

Neutrino oscillations: Measuring θ_{13} including its sign

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In neutrino phenomenology, terms in the oscillation probabilities linear in $\sin\theta_{13}$ lead naturally to the question “How can one measure θ_{13} including its sign?” Here we demonstrate analytically and with a simulation of neutrino data that $\mathcal{P}_{e\mu}$ and $\mathcal{P}_{\mu\mu}$ at $L/E = 2\pi/\Delta_{21}$ exhibit significant linear dependence on θ_{13} in the limit of vacuum oscillations. Measurements at this particular value of L/E can thus determine not only θ_{13} but also its sign, if CP violation is small.

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To incorporate neutrino oscillations, the standard model is conventionally extended by adding a mass term and a mixing matrix. This theory of three-flavor neutrino oscillations has been successful in accommodating the results of neutrino oscillation experiments, save those of the LSND experiment [1]. The theory contains six independent parameters: three mixing angles θ_{jk} , two independent mass-squared differences $\Delta_{jk} := m_j^2 - m_k^2$, and one Dirac CP phase δ . Reference [2], for example, provides a summary of the current knowledge of the values of these parameters and describes future experiments. The long baseline (LBL) K2K experiment [3] is in the process of confirming the results of the Super-K atmospheric experiment [4] by measuring the parameters Δ_{32} and θ_{23} , independent of atmospheric neutrino flux models. Future LBL experiments MINOS [5], OPERA [6], and ICARUS [7] will improve upon the bounds for these parameters. Additionally, a global analysis of these LBL experiments could provide a lower bound on the magnitude of θ_{13} [8]. Future LBL experiments might also resolve the question of mass hierarchy and the level, if any, of CP violation in the neutrino sector [9].

We here examine the related question of how to best measure θ_{13} , including its sign. In Ref. [10], we have shown that even in the presence of matter effects [11], neutrino oscillations can be uniquely and completely parametrized with the following bounds on the angles: the CP phase δ lies in the range $[0, \pi)$; θ_{13} lies in the range $[-\pi/2, \pi/2]$; and the remaining mixing angles lie within the first quadrant. This choice of bounds has two advantages. First, present experiments limit θ_{13} to a small asymmetric region about zero [12,13]. Other choices would break this region into two disconnected regions. Secondly, the CP violating phase is restricted to the first two quadrants; this range is sufficient to characterize all CP violating effects. Terms proportional to $\cos\delta$ are thus able to be used to uniquely determine its value, assuming we have knowledge of all the other parameters.

In what follows, we analytically examine the terms of the neutrino oscillation probability formulas that are first order (linear) in θ_{13} . These terms are proportional to either $\sin\delta$ or $\cos\delta$, as indicated in Refs. [2,14]. It has been suggested [14,15] that the presence of such terms, in part, can explain the excess of electronlike events in the Super-K atmospheric experiment [4]. In this work, we find that experiments that lie in the oscillatory region for the small (solar) mass-squared difference, an L/E on the order of 10^4 m/MeV, are sensitive

to the linear θ_{13} terms. We further find that by judicious choice of the value of L/E the effects of the CP violating phase δ can be suppressed if δ is near zero or π , while at the same time the effect of the linear term in θ_{13} is maximized. Subsequently, we utilize a simulation [13] of the existing neutrino oscillation data to further examine the ability of new data to determine θ_{13} , including its sign. We assume that either CP is conserved or that our choice of the value L/E has provided sufficient suppression of the CP violating terms. Finally, we summarize our conclusions and provide some thoughts on needed future theoretical work.

Here, we provide explicit analytic expressions for three neutrino oscillations valid for the incoherent limit of the atmospheric mass-squared difference. We confine our discussion to vacuum oscillations. For LBL experiments through the earth, we indicate which values of L and E yield the cleanest measurement of θ_{13} by avoiding significant contributions from matter effects [11]. Additionally, we indicate qualitatively the consequences of straying outside these energies and baselines. We use these analytic expressions to examine where the effects of the linear terms can best be seen. As the magnitude of θ_{13} and the mass-scale ratio $\alpha := |\Delta_{21}|/|\Delta_{32}|$ are known to be small, one may expand the oscillation probability formulas about these parameters. In these perturbations (cf. [16]), terms that are linear in θ_{13} are suppressed by a factor of $\alpha \sim 0.03$. From this, one might conclude that effects relevant to the sign of θ_{13} are forever relegated to the realm of the unobservable. Here, this is not the case as we look beyond the valid region of these expansions.

