Influence of the α-d motion in ^{6}Li on Trojan horse applications

R. G. Pizzone,[1] C. Spitaleri,[1,2] S. Cherubini,[1,2] M. La Cognata,[1,2] L. Lamia,[1,2] D. Miljanič,[3] A. Musumarra,[1,2] S. Romano,[1,2] A. Tumino,[1,2] S. Tudisco,[1,2] and S. Typel[4]

[1]*Laboratori Nazionali del Sud-INFN, Catania, Italy*

[2]*Dipartimento di Metodologie Fisiche e Chimiche per l'Ingegneria, Università di Catania, Catania, Italy*

[3]*Institut Ruder Bošković, Zagreb, Croatia*

[4]*Gesellschaft für Schwerionenforschung mbH, Theorie, Darmstadt, Germany*

(Received 1 December 2004; published 31 May 2005)

The α-d cluster structure of ^{6}Li has been extensively investigated in the past few decades. In particular the width of the α momentum distribution in ^{6}Li has been studied as a function of the transferred momentum. These investigations are now reviewed and updated after recent experiments. Trojan horse method applications are also discussed because the momentum distribution of the spectator particle inside the Trojan horse nucleus is a necessary input for this method. The impact of the width (FWHM) variation in the extraction of the astrophysical $S(E)$ factor is discussed for the ^{6}Li$(d,\alpha)^4$He reaction.

DOI: 10.1103/PhysRevC.71.058801 PACS number(s): 26.20.+f, 21.10.Pc, 24.50.+g, 25.70.Hi

In the 1970s and 1980s, significant theoretical and experimental efforts were devoted to the study of ^{6}Li since it was considered the simplest nucleus that could be described in the framework of cluster models. Its low dissociation energy ($E_b = 1.47$ MeV) into a deuteron and an α particle suggested a description in terms of these two clusters: $^6\text{Li} = \alpha \oplus d$. The study of the relative α-d motion was mainly carried out through quasifree reactions [1]. The experimental momentum distribution of the α cluster was studied as a function of the transferred momentum [2] and the intercluster wave function $\chi(\text{r})$ turned out to be well described by several functional forms (e.g., by the Hankel function).

These studies have been recently updated because of the importance of weakly bound nuclei (Trojan horse nuclei; hereafter TH nuclei) in the framework of the Trojan horse method (THM). The main features of this method are extensively discussed elsewhere [3–10]. Basically the method allows us to measure the bare-nucleus two-body cross sections [or equivalently the bare-nucleus astrophysical $S(E)$ factors] by means of quasifree three-body reactions.

Since the extraction of the bare-nucleus $S(E)$ factor uses the momentum distribution of the spectator cluster inside the TH nucleus, it is important to evaluate the impact of the uncertainty of the momentum distribution width on the final result.

The present paper can be regarded as part of an experimental as well as theoretical work aimed at analyzing the behavior of the momentum distribution width in the TH nuclei as a function of the transferred momentum. In particular, this work focuses on the investigation of the α-d momentum distribution within ^{6}Li. We have reanalyzed our previous data [2] and have derived new results from the recent ^{6}Li$(^6\text{Li},\alpha\alpha)^4$He three-body experiments. This study evaluates the dependence of the THM astrophysical factor on the momentum distribution width (FWHM), which might introduce additional uncertainties. The $S(E)$ factor for the ^{6}Li$(d,\alpha)^4$He two-body reaction extracted via the THM from the reaction ^{6}Li$(^6\text{Li},\alpha\alpha)^4$He has been reanalyzed according to this perspective.

A typical three-body reaction can be written as

$$a + A \to C + c + s,$$

where, in the case of a quasifree process, a is the projectile, A is the TH nucleus, s is the spectator, and c and C are the detected ejectiles. We will assume that the quasifree breakup takes place in $A = x \oplus s$, with x as a participant to the $a(x,c)C$ reaction.

