Triaxiality and the determination of the cubic shape parameter K_3 from five observables

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The absolute and the relative quadrupole shape invariants q_3 and K_3 provide a model independent measure of triaxiality for β-rigid nuclei. We will show that one can obtain approximate values of q_3 and K_3 by analytical formulas involving only few observables. These formulas allow one to get a—model independent—error for these approximate values. The approximations which are made will be shown to hold within a few percent both in the rigid triaxial rotor model and the interacting boson model. The shape parameter K_3 is given for an exemplary set of nuclei and is translated into effective values of the geometrical deformation parameters β and γ.

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I. INTRODUCTION

A basic property of the nucleus is its geometric shape. Therefore, the nuclear shape, whether it is spherical, prolate, oblate, axially symmetric, or triaxial, is a key property of the ground state, as well as of excited states of the nucleus, where it is nowadays investigated, e.g., by means of wobbling in superdeformed bands or chirality (see, e.g. [1–5]) in odd nuclei. Earlier, large triaxialities were found for low-lying states, e.g., in the odd ^{125,127}Xe isotopes [6,7]. Quantifying the nuclear shape, one usually turns to the well-known geometric deformation parameters β and γ. These are deduced from a comparison of data with, e.g., the Davydov-Fillipov model of a rigid triaxial rotor (RTRM) [8]. This approach incorporates a major problem. A rigid rotor model cannot account for vibrations of the nuclear shape, which is a strong limitation. But, even if a model is able to describe also vibrations in the deformation parameters as, e.g., by the Bohr Hamiltonian [9], the geometric interpretation of the interacting boson model (IBM) [10,11], or the GCM [12], the problem arises that, in general, the shape parameters β and γ do not have fixed values, because the nuclei do not in general have a rigid shape, but they are vibrating. Thus, it is useful to consider alternative parameters related to the shape of a nucleus, namely quadrupole shape invariants [13–15], which are model independent, and which are direct observables. In this paper we will discuss mainly the quadratic and cubic shape parameters q_2 and q_3. We will focus on the relative cubic shape parameter $K_3 = q_3/q_2^{3/2}$, which is independent of the nuclear radius R_0 and the charge e. We will show that it is possible to obtain q_2, q_3, and K_3 with good accuracy from only few data using a well-defined truncation scheme. The cubic shape parameter K_3 is related to triaxiality and will be given for a variety of nuclei. Its connection to the geometrical deformation parameters will be discussed.

Quadrupole shape invariants were introduced by Kumar [13] and widely used by Cline and co-workers, e.g. [14,16,17]. They are expectation values in a given nuclear eigenstate of higher order moments of the $E2$ transition operator, which is usually taken to be the quadrupole operator. Considering the ground state they are defined as

$$q_2 = e^2\langle 0_1^+|(Q\cdot Q)|0_1^+\rangle, \tag{1}$$

$$q_3 = \sqrt{\frac{35}{2}}e^3\langle 0_1^+|[QQQ]^{(0)}|0_1^+\rangle, \tag{2}$$

$$q_4 = e^4\langle 0_1^+|(Q\cdot Q)(Q\cdot Q)|0_1^+\rangle, \tag{3}$$

where the dot denotes a scalar product and brackets denote tensorial coupling, Q is the quadrupole operator, and e the elementary electric charge. Higher order moments can also be defined and are related to fluctuations in q_2 and q_3. The moments q_2 and q_3 can be written in terms of averages of geometrical deformation parameters as

$$q_2 = e^2Q_0^2\langle\beta^2\rangle = e^2Q_0^2\beta_{\rm eff}^2 \quad \text{and} \quad q_3 = e^3Q_0^3\langle\beta^3\cos(3\gamma)\rangle. \tag{4}$$

with

$$Q_0 = 3ZR_0^2/(4\pi). \tag{5}$$

These quadrupole shape invariants can be renormalized to the second order invariant q_2 by [15,18]

$$K_n = \frac{q_n}{{q_2}^{n/2}}, \tag{6}$$

omitting the nuclear radius or the electric charge in this form. These quantities can in principle be obtained directly from data, but this is difficult in praxis because a large number of $E2$ matrix elements including signs is involved in expressions (1)–(3). This can be seen expanding the invariants q_n into sums over $E2$ matrix elements using Racah algebra and the Wigner-Eckert theorem, which is shown here for q_2 and q_3:

$$q_2 = e^2\sum_i\langle 0_1^+||Q||2_i^+\rangle\langle 2_i^+||Q||0_1^+\rangle, \tag{7}$$

$$q_3 = \sqrt{\frac{7}{10}}e^3\sum_{i,j}\langle 0_1^+||Q||2_i^+\rangle\langle 2_i^+||Q||2_j^+\rangle\langle 2_j^+||Q||0_1^+\rangle. \tag{8}$$

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These sums have been referred to as Cline-Flaum sum rules [19]. An evaluation of q_2 and q_3 using extensive sets of experimental quadrupole matrix elements from multiple Coulomb excitation has been done for some nuclei by D. Cline and co-workers, even for various states of the ground state band and the γ band, e.g., in [14,16,17,20–22]. Of course, the existence of such extensive data sets is the favorable, but it is not the general case. Thus, there is interest to obtain the shape invariants from the most restricted sets of data. This can be done by various truncation schemes. These schemes allow also to establish theoretical errors by using various models, and to check for the quality of made approximations within these models. Our objective will be to obtain q_3 and K_3 only for the 0^+ ground state of even-even nuclei. But we want to establish relations which yield q_3 and K_3 from few observables with small theoretical errors. Besides giving the relations themselves, the calculation of these theoretical errors is a focus of the present paper.