We use the standard representation [17] of the three-neutrino mixing matrix with the notation $c_{jk} = \cos\theta_{jk}$, $s_{jk} = \sin\theta_{jk}$, and δ is the CP violating phase. In a three-neutrino theory, the probability that a neutrino with relativistic energy E and flavor α will be detected a distance L away as a neutrino of flavor β is given by

$$\begin{aligned} \mathcal{P}_{\alpha\beta}(L/E) = & \delta_{\alpha\beta} - 4 \sum_{\substack{j < k \\ j,k=1}}^3 \text{Re}(U_{\alpha j} U_{\alpha k}^* U_{\beta k} U_{\beta j}^*) \sin^2 \varphi_{jk} \\ & + 2 \sum_{\substack{j < k \\ j,k=1}}^3 \text{Im}(U_{\alpha j} U_{\alpha k}^* U_{\beta k} U_{\beta j}^*) \sin 2\varphi_{jk}, \end{aligned} \quad (1)$$

where $\varphi_{jk} := \Delta_{jk} L/4E$ with $\Delta_{jk} := m_j^2 - m_k^2$.

Examining the terms that are linear in $\sin \theta_{13}$ motivates us to consider the limit in which the oscillations due to the mass-squared differences $|\Delta_{32}| \cong |\Delta_{31}| \sim 10^{-3} \text{ eV}^2$ are incoherent while the oscillations due to $|\Delta_{21}| \sim 10^{-5} \text{ eV}^2$ are still relatively coherent. In this limit, we may take

$$\sin^2 \varphi_{23} = \sin^2 \varphi_{13} = \frac{1}{2}, \quad \sin 2\varphi_{23} = \sin 2\varphi_{13} = 0. \quad (2)$$

The oscillation probabilities $\mathcal{P}_{e\mu}$, $\mathcal{P}_{\mu\mu}$, and $\mathcal{P}_{\mu\tau}$, in the limit of incoherent atmospheric oscillations, are then given by

$$\begin{aligned} \mathcal{P}_{e\mu} = & \left[\frac{1}{2} \sin 2\theta_{12} \cos 2\theta_{12} \sin 2\theta_{13} c_{13} \sin 2\theta_{23} c_{\delta} \right. \\ & \left. + \sin^2 2\theta_{12} c_{13}^2 (c_{23}^2 - s_{13}^2 s_{23}^2) \right] \sin^2 \varphi_{12} \\ & + \frac{1}{2} \sin^2 2\theta_{13} s_{23}^2 + 2J \sin 2\varphi_{12}, \end{aligned} \quad (3)$$

$$\begin{aligned} \mathcal{P}_{\mu\mu} = & 1 - \left[\sin 2\theta_{12} \cos 2\theta_{12} \sin 2\theta_{13} c_{13} \sin 2\theta_{23} s_{23}^2 c_{\delta} \right. \\ & + 2 \sin 2\theta_{12} \cos 2\theta_{12} s_{13} \sin 2\theta_{23} \cos 2\theta_{23} c_{\delta} \\ & + (1 - \sin^2 2\theta_{12} c_{13}^2) s_{13}^2 \sin^2 2\theta_{23} \\ & \left. + \sin^2 2\theta_{12} (c_{23}^2 - s_{13}^2 s_{23}^2)^2 \right] \sin^2 \varphi_{12} \\ & - 2c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2), \end{aligned} \quad (4)$$

$$\begin{aligned} \mathcal{P}_{\mu\tau} = & \left[\sin 2\theta_{12} \cos 2\theta_{12} s_{13} (1 + s_{13}^2) \sin 2\theta_{23} \cos 2\theta_{23} c_{\delta} \right. \\ & \times (1 - \sin^2 2\theta_{12} c_{13}^2) s_{13}^2 \sin^2 2\theta_{23} \\ & - \frac{1}{4} \sin^2 2\theta_{12} (1 + s_{13}^2)^2 \sin^2 2\theta_{23} \\ & \left. + \sin^2 2\theta_{12} s_{13}^2 \right] \sin^2 \varphi_{12} \\ & + \frac{1}{2} \sin^2 2\theta_{23} c_{13}^4 + 2J \sin 2\varphi_{12}, \end{aligned} \quad (5)$$

with

$$J = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{13} c_{13} \sin 2\theta_{23} s_{\delta}. \quad (6)$$