In the framework of the impulse approximation (IA) the theoretical triple-differential cross section is given by [11,12]

$$\frac{d^3\sigma}{dE_1 d\Omega_c d\Omega_C} \propto (KF)\left(\frac{d\sigma}{d\Omega}^{\text{off}}_{\text{c.m.}}\right)|\Phi(\vec{p}_s)|^2, \tag{1}$$

where

(i) $(d\sigma/d\Omega)^{\text{off}}_{\text{c.m.}}$ is the off-energy-shell differential cross section for the two-body $a(x,c)C$ reaction at the center-of-mass energy $E_{\text{c.m.}}$ given in postcollision prescription by [13]

$$E_{\text{c.m.}} = E_{c-C} - Q_{2b}, \tag{2}$$

with Q_{2b} the two-body Q value of the $a(x,c)C$ reaction and E_{c-C} the relative energy between the outgoing particles c and C;

(ii) KF is a kinematical factor; and

(iii) $|\Phi(\vec{p}_s)|^2$ is the momentum distribution of the spectator.

The $|\Phi(\vec{p}_s)|^2$ factor in Eq. (1) is the square of the Fourier transform of the intercluster wave function $\chi(\vec{r})$:

$$\Phi(\vec{p}_s) = (2\pi\hbar)^{-3/2}\int e^{-i\vec{p}_s\cdot\vec{r}/\hbar}\chi(\vec{r})d^3r. \tag{3}$$

As already mentioned the intercluster motion is well described by the Hankel function with cutoff radius R_c [1].

In the standard THM approach the two-body cross section is obtained by inverting Eq. (1), so that

$$\left(\frac{d\sigma}{d\Omega}^{\text{off}}_{\text{c.m.}}\right) \propto \frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2}\Big/[(KF)|\Phi(\vec{p}_s)|^2],$$

where the product $KF \cdot |\Phi(\vec{p}_s)|^2$ is calculated by means of a Monte Carlo simulation and the three-body cross section is measured. The intercluster wave function for a s-wave motion between the two clusters can be written as

$$\chi(\vec{r}) = R(r) Y_{00}(\theta, \phi). \tag{4}$$

It was shown in previous works [1] that the width $w(q_t)$ of the momentum distribution is not sensitive to the choice of the wave function if a cutoff radius $R_c > 2$ fm is introduced. The role of the cutoff radius is to take into account reabsorption effects, or even to simulate the inadequacy of the cluster model for small intercluster distances [1]. The introduction of R_c in the intercluster wave function is such that

$$R(r) = 0 \text{ for } r < R_c.$$

This ensures that the main features in the shape of momentum distributions for spectator momenta value lower than 100 MeV/c, calculated in the plane wave impulse approximation (PWIA) or in the distorted wave impulse approximation (DWIA), are essentially the same [14–17]. The PWIA has been widely used because of its simplicity and since it predicts reasonably well the shape of the experimental momentum distribution in the region away from its zeros [16]. Since in the experimental TH applications one generally selects events with low momentum, this approximation is well justified.

In previous works [2,18] it has been shown that the $w(q_t)$ for the α-d momentum distribution in ^{6}Li slowly increases with increasing transferred momentum q_t. This can be defined [2] for quasifree processes as

$$\vec{q}_t = \vec{p}_a - \frac{\vec{p}_c + \vec{p}_C}{2}, \tag{5}$$

where $\vec{p}_a$, $\vec{p}_c$, and $\vec{p}_C$ are the momenta of the projectile and ejectiles c and C, respectively. Thus $\vec{q}_t$ is defined as the mean transferred momentum between the momentum transferred to c, $\vec{q}_{tc}$, and to C, $\vec{q}_{tC}$ [i.e., $\vec{q}_t = 0.5(\vec{q}_{tc} + \vec{q}_{tC})$].

A width (FWHM) of around 73 MeV/c corresponds to values of the transferred momentum larger than 300 MeV/c, in agreement with the asymptotic value predicted by both PWIA and DWIA. The narrowing of the momentum distribution with decreasing q_t was interpreted as related to distortion or absorption effects [19,20].

