Comment on "Width of ^{12}O(g.s.)"

F. C. Barker
Department of Theoretical Physics, Research School of Physical Sciences and Engineering, The Australian National University, Canberra, Australian Capital Territory 0200, Australia
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Values calculated by Fortune and Sherr [Phys. Rev. C **68**, 034309 (2003)] for the width of the ^{12}O ground state because of either sequential decay through ^{11}N(g.s.) or ^{2}He decay are appreciably larger than earlier estimates. This may be at least in part because of incorrect normalization of their convolution functions.

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Fortune and Sherr [1] (hereafter referred to as FS) have calculated contributions to the width of the ^{12}O(g.s.) from sequential decay through the ^{11}N(g.s.) and from simultaneous (^{2}He) decay. FS say that their calculation of the sequential width is in the manner of Refs. [2,3], but that the R-matrix formulation used there is replaced by one based on widths calculated in a Woods-Saxon well. The FS calculation of the ^{2}He width involves convolution with a profile function different from the density-of-states function used in Ref. [3]. They find each contribution to be about 60 keV. These are appreciably larger than previous estimates of 10–15 keV for sequential decay [2] and $\lesssim$5 keV for ^{2}He decay [3]. A quite different method of calculation [4] found a total width of about 60 keV.

For sequential decay through ^{11}N(g.s.) at $E_p = 1.4$ MeV, with the spectroscopic factors $\mathcal{S}_1$ and $\mathcal{S}_2$ for the two stages of the decay each equal to 1.0, FS find the convoluted width $\Gamma_{\rm sp} = 74$ keV. With the R-matrix formalism of Ref. [2], with $Q_{1p} = 1.4$ MeV and $\mathcal{S}_1 = \mathcal{S}_2 = 1.0$, we find $\Gamma_{\rm sp} = 32$ keV. These values could be different because different convolution functions ρ are used in the two calculations and because the energy E_p used by FS is defined differently from the energy Q_{1p} used in the R-matrix calculation. In the latter, ρ is given by the one-level approximation (Eq. (2) of Ref. [2]) and Q_{1p} is the resonance energy E_r [from Eqs. (2) and (9) of Ref. [2]]. This is the energy at which the ^{10}C + p s-wave resonant phase shift β passes through 90° (here $\beta = \delta + \phi$, with δ the total phase shift and $-\phi$ the hard-sphere phase shift). FS do not say what they use for ρ, nor how they define E_p, but probably ρ is proportional to $d\delta/dE$ as a function of energy, and E_p is the energy at which ρ is a maximum [5]. From the R-matrix parameter values that give $E_r = 1.4$ MeV, we calculate δ and find that $d\delta/dE$ is a maximum at $E_p = 1.08$ MeV. Thus the R-matrix calculation of the width is effectively using a lower energy for the ^{11}N(g.s.) than did FS; for comparison with the Woods-Saxon calculation with $E_p = 1.4$ MeV, the R-matrix calculation should use $E_r > 1.4$ MeV, which would lead to a still smaller value of $\Gamma_{\rm sp}$ and a greater discrepancy with the FS value. (In an R-matrix calculation, to make $d\delta/dE$ peak at 1.4 MeV, we require $E_r = 2.26$ MeV, leading to $\Gamma_{\rm sp} = 8.8$ keV, but this value of E_r seems unreasonably high [6].)

With their estimated values $\mathcal{S}_1 = 1.04$ and $\mathcal{S}_2 = 0.75$, FS multiply their value of $\Gamma_{\rm sp}$ by $\mathcal{S}_1\mathcal{S}_2$ to get a sequential contribution to the ^{12}O width of 58 keV. In the R-matrix calculation, if $\mathcal{S}_2$ is different from 1.0, the width of the density-of-states function ρ is changed, but this has little effect on the value of the observed width Γ (see Fig. 1 of Ref. [2]); for $\mathcal{S}_2 = 0.75$, we find $\Gamma = 31$ keV. For $\mathcal{S}_1 = 1.04$, one has $\Gamma \approx 1.04 \times 31\,{\rm keV} = 32\,{\rm keV}$ (the smaller estimates in Ref. [2] were based on $\mathcal{S}_1 = 0.4$).

