Rotating twin stars and signature of quark-hadron phase transition

Abhijit Bhattacharyya,[1] Sanjay K. Ghosh,[2] Matthias Hanauske,[3] and Sibaji Raha[2]
[1]*Department of Physics, Scottish Church College, 1 & 3, Urquhart Square, Kolkata 700 006, India*
[2]*Department of Physics, Bose Institute, 93/1, A.P.C. Road, Kolkata 700 009, India*
[3]*Institut für Theoretische Physik, J. W. Goethe-Universitat, D-60054 Frankfurt am Main, Germany*
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The quark-hadron phase transition in a rotating compact star has been studied. The NLZ (nonlinear consistent zero point energy) model has been used for the hadronic sector and the MIT bag model has been used for the quark sector. It has been found that rotating twin star (third family) solutions are obtained up to $\Omega \approx 4000\,\mathrm{s}^{-1}$. Stars that are rotating faster than this limit do not show twin star solution. A backbending in the moment of inertia is also observed in the supermassive rest mass sequences. The braking index is found to diverge for a star having a larger region of mixed phase.

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The properties of strongly interacting matter at extreme conditions have been an intense area of research for quite some time now. The results from relativistic heavy-ion collision experiments [1] have enriched our knowledge of matter at high temperatures and small densities. Conversely, interesting physical phenomena are also expected to occur at high densities and small/zero temperatures. Nature has provided us with the best possible laboratory to study strongly interacting matter at supernuclear densities in the form of astrophysical compact stars [2]. The matter density near the core of a compact star can be even 10 times that of normal nuclear matter. At such high densities different exotic phase transitions may take place in the strongly interacting matter [1]. A large number of works have been done to study the quark-hadron phase transition, kaon condensation, hyperonic phase transition, and so on [3] at high densities and their correlation with the observable properties of a compact star.

For a static (nonrotating) compact star, the mass-radius relationship may be a possible signature of phase transition. However, the situation is a little different for a rotating star. It has been argued by Glendenning *et al.* [4] that one of the important quantities to probe, to study the phase transition, is the braking index. The central density of a rapidly rotating pulsar increases with time as it spins down, and when a certain central density is achieved, a quark phase may appear at the core of the star. Glendenning *et al.* [4] found that when the pure quark core appears, the star undergoes a brief era of spin-up and the braking index shows an anomalous behavior. Furthermore, the moment of inertia as a function of angular velocity shows a "backbending."

Motivated by the above results, some authors have studied the backbending phenomenon by using a different equation of state (EOS) and also with different rotating star codes. Recently, Spyrou and Stergioulas [5] have published an interesting result. They have studied the rest mass sequences for a particular EOS (with a quark-hadron phase transition) using their code "rns" and concluded that backbending is observed only for the supermassive sequences.

Recently some of us have used the NLZ model for the hadronic sector and the MIT bag model for the quark sector to look at the quark-hadron phase transition in static compact stars [6]. In that work it was found that there was a solution for the third family of stars known as twin stars [7]. Here we employ the same EOS for a rapidly rotating star. The basic motivation is to study the fate of the twin stars in a rotating model and also to study the possible signatures of these stars. First we describe the model that we use here, then the general relativistic features of a rotating star are briefly outlined, and at the end we discuss the results.

The Lagrangian density for the hadronic model is [8] as follows:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{YY} + \mathcal{L}_l, \quad (1)$$

