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Residual interaction effects on deeply bound pionic states in Sn and Pb isotopes

N. Nose-Togawa,[1] H. Nagahiro,[1] S. Hirenzaki,[2] and K. Kume[2]
[1]*Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567–0047, Japan*
[2]*Department of Physics, Nara Women's University, Nara 630–8506, Japan*
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We have studied the residual interaction effects theoretically on the deeply bound pionic states in Pb and Sn isotopes. We need to evaluate the residual interaction effects carefully to deduce the nuclear medium effects for pion properties, which are believed to provide valuable information on nuclear chiral dynamics. The *s*- and *p*-wave πN interactions are used for the pion-nucleon residual interactions. We show that the complex energy shifts are around $[(10\text{–}20) + i(2\text{–}7)]$ keV for 1*s* states in Sn, which should be taken into account in the analyses of the high precision data of deeply bound pionic 1*s* states in Sn isotopes.

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Deeply bound pionic states in heavy nuclei were predicted to be quasistable by Friedman and Soff [1] and Toki and Yamazaki [2] independently. According to the theoretical predictions for the formation reactions [3,4], the deeply bound pionic $2p$ states are observed in ^{207}Pb nucleus experimentally in $(d,^3\mathrm{He})$ missing mass spectra [5,6]. After this discovery, precise data of the deeply bound pionic $1s$ and $2p$ states in ^{205}Pb were also observed [7,8]. Furthermore, Umemoto *et al*. predicted that the Sn isotopes are ideal target nuclei to observe $1s$ pionic states and to deduce the isotope shifts of the pionic atoms [9]. Recently, K. Suzuki *et al*. performed the experiments of the $(d,^3\mathrm{He})$ reactions on the Sn targets and succeeded to observe deeply bound $1s$ pionic states in Sn isotopes quite precisely [10]. Experimental errors for the binding energies of the $1s$ states are around $\Delta E \sim 20$ keV.

From these experiments, we can study the s-wave part of the pion-nucleus interaction, which is very interesting because the *s*-wave strength is expected to provide information on the pion mass excess and pion decay constant f_π in the nuclear medium through the Tomozawa-Weinberg theorem [11,12]. The f_π^2 is the order parameter of chiral symmetry breaking of quantum chromodynamics (QCD) and is connected to the quark condensate through the Gell-Mann-Oakes-Renner relation [13]. Thus, it is very interesting to determine the *s*-wave potential parameters from deeply bound pionic atoms. For this purpose, it is required to observe the pionic $1s$ states because these states depend predominately on the *s*-wave potential [9] and, furthermore, the $1s$ states in heavy nuclei $(N > Z)$ provide key information on the isovector part of the *s*-wave potential. As described above, K. Suzuki *et al*. performed the experiment and obtained excellent new data of the deeply bound pionic $1s$ states in Sn isotopes [10], which are very suited for the purpose.

However, because we make use of the single neutron pickup $(d,^3\mathrm{He})$ reactions, the final pionic states are the pion plus neutron-hole state $[\pi \otimes n^{-1}]_J$ with respect to the target nuclei [4,7,9]. So far all theoretical calculations and analyses of the data, except for Ref. [14], postulate that the residual interaction effects are small enough and can be neglected. This is actually true for pionic atoms in ^{207}Pb case because experimental errors are significantly larger than the estimated residual interaction effects [14]. However, in the present cases for the $1s$ states in Sn isotopes, it is not obvious whether the effects are negligible because the experimental errors for Sn cases are comparable to the calculated residual interaction effects for ^{207}Pb [14]. Thus, it is very important to evaluate the residual interaction effects for $1s$ states in Sn isotopes to deduce physical quantities related to pion behaviors in the nuclear medium from the observed spectra. In this report we evaluate the residual interaction effects on pionic states in ^{207}Pb, ^{205}Pb, and Sn isotopes by taking into account both *s*-wave and *p*-wave π-N interactions. We will also describe the theoretical formula to evaluate the residual interaction effects.