As we are interested in the sign of θ_{13} , we isolate those terms that are odd with respect to θ_{13}

$$\begin{aligned} \mathcal{P}_{e\mu}(\theta_{13}) - \mathcal{P}_{e\mu}(-\theta_{13}) = & 4J \sin 2\varphi_{12} \\ & + \sin 2\theta_{12} \cos 2\theta_{12} \sin 2\theta_{13} c_{13} \\ & \times \sin 2\theta_{23} c_{\delta} \sin^2 \varphi_{12}, \end{aligned} \quad (7)$$

$$\begin{aligned} \mathcal{P}_{\mu\mu}(\theta_{13}) - \mathcal{P}_{\mu\mu}(-\theta_{13}) = & -4(c_{23}^2 - s_{13}^2 s_{23}^2) \\ & \times \sin 2\theta_{12} \cos 2\theta_{12} s_{13} \\ & \times \sin 2\theta_{23} c_{\delta} \sin^2 \varphi_{12}, \end{aligned} \quad (8)$$

$$\begin{aligned} \mathcal{P}_{\mu\tau}(\theta_{13}) - \mathcal{P}_{\mu\tau}(-\theta_{13}) = & 4J \sin 2\varphi_{12} + 2 \sin 2\theta_{12} \\ & \times \cos 2\theta_{12} s_{13} (1 + s_{13}^2) \sin 2\theta_{23} \\ & \times \cos 2\theta_{23} c_{\delta} \sin^2 \varphi_{12}. \end{aligned} \quad (9)$$

Note that the sign of θ_{13} exhibits the maximal effect whenever $\sin^2 \varphi_{12}$ is maximal for $c_{\delta} \sim 1$. This occurs whenever $\varphi_{12} = (2n + 1)\pi/2$ or, in other terms, for $L/E = 2(2n + 1)\pi/\Delta_{12}$. These oscillations will be more coherent for the smaller values of n .

This choice is fortuitous in that it also removes from Eqs. (7) and (9) terms dependent on s_{δ} as $\sin 2\varphi_{21} = 0$ whenever φ_{21} is an odd-integer multiple of $\pi/2$. This removes the CP violating terms from consideration. The remaining terms are modulated by c_{δ} . Thus for δ near zero or π , we would

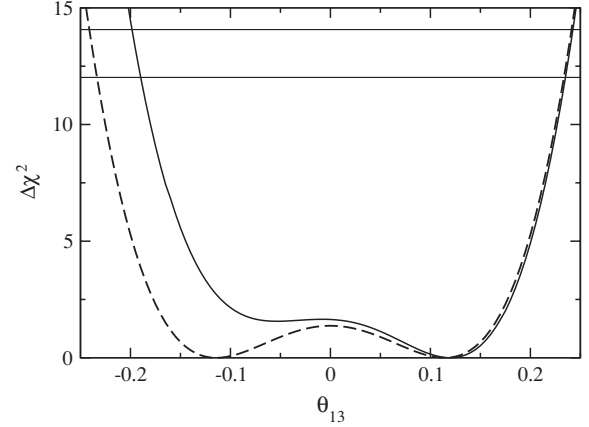


FIG. 1. Value of $\Delta\chi^2 = \chi^2 - \chi_{\min}^2$ versus θ_{13} as extracted from the world's neutrino oscillation data using the simulation of Ref. [13]. All other parameters other than θ_{13} are varied. There are two minima, $\theta_{13} = 0.11$ and -0.04 . Dashed line represents the results of the “one mass-squared dominance” approximation. Horizontal lines represent the 90 and 95% confidence levels.

have a clean measurement of θ_{13} . Also, should CP violation be found to be maximal, then the terms involving c_{δ} vanish.