The explanation of this behavior (see Fig. 2) is the following. In the scattering of electromagnetic probes (e.g., electron scattering) the momentum transfer to the center of mass of the probe is the same as the momentum transfer to the nucleus. However, because of strong absorption, this relationship is not valid for nucleus-nucleus scattering. The momentum transfer to nuclear excitation is spread up to a value close to the cm momentum transfer. This means that after a certain value q_t^*, all values of the internal momentum distribution inside ^{6}Li are probed [21]. However, for small values of q_t large values of p_s, (i.e., $p_s \geqslant q_t^*$) are suppressed, thus narrowing the momentum distribution probed in the scattering.

We have improved these results for the ^{6}Li nucleus with new data taken from recent experiments [3,22]. $|\Phi(\vec{p}_s)|^2$ was calculated by using a Hankel function for the s state given

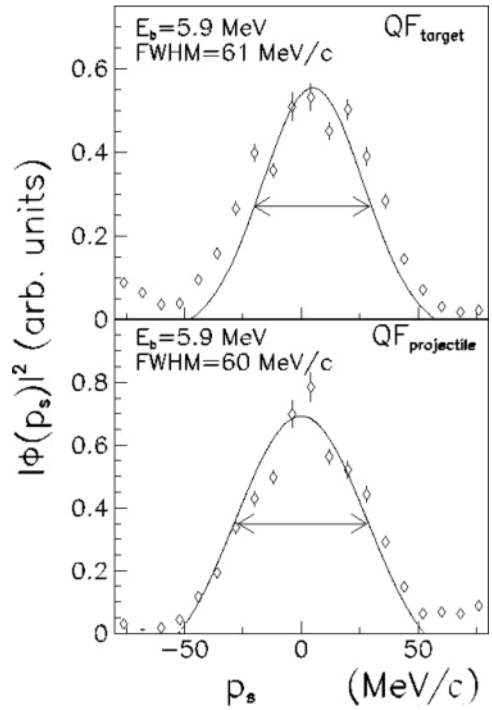

FIG. 1. Experimental momentum distribution for the α particle inside ^{6}Li derived according to the guidelines given in the text for the ^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He reaction. The upper and lower parts refer to the target and projectile breakup cases, respectively.

by [23]

$$|\Phi(\vec{p}_s)|^2 = N \frac{1}{\left(k_s^2 + \beta^2\right)^2} \left[\frac{\sin k_s R_c}{k_s} + \frac{\cos k_s R_c}{\beta}\right] \tag{6}$$

with $k_s = p_s/\hbar$, R_c the cutoff radius, and β defined by means of the α-d binding energy E_B in ^{6}Li, according to $\beta = (2\mu E_B/\hbar^2)^{1/2}$, where μ is the reduced mass.

The width of the momentum distribution was extracted for the ^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He reaction at different energies. The experimental details of these experiments are reported elsewhere [3].

The standard procedure for the extraction of the momentum distribution is described in [10]. As shown in Eq. (1) the PWIA allows us to directly relate the three-body differential cross section to the spectator momentum distribution. Thus this equation can be applied to derive $|\Phi(\vec{p}_s)|^2$ once the other quantities are known. Although KF can be easily calculated, the energy behavior of $(d\sigma/d\Omega)^{\text{off}}_{\text{c.m.}}$, which is the quantity one wants to obtain in TH applications, is generally not known. What is usually done is to select coincidence events in the $E_{\text{c.m.}}$ versus p_s plot in such a narrow energy range (∼100 keV) that, when projected onto the p_s axis, $(d\sigma/d\Omega)^{\text{off}}_{\text{c.m.}}$ for those events is nearly constant.

Thus dividing the triple differential cross section by the kinematical factor one obtains directly the shape of the experimental momentum distribution in arbitrary units. Experimental results referring to the ^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He process are reported in Fig. 1; the upper and lower parts refer to the case of target and projectile breakup, respectively. In both cases the appropriate

TABLE I. Transferred momentum for different quasi-free reactions and beam energies together with the measured width (FWHM) and the fitted R_c, according to the text.

