüber, and U. van Kolck, Phys. Rev. C **59**, 53 (1999).
[18] S. C. Pieper, K. Varga, and R. B. Wiringa, Phys. Rev. C **66**, 044310 (2002).
[19] V. Pascalutsa and D. R. Phillips, Phys. Rev. C **67**, 055202 (2003); **68**, 055205 (2003).
[20] D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001(R) (2003); E. Epelbaum, W. Glöckle, and U.-G. Meißner, Nucl. Phys. **A747**, 362 (2005).
[21] E. Epelbaum, A. Nogga, W. Glöckle, H. Kamada, U.-G. Meißner, and H. Witała, Eur. Phys. J. A **15**, 543 (2002).