where

$$\begin{aligned}\mathcal{L}_0 = &\sum_B \overline{\psi}_B(i\not\partial - m_B)\psi_B + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - U(\sigma)\\ &- \frac{1}{4}G^{\mu\nu}G_{\mu\nu} + U(\omega) - \frac{1}{4}\vec{B}^{\mu\nu}\vec{B}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{R}^\mu\vec{R}_\mu\\ &- \sum_B \overline{\psi}_B(g_{\sigma B}\sigma + g_{\omega B}\omega^\mu\gamma_\mu + g_\rho\vec{R}^\mu\gamma_\mu\vec{\tau}_B)\psi_B, \quad (2)\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{YY} = &\frac{1}{2}\left(\partial^\mu\sigma^*\partial_\mu\sigma^* - -m_{\sigma^*}^2\sigma^{*2}\right) - \frac{1}{4}S^{\mu\nu}S_{\mu\nu}\\ &+ \frac{1}{2}m_\phi^2\phi^\mu\phi_\mu - \sum_B \overline{\psi}_B(g_{\sigma^*B}\sigma^* + g_{\phi B}\phi^\mu\gamma_\mu)\psi_B,\end{aligned} \quad (3)$$

$$\mathcal{L}_l = \sum_{l=e,\mu} \overline{\psi}(i\not\partial - m_l)\psi_l. \quad (4)$$

In the above equations the $\sum_B$ runs over all the baryons (p, n, Λ, Σ^0, Σ^+, Σ^-, Ξ^0, and Ξ^-) and the $\sum_l$ runs over all the leptons. The piece of the Lagrangian $\mathcal{L}_{YY}$ is responsible for the hyperon-hyperon interactions [8]. The meson fields are σ, ω, $\vec{R}(\rho)$, σ^* [$f_0(975)$], and ϕ. The U_σ and U_ω are the σ and ω meson potentials [8–10].

The details of the coupling constants, parameter sets used here, and the equation of state are given in Ref. [6]. For the quark sector the standard noninteracting MIT bag model [11] has been used. Starting from the two models for the hadronic and quark sectors, mentioned above, a first-order deconfinement phase transition is obtained that proceeds via a mixed

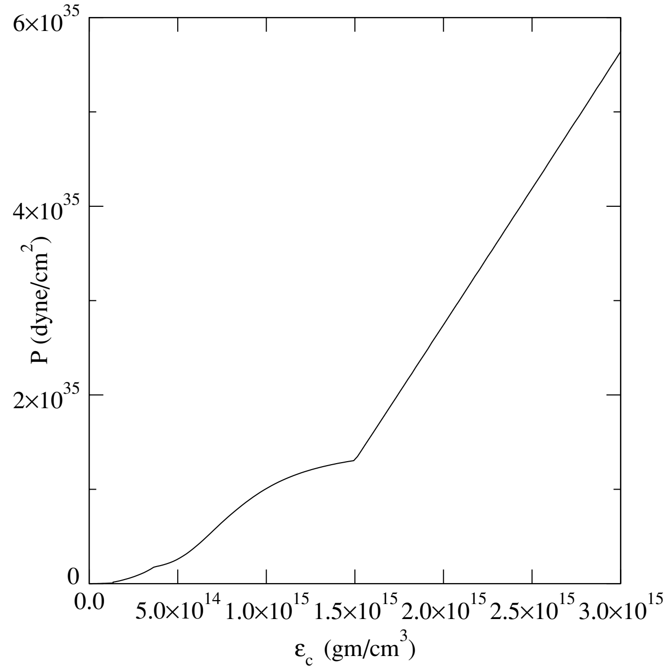

FIG. 1. Equation of state of the model considered here.