II. APPROXIMATIONS

The basic idea is to invoke the Q-phonon scheme for truncation as has been discussed in [23]. The Q-phonon scheme was originally suggested by Otsuka [24], and was developed by a Köln-Tokyo collaboration, e.g. [25–28]. The Q-phonon scheme implies that the wave functions of low-lying states are exhausted by only a few multiple Q-phonon configurations, where a Q phonon itself is an excitation by the quadrupole operator. The 2_1^+ state in an even-even nucleus is dominantly a one-Q-phonon state. It was shown [26,27] that the Q-phonon scheme holds with good accuracy for the lowest levels of collective nuclei. Here, as we will consider only the very lowest states, we keep within the nonorthogonalized Q-phonon scheme [28,29], which will be shown to be sufficient for our purpose. The Q-phonon scheme gives a simple selection rule, namely, that an $E2$ transition between two states may change the number of Q phonons in first order only by one, i.e., $\Delta Q = 1$ ($\Delta Q = 0$ is allowed only for the $E2$ moment of a given state). Neglecting all Q-forbidden transition matrix elements with $\Delta Q \geqslant 2$ gives the first order approximation. We will denote quantities given in this first order approximation by a superscript (1). This leads to a drastic truncation in the matrix elements needed in the expansions in Eqs. (7), (8), e.g., q_2 as given in Eq. (7) is approximated by

$$q_2 \approx q_2^{(1)} = e^2 \langle 2_1^+ || Q || 0_1^+ \rangle^2 = B(E2; 0_1^+ \to 2_1^+), \tag{9}$$

because transitions from the two-phonon 2_2^+ state or even higher-lying 2^+ states to the ground state have $\Delta Q > 1$ and are thus Q-forbidden in first order. Equation (9) reflects the well-known fact that in most even-even nuclei the largest part of the $E2$ excitation strength is concentrated in the first excited 2^+ state. In the rigid rotor this $B(E2)$ value is known to be directly proportional to the squared β deformation by Eq. (4). In the case of nonrigid β deformation Eq. (4) defines an effective deformation parameter β_{eff} or, making use of the approximation (9), an approximate $\beta_{\text{eff}}^{(1)}$.

Using the Q-phonon scheme in first order for the truncation of q_3 one obtains

$$q_3^{(1)} = \sqrt{\frac{7}{10}} e^3 \langle 2_1^+ || Q || 0_1^+ \rangle^2 \cdot \langle 2_1^+ || Q || 2_1^+ \rangle. \tag{10}$$

Then, approximating the K_3 parameter following its definition in Eq. (6) ($n = 3$) results in

$$K_3^{(1)} = \frac{q_3^{(1)}}{\left(q_2^{(1)}\right)^{3/2}} = \sqrt{\frac{7}{10}} \frac{\langle 2_1^+ || Q || 2_1^+ \rangle}{\langle 2_1^+ || Q || 0_1^+ \rangle}, \tag{11}$$

which is calculated from the ratio of the quadrupole moment of the 2_1^+ state and its $E2$ matrix element to the ground state. It turns out, e.g., checking this approximation within the rigid triaxial rotor model or the IBM-1, that the first order truncation of the sum given in Eq. (8) is not sufficient for a good approximation to K_3, as we will show in Secs. III and IV. Therefore, we used a second order approximation, allowing in each term of the sum *at most one* Q-forbidden matrix element with $\Delta Q = 2$. Doing so, we give a second order approximation for q_3 as

$$\begin{aligned} q_3^{(2)} = q_3^{\text{appr.}} = \sqrt{\frac{7}{10}} e^3 [& \langle 2_1^+ || Q || 0_1^+ \rangle^2 \langle 2_1^+ || Q || 2_1^+ \rangle \\ & + 2 \langle 0_1^+ || Q || 2_2^+ \rangle \langle 2_2^+ || Q || 2_1^+ \rangle \langle 2_1^+ || Q || 0_1^+ \rangle]. \end{aligned} \tag{12}$$

This expression involves only four $E2$ matrix elements. In the following we will always denote the second order approximation with the superscript "appr." instead of (2), as it is the only one we use. Note that the approximation to q_2 in second order approximation is the same as in first order, as a Q-forbidden matrix element would always appear squared and such terms are not included in this approximation, and we get

$$q_2^{\text{appr.}} = q_2^{(1)} \quad \text{and} \quad \beta_{\text{eff}}^{\text{appr.}} = \beta_{\text{eff}}^{(1)}. \tag{13}$$

Dividing $q_3^{\text{appr.}}$ from Eq. (12) by $q_2^{\text{appr.}}$, we get a second order approximation for K_3 that includes only four different $E2$ matrix elements, involving the lowest two excited 2^+ states and the ground state.

A problem that appears in Eq. (12) is that the signs of the $E2$ matrix elements are needed, which are not known in many cases. Usually we know at most the $B(E2)$ values which are

$$B(E2; J_i \to J_f) = \frac{1}{2J_i + 1} e^2 \langle J_f || Q || J_i \rangle^2. \tag{14}$$

This lack of knowledge in the signs can be avoided by using a relation between the signs of four matrix elements, which was suggested, e.g., by Kumar [30] for vibrational and rotational nuclei, and shown to hold in the rather general parameter space of the IBM-1 by Jolos and von Brentano [31]:

$$\begin{aligned} \text{sign}(\langle 2_1^+ || Q || 2_1^+ \rangle) = & -\text{sign}(\langle 0_1^+ || Q || 2_2^+ \rangle \\ & \times \langle 2_2^+ || Q || 2_1^+ \rangle \langle 2_1^+ || Q || 0_1^+ \rangle). \end{aligned} \tag{15}$$

This relation gives the relative phase of the two terms in Eq. (12). There is still an overall sign of K_3, which is the sign of the quadrupole moment of the 2_1^+ state, deciding between prolate and oblate deformation. Then, the second

order approximation for $K_3^{\text{appr.}}$ is

$$K_3^{\text{appr.}} = \sqrt{\frac{7}{10}}\text{sign}(Q(2_1^+))\left[\sqrt{\frac{B(E2;2_1^+\to 2_1^+)}{B(E2;2_1^+\to 0_1^+)}} - 2\frac{\sqrt{B(E2;2_2^+\to 0_1^+)\cdot B(E2;2_2^+\to 2_1^+)}}{B(E2;2_1^+\to 0_1^+)}\right], \quad (16)$$

where we use an alternative but useful definition of the squared quadrupole moment following Eq. (14),

$$B(E2;2_1^+\to 2_1^+) = \frac{1}{5}e^2\langle 2_1^+||Q||2_1^+\rangle^2 = \frac{35}{32\pi}Q(2_1^+)^2. \quad (17)$$