We consider the pionic states whose Hamiltonian is expressed as follows:

$$H = \sum_i \omega_i c_i^\dagger c_i + \sum_i \varepsilon_i a_i^\dagger a_i + \sum_{ijk\ell} V_{ji,\ell k} c_j^\dagger a_i^\dagger a_k c_\ell, \qquad (1)$$

where the $c^\dagger$ (c) and $a^\dagger$ (a) are creation (annihilation) operators of the pion and the nucleon respectively. The indices characterize their quantum numbers. In Eq. (1), ω_i is the pion binding energy, ε_i the single-nucleon energy, and $V_{ji,\ell k}$ indicate the matrix elements of the pion-nucleon residual interaction.

Because we make use of the single-neutron pickup $(d,^3$ He) reaction, each final state is the pion-plus-single-neutron-hole state with respect to the target nucleus. To calculate the residual-interaction effects between pion and the neutron hole, we introduce a neutron-hole creation (annihilation) operator $b^\dagger(b)$, which are defined as follows:

$$a_{jm}^\dagger = (-1)^{j-m} b_{j-m}. \qquad (2)$$

Here, we show the angular momentum quantum numbers explicitly, with the isospin indices abbreviated. The third term in Eq. (1) can be rewritten as follows:

$$\sum_{ijk\ell} V_{ji,\ell k} c_j^\dagger a_i^\dagger a_k c_\ell \longrightarrow \hat{V} = \sum_{ijk\ell} \overline{V_{jk,\ell i}} c_j^\dagger b_k^\dagger b_i c_\ell, \qquad (3)$$

where we discarded the core contribution, which is already included in the second term of Eq. (1), and $\overline{V_{jk,\ell i}}$ are the interaction matrix elements between pion and the nucleon hole, which correspond to the Pandya transformation of the pion-nucleon interaction.

The state of pionic atom with a single neutron-hole state can be expressed as follows:

$$|\pi, N_\alpha; J\rangle = \left(c_\pi^\dagger \otimes b_{N_\alpha}^\dagger\right)^J |0\rangle, \qquad (4)$$

where the suffixes π and N_α specify the quantum numbers of the pion and the neutron-hole respectively, and J is the total angular momentum of the pion-nucleus system. The matrix elements of the Hamiltonian with respect to these states are expressed as follows:

$$\langle \pi', N_\beta; J|H|\pi, N_\alpha; J\rangle = (\omega_\pi - \varepsilon_\alpha)\delta_{\pi,\pi'}\delta_{\alpha,\beta} \times \langle \pi', N_\beta; J|\hat{V}|\pi, N_\alpha; J\rangle, \quad (5)$$

where ω_π is the eigenenergy of the pionic state specified as π and ε_α is the separation energy of neutron from the target nucleus.

As the residual interaction, we consider the following pion-nucleon interaction:

$$V = -\frac{2\pi}{m_\pi}[b_0 + b_1 \boldsymbol{\tau}\cdot\mathbf{I} + (c_0 + c_1\boldsymbol{\tau}\cdot\mathbf{I})\nabla\cdot\nabla]\delta(\mathbf{r}). \quad (6)$$

Here, we have taken into account both the s- and the p-wave pion-nucleon interaction. The gradient operators act on the right- and the left-hand-side pion wave functions, respectively. In Ref. [14], we reported the results with the s-wave contribution only. We fix the parameters as $b_0 = -0.0283m_\pi^{-1}$, $b_1 = -0.12m_\pi^{-1}$, $c_0 = 0.223m_\pi^{-3}$, $c_1 = 0.25m_\pi^{-3}$, which are taken from Ref. [15]. The pion-nucleon interaction adopted here is consistent with the pion-nucleus optical potential used to calculate the pion wave functions. By folding the pion-nucleon interaction with the nuclear density, we obtain exactly the same real part of the pion-nucleus optical potential except for the small corrections coming from the transformation of the center-of-mass coordinates. As for the imaginary part, we simply assume the pion-nucleon residual interaction is real and has no absorptive effects because two nucleon degrees of freedom are necessary at least in absorptive processes. The effects of the pion absorption by the core nucleus are incorporated phenomenologically as the density quadratic term in the imaginary parts of the pion-nucleus optical potential as usual. In this theoretical framework, we do not evaluate the absorptive effects because of processes, including both nucleon-hole and nucleon-particle degrees of freedom simultaneously, which we expect to be small.