A consequence of removing the dependence on J is that CP violating effects are suppressed. At these local values of L/E , we have $\mathcal{P}_{\alpha\beta} = \mathcal{P}_{\bar{\alpha}\bar{\beta}}$, where $\bar{\alpha}$ indicates an antineutrino of flavor α ; assuming CPT is invariant, this can be expressed as $\mathcal{P}_{\alpha\beta} = \mathcal{P}_{\beta\alpha}$. This supports our previous statement that it is sufficient to only consider $\mathcal{P}_{e\mu}$, $\mathcal{P}_{\mu\mu}$, and $\mathcal{P}_{\mu\tau}$ in regards to their dependence on the sign of θ_{13} . These probabilities in addition to \mathcal{P}_{ee} , which is a function of θ_{13}^2 , will give us all the other oscillation probabilities at this value of L/E . The remaining probabilities are

$$\mathcal{P}_{e\tau} = 1 - \mathcal{P}_{ee} - \mathcal{P}_{e\mu}, \quad (10)$$

$$\mathcal{P}_{\tau\tau} = 1 - \mathcal{P}_{\mu\tau} - \mathcal{P}_{e\tau}, \quad (11)$$

so that the dependence of $\mathcal{P}_{e\tau}$ on the sign of θ_{13} can be surmised from the statements made concerning $\mathcal{P}_{e\mu}$ and, likewise, the behavior of $\mathcal{P}_{\tau\tau}$ can be surmised from $\mathcal{P}_{\mu\tau}$ and $\mathcal{P}_{e\mu}$. In what follows, we will assume that any CP violation is small so that we may set the phase equal to 0.

We have previously performed [12] a simulation, assuming no CP violation, of the world's neutrino oscillation data. An analysis that includes more recent data [13], in preparation, produces $\Delta\chi^2 := \chi^2 - \chi_{\min}^2$ as pictured in Fig. 1. Included in the analysis are data for neutrinos from the sun [18], from reactors [19], atmospheric neutrinos [20], and beam-stop neutrinos [3]. For one standard deviation, the analysis bounds θ_{13} to lie within $[-0.17, 0.22]$ with two minima located at $\theta_{13} = 0.11$ and -0.04 . For the absolute minimum $\theta_{13} = 0.11$, we find $\theta_{12} = 0.48$, $\theta_{23} = 0.80$, $\Delta_{21} = 7.7 \times 10^{-5} \text{ eV}^2$, and $\Delta_{32} = 2.6 \times 10^{-3} \text{ eV}^2$.

Since \mathcal{P}_{ee} is a function of $\sin^2 \theta_{13}$, the asymmetry seen in Fig. 1 must arise from the atmospheric and K2K data, which involve $\mathcal{P}_{e\mu}$ and $\mathcal{P}_{\mu\mu}$. If we employ the “one mass-squared dominance” approximation, as is often done, we find the

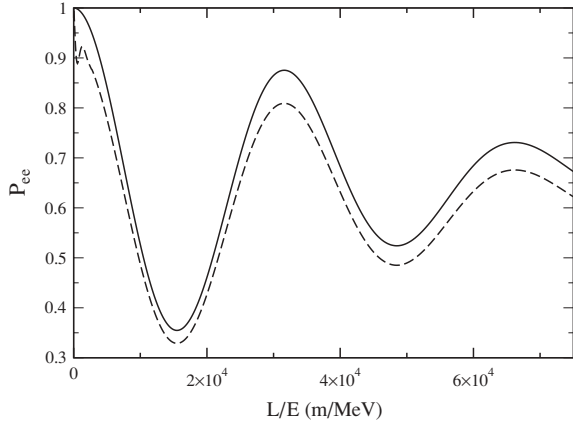


FIG. 2. \mathcal{P}_{ee} versus L/E . Solid curve corresponds to the mixing parameters for a fit with $\theta_{13} = 0$. Dashed curve represents the two curves given by $\theta_{13} = \pm 0.2$, which are identical.

dashed curve in Fig. 1. This approximation gives oscillation probabilities that are a function of $\sin^2 \theta_{13}$.