For ^{2}He decay, FS find $\Gamma_{\rm sp} = 227$ keV for $\epsilon = 0$ (where ϵ is the relative energy of the two protons, called U in Ref. [3]), and $\Gamma_{\rm sp} = 15.3$ keV for $\epsilon = 700$ keV [these are for $n = 2$, corresponding to the $(sd)^2$ component of the ^{12}O(g.s.)]. These values obtained from the Woods-Saxon calculation agree reasonably with the values from the R-matrix formalism of 214 and 14.1 keV respectively. As convolution function ρ, FS use the 0° ϵ dependence of Okamura [7]. Okamura also gives the ϵ dependence at five other angles and as obtained in two different approximations, including the Migdal-Watson formalism. Of these, the 0° dependence is the one that peaked at the lowest energy ($\epsilon \approx 540$ keV), and consequently it leads to the largest contribution to the ^{12}O width. FS find the convoluted width $\Gamma_{\rm sp} = 43.6$ keV for $n = 2$ [for $n = 1$, corresponding to the p^2 component of the ^{12}O(g.s.), $\Gamma_{\rm sp} = 30.5$ keV]. From the R-matrix convolution [3] we obtain, however, a convoluted width of $\Gamma_{\rm sp} = 5.6$ keV for $n = 2$ and 3.7 keV for $n = 1$. These are about 12% of the FS values.

To see if this difference can be attributed to the R-matrix ρ peaking at a higher energy ($\epsilon \approx 760$ keV) than the Okamura 0°ϵ dependence (but close to where the Migdal-Watson approximation peaks), we attempt to fit the Okamura function with the form of ρ used in Refs. [2,3]. We could not get a reasonable fit using a phase-shift δ obtained from the hard-core effective-range formula [3]. With an R-matrix one-level approximation [2], a reasonable fit can be obtained but only for a very large value of the channel radius ($a \approx 30$ fm). We then find $\Gamma_{\rm sp} = 16.7$ keV for $n = 2$ and $\Gamma_{\rm sp} = 11.2$ keV for $n = 1$, which are still less than 40% of the FS values.

By using the opening angle spectrum measured by Kryger *et al.* [8], FS again find that the contributions from ^{2}He decay and sequential decay are comparable (27–46% of the total and 73–54% respectively). FS suggest that there is destructive interference between the ^{2}He and sequential amplitudes at the three smallest angles and constructive interference at the larger angles. These branching ratios mean that the ^{2}He contribution is dominant at the smaller angles. If we assume such interference (although it is surprising that either of the amplitudes would change sign at an angle near which its magnitude is reasonably large), and if we assume ^{2}He dominance at small angles, we find a best fit to the measured spectrum (χ^2/degrees of freedom $\approx$10) with a ^{2}He branching ratio of about 13%, somewhat below the FS range. (A much better fit is obtained if the sequential decay is dominant at small angles, giving χ^2/degrees of freedom ≈ 1.3 and a ^{2}He branching ratio of 0.5%, which is consistent with the experimental upper limit [8]; however, the assumption about interference on which this is based is very dubious.)

We have still not accounted for the differences between the FS and R-matrix values of Γ_{sp} for the sequential decay (74 keV compared with 32 or 8.8 keV) and for the ^{2}He decay (43.6 keV compared with 5.6 or 16.7 keV and 30.5 keV compared with 3.7 or 11.2 keV). We note that in both Refs. [2] and [3], the density-of-states function ρ is normalized by the following:

$$\int_0^\infty \rho(U)\, dU = 1. \tag{1}$$

Okamura [7] gives his 0° ϵ dependence only for $\epsilon \leqslant 3.0$ MeV. For our R-matrix approximation to the Okamura function, with ρ normalized by Eq. (1), we find the following:

$$\int_0^{3.0} \rho(U)\, dU \approx 0.69. \tag{2}$$

If FS normalized their convolution function to unity over the range 0–3.0 MeV only, they would obtain Γ_{sp} values for ^{2}He decay that are too large by about 45%. If they had taken the upper limit of the normalization integral as the $2p$ decay Q value ($Q_{2p} = 1.78$ MeV), similar to what had been done at least once previously [9], their widths would have been too large by nearly 90%. If, in the sequential decay, we take the upper limit of our normalization integral for ρ as Q_{2p}, we find that Γ_{sp} increases by about 160%. It may be that the differences between the FS and R-matrix values can be attributed, at least in part, to such incorrect normalizations.

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