The approximation formula for K_3 given in Eq. (16) is one key result of this work. It allows us to measure the observable $K_3^{\text{appr.}}$ directly and in a model independent way from only few data. These are four absolute $B(E2)$ values, namely $B(E2;2_1^+\to 0_1^+)$, $B(E2;2_2^+\to 0_1^+)$, $B(E2;2_2^+\to 2_1^+)$, and $B(E2;2_1^+\to 2_1^+)$, and the sign of the quadrupole moment of the 2_1^+ state, which we consider here as a fifth observable, as it cannot be obtained from $B(E2)$ values alone. This method to determine K_3 is of particular interest because K_3 is closely connected to the triaxiality of nuclei, i.e., to γ deformation. For axial symmetry $K_3 = -1$ for prolate ($\gamma = 0^\circ$) and $K_3 = +1$ for oblate ($\gamma = 60^\circ$) nuclei, while K_3 drops to zero at a maximum triaxiality of $\gamma = 30^\circ$. This holds for geometrical models like the Davydov-Fillipov triaxial rotor model, as well as for the dynamical symmetries of the IBM-1. One major difference between these two models is that the IBM-1 describes nonrigid β and γ deformation, e.g., in the U(5) vibrational limit and the O(6) limit of γ-soft nuclei, in both of which K_3 vanishes. In the SU(3) and $\overline{\text{SU(3)}}$ dynamical symmetries of the IBM-1, which correspond to the prolate and oblate axially symmetric rigid rotors, respectively, the same values for K_3 are derived as in the geometrical model. In the following we will check to which extent

$$K_3 \approx K_3^{\text{appr.}} \quad (18)$$

holds, using as test models the RTRM of Davydov and Fillipov and the IBM-1. Of course, in the models one can calculate both, K_3 and $K_3^{\text{appr.}}$, and their difference $\Delta K_3^{\text{appr.}} = |K_3 - K_3^{\text{appr.}}|$, which one can call the model dependent theoretical error of $K_3^{\text{appr.}}$. It is worthwhile to note that $q_3^{\text{appr.}}$ and $K_3^{\text{appr.}}$ are derived in a model independent way, whereas the theoretical error, $\Delta K_3^{\text{appr.}}$, is a model dependent quantity. In the following Secs. III and IV we discuss the calculation of the error.

III. ERROR ANALYSIS IN THE RTRM

The Hamiltonian of the Davydov-Fillipov rotor model is

$$H_{\text{geo}} = A_1 J_1^2 + A_2 J_2^2 + A_3 J_3^2, \quad (19)$$

where J_n are the projections of the spin J on the three symmetry axes, and where the parameters A_k are connected to the moments of inertia Θ_k by

$$A_k = \frac{\hbar^2}{2\Theta_k}. \quad (20)$$

The moments of inertia can further be written in terms of the geometrical deformation parameters β and γ,

$$\Theta_k = 4B\beta^2\sin^2\left(\gamma - \frac{2k}{3}\pi\right). \quad (21)$$

The $E2$ transition operator is given by

$$T(E2)_{\text{geo}} = eQ_{2\mu} = eQ_0\beta\left[D^{2*}_{\mu 0}\cos(\gamma) + \frac{1}{\sqrt{2}}\left(D^{2*}_{\mu 2} + D^{2*}_{\mu -2}\right)\sin(\gamma)\right], \quad (22)$$

where the $D^2_{\mu\nu}$ are the Wigner D matrices and Q_0 is given by Eq. (5). We stress that this model with rigid β and γ deformations is applicable to only a limited number of nuclei. Nevertheless, apart from our discussion of the ground state deformation, the model is often applied to highly excited and strongly and also superdeformed bands as well, for which, in principle, our approach of K parameters may also apply.

In our calculations we vary the parameter γ over the range ($\gamma \in [0^\circ, 30^\circ]$), covering the range of prolate axially symmetric and triaxial structures inherent to the model. The results for $\gamma \in [30^\circ, 60^\circ]$ are fully symmetric to those given and thus omitted. The choice of β is arbitrary, as in the β-rigid case K_3 is independent of β and is given by

$$K_3 = -\frac{\beta^3\cos(3\gamma)}{(\beta^2)^{3/2}} = -\cos(3\gamma). \quad (23)$$

In a similar way one defines an approximate deformation $\gamma^{\text{appr.}}$ from $K_3^{\text{appr.}}$ by

$$K_3^{\text{appr.}} = -\cos(3\gamma^{\text{appr.}}). \quad (24)$$

In order to avoid a division by zero, we use the ratio

$$R_{\text{geo}}^{K_3} = \frac{1 + |K_3^{\text{appr.}}|}{1 + |K_3|} \quad (25)$$

as a measure of the quality of the approximation (18). The solid curve in the left panel of Fig. 1 shows the quantity $R_{\text{geo}}^{K_3}$ versus the deformation parameter γ, calculated numerically using the code DAVIDOV [32]. In the axially symmetric limit at $\gamma = 0^\circ$ the approximation is exact. This also holds for the case of maximum triaxiality at $\gamma = 30^\circ$, while $R_{\text{geo}}^{K_3}$ is small for all intermediate cases with a deviation from one of 8% in maximum. The dashed curve represents the same calculation, but using the first order approximation $K_3^{(1)}$ from Eq. (11). The deviation of the first order approximation is clearly much larger with a maximum of about 30%, showing that the use of a second order approximation is unavoidable for transitional nuclei. On the right hand side of Fig. 1 the corresponding absolute deviation of $\gamma^{\text{appr.}}$ derived from Eq. (24) from the real γ values in the model is shown as a solid curve. The maximum deviation is below 3.5° at $\gamma \approx 15^\circ$. Again, the deviation is much larger using only the first order approximation, given as a dashed curve.

