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Spin-orbit transition interactions

Jacques Raynal
4 rue du Brief, F-91380 Chilly-Mazarin, France
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The spin-orbit two-body interaction has been used for a long time in nuclear structure studies and is unique. In contrast, two different expressions are used in nuclear scattering studies. One of them is stronger when the eigenvalues of $(\boldsymbol{\ell}\cdot\boldsymbol{\sigma})$ of the two particles involved are opposite to each other, such as the interaction used in nuclear structure studies for an even parity excitation. This expression of the spin-orbit interaction can be qualified as "normal" in view of the similar behavior in these two applications. The other expression of the spin-orbit interaction involves the sum of these two eigenvalues.

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The spin-orbit two-body interaction has been used for a long time in nuclear structure studies and is unique. It can be used also in the microscopic description of inelastic nucleon–nucleus scattering: in the intermediate step of such a calculation (after integration over the coordinates of one of the interacting nucleons), it can be considered as a pattern for a macroscopic approach. For "natural parity" excitations—the only ones that have a macroscopic equivalent—the interaction is expressed primarily in term of the difference of the eigenvalues of $(\boldsymbol{\ell}\cdot\boldsymbol{\sigma})$ [1] and is stronger when $(\ell - j) = -(\ell' - j')$ than in the opposite case: we qualify this behavior as "normal." On the contrary, two different expressions are used in nuclear scattering studies. One of them [2,3] is stronger when the eigenvalues of $(\boldsymbol{\ell}\cdot\boldsymbol{\sigma})$ for the two particles involved have opposite signs such as the two-body interaction after integration over one variable. This expression of the spin-orbit interaction can be qualified as "normal" in view of this similar behavior. The other expression of the spin-orbit interaction [4] involves only the half-sum of these two eigenvalues.

In a recent article [4], the authors describe in their Appendix B the interaction they use. In the description of the potential matrix element $V_{cc'}(r)$, where c and c' denote the quantum numbers of the bra and the ket, the authors write on the second line of page 92:

$$-\frac{1}{2\alpha r}W_{ls}\{[\boldsymbol{\ell}\cdot\boldsymbol{s}]_{c'}+[\boldsymbol{\ell}\cdot\boldsymbol{s}]_{c}\}, \tag{1}$$

which, with the two following lines, deals with the spin-orbit interaction. This is an arbitrary generalization of the spin-orbit potential of the optical model, which is as follows:

$$-2\left(\frac{\hbar}{m_\pi c}\right)^2\frac{1}{r}\left\{\frac{d}{dr}V(r)\right\}[\boldsymbol{\ell}\cdot\boldsymbol{s}]. \tag{2}$$

A symmetrized form of this expression was used when first asymmetry measurements in inelastic scattering with polarized proton beams became available. The radial dependence of Eq. (2) is what is obtained by elimination of the small component of a Dirac radial equation, $V(r)$ being the difference between scalar and tensor potentials. The factor in front, depending on the pion mass m_π is 2, such that the definition of the spin-orbit interaction is $4(\boldsymbol{l}\cdot\boldsymbol{s}) = 2(\boldsymbol{l}\cdot\boldsymbol{\sigma})$ for particles with spin 1/2: for convenience, $\boldsymbol{\sigma}$ is used instead of $\boldsymbol{s}$ in the following.

Going back to the "full Thomas term" obtained for the spin-orbit by transforming a Dirac equation into a Schrödinger equation, J. S. Blair and H. Sherif [2,3] used the following expression:

$$\nabla\{V(\boldsymbol{r})\}\times\frac{\nabla}{i}\cdot\boldsymbol{\sigma} \tag{3}$$

in computations for nucleon inelastic scattering. To show the derivation of its expression in terms of the eigenvalues of $(\boldsymbol{\ell}\cdot\boldsymbol{\sigma})$ the "full Thomas form" for a multipole (λ,μ) can be written as follows:

$$V^{\rm LS}_{\lambda,\mu}(r) = \left(\nabla V_\lambda(r)Y^\mu_\lambda(\hat{r})\right)\times\frac{\nabla}{i}\cdot\boldsymbol{\sigma} \tag{4}$$