To demonstrate the relative size of the effect of the sign of θ_{13} , we choose some realistic values for the mixing angles: $\theta_{12} = 0.56$ and $\theta_{23} = 0.78$. The first two peaks of $\sin^2 \varphi_{12}$ occur around $L/E = 1.6 \times 10^4$ m/MeV and 4.8×10^4 m/MeV. For such values of L/E , the oscillations due to Δ_{32} and Δ_{31} would be incoherent. It is clear from Eq. (9) that the screening effect of maximal mixing for θ_{23} results in independence of the sign of θ_{13} for $\mathcal{P}_{\mu\tau}$. For the remaining two oscillation channels, we have a sizable effect. The oscillation probabilities evaluated at the one-standard-deviation points for θ_{13} , -0.17 and 0.22 , are

$$\mathcal{P}_{e\mu}(\theta_{13} = -0.17) = 0.35, \quad \mathcal{P}_{e\mu}(\theta_{13} = 0.22) = 0.50, \quad (12)$$

$$\mathcal{P}_{\mu\mu}(\theta_{13} = -0.17) = 0.37, \quad \mathcal{P}_{\mu\mu}(\theta_{13} = 0.22) = 0.22. \quad (13)$$

The relative differences are more appropriate quantities to consider; for $\mathcal{P}_{\mu\mu}$ we have the most significant effect

$$\frac{\mathcal{P}_{\mu\mu}(\theta_{13} = 0.22) - \mathcal{P}_{\mu\mu}(\theta_{13} = -0.17)}{\mathcal{P}_{\mu\mu}(\theta_{13} = 0.22) + \mathcal{P}_{\mu\mu}(\theta_{13} = -0.17)} = -0.25, \quad (14)$$

while the effect is still large for $\mathcal{P}_{e\mu}$

$$\frac{\mathcal{P}_{e\mu}(\theta_{13} = 0.22) - \mathcal{P}_{e\mu}(\theta_{13} = -0.17)}{\mathcal{P}_{e\mu}(\theta_{13} = 0.22) + \mathcal{P}_{e\mu}(\theta_{13} = -0.17)} = 0.17. \quad (15)$$

We compare the one-sigma extremes of θ_{13} in order to demonstrate the potential size of the effect.

The previous analytic work tells us where to look if we wish to observe the terms in the oscillation formulas that are linear in $\sin \theta_{13}$. We investigate this further by utilizing the analysis from Ref. [13]. We proceed by fixing all of the oscillation parameters except θ_{13} to their values given by an analysis in which θ_{13} is set to zero.

In Figs. 2–4, we present the oscillation probabilities \mathcal{P}_{ee} , $\mathcal{P}_{e\mu}$, and $\mathcal{P}_{\mu\mu}$ as a function of L/E , respectively. In all cases, we assume a Gaussian spread in energy of 20%. In all

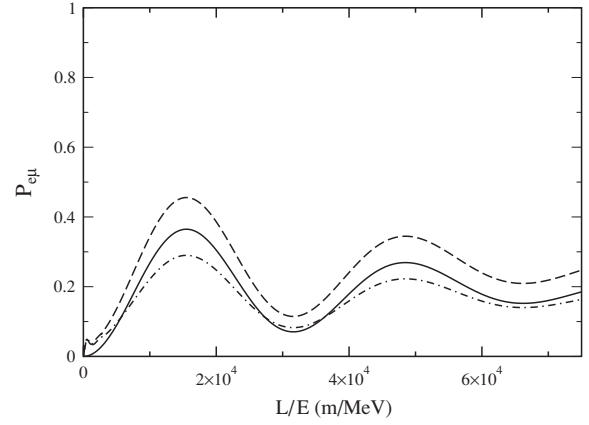


FIG. 3. $\mathcal{P}_{e\mu}$ versus L/E . Solid curve corresponds to the mixing parameters for a fit with $\theta_{13} = 0$. Dashed (dot-dashed) curve corresponds to $\theta_{13} = 0.2$ (-0.2).

cases, curves are presented for $\theta_{13} = 0$ and $\theta_{13} = \pm 0.2$. In Fig. 2 for \mathcal{P}_{ee} there are only two curves, as \mathcal{P}_{ee} is a function of $\sin^2 \theta_{13}$. Thus the curves for $\theta_{13} = \pm 0.2$ are identical. The curve is presented for completeness and to note that there is a measurable dependence of \mathcal{P}_{ee} on the magnitude of θ_{13} near the peak at $L/E = 3.2 \times 10^4$ m/MeV, near the location of KamLAND as indicated in Ref. [13].