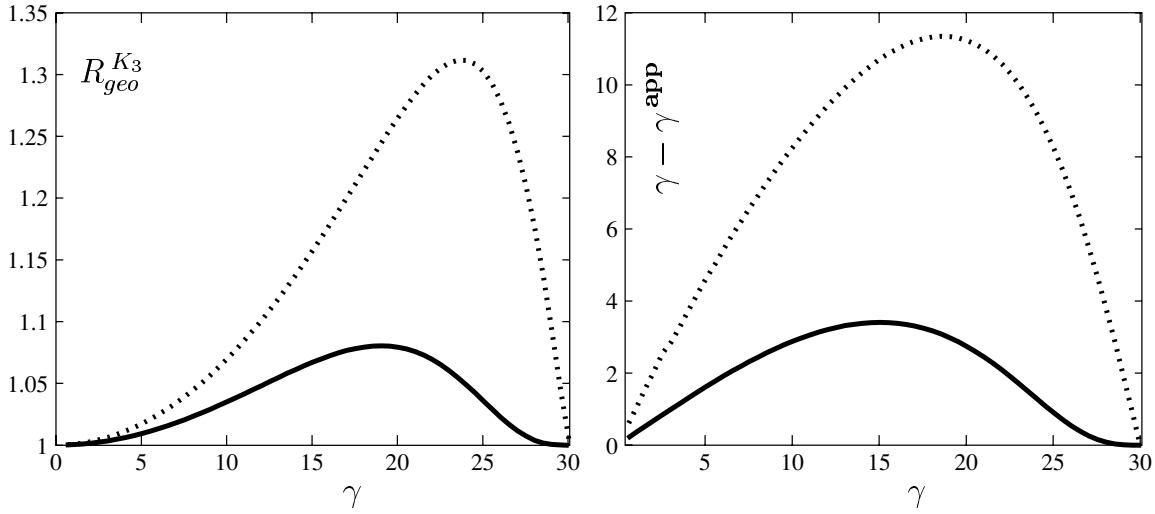

FIG. 1. $R_{\mathrm{geo}}^{K_3}$ calculated for all values of γ (solid line on the left hand side), showing that the second order approximation Eq. (18) holds well in the rigid triaxial rotor model. The approximation does not seriously change the value of γ (solid line on the right hand side). The dashed lines give the values derived from the use of only the first order approximation.

IV. ERROR ANALYSIS IN THE IBM-1

Now, we calculate $K_3^{\mathrm{appr.}}$ in the IBM-1, within the extended consistent Q formalism (ECQF) [33,34] using the Hamiltonian [18]

$$H_{\mathrm{IBM}} = (1-\zeta)n_d - \frac{\zeta}{4N} Q^\chi \cdot Q^\chi, \tag{26}$$

which depends on only two structural parameters, ζ and χ, and where we omit an overall energy scale. The $E2$ transition operator in the ECQF is chosen to be proportional to the quadrupole operator in the Hamiltonian,

$$T(E2)_{\mathrm{IBM}} = e_B Q^\chi = e_B[(s^+\tilde{d} + d^+ s) + \chi(d^+\tilde{d})], \tag{27}$$

where e_B is the effective boson charge, and $n_d = (d^+\tilde{d})$ is the boson number operator. Varying the values of ζ and χ in the Casten triangle over the full range of symmetries ($\zeta \in [0,1]$, $\chi \in [-\sqrt{7}/2, \sqrt{7}/2]$), one covers the dynamical symmetry limits of the IBM, namely U(5) ($\zeta = 0$, χ), the prolate (oblate) SU(3) ($\overline{\mathrm{SU(3)}}$) ($\zeta = 1$, $\chi = \mp\sqrt{7}/2$), and O(6) ($\zeta = 1$, $\chi = 0$), as well as the transitional structures in between. In analogy to Eq. (25), we define the ratio

$$R_{\mathrm{IBM}}^{K_3} = \frac{1 + \left|K_3^{\mathrm{appr.}}\right|}{1 + |K_3|}, \tag{28}$$

which has been calculated over the full parameter space using the code PHINT [35]. Again, $K_3^{\mathrm{appr.}}$ is defined by Eq. (16). The results are shown in the top part of Fig. 2 for a boson number $N = 10$. They are given only for $\chi < 0$, because the results for $\pm\chi$ are fully symmetric, as the change in sign is equivalent to the symmetry transformation $d \to -d$ (keeping $s \to s$). The use of positive χ values corresponds to the choice of $\gamma > 30^\circ$ in the geometrical model. Deviations of $K_3^{\mathrm{appr.}}$ from the exact K_3 values are small in all cases, the deviation of $R_{\mathrm{IBM}}^{K_3}$ from 1 is below 7%. For comparison, Fig. 3 shows $R_{\mathrm{IBM}}^{K_3(1)}$, which is defined analog to Eq. (28), but where the first order approximation, i.e., Eq. (11), is used. Like in the geometrical model it is seen, that the deviations from the exact value of K_3 are much larger in the first order approximation. Thus, in general it is necessary to use $K_3^{\mathrm{appr.}}$ in the second approximation from Eq. (16).

The deviation $R_{\mathrm{IBM}}^{K_3}$ peaks in a region around SU(3). A reason for this behavior is found by a close look at this region. The middle part of Fig. 2 shows the values of K_3 and $K_3^{\mathrm{appr.}}$ on the U(5)–SU(3) (left) and O(6)–SU(3) (right) transition legs. It is obvious that the maximum deviation of $R_{\mathrm{IBM}}^{K_3}$ from unity appears in those regions, in which K_3 changes most rapidly. These are exactly those regions that are connected to the shape/phase transition between spherical and axially symmetric nuclei, or between prolate and oblate deformations, as discussed, e.g., in [36–43]. This means that in the IBM the approximation $K_3^{\mathrm{appr.}}$ misses the exact value of K_3 somewhat when leaving the rotational limit. However, overall deviations of $R_{\mathrm{IBM}}^{K_3}$ from unity are small and the approximation (18) is well fulfilled.

From comparison with the geometrical model an effective γ deformation can be defined [18] from K_3 by

$$K_3 = -\frac{\langle\beta^3\cos(3\gamma)\rangle}{\langle\beta^2\rangle^{3/2}} = -\cos(3\gamma_{\mathrm{eff}}), \tag{29}$$

and an approximate value $\gamma_{\mathrm{eff}}^{\mathrm{appr.}}$ can be defined analogously from $K_3^{\mathrm{appr.}}$. The differences between the exact and the approximate γ values, $\gamma_{\mathrm{eff}} - \gamma_{\mathrm{eff}}^{\mathrm{appr.}}$ are included in the bottom part of Fig. 2 and show good agreement. The deviation of $\gamma_{\mathrm{eff}}^{\mathrm{appr.}}$ from γ_{eff} is always smaller than 2.5°.

These effective γ values are not and cannot be equivalent to those given by Eqs. (23), (24), because K_3 is not generally independent of β deformation and fluctuations in β occur, especially for vibrational nuclei. Moreover, in case of rigid β [on the SU(3)–O(6) transitional line] K_3 is a measure of $\langle\cos(3\gamma)\rangle$, while in case of rigid γ it is a measure of $\langle\beta^3\rangle/\langle\beta^2\rangle^{3/2}$. The effect of a β vibration is only effectively taken out in the translation to the geometric model by Eq. (29).

