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Onset of classical QCD dynamics in relativistic heavy ion collisions

Dmitri Kharzeev,[1] Eugene Levin,[2] and Marzia Nardi[3]
[1]*Department of Physics, Brookhaven National Laboratory, Upton, New York 11973-5000, USA*
[2]*HEP Department, School of Physics, Raymond and Beverly Sackler Faculty of Exact Science, Tel Aviv University, Tel Aviv 69978, Israel*
[3]*Dipartimento di Fisica Teorica dell'Università di Torino and INFN, Sezione di Torino, via P. Giuria 1, I-10125 Torino, Italy*
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The experimental results on hadron production obtained recently at RHIC offer a new prospective on the energy dependence of the nuclear collision dynamics. In particular, it is possible that parton saturation—the phenomenon likely providing initial conditions for the multiparticle production at RHIC energies—may have started to set in central heavy ion collisions already around the highest CERN SPS energy. We examine this scenario, and make predictions based on high density QCD for the forthcoming $\sqrt{s} = 22$ GeV run at RHIC.

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High energy nuclear collisions allow us to test QCD at the high parton density strong color field frontier, where the highly nonlinear behavior is expected. Already after one year of RHIC operation, a wealth of new experimental information on multiparticle production has become available [1–4]. It appears that the data on hadron multiplicity and its energy, centrality, and rapidity dependence so far are consistent with the approach [5,6] based on the ideas of parton saturation [7,8] and semiclassical QCD ("the color glass condensate") [9,10]. The centrality dependence of transverse mass spectra appears to be consistent with this scenario as well [11].

Strictly speaking, the use of classical weak coupling methods in QCD can be justified only when the "saturation scale" Q_s^2 [7,9], proportional to the density of the partons, becomes very large, $Q_s^2 \gg \Lambda_{\rm QCD}^2$ and $\alpha_s(Q_s^2) \ll 1$. At RHIC energies, the value of saturation scale in Au-Au collisions varies in the range of $Q_s^2 = 1$–2 GeV^2 depending on centrality. At these values of Q_s^2, we are still at the borderline of the weak coupling regime. However, the apparent success of the saturation approach seems to imply that the transition to semiclassical QCD takes place already at RHIC energies.

This may shed new light on the mechanism of hadron production at lower energies, perhaps including the energy of CERN SPS. Indeed, extrapolating down in energy using the formulas of [6] yields for the saturation scale in central Pb-Pb collisions at SPS energy of $\sqrt{s} = 17$ GeV the value of $Q_s^2 \approx 1.2$ GeV^2.[1] The same average value at a RHIC energy of $\sqrt{s} = 130$ GeV is reached in peripheral Au-Au collisions at impact parameter $b \approx 9$ fm and an average number of participants of $N_{\rm part} \approx 90$. At $N_{\rm part} < 100$, and impact parameters $b > 9$ fm, reconstruction of the geometry of the collision and the extraction of the number of participants face sizable uncertainties, and no firm conclusion on the applicability of the saturation approach can be drawn from the data. Given this uncertainty, one may consider two different scenarios:

(1) The onset of saturation occurs somewhere in the RHIC energy range, below $\sqrt{s} = 130$ GeV but above $\sqrt{s} = 17$ GeV; the mechanisms of multiparticle production at RHIC and SPS energies are thus totally different;
(2) Saturation sets in central heavy ion collisions already around the highest SPS energy. The second scenario would, in particular, have important implications for interpretation of the SPS results.

It should be possible to distinguish between these two scenarios by extrapolating the results of Refs. [5,6] down in energy and comparing them to the data. In fact, very soon RHIC will collect data at the energy of $\sqrt{s} = 22$ GeV, not far from the highest SPS energy of $\sqrt{s} = 17$ GeV. In this paper, we make predictions for hadron production at this energy based on the saturation scenario. It should be stressed that *a priori* there is no solid reason to expect this approach to work at low energies; we provide these predictions to make it possible to decide between scenarios (1) and (2) listed above based on the data when they become available.