In Fig. 3, we verify two facts as derived analytically previously. First, there is a significant linear dependence of $\mathcal{P}_{e\mu}$ on $\sin \theta_{13}$. Secondly, the dependence is maximal at $\varphi_{12} = \pi/2$ and $3\pi/2$. The optimal value of L/E to measure θ_{13} , including its sign, is thus $L/E = 2\pi/\Delta_{21}$. The linear term in θ_{13} clearly dominates near the maximum of the oscillation. However, the constant term in Eq. (3) proportional to $\sin^2 \theta_{13}$ becomes relevant whenever the probability is minimal.

In Fig. 4, we see that $\mathcal{P}_{\mu\mu}$ is a somewhat better quantity to measure than $\mathcal{P}_{e\mu}$, as here the linear term is even more dominant. Again, the linear dependence on θ_{13} is maximal at $\varphi_{12} = \pi/2$ and $3\pi/2$. Figure 1 demonstrates the inadequacy of parametrizing oscillation parameters as a function of $\sin^2 \theta_{13}$. Figures 3 and 4 demonstrate this more dramatically as the

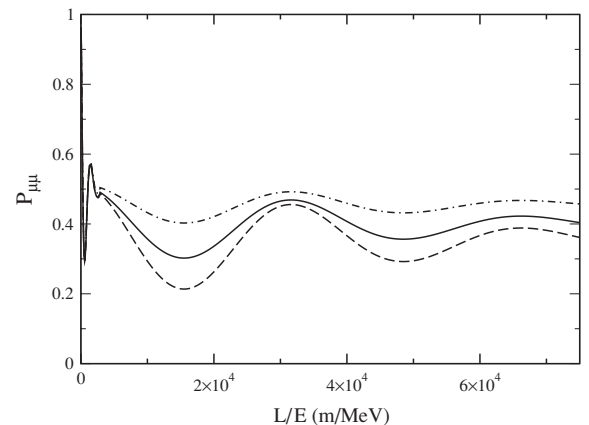


FIG. 4. Same as Fig. 3, except the muon survival probability $\mathcal{P}_{\mu\mu}$ is presented.

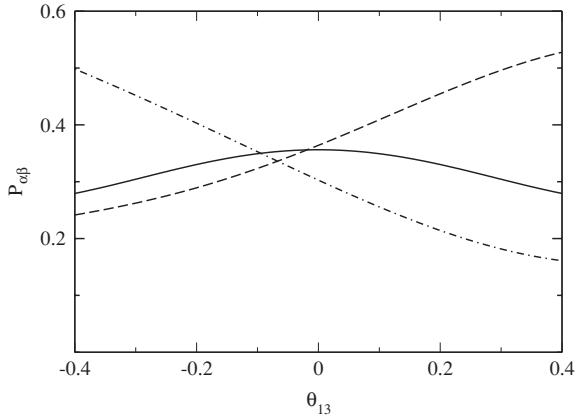


FIG. 5. The oscillation probabilities versus the mixing angle θ_{13} with the other parameters fixed at their value from a fit with $\theta_{13} = 0$. Solid curve is \mathcal{P}_{ee} ; dashed curve, $\mathcal{P}_{e\mu}$; dot-dashed curve, $\mathcal{P}_{\mu\mu}$. We take $L/E = 1.6 \times 10^4$ m/MeV where the effect of the linear terms in θ_{13} are maximized.

linear term in θ_{13} is found to dominate the quadratic term for $\mathcal{P}_{e\mu}$ and $\mathcal{P}_{\mu\mu}$ in this region of L/E .

The ideal measurement would be to determine $\mathcal{P}_{\mu\mu}$ over a range for L/E from approximately 1.6×10^3 to 1.6×10^4 m/MeV. The lower end of this range provides an overall calibration point where the effect of nonzero θ_{13} is small, whereas the upper end is the point where the effect is largest. To avoid the further complications of the earth's Mikheev-Smirnov-Wolfenstein (MSW) effect, the energy should be less than about 100 MeV, the energy of the MSW resonance in the mantle of the earth. For an upper limit on the energy of 50 MeV, this sets an ideal value $L = 80$ km and a range in energy for the muon neutrinos of 5 to 50 MeV. For a smaller value of L , the range of the neutrino energies would have to be proportionally smaller. More likely one's neutrino source would produce neutrinos with energies of at least a few hundred MeV. Here, matter effects become significant, though the qualitative features of the curves in Figs. 3 and 4 can be salvaged. The most notable changes for oscillations through the mantle in this scenario are the increased frequency of oscillations, necessitating a suitable change in the baseline and the adjustment of the matter mixing angle θ_{12}^m . However, the difference between the matter mixing angle θ_{12}^m and the vacuum value of the mixing angle can be minimized by choosing a neutrino energy twice that of the resonance energy. For such a situation, the qualitative features of the oscillations presented above remain the same.