Reaction	E_{beam} (MeV)	q_t (MeV/c)	$w(q_t)$ (MeV/c)	Ref.
^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He	2.1	71	49 ± 4	
	2.7	50	43 ± 4	
	2.7	87	53 ± 4	
	3.6	96	40 ± 5	[26]
	4.2	107	40 ± 5	[26]
	4.7	115	45 ± 5	[26]
	5.7	125	49 ± 5	[26]
	5.9	136	61 ± 5	[3]
	6.7	138	63 ± 5	[26]
	44	398	72 ± 15	
^{3}He(^{6}Li, pα)^{4}He	5	83	40 ± 3	[22]
	6	90	55 ± 3	[22]

Hankel function [Eq. (6)] was used to fit the data (solid line in the figure) and the width (FWHM) was easily derived. The cutoff radius R_c was considered as a free parameter.

Other measurements for the ^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He reaction were performed at different beam energies. With the same procedure the $w(q_t)$ estimates were deduced. Details for all the performed experiments are reported in Table I.

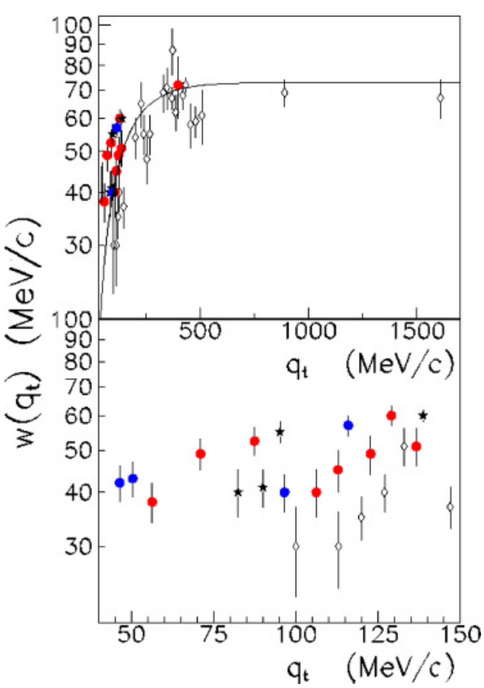

FIG. 2. (Color online) Width (FWHM) for the α momentum distribution inside ^{6}Li as a function of the transferred momentum q_t (see text). Diamonds represent previous results by [2,18], full dots and stars are new data from the ^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He (blue projectile breakup, red target breakup) and ^{3}He(^{6}Li, $p\alpha$)^{4}He experiments [3,7,22,24]. The solid line represents an empirical fit described in the text. On the lower picture only the low transferred momenta region is represented.

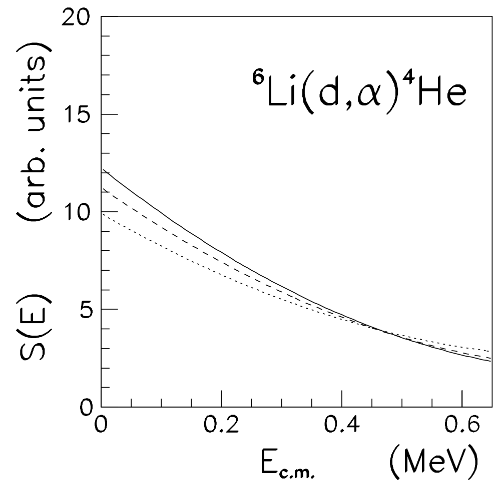

FIG. 3. Experimental ^{6}Li$(d,\alpha)^4$He $S(E)$ factor, extracted via the THM, for different choices of the $w(q_t)$ for the α momentum distribution inside ^{6}Li. The solid line represents the case of $w(q_t) = 70$ MeV/c, the dashed line is for $w(q_t) = 61$ MeV/c, and dotted line is for $w(q_t) = 50$ MeV/c.

Figure 2 shows the trend of the width for the α-d relative motion as a function of the transferred momentum q_t. Previous data from [2,18] are presented as diamonds and our ^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He recent results at different energies [3,7] are shown as full dots. The stars refer to the ^{3}He(^{6}Li, $p\alpha$)^{4}He [24] experiment. It is evident how, at low q_t, the $w(q_t)$ smoothly increases until the predicted PWIA asymptotic value (73 MeV/c) [16,19] is reached around 250–300 MeV/c.