The zeroth-order term of [4] is in $V_{0,0}(r)$, the first-order term in the deformation parameter β_2 is in $V_{2,0}(r)$; the second-order terms in β_2 contribute to $V_{00}(r)$, $V_{20}(r)$, and $V_{40}(r)$. Using the following identities:

$$\begin{aligned}\nabla &= \frac{\boldsymbol{r}}{r}\frac{d}{dr} - i\frac{\boldsymbol{r}\times\boldsymbol{\ell}}{r^2},\\ i\boldsymbol{\sigma}\cdot(\boldsymbol{A}\times\boldsymbol{B}) &= (\boldsymbol{\sigma}\cdot\boldsymbol{A})(\boldsymbol{\sigma}\cdot\boldsymbol{B}) - (\boldsymbol{A}\cdot\boldsymbol{B}),\\ (\boldsymbol{\sigma}\cdot\nabla) &= \frac{(\boldsymbol{\sigma}\cdot\boldsymbol{r})}{r}\left(\frac{d}{dr}-\frac{(\boldsymbol{\ell}\cdot\boldsymbol{\sigma})}{r}\right),\\ (\boldsymbol{\sigma}\cdot\boldsymbol{\ell})(\boldsymbol{\sigma}\cdot\boldsymbol{r}) &= -(\boldsymbol{\sigma}\cdot\boldsymbol{r})(\boldsymbol{\sigma}\cdot\boldsymbol{\ell}),\\ (\boldsymbol{\sigma}\cdot\boldsymbol{r})^2 &= r^2,\end{aligned} \tag{5}$$

$V^{\rm LS}_{\lambda,\mu}(r)$ can be written as follows:

$$\begin{aligned}V^{\rm LS}_{\lambda,\mu}(r) = &-\left(\left[\frac{d}{dr}+\frac{(\boldsymbol{\ell}\cdot\boldsymbol{\sigma})}{r}\right]V_\lambda(r)Y^\mu_\lambda(\hat{r})\right)\\ &\times\left[\frac{d}{dr}-\frac{(\boldsymbol{\ell}\cdot\boldsymbol{\sigma})}{r}\right]+\left(\frac{d}{dr}V_\lambda(r)Y^\mu_\lambda(\hat{r})\right)\frac{d}{dr}\\ &-\left(\frac{\boldsymbol{r}\times\boldsymbol{\ell}}{r^2}V_\lambda(r)Y^\mu_\lambda(\hat{r})\right)\frac{\boldsymbol{r}\times\boldsymbol{\ell}}{r^2}.\end{aligned} \tag{6}$$

The terms with two derivatives cancel each other. The operator $(\boldsymbol{\ell}\cdot\boldsymbol{\sigma})$ acting on $Y^\mu_\lambda(\hat{r})$ can be replaced by $(\boldsymbol{\ell}\cdot\boldsymbol{\sigma})_c - (\boldsymbol{\ell}\cdot\boldsymbol{\sigma})_{c'}$ because $\boldsymbol{\ell}_c = \boldsymbol{\ell}_{c'} + \boldsymbol{\lambda}$. The last term can be simplified, using the relation:

$$(\boldsymbol{A}\times\boldsymbol{B})\cdot(\boldsymbol{C}\times\boldsymbol{D}) = (\boldsymbol{A}\cdot\boldsymbol{C})(\boldsymbol{B}\cdot\boldsymbol{D}) - (\boldsymbol{B}\cdot\boldsymbol{C})(\boldsymbol{A}\cdot\boldsymbol{D}) \tag{7}$$

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which replaces the two cross products by $r^2(\boldsymbol{\lambda}\cdot\boldsymbol{\ell})$. But, as $(\boldsymbol{\ell}\cdot\boldsymbol{\ell}) = (\boldsymbol{\ell}\cdot\boldsymbol{\sigma})^2 + (\boldsymbol{\ell}\cdot\boldsymbol{\sigma})$:

$$2(\boldsymbol{\lambda}\cdot\boldsymbol{\ell}_i) = ([\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_c - [\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'}) \times ([\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_c + [\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'} + 1) - \lambda(\lambda+1). \quad (8)$$

With these manipulations, the result is obtained as follows:

$$V^{\mathrm{LS}}_{\lambda,\mu}(r) = Y^{\mu}_{\lambda}(\hat{r})\Bigg[\frac{dV_\lambda(r)}{dr}[\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'} + \frac{V_\lambda(r)}{r}([\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'} - [\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_c)\frac{d}{dr} + \frac{V_\lambda(r)}{2r^2}\big\{\lambda(\lambda+1) - ([\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_c - [\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'})([\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_c - [\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'} \pm 1)\big\}\Bigg], \quad (9)$$

where ± 1 is $+1$ in this tridimensional derivation and is -1 if the wave functions are multiplied by r as usual. Note that there are three form factors:

1. $\frac{1}{r}\frac{d}{dr}V_\lambda(r)$ which is the only one for elastic scattering and is multiplied only by the eigenvalue for the ket.
2. $\frac{V_\lambda(r)}{2r^2}$ which is, divided by r^2, the true spin-orbit multipole which disappears in elastic scattering for which $\lambda = 0$ and $[\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'} = [\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_c$.
3. $\frac{V_\lambda(r)}{r}\frac{d}{dr}$ which is the form factor multiplying the derivative of the ket radial function; integrating by part shows that the whole is symmetric in c and c'.