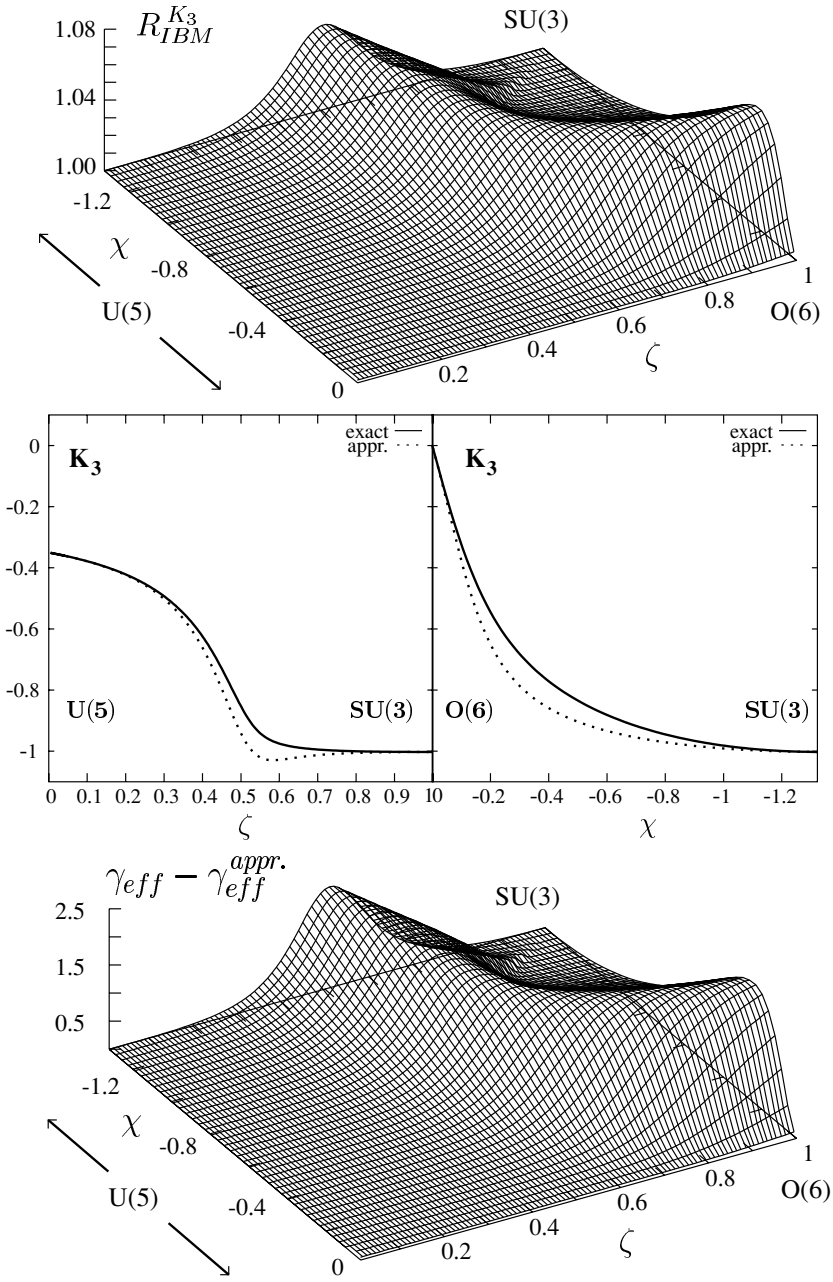

FIG. 2. Top panel: $R^{K_3}_{\rm IBM}$ calculated over the whole IBM parameter space confirms a good fulfillment of the second order approximation to K_3. Middle panel: The two transitional legs for fixed values of $\chi = -\sqrt{7}/2$ (left) and $\zeta = 1$ (right). The approximation misses slightly the phase-/shape-transitional parameter region. Bottom panel: Effective γ deformations calculated from K_3 and $K_3^{\rm appr.}$ calculated over the whole parameter range. All calculations are for $N = 10$ bosons.

However, if fluctuations in β are small, which is the case past the phase transition towards deformed nuclei (typically for $\zeta > 0.6$), a factorization of the averages over β and $\cos(3\gamma)$ should work, and we can assume

$$\langle \beta^3 \cos(3\gamma)\rangle = \langle \beta^3\rangle \langle \cos(3\gamma)\rangle \quad \text{and} \quad \frac{\langle \beta^3\rangle}{\langle \beta^2\rangle^{3/2}} = 1, \tag{30}$$

making $\gamma_{\rm eff}$ comparable to the geometrical γ deformation.

V. K_3 FOR VARIOUS NUCLEI

A. Direct measure of K_3

For the two considered models we have shown that $K_3^{\rm appr.}$ is a good approximation to the value of the cubic shape parameter K_3. Thus we assume this to hold also in other collective models such as the GCM or the Bohr Hamiltonian, and quite general for collective nuclei. Only few observables have to be obtained in order to derive $K_3^{\rm appr.}$, namely the lifetime of the 2_1^+ state, the lifetime and the branching ratio of the 2_2^+ state, and the quadrupole moment of the 2_1^+ state. For the $2_2^+ \to 2_1^+$ transition also the $E2/M1$ multipole mixing ratio needs to be known, which can be assumed to be in favor of $E2$ radiation in many cases. Besides the modulus of K_3 also its sign is interesting, which is obtained from the sign of the quadrupole moment of the 2_1^+ state. This quadrupole moment itself is not easy to obtain, therefore it is a challenge to measure triaxiality. Especially for vibrational or γ-soft nuclei, where the quadrupole moment is small, high quality data are needed. Thus, an approximate value of K_3 is so far known