These data confirm the behavior already discussed in [2]. A fit on the data was performed by using the following function:

$$w(q) = f_0(1 - e^{-q_t/q_0}), \quad (7)$$

where f_0 represents the asymptotic width value of 73 MeV/c and q_0 is the fitting parameter, set to 122 ± 3.5 MeV/c.

Until now the determination of the TH $S(E)$ factor was performed by assuming the FWHM asymptotic value in Eg. (1). These new results for ^{6}Li confirm that the $w(q_t)$ could be significantly lower than this value, especially for low q_t.

This variation was taken into account in a reanalysis of the quasifree ^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He data at 5.9 MeV [3]. The corresponding transferred momentum is 136 MeV/c, whereas the measured width and the value calculated with the fitting function 7 are, respectively, 61 ± 5 MeV/c and 50 MeV/c. The Monte Carlo simulation to evaluate the $KF \cdot |\Phi(\vec{p}_s)|^2$ product was performed for three choices of $w(q_t)$: 50 Mev/c [the value from Eq. (7)], 61 Mev/c (the experimental value), and 73 MeV/c (the theoretical value). The corresponding energy trends of the $S(E)$ factor are reported in arbitrary units in Fig. 3.

The $S(E)$ factor undergoes a variation of about 6% by changing the width with respect to the previous determination. However, in that experiment [3,25] the $S(E)$ factor was affected by a total error of about 15%. This work updates the previous TH value of $S(E = 0)$ from $S(0) = 16.9 \pm 2.5$ MeV·b to $S(0) = 15.9 \pm 2.7$ MeV·b after adding the error arising from the $w(q_t)$ variation.

In conclusion, the FWHM for the momentum distribution of α-d system in ^{6}Li was investigated as a function of the transferred momentum. Previous results [2] were updated using new experimental ^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He data and were confirmed.

The dependence of the width (FWHM) was parametrized in terms of the transferred momentum, thus allowing, once q_t is fixed, an a priori evaluation of its approximate value. In such a way it is possible to optimize the selection of the experimental regions where the sequential processes are expected to be negligible.

More precise calculations of the $(d\sigma/d\Omega)^{\rm off}_{\rm c.m.}$ cross section from $d^3\sigma/(dE_1 d\Omega_1 d\Omega_2)$ are thus possible.

This procedure was applied in the case of the ^{6}Li(d, α)^{4}He reaction. The uncertainty of 6% in the $S(E)$ factor is significantly lower than the total error (∼15%) and has been taken into account.

The authors would like to thank Prof. C. A. Bertulani for a careful reading of the manuscript and for fruitful comments.