Except for the first, the form factors differ from the ones of Eq. (1). However, as the interaction of Eq. (1) is larger when the eigenvalues for c and c' are of the same sign and the coefficients of the second and third form factors above are larger in the opposite case, one can guess that the effect should be quite different.

The hermiticity of Eq. (9) can be written as follows:

$$\big(\langle\phi_c(r)|V^{\mathrm{LS}}_{\lambda,\mu}(r)\big)|\phi_{c'}(r)\rangle - \langle\phi_c(r)|\big(V^{\mathrm{LS}}_{\lambda,\mu}(r)|\phi_{c'}(r)\rangle\big) = 0, \quad (10)$$

where $[\langle\phi_c(r)|V^{\mathrm{LS}}_{\lambda,\mu}(r)]$ means that the operator acts on the left with a change of sign for d/dr (third form factor listed above). Using wave functions multiplied by r, Eq. (9) is $([\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_c - [\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'})$ multiplied by the following:

$$\int_0^\infty \left(\left[\frac{dV_\lambda(r)}{dr} - \frac{V_\lambda(r)}{r^2}\right]\phi_c(r)\phi_{c'}(r) + \frac{V_\lambda(r)}{r}[\phi'_c(r)\phi_{c'}(r) + \phi_c(r)\phi'_{c'}(r)]\right)dr = \left|\frac{V_\lambda(r)}{r}\phi_c(r)\phi_{c'}(r)\right|_0^\infty, \quad (11)$$

which vanishes in the applications because the wave functions $\phi(r)$ vanish at the origin and the form factor $V_\lambda(r)$ vanishes at infinity.

The use of the spin-orbit deformation given by Eq. (9) in coupled channel calculation is more difficult [5]. It was the subject of codes ECIS ("Equations Couplées en Itérations Séquentielles") from ECIS68 to ECIS03 [6]. To compare results obtained with the interactions given by Eq. (1) and Eq. (9), the spin-orbit interaction is parameterized as follows:

$$\frac{1}{r}\frac{dV_\lambda(r)}{dr}(z_1 + z_3[\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'} + z_4[\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_c) + \frac{V_\lambda(r)}{r} \times z_6([\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'} - [\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_c)\frac{d}{dr} + \frac{V_\lambda(r)}{2r^2}z_5[z_2\lambda(\lambda+1) - ([\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_c - [\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'})([\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_c - [\boldsymbol{\ell}\cdot\boldsymbol{\sigma}]_{c'} \pm 1)] \quad (12)$$

in all these codes. The coupling of Eq. (1) is obtained by setting

$$z_1 = z_2 = z_5 = z_6 = 0, \qquad z_3 = z_4 = \tfrac{1}{2}, \quad (13)$$

the coupling of Eq. (9) multiplied by a parameter x (with $x = 1$ for the "unparametrized" case) is given by the following:

$$z_1 = z_4 = 0, \qquad z_2 = 1, \qquad z_3 = z_5 = z_6 = x. \quad (14)$$

The parameter x allows to increase the strength of the spin-orbit transition without deforming its form factor in the rotational model. Equation (12) is hermitian only if $z_6 = z_5 = z_3 - z_4$, as verified by Eq. (13) and Eq. (14); it allows the mixture of the two interactions with two more ones, the spin independent with the two nonderivative form factors.

For people who do not want to consider Dirac equation at low energy, there is another justification of Eq. (9) based on the nucleon-nucleon interaction [1,7,8]. The most recent publication of this topic can be found in Ref. [9], formulas (4.44) to (4.50). A natural parity excitation involves seven form factors, of which two are for a derivative term. At the zero-range limit [7,8] of the two-body spin-orbit interaction, these form factors can be expressed with the product of a particle and a hole function. Assuming that the sum of the eigenvalues of $\boldsymbol{\ell}\cdot\boldsymbol{s}$ for the particles and holes vanishes (and also the sum of products of particle by the derivative of hole functions) the result differs by a factor 2 in front of $\lambda(\lambda+1)$ from Eq. (9) with the product of the particle and the hole functions as V_λ. This approach, with the most general consideration of the two–body interaction has been the subject of a series of codes, from DWBA70 to DWBA98 [10].

Both forms, Eq. (1) and Eq. (9), of the spin-orbit coupling have been used with varying degrees of success to interpret the scattering of spin 1/2 projectiles from nuclei for over 30 years. Our focus here has been to stress that (a) it is the coupling for inelastic scattering given by Eq. (9) that is linked closely to the underlying nucleon-nucleon interaction (and/or to the Dirac equation) and (b) more specifically, at the partial-wave level these two forms of spin-orbit coupling are qualitatively different. I thank the Service de Physique Théorique de Saclay for allowing me to follow this kind of problems after my retirement. I also thank H. Sherif for reading the manuscript and for helpful correspondence.

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