The interaction matrix elements between the pion and the nucleon hole are expressed as follows:

$$\begin{aligned}\langle \pi', N_\beta; J|\hat{V}|\pi, N_\alpha; J\rangle &= -\frac{1}{2m_\pi}(-1)^{-J+j_\alpha+j_\beta+1/2} \\ &\times \sqrt{(2j_\alpha+1)(2j_\beta+1)(2\ell_\alpha+1)(2\ell_\beta+1)(2\ell'_\pi+1)(2\ell_\pi+1)} \\ &\times \sum_L (-1)^L \begin{Bmatrix} \ell'_\pi & j_\beta & J \\ j_\alpha & \ell_\pi & L \end{Bmatrix} \begin{Bmatrix} \ell_\alpha & j_\alpha & \frac{1}{2} \\ j_\beta & \ell_\beta & L \end{Bmatrix} \\ &\times (\ell_\beta 0 \ell_\alpha 0 \mid L0)(\ell_\pi 0 \ell'_\pi 0 \mid L0) \\ &\times \Bigg[(b_0+b_1)\int_0^\infty dr r^2 R^*_{\ell_\beta}(r)R_{\ell_\alpha}(r)R_{\ell'_\pi}(r)R_{\ell_\pi}(r) \\ &+ (c_0+c_1)\int_0^\infty dr r^2 R^*_{\ell_\beta}(r)R_{\ell_\alpha}(r)\Bigg\{\left(\frac{dR_{\ell'_\pi}(r)}{dr}\right)\left(\frac{dR_{\ell_\pi}(r)}{dr}\right) \\ &+ \frac{\ell_\pi(\ell_\pi+1)+\ell'_\pi(\ell'_\pi+1)-L(L+1)}{2}\frac{R_{\ell'_\pi}(r)R_{\ell_\pi}(r)}{r^2}\Bigg\}\Bigg], \end{aligned} \quad (7)$$

TABLE I. Nuclear density parameters used in the present calculations.

Nucleus	$r_p(=r_n)$[fm]	a_p[fm]	a_n[fm]
^{116}Sn	5.417	0.5234	0.5837
^{120}Sn	5.459	0.5234	0.6014
^{124}Sn	5.491	0.5234	0.6175
^{132}Sn	5.548	0.5234	0.6487
^{206}Pb	6.631	0.5234	0.6389
^{208}Pb	6.647	0.5234	0.6439

where $R_{\ell_\pi}(r)$ and $R_{\ell_\alpha}(r)$ are the radial wave function of the pion and the neutron hole, respectively. We consider the pionic orbits of the $1s$, $2s$, $2p$, $3s$, $3p$, and $3d$, states which are obtained by solving the Klein-Gordon equation numerically. Because the Klein-Gordon equation includes the complex optical potential that makes the Hamiltonian non-Hermite and makes the eigenenergies complex, and hence we normalize the pionic wave function on the proper orthonormal condition according to the prescription in Ref. [16].

For the proton and the neutron distributions, we use the two-parameter Fermi type density distribution as follows:

$$\rho_{p(n)} = \frac{\rho_0}{1+\exp[(r - r_{p(n)})/a_{p(n)}]} \quad (8)$$

and assume the same radius parameters of the proton and the neutron. These radius parameters and the proton diffuseness parameter are taken from the experimental values in Ref. [17]. For the diffuseness parameter of the neutron we adopt the values in Ref. [18]. These density parameters are compiled in Table I.