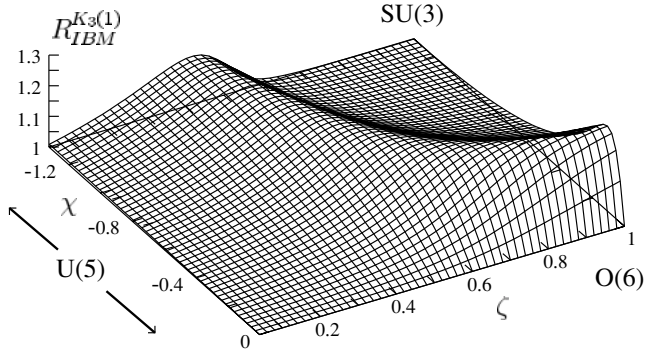

FIG. 3. $R_{\mathrm{IBM}}^{K_3(1)}$, the analog to the top panel of Fig. 2, but using only the first order approximation. Deviations from unity are much larger than in the second approximation.

for a number of nuclei in or near the valley of stability only. For a set of nuclei that belong to various symmetry regions the $K_3^{\mathrm{appr.}}$ parameter has been calculated from tabulated data. If matrix elements including signs were known we used them and calculated $K_3^{\mathrm{appr.}}$ from Eqs. (9), (12), else $B(E2)$ values were used and $K_3^{\mathrm{appr.}}$ calculated from Eq. (16). Data, that means $E2$ matrix elements or $B(E2)$ values, used for our calculations are listed in Table I. Results are given in Table II, together with effective γ-deformation parameters derived from Eq. (29), and effective β deformations from Eqs. (4), (9). For the Os, Pt, and Pd isotopes, and for ^{114}Cd, data from Coulomb excitation experiments have been used, yielding signs of the matrix elements which are in agreement with the sign relation given in Eq. (15), except as for ^{194}Pt as discussed below. For some of the nuclei Cline and co-workers found similar values, while we use Eqs. (12) and (16) to derive $K_3^{\mathrm{appr.}}$ in a consistent way.

Typical rotational nuclei like the heavier Gd or Dy isotopes show K_3 values close to -1 as it is expected for prolate deformed axially symmetric shapes. Also ^{152}Sm and ^{154}Gd, which are attributed [37,54] to be close to the critical point symmetry X(5) proposed by Iachello [36], show such values. In these nuclei, as well as in the adjacent ^{156}Gd, the 2_2^+ state is of a different nature than in the other listed nuclei, namely it is the 2^+ state built on the lowest excited 0^+ state instead of the head of a γ band, which is in these cases identified as the third excited 2^+ state. For consistency, in ^{152}Sm and ^{154,156}Gd, we took the 2_3^+ state into account for the determination of $K_3^{\mathrm{appr.}}$, instead of the 2_2^+ state (for which an additional term that would contribute to the K_3 value is about an order of magnitude smaller than that involving the 2_3^+ state and, hence, can be neglected).

The K_4 parameter obtained from q_4 of Eq. (3) and Eq. (6), which can be approximated in a similar way [23,31], gives a direct measure for β softness. One finds that $K_4^{\mathrm{appr.}} = 1$ for β-rigid nuclei and $K_4^{\mathrm{appr.}} \sim 1.4$ for vibrators. For ^{152}Sm and ^{154}Gd one finds $K_4^{\mathrm{appr.}}$ values of 1.02(3) [55] and 1.088(26) [54], respectively, which, in combination with $K_3^{\mathrm{appr.}}$, meets the expectations for the vibrator to well-deformed rotor transition, more on the rotational side of the phase transition. Especially ^{152}Sm seems to be on the rotor side (where $K_3^{\mathrm{appr.}} = -1$ and $K_4^{\mathrm{appr.}} = 1$) of the phase transition, which seemingly conflicts with the interpretation of this nucleus as being close to the phase transitional point. This may be related to the systematical error made in the approximations for K_3 that maximizes exactly in the transitional region in the IBM, which may be reflected also in data. However, the systematical error made in

TABLE I. $E2$ data used for our derivation of $K_3^{\mathrm{appr.}}$. The used $E2$ matrix elements or $B(E2)$ values are listed with the references they have been taken from. As errors always the relevant last digits are given. For the nuclei marked with an asterisk the 2_3^+ state was taken into account instead of the 2_2^+ state for the corresponding transition, as explained in the text.

	Data from	$\langle 2_1^+ \Vert Q \Vert 2_1^+ \rangle$ [e b]	$\langle 0_1^+ \Vert Q \Vert 2_1^+ \rangle$ [e b]	$\langle 0_1^+ \Vert Q \Vert 2_2^+ \rangle$ [e b]	$\langle 2_1^+ \Vert Q \Vert 2_2^+ \rangle$ [e b]
^{188}Os	[16]	-1.73^{+19}_{-5}	$+1.585(10)$	$+0.483^{+2}_{-9}$	$+0.865(11)$
^{190}Os	[16]	-1.25^{+22}_{-13}	$+1.530^{+20}_{-11}$	$+0.444^{+9}_{-7}$	$+1.065^{20}_{-37}$
^{192}Os	[16]	-1.21^{+6}_{-17}	$+1.456^{+8}_{-9}$	$+0.430^{+8}_{-4}$	$+1.230^{34}_{-16}$
^{194}Pt	[16]	$+0.54^{+8}_{-6}$	$+1.208^{+49}_{-17}$	$+0.0888(12)$	$+1.517^{11}_{-18}$
^{196}Pt	[50]	$+0.82(10)$	$+1.1697(13)$	–	$+1.36(1)$
^{106}Pd	[51]	-0.72^{+6}_{-7}	$+0.79(4)$	$-0.114(6)$	$-0.76(4)$
^{108}Pd	[51]	-0.81^{+4}_{-9}	$+0.87^{+6}_{-4}$	$-0.098(5)$	$-0.88(4)$
^{114}Cd	[53]	-0.36^{+1}_{-3}	$+0.714(21)$	$+0.091(3)$	$+0.684(21)$
		$Q(2_1^+)$ [e b]	$B(E2; 2_1^+ \to 0_1^+)$ [e^2 b^2]	$B(E2; 2_{2(\gamma)}^+ \to 0_1^+)$ [e^2 b^2]	$B(E2; 2_{2(\gamma)}^+ \to 2_1^+)$ [e^2 b^2]
^{152}Sm*	[49]	$-1.702(17)$	0.694(14)	0.017(1)	0.045(2)
^{154}Gd*	[48]	$-1.82(4)$	0.775(5)	0.028(2)	0.060(6)
^{156}Gd*	[44]	$-1.93(4)$	0.933(25)	0.023(1)	0.036(1)
^{158}Gd	[45]	$-2.01(4)$	1.005(30)	0.030(4)	0.0173(15)
^{160}Gd	[46]	$-2.08(4)$	1.038(8)	0.020(1)	0.037(1)
^{164}Dy	[47]	$-2.08(15)$	1.114(16)	0.021(2)	0.043(4)
^{112}Cd	[52]	$-0.37(4)$	0.097(1)	0.0019(3)	0.048(10)

TABLE II. Approximate $K_3^{\text{appr.}}$ values, the effective approximate β- and γ-deformation parameters derived from our approach are listed for a set of nuclei. For β deformations, errors are omitted as they are in the order of per mil or smaller, and the systematic error made by assuming $R_0 = 1.2$ fm for the nuclear radius is presumably larger. The last two columns give upper and lower limits for the value of K_3 fitted to the observables (33), (34) as described in Sec. B.