In Ref. [6] we derived a simple analytical scaling formula, describing the energy, centrality, rapidity, and atomic number dependences of hadron multiplicities in high energy nuclear collisions:

$$\frac{dN}{dy} = c\, N_{\rm part} \left(\frac{s}{s_0}\right)^{\frac{\lambda}{2}} e^{-\lambda|y|} \left[\ln\left(\frac{Q_s^2}{\Lambda_{\rm QCD}^2}\right) - \lambda|y|\right] \times \left[1 + \lambda|y|\left(1 - \frac{Q_s}{\sqrt{s}} e^{(1+\lambda/2)|y|}\right)^4\right], \qquad (1)$$

with $Q_s^2(s) = Q_s^2(s_0)\,(s/s_0)^{\lambda/2}$. Once the energy-independent constant $c \sim 1$ and $Q_s^2(s_0)$ are determined at some energy s_0, Eq. (1) contains no free parameters. (The value of λ, describing the growth of the gluon structure functions at small x can be determined in deep-inelastic scattering; the HERA data are fitted with $\lambda \simeq 0.25$–0.3 [12].) At $y = 0$ the expression (1) coincides with the one derived in Ref. [5], and extends it to describe the rapidity and energy dependences.

Using the value of $Q_s^2 \simeq 2.05$ GeV^2 [5] extracted at $\sqrt{s} = 130$ GeV and $\lambda = 0.25$ [12] used in Ref. [6], Eq. (1) leads to the following approximate formula for the energy dependence

[1]We use $Q_s^2 \propto s^{\lambda/2}$ with $\lambda \approx 0.25$–0.3 as it follows from the scaling behavior of the HERA data [12]; see below.

of charged multiplicity in central Au-Au collisions:

$$\left\langle \frac{2}{N_{\rm part}} \frac{dN_{\rm ch}}{d\eta} \right\rangle_{\eta<1} \approx 0.87 \left(\frac{\sqrt{s}\ ({\rm GeV})}{130} \right)^{0.25} \times \left[3.93 + 0.25 \ln \left(\frac{\sqrt{s}\ ({\rm GeV})}{130} \right) \right]. \quad (2)$$

At $\sqrt{s} = 130$ GeV, we estimate from Eq. (2) $2/N_{\rm part}\, dN_{\rm ch}/d\eta\,|_{\eta<1} = 3.42 \pm 0.15$, to be compared to the average experimental value of $2/N_{\rm part}\, dN_{\rm ch}/d\eta\,|_{\eta<1} = 3.37 \pm 0.12$ [1–4]. At $\sqrt{s} = 200$ GeV, one gets $2/N_{\rm part}\, dN_{\rm ch}/d\eta\,|_{\eta<1} = 3.91 \pm 0.15$, to be compared to the PHOBOS value [1] of $2/N_{\rm part}\, dN_{\rm ch}/d\eta\,|_{\eta<1} = 3.78 \pm 0.25$. Finally, at $\sqrt{s} = 56$ GeV, we find $2/N_{\rm part}\, dN_{\rm ch}/d\eta\,|_{\eta<1} = 2.62 \pm 0.15$, to be compared to [1] $2/N_{\rm part}\, dN_{\rm ch}/d\eta\,|_{\eta<1} = 2.47 \pm 0.25$. Having convinced ourselves that our result (2) describes the experimentally observed energy dependence of hadron multiplicity in the entire interval of existing measurements at RHIC within error bars, we can extrapolate it to the small energy of $\sqrt{s} = 22$ GeV and make a prediction:

$$\left\langle \frac{2}{N_{\rm part}} \frac{dN_{\rm ch}}{d\eta} \right\rangle_{\eta<1} = 1.95 \pm 0.1; \quad \sqrt{s} = 22\ {\rm GeV}. \quad (3)$$