Finally, in Fig. 5, we present the values of \mathcal{P}_{ee} , $\mathcal{P}_{e\mu}$, and $\mathcal{P}_{\mu\mu}$ as a function of θ_{13} . The other parameters are fixed at their values for a fit with $\theta_{13} = 0$. We use a value of $L/E = 1.6 \times 10^4$ m/MeV, chosen to maximize the relative importance of the linear terms in θ_{13} . Again, \mathcal{P}_{ee} is quadratic in θ_{13} . However, $\mathcal{P}_{e\mu}$ and $\mathcal{P}_{\mu\mu}$ are nearly linear in θ_{13} over this rather large range of $-0.4 \leq \theta_{13} \leq 0.4$. The near linearity reinforces our observation that the measurement of $\mathcal{P}_{e\mu}$ and $\mathcal{P}_{\mu\mu}$ at $L/E = 2\pi/\Delta_{21}$ is a way of determining θ_{13} , including its sign.

The phenomenology of neutrino oscillations, in the absence of CP violation and various exotica such as a fourth sterile neu-

trino, has been performed in the context of determining three mixing angles and two mass-squared differences. Historically, the results were presented in terms of $\sin^2 \theta_{13}$, which yields an upper limit. When the bounds on the mixing angles were explicitly quoted, they were all stated to be bounded by $\pi/2$. In Ref. [21], it was shown that in the physical case which necessarily includes the MSW matter effect, a second branch corresponding to $\delta = \pi$ must also be included. In Ref. [10], we extended the derivation, again in the physical case which necessarily includes the MSW matter effect, to show that only the $\delta = 0$ branch is required if the mixing angle θ_{13} is allowed to vary from $-\pi/2$ to $+\pi/2$. There are two advantages to this convention. First, the allowed region for θ_{13} consists of a single region that extends on either side of 0, rather than two disjoint regions, one for $\delta = 0$ and one for $\delta = \pi$. Secondly, in the presence of CP violation, the CP phase is bounded by 0 and π . Thus, a measurement that is sensitive to $\cos \delta$ could uniquely determine the quadrant in which δ lies.

The natural question that arises from this formal work is whether you can measure θ_{13} including its sign. In Ref. [12] we demonstrated that for a model analysis of the world's data, the χ^2 space was asymmetric in θ_{13} , thus demonstrating that such a measurement might be possible. A more recent analysis [13], which includes recent data, verified this. Here we address the question directly and demonstrate that the answer to our question is yes—the mixing angle θ_{13} does appear linearly in the oscillation probabilities at a level where it dominates the quadratic term for the correctly chosen oscillation probabilities when measured at the correct value of L/E .

We find that this is true for measurements of $\mathcal{P}_{e\mu}$ and $\mathcal{P}_{\mu\mu}$ at $L/E = 2\pi/\Delta_{21}$. This fortuitously also corresponds to a value of L/E where the contribution from the CP violating effects is minimal. A consequence of this work is that it reinforces our earlier thesis that parametrizing neutrino oscillation probabilities as a function of $\sin^2 \theta_{13}$ is inadequate.

There remains additional work to be done. We find that present data indicate a correlation between the allowed value of θ_{13} and θ_{23} when the linear terms are included in the analysis. We are investigating the implications for θ_{13} that will result from measurements of θ_{23} . We have assumed no CP violation in our model analysis. Since the CP phase and θ_{13} are interrelated, this needs further clarification. For cleanliness of interpretation, we propose to conduct experiments in a region of E where the earth's MSW effect is small. We are examining in a quantitative way how the earth's MSW effect might modify an analysis should this be necessary. We, like others, have excluded the LSND [1] experiments. If MiniBoone verifies the LSND results, then a whole new physics will be needed to reach a consistent understanding of neutrino oscillations.

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