	Data from	$K_3^{\text{appr.}}$	$\beta_{\text{eff}}^{\text{appr.}}$	$\gamma_{\text{eff}}^{\text{appr.}}$	K_3^{fit}	
					upper	lower
^{156}Gd	[44]	$-0.93(3)$	0.339	7(2)	-0.86	-0.98
^{158}Gd	[45]	$-0.95(3)$	0.349	6(2)	-0.83	-1.00
^{160}Gd	[46]	$-0.96(2)$	0.351	5(2)	-0.85	-1.00
^{164}Dy	[47]	$-0.93(8)$	0.347	7(4)	-0.77	-1.00
^{154}Gd	[48]	$-0.93(3)$	0.310	7(2)	-0.77	-0.84
^{152}Sm	[49]	$-0.94(2)$	0.307	6(1)	-0.54	-0.83
^{188}Os	[16]	-0.63^{+12}_{-3}	0.185	17^{+3}_{-2}	-0.65	-0.76
^{190}Os	[16]	-0.35^{+14}_{-9}	0.177	23^{+3}_{-2}	-0.53	-0.75
^{192}Os	[16]	-0.28^{+4}_{-12}	0.167	25^{+1}_{-2}	-0.48	-0.71
^{194}Pt	[16]	$+0.53^{+7}_{-6}$	0.143	41^{+2}_{-1}	0.08	0.14
^{196}Pt	[50]	$+0.59(9)$	0.129	42(2)	0.00	0.02
^{106}Pd	[51]	-0.53^{+8}_{-9}	0.230	19^{+2}_{-2}	-0.41	-0.55
^{108}Pd	[51]	-0.59^{+6}_{-11}	0.242	18^{+2}_{-3}	-0.23	-0.30
^{112}Cd	[52]	$-0.42(8)$	0.181	22(2)	-0.57	-0.83
^{114}Cd	[53]	-0.22^{+2}_{-4}	0.184	25.8^{+3}_{-8}	-0.36	-0.59

the determination of K_4 should be smaller in that region [31] and a problem remains.

The approximate values of the shape parameters K_3 and K_4 as measured for ^{152}Sm and ^{154}Gd can be compared to those derived within the IBM-1 at the critical point of the vibrator-rotor phase transition, ζ_c ($\chi = -\sqrt{7}/2$ is fixed). The definition of this critical value of the IBM parameter ζ is not trivial due to the finite size of the system and the subsequent smearing of the critical point, which is hence not well defined in such systems. However, a critical point may be defined following a procedure described in [40] within the IBM-1, resulting in $\zeta_c \approx 0.54$ for boson numbers of $N = 10, 11$, corresponding to ^{152}Sm and ^{154}Gd, respectively. At that point, the critical values of $K_3^{\text{appr.}}$ and $K_4^{\text{appr.}}$ are $K_{3,c}^{\text{appr.}} \approx -1.02$ and $K_{4,c}^{\text{appr.}} \approx 1.12$, respectively. The values of the measured $K_3^{\text{appr.}}$ parameters indicate that this choice of the model parameters is not ideal. $K_4^{\text{appr.}}$ which is more sensitive to the vibrator-rotor transition clearly shows a deviation from its critical value in the case of ^{152}Sm, while the agreement for ^{154}Gd is much better.

The K_3 values of the Os isotopes show an evolution from the axially symmetric rotor towards O(6) symmetry with a maximum effective triaxiality of $\gamma_{\text{eff}} = 30^\circ$. Note, that here one talks of *effective* γ deformation, as the nucleus does not have a rigid triaxiality. The more vibrational Pd and Cd isotopes show moderate values of K_3 with relatively large errors due to the quadrupole moments. Nonzero values are not a contradiction to a more U(5) like structure as they may emanate from finite N effects (see [18]).

A surprising conflict appears for ^{196}Pt, which is usually taken as a prime example of O(6) symmetry [56], as well as for the neighboring ^{194}Pt. Both nuclei show rather large, positive quadrupole moments [16] and thus have quite large values of $K_3^{\text{appr.}}$. In addition, for ^{194}Pt the sign relation in Eq. (15) is not valid. This may be related to the fact that the matrix element $\langle 0_1^+ || Q || 2_2^+ \rangle$ is exceptionally small in this case, and the actual value and sign of the matrix element may depend on effects not incorporated by the IBM-1 in which the sign relation (15) was tested. For ^{196}Pt even only an upper limit for this matrix element is known, which is negligible for our needs. In Table II we used the measured signs from [16] as input for Eq. (12), which are in favor of somewhat larger values of $K_3^{\text{appr.}}$ for ^{194}Pt. The finite positive value of $K_3^{\text{appr.}}$ shows that both Pt isotopes have oblate deformation, with a considerable deviation of $K_3^{\text{appr.}}$ from the γ-soft expectation value, $K_3[O(6)] = 0$. Other observables like the branching ratio of the 2_2^+ state or energies agree much better with the O(6) predictions. Therefore, values of K_3 derived from an IBM fit (see below) agree much better with $K_3 = 0$. One cannot argue that this deviation is due to the dependence on β fluctuations [compare Eq. (29)]. On the SU(3)–O(6) transition line, no β fluctuations are allowed, and indeed, the shape invariant K_4 approximately equals 1 (see [23]) for both nuclei, which pinpoints β rigidity. Again, this may be related to the maximal systematical error close to O(6) seen from Fig. 2. But, even if the value of K_3 is overpredicted from the approximation, a deviation from O(6) remains. However, we want to stress that these values, e.g., $\gamma_{\text{eff}} = 42^\circ$ instead of $\gamma_{\text{eff}} = 30^\circ$ for ^{196}Pt, still indicate a strong triaxiality. It is only the quantitative value of γ_{eff} which is in doubt, and which may suggest the use of a different Hamiltonian for the description of these nuclei, e.g., involving higher terms in the IBM-1, breaking