For the neutron-hole states, we have taken into account the orbits $p^{-1}_{1/2}, f^{-1}_{5/2}, p^{-1}_{3/2}, i^{-1}_{13/2}$ for ^{205,207}Pb and $d^{-1}_{3/2}, s^{-1}_{1/2}, h^{-1}_{11/2}, g^{-1}_{7/2}, d^{-1}_{5/2}$ for ^{115,119,123,131}Sn. These states are calculated using a potential of Woods-Saxon form in Ref. [19]. The neutron separation energies ε_α are determined from experimental data as far as possible. We can disregard the spreading widths of the neutron-hole states that are considerably narrower than the width of the pionic states and little affect the results here. For ^{207}Pb, the separation energies are obtained from the *Table of Isotopes* [20] as the excited energies of the levels coupled to the neutron pickup reactions. For open shell nuclei ^{205}Pb and ^{115,119,123}Sn, we adopt the excited states observed in single-neutron pickup reactions [21,22] and use the observed excitation energies to deduce the neutron separation energies. In the case that there exist plural states assigned to the same spin and parity, we choose the level which has a larger spectroscopic factor. As for ^{131}Sn, we use the separation energies deduced from the systematics in Ref. [23] because no data of the neutron pickup reactions are available. We diagonalize the matrix elements of the whole Hamiltonian expressed in Eq. (5). Then, we can calculate the complex energy shifts defined as follows:

$$\Delta E \equiv E(\pi, N_\alpha; J) - (\omega_\pi - \epsilon_\alpha), \quad (9)$$

TABLE II. Calculated complex energy shifts because of the residual interaction in ^{207}Pb. The results are shown in units of kilo-electron-volts for $[(1s)_\pi \otimes j_n^{-1}]_J$ and $[(2p)_\pi \otimes j_n^{-1}]_J$, including the s-wave and the p-wave parts of pion neutron-hole residual interaction. The values in the parentheses are the results obtained only with the s-wave residual interaction. Experimental errors are taken from Ref. [6].

$1s$		$2p$	
		$J = 1/2$	$-7.7 - 2.1i$
$p_{1/2}^{-1}$	$-13.3 - 2.9i$		$(-9.4 - 2.7i)$
	$(-14.2 - 3.1i)$	$J = 3/2$	$-7.7 - 2.1i$
			$(-9.3 - 2.7i)$
		$J = 1/2$	$-15.8 - 4.4i$
			$(-17.6 - 5.0i)$
$p_{3/2}^{-1}$	$-12.9 - 2.9i$	$J = 3/2$	$-0.16 + 0.2i$
	$(-13.8 - 3.1i)$		$(-1.7 - 0.5i)$
		$J = 5/2$	$-8.9 - 2.4i$
			$(-10.1 - 3.0i)$
		$J = 3/2$	$-13.9 - 4.6i$
			$(-15.9 - 5.2i)$
$f_{5/2}^{-1}$	$-13.1 - 3.5i$	$J = 5/2$	$0.90 + 0.5i$
	$(-14.1 - 3.6i)$		$(-0.8 - 0.3i)$
		$J = 7/2$	$-9.5 - 3.1i$
			$(-11.4 - 3.8i)$
		$J = 11/2$	$-13.4 - 7.6i$
			$(-17.2 - 7.4i)$
$i_{13/2}^{-1}$	$-14.8 - 6.3i$	$J = 13/2$	$2.1 + 1.0i$
	$(-17.2 - 5.7i)$		$(-0.2 - 0.1i)$
		$J = 15/2$	$-11.3 - 6.5i$
			$(-14.9 - 6.4i)$
Exp. error		± 20 (stat.) ± 120 (sys.)	
		$\pm 30i$ (stat.) $\pm 30i$ (sys.)	