[1] M. Lattuada, F. Riggi, C. Spitaleri, and D. Vinciguerra, Il Nuovo Cimento A **83**, 151 (1984) and references therein.
[2] S. Barbarino, M. Lattuada, F. Riggi, C. Spitaleri, and D. Vinciguerra, Phys. Rev. C **21**, 1104 (1980).
[3] C. Spitaleri, S. Typel, R. G. Pizzone, M. Aliotta, S. Blagus, M. Bogovac, S. Cherubini, P. Figuera, M. Lattuada, M. Milin, D. Miljanić, A. Musumarra, M. C. Pellegriti, D. Rendić, C. Rolfs, S. Romano, N. Soić, A. Tumino, H. H. Wolter, and M. Zadro *et al.*, Phys. Rev. C **63**, 055801 (2001).
[4] M. Lattuada, R. G. Pizzone, S. Typel, P. Figuera, D. Miljanić, A. Musumarra, M. G. Pellegriti, C. Rolfs, C. Spitaleri, and H. H. Wolter, Astrophys. J. **562**, 1076 (2001).
[5] F. Strieder, C. Rolfs, C. Spitaleri, and P. Corvisiero, Naturwissenschaften **88**, 461 (2001).
[6] C. Spitaleri, in *Problems of Fundamental Physics,* II, edited by R. Cherubini, P. Dal Piaz, and B. Minetti (World Scientific, Singapore), (1990).
[7] S. Cherubini, V. N. Kondratyev, M. Lattuada, C. Spitaleri, D. Miljanić, M. Zadro, and G. Baur, Astrophys. J. **457**, 855 (1996).
[8] C. Spitaleri, M. Aliotta, S. Cherubini, M. Lattuada, D. Miljanić, S. Romano, N. Soić, M. Zadro, and R. A. Zappalà, Phys. Rev. C **60**, 055802 (1999).
[9] A. Tumino, C. Spitaleri, A. Di Pietro, P. Figuera, M. Lattuada, A. Musumarra, M. G. Pellegriti, R. G. Pizzone, S. Romano, C. Rolfs, S. Tudisco, and S. Typel *et al.*, Phys. Rev. C **67**, 065803 (2003).
[10] C. Spitaleri, L. Lamia, A. Tumino, R. G. Pizzone, S. Cherubini, A. Del Zoppo, P. Figuera, M. La Cognata, A. Musumarra, M. G. Pellegriti, A. Rinollo, C. Rolfs, S. Romano, and S. Tudisco *et al.*, Phys. Rev. C **69**, 055806 (2004).
[11] G. F. Chew, Phys. Rev. **80**, 196 (1950).
[12] G. F. Chew and G. C. Wick, Phys. Rev. **85**, 636 (1952).
[13] M. Zadro, D. Miljanic, C. Spitaleri, G. Calvi, M. Lattuada, and F. Riggi, Phys. Rev. C **40**, 181 (1989).
[14] P. G. Roos, D. A. Goldberg, N. S. Chant, R. Woody III, and W. Reichart, Nucl. Phys. **A257**, 317 (1976).
[15] M. Jain, P. G. Roos, H. G. Pugh, and H. D. Holmgrem, Nucl. Phys. **A153**, 49 (1970).
[16] P. G. Roos, N. S. Chant, A. A. Cowley, D. A. Goldberg, H. D. Holmgren, and R. Woody III, Phys. Rev. C **15**, 69 (1977).
[17] R. Ent, H. P. Blok, J. F. A. van Hienen, G. van der Steenhoven, J. F. J. van den Brand, J. W. A. den Herder, E. Jans, P. H. M. Keizer, L. Lapikás, E. N. M. Quint, and P. K. A. de Witt Huberts *et al.*, Phys. Rev. Lett. **57**, 2367 (1986).
[18] M. Lattuada, F. Riggi, C. Spitaleri, D. Vinciguerra, C. Sutera, and A. Pantaleo, Il Nuovo Cimento A **71**, 429 (1982).
[19] J. W. Watson, H. G. Pugh, P. G. Roos, D. A. Goldberg, R. A. J. Riddle, and D. I. Bonbright, Nucl. Phys. **A172**, 513 (1971).
[20] M. S. Hussein and K. W. McVoy, Nucl. Phys. **A445**, 124 (1985).
[21] C. A. Bertulani and G. Baur, Nucl. Phys. **A480**, 615 (1988).
[22] M. La Cognata, A. Musumarra, C. Spitaleri, A. Tumino, C. Bonomo, S. Cherubini, P. Figuera, L. Lamia, M. G. Pellegriti, A. Rinollo, R. G. Pizzone, C. Rolfs, S. Romano, D. Schürmann, F. Strieder, S. Tudisco, and S. Typel, in press on Nuclear Physics A (2005).
[23] J. Kasagi, T. Nakagawa, N. Sekine, and T. Tohei, Nucl. Phys. **A239**, 233 (1975).
[24] M. Zadro, Dj. Miljanić, M. Lattuada, F. Riggi, and C. Spitaleri, Nucl. Phys. **A474**, 373 (1987).
[25] A. Musumarra, R. G. Pizzone, S. Blagus, M. Bogovac, P. Figuera, M. Lattuada, M. Milin, D. Miljanìc, M. G. Pellegriti, D. Rendic, C. Rolfs, N. Soic, C. Spitaleri, S. Typel, H. H. Wolter, and M. Zadro *et al.*, Phys. Rev. C **64**, 068801 (2001).
[26] M. Lattuada, F. Riggi, D. Vinciguerra, C. Spitaleri, G. Vourvopoulos, D. Miljanić, and E. Norbeck, Z. Phys. A **330**, 183 (1986).