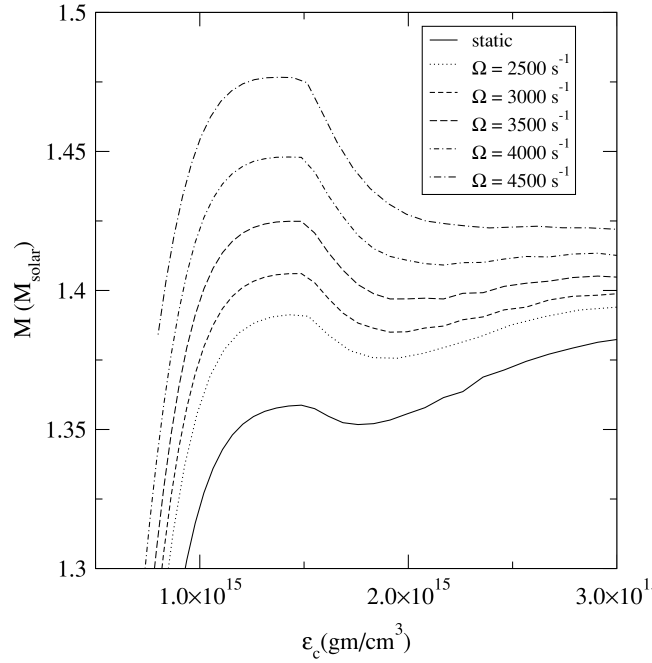

FIG. 2. Mass-ϵ_c plots for different Ω.

phase. At zero temperature, in the presence of two conserved charges, the mixed phase is constructed following the Gibbs criterion [4].

In the quark sector, we have taken light quark masses to be zero, the strange quark mass to be 150 MeV, and $B^{1/4} = 180$ MeV (which gives twin star solutions).

The EOS is plotted in Fig. 1. From the figure one can see that the phase transition starts at around $\epsilon_c = 4 \times 10^{14}$ gm/cm^3 and it ends around $\epsilon_c = 1 \times 10^{15}$ gm/cm^3. We would like to mention at this stage that because of the phase transition, there is a substantial change in the slope (sound speed) of the EOS.

The EOS obtained above is used to solve Einstein's equations for the rotating stars. Following the procedure adopted by Komatsu *et al.* [12], one can derive the solutions of Einstein's equations. In this work we briefly outline some of the steps only. The metric for a stationary rotating star can be written as [13] follows:

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2) + e^{\gamma-\rho}r^2 \sin^2\theta(d\phi - \omega dt)^2, \quad (5)$$

where α, γ, ρ, and ω are the gravitational potentials that depend on r and θ only. Einstein's equations for the three potentials γ, ρ, and ω have been solved by Komatsu *et al.* [12] using Green's function technique. The fourth potential α has been determined from other potentials. All the physical quantities can be calculated from these potentials [13].

Solution of the potentials, and hence the calculation of physical quantities, is numerically quite an involved process. There are several numerical codes in the community for this purpose. In this work we have used the "rns" code. This code developed by Stergioulas is very efficient in calculating the rotating star properties. The details of the code may be obtained in Refs. [12,13]. In the present work, this code has been used to study the rest mass sequences and the Ω sequences for both normal and supermassive stars.

Generally, the mass-ϵ_c curve of neutron stars shows a maximum beyond which no stable configuration is supposed to exist. It was shown by Harrison *et al.* [14] that for a smooth equation of state, no stable stellar configuration exists for central densities above that corresponding to the maximum mass limit. Recently Glendenning *et al.* [15] have found that if there is a first-order phase transition inside the neutron star, this inference may not be valid. As a result, a stable configuration for a third family of stars, with higher central densities, may arise. In the present work also, we have found the existence of stars that belong to the third family, known as twin stars. These results may be seen in Fig. 2. Here we have plotted the variation of mass with ϵ_c for different values of Ω. However, as we increase Ω, the third family solution becomes less probable, as can be seen from the plots, and it vanishes just after $\Omega = 4000\,\mathrm{s}^{-1}$. This is natural as with increasing angular velocity, phase boundary moves outward, matter gets redistributed in a larger radius, and the twin star solution vanishes. To illustrate it we have plotted the density profiles in Fig. 3. In this figure we have plotted the density profiles of twin stars for both a static star ($M = 1.355M_\odot$) and a star rotating ($M = 1.412M_\odot$) with an angular velocity $\Omega = 4000\,\mathrm{s}^{-1}$. We can see from the plots that as we increase the frequency of rotation the phase boundary moves toward the surface. For example, in the static case the mixed phase ends at around $R/R_e = 0.55$, whereas for the rotating case it ends at around $R/R_e = 0.64$; R_e being the equatorial radius. This is probably the best way to understand the disappearance of twin stars at high frequency.