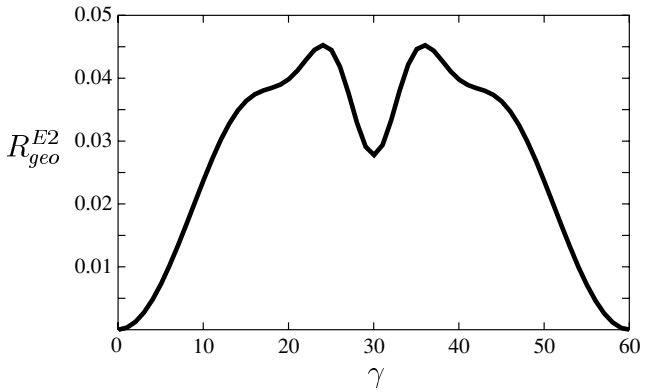

FIG. 4. $R^{E2}_{\rm geo}$ calculated for all values of γ. The $E2$ relation (32) holds well in the geometrical model.

the ECQF, or introducing triaxiality by the proton-neutron degree of freedom as proposed in [57]. Within the RTRM the problem was recently approached by considering a mixing of the two 2^+ states of the model [58].

We stress that if one uses only the first order approximation, the value of K_3 is missed for transitional nuclei like the Os isotopes, for which the transition $2^+_2 \to 0^+_1$ is sizeable, e.g., $K_3^{\rm appr.} = -0.70^{+0.03}_{-0.10}$ for ^{192}Os in the first order approximation, underestimating triaxiality, which is revealed in the second order approximation giving $K_3^{\rm appr.} = -0.28^{+0.04}_{-0.12}$.

Note, that the electric quadrupole moment of the 2^+_1 state, $Q(2^+_1)$, is usually not easy to access experimentally. A relation was derived in [23] within the first order Q-phonon truncation, and tested within the IBM-1 and data. Recently we found that the relation for infinite boson number was already given in 1975 in [59,60]. The relation gives a way to approximately determine $|Q(2^+_1)|$ or $B(E2; 2^+_1 \to 2^+_1)$, respectively:

$$B(E2; 2^+_1 \to 2^+_1)^{(1)} = B(E2; 4^+_1 \to 2^+_1) - B(E2; 2^+_2 \to 2^+_1). \tag{31}$$

The second approximation would affect the truncation of the sum given in Eq. (7) in [23] and thus the relation, however, in that work it was shown that the first truncation is sufficient. The relation may be used to obtain the quadrupole moment as an input for K_3, but it will give a large uncertainty especially for vibrational or γ-soft nuclei, which have a small quadrupole moment. So far the relation (31) was only checked in the IBM [23]. Figure 4 shows the deviation

$$R^{E2}_{\rm geo} = 1 - \frac{B(E2; 4^+_1 \to 2^+_1)}{B(E2; 2^+_1 \to 2^+_1) + B(E2; 2^+_2 \to 2^+_1)} \tag{32}$$

from the real value calculated within the rigid triaxial rotor model. Also in the geometrical model the agreement is good.

B. Fit procedure for K_3

In cases where not all of the needed data are present, one may follow another procedure, fitting parameters of a model to the available data for one nucleus, and calculating K_3 from the model. Here we used the simple two parameter Hamiltonian of the IBM given in Eq. (26). The two parameters were fitted to the energy ratio

$$R_{4/2} = E(4^+_1)/E(2^+_1) \tag{33}$$

and the $B(E2)$ ratio

$$B(E2; 2^+_2 \to 0^+_1)/Bg(E2; 2^+_2 \to 2^+_1), \tag{34}$$

that are sensitive to changes in structure over wide parameter regions. For the reproduction of the energy ratio an error of 2% was allowed, while for the $B(E2)$ ratio the experimental errors were taken into account, resulting in an allowed parameter range of ζ and χ, in which K_3 takes various values within a certain range, with an upper and a lower limit. In Table II we denote values of K_3 obtained from the fit as $K_3^{\rm fit}$, and give the upper and lower limits allowed from the experimental errors. These values can be compared with the measured $K_3^{\rm appr.}$. The values agree reasonably well in most cases, considering the simplicity of the Hamiltonian and the arbitrary choice of the two observables used in the fit. Note, that other observables can and should be used for the fit. In some cases the simple Hamiltonian used cannot describe all features of a given nucleus, as, e.g., for the Pt nuclei. Therefore, the Hamiltonian (26) may be extended or another model used. The description given here shall just serve as an example on how to obtain values for $K_3^{\rm appr.}$ from fitting parameters of a model to data. The choice of the fitted observables and the model itself cannot be generalized.

VI. CONCLUSIONS

To conclude, we discussed measures of triaxiality. In this respect we considered in particular the absolute and relative cubic shape parameters q_3 and K_3. The approximative $K_3^{\rm appr.}$ was introduced as a direct, model independent observable, which is, if β is rigid, a measure of triaxiality, while $K_3^{\rm appr.}$ is more general an observable in all structural limits and the regions between them. $K_3^{\rm appr.}$ was shown to be a good approximation to the exact value of K_3, and can with good accuracy be obtained from only four matrix elements, or from four $B(E2)$ values, one of them equivalent to the modulus of the quadrupole moment $Q(2^+_1)$, and the sign of $Q(2^+_1)$. This, manifested in Eq. (16), is the key result of this work, together with a test of the accuracy of the approximation within the IBM and the RTRM. The need of the second order approximation within the Q-phonon scheme was shown in both models, especially in transitional regions. Effective values of β and γ deformation in the ground state, derived from $q_2^{\rm appr.}$ and $K_3^{\rm appr.}$, respectively, have been deduced from data. For vibrational nuclei geometrical deformation parameters cannot be given, while q_2 and K_3 are always well-defined properties of the ground state. Finally, we proposed a way how to derive $K_3^{\rm appr.}$ from a model fitting data.

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