TABLE III. Calculated complex energy shifts because of the residual interaction in ^{205}Pb. The results are shown in units of kilo-electron-volts for $[(1s)_\pi \otimes j_n^{-1}]_J$ and $[(2p)_\pi \otimes j_n^{-1}]_J$, including the s-wave and the p-wave parts of pion neutron-hole residual interaction. Experimental errors are taken from Ref. [8].

$1s$		$2p$	
$p_{1/2}^{-1}$	$-13.6 - 3.1i$	$J = 1/2$	$-8.3 - 2.5i$
		$J = 3/2$	$0.4 + 0.2i$
		$J = 1/2$	$-15.7 - 4.4i$
$p_{3/2}^{-1}$	$-13.2 - 3.1i$	$J = 3/2$	$-0.1 + 0.3i$
		$J = 5/2$	$-9.1 - 2.5i$
		$J = 3/2$	$-22.6 - 7.3i$
$f_{5/2}^{-1}$	$-13.5 - 3.7i$	$J = 5/2$	$0.9 + 0.6i$
		$J = 7/2$	$-9.8 - 3.3i$
		$J = 11/2$	$-13.9 - 8.0i$
$i_{13/2}^{-1}$	$-15.4 - 6.6i$	$J = 13/2$	$2.2 + 1.1i$
		$J = 15/2$	$-11.7 - 6.8i$
Exp. error	$+86i$ ± 61 $-77i$		$+30i$ ± 45 $-31i$

where $E(\pi, N_\alpha; J)$ are the corresponding eigenenergies of the pion-nucleus system.

We will now show the numerical results of the residual interaction effects for the pionic atoms. As we explained above, we include six pionic states and 4(5) neutron states for Pb(Sn) isotopes in the present calculation to evaluate the matrix elements. Because the residual interaction effects are larger for deeper bound pionic states, we show the numerical results for pionic $1s$ and $2p$ states.

In Table II, we show the complex energy shifts of the pionic states on ^{207}Pb. To see the contributions from πN p-wave interactions, which are newly included in present work, we show both results with only s-wave interaction and with s- and p-wave residual interactions. Here, because we have used more realistic neutron wave functions and nuclear density distributions than those used in Ref. [14], the present results are slightly different from those in the previous work. In Table II, the results only with the s-wave residual interaction are written in the parentheses and have the same negative sign for all configurations, which means that the s-wave residual interaction effects make the bound states deeper and the level widths wider. This fact can be understood intuitively as the result of the lack of the repulsive s-wave interaction from the removed one neutron. The calculated results with both s- and p-wave residual interactions are also shown in the same table. We find that the p-wave interaction has the opposite effects to the s-wave interaction in general and the complex energy shifts become less attractive and absorptive in almost all configurations except for a few cases. This tendency also can be understood as the result of the missing attractive p-wave interaction from one picked-up neutron. In this case with ^{207}Pb nuclei, the calculated shifts are reasonably smaller than the experimental errors and we think we can safely neglect the residual interaction effects as concluded in Ref. [14].

We show the calculated results in Table III for pionic atoms in ^{205}Pb with the experimental errors reported in Ref. [8]. In this case, the largest shifts appears for $[(2p)_\pi \otimes f_{5/2}^{-1}]_{3/2}$ configuration and is around half of the corresponding experimental error for the real part. However, this configuration only has minor contribution to the formation cross section [7,8]. The dominant contribution to the formation process of pionic $2p$ state is from $[(2p)_\pi \otimes p_{3/2}^{-1}]$ configurations and the residual interaction shifts for this configuration are evaluated to be around 1/3 or less of the experimental error in real part. The each level corresponding to different total angular momentum J has different energy shifts and splits by a few kilo-electron-volts, which will be seen as a broadening of the resonance peak because each level overlaps because of their large natural widths. As for the pionic $1s$ state, the residual-interaction effects are around $1/4 \sim 1/5$ of experimental error for the real part and smaller by about 10 for the imaginary part. Therefore, we can also conclude that the residual interaction effects can be neglected safely for pionic atoms in ^{205}Pb case.