It is also interesting to note that formula (2), when extrapolated to very high energies, predicts for the LHC energy a value substantially smaller than found in other approaches:

$$\left\langle \frac{2}{N_{\rm part}} \frac{dN_{\rm ch}}{d\eta} \right\rangle_{\eta<1} = 10.8 \pm 0.5; \quad \sqrt{s} = 5500\ {\rm GeV}, \quad (4)$$

corresponding only to a factor of 2.8 increase in multiplicity between the RHIC energy of $\sqrt{s} = 200$ GeV and the LHC energy of $\sqrt{s} = 5500$ GeV (numerical calculations show that when normalized to the number of participants, the multiplicity in central Au-Au and Pb-Pb systems is almost identical).

Let us now turn to the centrality dependence. Our method has been described in detail before [5,6]. We first use the Glauber approach to reconstruct the geometry of the collision, and then apply semiclassical QCD to evaluate the multiplicity of produced gluons at a given centrality and pseudorapidity. The Glauber formalism (see [5,13] for details) allows us to evaluate the differential cross of inelastic nucleus–nucleus interaction at a given (pseudo)rapidity η:

$$\frac{d\sigma}{dn} = \int d^2b \mathcal{P}(n; b)[1 - P_0(b)]; \quad (5)$$

$P_0(b)$ is the probability of no interaction among the nuclei at a given impact parameter b:

$$P_0(b) = [1 - \sigma_{NN} T_{AB}(b)]^{AB}, \quad (6)$$

where σ_{NN} is the inelastic nucleon–nucleon cross section, and $T_{AB}(b)$ is the nuclear overlap function for the collision of nuclei with atomic numbers A and B; we have used the three–parameter Woods–Saxon nuclear density distributions [14]. For $\sqrt{s} = 22$ GeV we use $\sigma_{NN} = 33 \pm 1$mb basing on the interpolation of existing pp data [15]. The correlation function $\mathcal{P}(n; b)$ has a Gaussian form described in [5,13]. The total cross section of inelastic hadronic Au-Au interactions computed in our approach at $\sqrt{s} = 22A$ GeV is $\sigma_{\rm tot} = 6.9 \pm 0.05$b.

The correspondence between a given centrality cut and the mean numbers of nucleon participants and nucleon–nucleon collisions can now be established by computing the average over the distribution (5), as described in Ref. [5]. At $\sqrt{s} = 22$ GeV for Au-Au collisions we find for the 0−6% centrality cut $\langle N_{\rm part}\rangle = 332 \pm 2$; $\langle N_{\rm coll}\rangle = 828 \pm 6$. For 15−25% centrality cut, we have $\langle N_{\rm part}\rangle = 179 \pm 2$ and $\langle N_{\rm coll}\rangle = 367 \pm 6$, while for the 35−45% cut, corresponding to rather peripheral collisions with the average impact parameter of $\langle b\rangle \approx 9.5$ fm, one gets $\langle N_{\rm part}\rangle = 77 \pm 2$ and $\langle N_{\rm coll}\rangle = 120 \pm 5$.

We now have the information about the geometry of the collision needed to proceed with our calculation of centrality dependence of hadron multiplicities in the semiclassical QCD approach. However, in applying this method at small energies and/or for peripheral collisions, we face a fundamental dilemma. In the semiclassical approach, the multiplicity of the produced gluons is proportional to $1/\alpha_s(Q_s^2)$ [this is, of course, the origin of the factor $\ln(Q_s^2/\Lambda_{\rm QCD}^2)$ in our formula (1)]. Once the saturation scale Q_s^2 becomes small, the result thus becomes sensitive to the behavior of the strong coupling in the infrared region. Taking a conservative viewpoint, this simply signals that the method ceases to be applicable. If we accept this, we have to stop and conclude in favor of scenario (1) described above.