For Sn isotopes, we have made similar calculation for ^{115}Sn, ^{119}Sn, and ^{123}Sn. In these cases together with ^{205}Pb

TABLE IV. Calculated complex energy shifts because of the residual interaction in ^{115,119,123}Sn. The results are shown in units of kilo-electron-volts for $[(1s)_\pi \otimes j_n^{-1}]_J$ and $[(2p)_\pi \otimes j_n^{-1}]_J$, including the s-wave and the p-wave parts of pion neutron-hole residual interaction. Experimental errors are taken from Ref. [10].

	^{115}Sn			^{119}Sn			^{123}Sn		
	$1s$	$2p$		$1s$	$2p$		$1s$	$2p$	
$s_{1/2}^{-1}$	-15.4	$J=1/2$	$-4.0-1.1i$	-13.5	$J=1/2$	$-5.2-2.0i$	-12.3	$J=1/2$	$-3.2-0.7i$
	$-4.2i$	$J=3/2$	$-4.0-1.1i$	$-3.3i$	$J=3/2$	$-3.8-1.1i$	$-2.4i$	$J=3/2$	$-3.5-0.8i$
$d_{3/2}^{-1}$	-15.9	$J=1/2$	$-9.1-3.1i$	-14.3	$J=1/2$	$-7.0-1.6i$	-12.8	$J=1/2$	$-8.1-2.5i$
	$-4.8i$	$J=3/2$	$0.3+0.3i$	$-3.7i$	$J=3/2$	$0.4+0.3i$	$-2.9i$	$J=3/2$	$0.2+0.1i$
		$J=5/2$	$-5.2-1.8i$		$J=5/2$	$-4.6-1.4i$		$J=5/2$	$-4.3-1.2i$
$g_{7/2}^{-1}$	-15.4	$J=5/2$	$-6.0-3.8i$	-13.0	$J=5/2$	$-5.5-3.3i$	-11.1	$J=5/2$	$-4.9-2.8i$
	$-7.3i$	$J=7/2$	$1.5+0.8i$	$-5.8i$	$J=7/2$	$1.3+0.7i$	$-4.6i$	$J=7/2$	$1.2+0.6i$
		$J=9/2$	$-4.4-2.9i$		$J=9/2$	$-3.9-2.4i$		$J=9/2$	$-3.5-2.0i$
$h_{11/2}^{-1}$	-18.3	$J=9/2$	$-7.7-4.0i$	-16.0	$J=9/2$	$-6.9-3.5i$	-14.1	$J=9/2$	$-6.3-3.0i$
	$-7.2i$	$J=11/2$	$1.7+0.8i$	$-6.0i$	$J=11/2$	$1.5+0.7i$	$-5.1i$	$J=11/2$	$1.4+0.6i$
		$J=13/2$	$-6.2-3.3i$		$J=13/2$	$-5.6-2.8i$		$J=13/2$	$-5.1-2.5i$
$d_{5/2}^{-1}$	-15.1	$J=3/2$	$-7.6-2.6i$	-13.6	$J=3/2$	$-7.1-2.2i$	-12.2	$J=3/2$	$-6.5-1.9i$
	$-4.8i$	$J=5/2$	$1.0+0.6i$	$-3.7i$	$J=5/2$	$0.9+0.4i$	$-2.8i$	$J=5/2$	$0.8+0.3i$
		$J=7/2$	$-5.0-1.7i$		$J=7/2$	$-4.6-1.4i$		$J=7/2$	$-4.3-1.2i$
Exp.	± 24			± 18			± 18		
		—			—			—	
error	$\pm 44i$			$\pm 40i$			$\pm 36i$		

case, the target nuclei are not closed and thus the description of the nuclear structure is much more complicated. The purpose of the present calculation is, however, not to make detailed comparison with the experiment but to estimate the size of the correction coming from the effects of the residual interaction between pion and the residual nucleus. Then, we simply assumed that the residual nucleus Ψ_r consists of a single-hole state with respect to the target nucleus Ψ_i as follows:

$$\Psi_r = C b_\alpha^\dagger \Psi_i, \tag{10}$$

and we simply assumed the constant $C = 1$ to estimate the largest possible residual interaction effects. The calculated results for the $^{115-123}$Sn are compiled in Table IV. As can be seen, the residual interaction shifts are comparable to the experimental errors of the real part for the pionic $1s$ states, which are the most important states and have dominant contributions for the formation reaction [9,10]. Typically, the real energy shifts are around 15 keV and the imaginary shifts are around 5 keV for pionic $1s$ states. The residual interaction effects slightly decrease for heavier Sn isotopes, because the binding energies of $1s$ pionic states are smaller and less bound for heavier Sn isotopes [9]. We also show the calculated results for the pionic states in ^{131}Sn in Table V, which has the single neutron-hole configuration with respect to the doubly closed-shell structure. The results are close to those of the other Sn isotopes.

In the pionic atom formation spectra of the $(d,^3\mathrm{He})$ reactions on Sn targets, the dominant configuration is $[(1s)_\pi \otimes s_{1/2}^{-1}]_{1/2}$ [9]. Thus, the residual interaction effects of this configuration should be considered carefully. As we can see in Table IV, the residual interaction effects on the $[(1s)_\pi \otimes s_{1/2}^{-1}]_{1/2}$ configuration are slightly smaller than experimental error for all isotopes. Hence, we could just manage to neglect the residual interaction effects again. However, the

TABLE V. Calculated complex energy shifts because of the residual interaction in ^{131}Sn. The results are shown in units of kilo-electron-volts for $[(1s)_\pi \otimes j_n^{-1}]_J$ and $[(2p)_\pi \otimes j_n^{-1}]_J$, including the s-wave and the p-wave parts of the pion neutron-hole residual interaction.

	$1s$	$2p$	
$s_{1/2}^{-1}$	$-10.5-1.3i$	$J=1/2$	$-3.2-0.6i$
		$J=3/2$	$-3.3-0.6i$
$d_{3/2}^{-1}$	$-10.4-2.1i$	$J=1/2$	$-7.1-2.0i$
		$J=3/2$	$0.2+0.0i$
		$J=5/2$	$-3.8-1.1i$
$g_{7/2}^{-1}$	$-6.5-1.6i$	$J=5/2$	$-3.0-1.2i$
		$J=7/2$	$0.9+0.4i$
		$J=9/2$	$-2.1-0.8i$
$h_{11/2}^{-1}$	$-9.6-2.6i$	$J=9/2$	$-4.6-1.8i$
		$J=11/2$	$1.1+0.4i$
		$J=13/2$	$-3.7-1.4i$
$d_{5/2}^{-1}$	$-9.9-1.9i$	$J=3/2$	$-5.8-1.5i$
		$J=5/2$	$0.6+0.2i$
		$J=7/2$	$-3.9-1.1i$

magnitude of the real energy shifts are more or less comparable to those of the experimental error and should be taken into account seriously in analyses of data with higher precision than presented in Ref. [10].

In summary, we have evaluated the complex energy shifts of the deeply bound pionic states because of the residual interaction in Pb and Sn isotopes. We have shown the numerical results that include both s-wave and p-wave πN residual interaction effects. For the open-shell nuclei, we have assumed a one-neutron-hole configuration as described in Eq. (10). The present results show that the sizes of the residual interaction effects are slightly smaller than the experimental errors presented in Ref. [10] for 1s pionic states in Sn isotopes. Hence, we could conclude that we can neglect the residual interaction effects in the analyses of data in Ref. [10] for deeply bound pionic 1s states in Sn as in the cases of Pb. However, the magnitude of the residual interaction effects are more or less comparable to the experimental errors in the latest data and the effects should be taken into account seriously in analyses of data with higher accuracy than Ref. [10]. We think that it is essentially important to study deeply bound pionic atoms in future to deduce quantitative information on nuclear chiral dynamics.

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