However, this is not necessarily correct—there is a solid body of evidence from jet physics that QCD coupling stays reasonably small, $\langle\alpha_s\rangle_{IR} = 0.4$–$0.6$ in the infrared region [16]. The "freezing" at small virtualities solution for the QCD coupling $\langle\alpha_s\rangle_{IR} \approx 0.43$ has been found by Gribov [17] as a consequence of "supercritical" screening of color charge by light quark–antiquark pairs. Matching QCD onto the chiral theory through scale anomaly leads to the coupling frozen in the infrared region, with magnitude $\langle\alpha_s\rangle_{IR} \approx 0.56$ [18]. "Freezing" solutions for the running coupling are repeatedly discussed; see Ref. [19] for a recent review. It is possible that the presence of relatively large scale in hadroproduction reflects the properties of QCD vacuum [20,21].

We will thus try to adopt an optimistic point of view and assume that the strong coupling indeed "freezes" below $Q_s^2 \approx 0.8$ GeV^2 at the value of $\langle\alpha_s\rangle_{IR} \approx 0.5$. In fact, one may even dare to go further—*assuming* the validity of semiclassical QCD approaches at low energies, the centrality dependence of hadron multiplicity may be used to glean information about the behavior of strong coupling in the infrared region.

To evaluate the resulting centrality dependence around $\eta = 0$ we use two different ansätze for the running coupling: (a) "smooth freezing" $\alpha_s \sim 1/\ln[(Q_s^2 + \Lambda^2)/\Lambda_{\rm QCD}^2]$, with $\Lambda = 0.8$ GeV^2; (b) "sudden freezing," when α_s is simply put equal to $\alpha_s(\Lambda^2)$ when $Q_s^2 < \Lambda^2$. The results are shown in Fig. 1 by solid [ansatz (a)] and dashed [ansatz (b)] lines. Note that even in the case of "sudden freezing," centrality dependence is smooth—this is because the fraction of the transverse area where the local value of Q_s^2 becomes smaller than the cutoff Λ^2 is a smooth function of centrality.

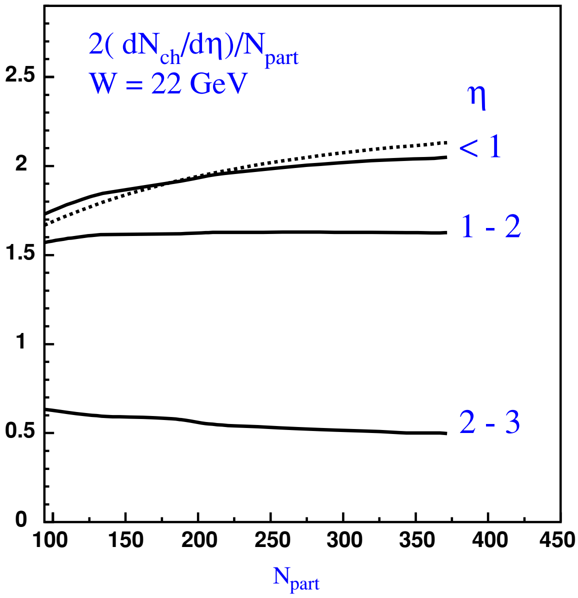

FIG. 1. (Color online) Centrality dependence of the charged multiplicity per participant pair at different pseudorapidity intervals at $\sqrt{s} = 22A$ GeV; see text for details.

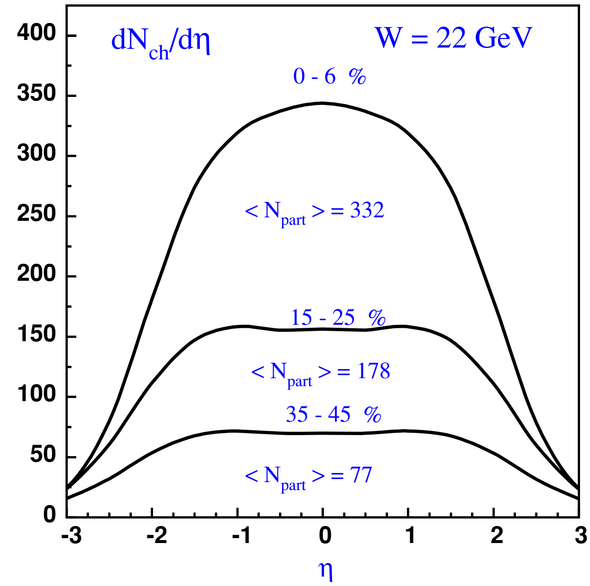

FIG. 2. (Color online) Pseudorapidity distribution of the charged multiplicity per participant pair in different centrality cuts at $\sqrt{s} = 22A$ GeV.

Let us now discuss rapidity dependence. Unfortunately, we have found that at at low energies $\sqrt{s} \sim 20$ GeV the expression (1) provides a poor approximation to the numerical result based on the general formula [7,22] used in Ref. [6]:

$$E\frac{d\sigma}{d^3p} = \frac{4\pi N_c}{N_c^2-1}\frac{1}{p_t^2} \times \int dk_t^2 \alpha_s \varphi_A\left(x_1, k_t^2\right) \varphi_A\left[x_2, (p-k)_t^2\right], \quad (7)$$

where $x_{1,2} = (p_t/\sqrt{s})\exp(\pm y)$ and $\varphi_A(x, k_t^2)$ is the unintegrated gluon distribution. This happens because at low energies the limited phase space suppresses the transverse momentum distribution of the produced gluons already below the saturation momentum Q_s. We thus have to evaluate the integral in Eq. (7) numerically (see Ref. [6] for the list of formulas needed for this computation).

To convert the computed rapidity distributions of gluons to the observed pseudorapidity distribution of hadrons, we follow the procedure of Ref. [6], assuming that the "local parton–hadron duality" (see Ref. [17] and references therein) in the space of emission angles θ, or, equivalently, in pseudorapidity $\eta = -\ln\tan(\theta/2)$. This corresponds to the physical assumption that once a gluon has been emitted along a certain direction, its final state interactions and fragmentation will not significantly change the direction of the resulting hadrons. The results of our calculations are presented in Figs. 1 and 2.

How reliable are our results, apart from the obvious leap of faith involved in the application of the semiclassical method at small energy? The least reliable of our predictions is the distribution in pseudorapidity; indeed, we have assumed that multiparticle production is dominated by gluons, and this is not necessarily so at $\sqrt{s} = 22$ GeV, where valence quarks may give essential contribution even at central rapidity. Also, since we are quite close to the fragmentation region at this energy, multiparton correlation effects can also play a role. The absolute value of multiplicity around $\eta = 0$ is more stable, but still may be affected by the contribution from quarks and deviations from $\sim x^{-\lambda}$ behavior of the nuclear gluon distribution at larger x.

However there does exist a prediction of our approach which is both robust and distinct: it is the rise with centrality of multiplicity per participant shown in Fig. 1 around $\eta \approx 0$. The shape of this dependence simply reflects the running of strong coupling, and is similar to the shape observed at the much higher energy of $\sqrt{s} = 130$ GeV: indeed, the logarithm in Eq. (1) is a slowly varying function, and the dependence of saturation scale on energy is quite weak. This prediction is in marked contrast both to the strong increase of the slope of centrality dependence with energy predicted in a two–component model [23] (for a recent development, see, however, Ref. [24]) and to the final–state saturation model [25], predicting a nearly constant, weakly decreasing, centrality dependence of multiplicity per participant.

Note added. After this manuscript had been submitted (in November of 2001), the experimental data on hadron multiplicities in Au-Au collisions at $\sqrt{s} \simeq 20$ GeV at RHIC were collected, analyzed, and published [26,27] (Ref. [27] also contains detailed comparisons to the previous data from CERN SPS at somewhat smaller energies, $\sqrt{s} \simeq 17$ GeV). As stated in these papers, the RHIC data are described well by our theoretical predictions.

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