The appearance of twin star solution has other interesting consequences as well. In Fig. 4 we have plotted the moment of inertia (I) as a function of Ω for different rest masses (i.e., for both normal and supermassive sequences). For the

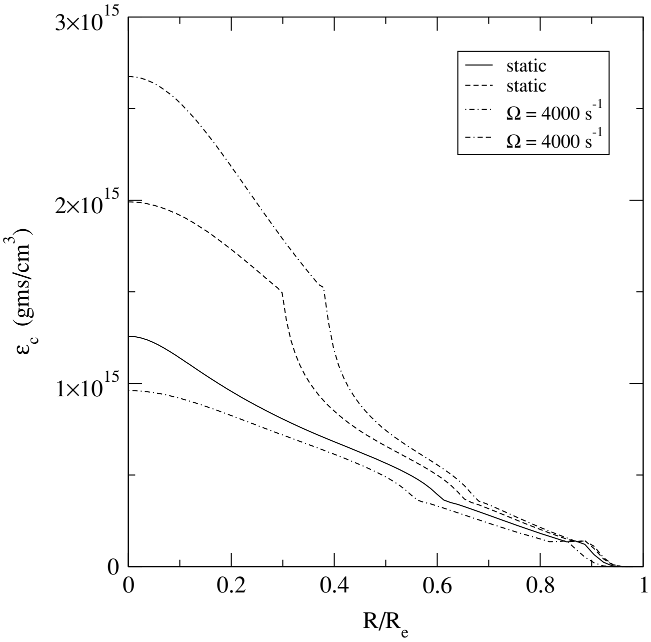

FIG. 3. Density profiles of static and rotating twin stars.

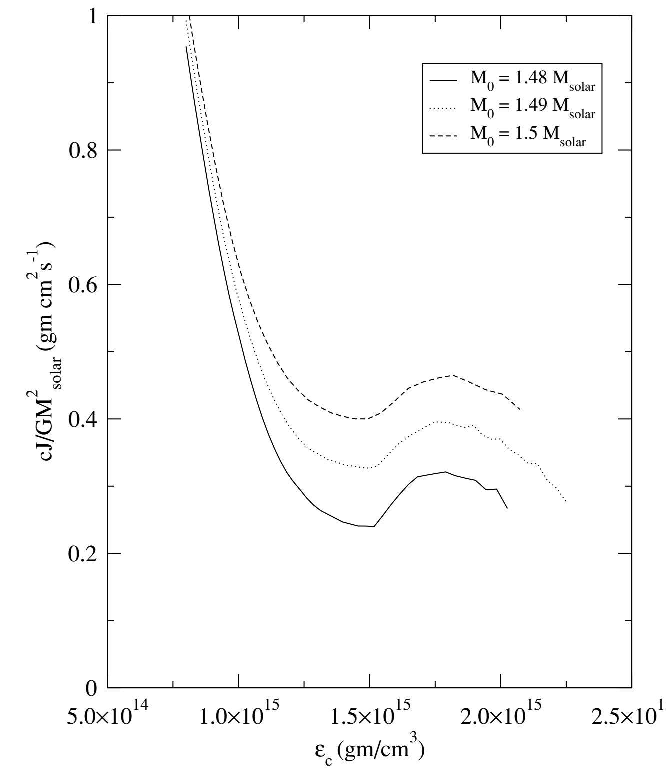

FIG. 5. J as a function of ϵ_c for different rest masses.

normal sequence the moment of inertia increases with the angular velocity monotonically. However, for a supermassive sequence, there are two different branches; for one branch of the curve the moment of inertia increases with Ω, whereas for the other the moment of inertia decreases with increase in Ω. This anomalous behavior of the moment of inertia is known as backbending and can be attributed to the phase transition from incompressible nuclear matter to highly compressible quark matter [4]. As pointed out by several authors, this feature could be a possible signature of the quark-hadron phase transition. Here we emphasize that even for a supermassive sequence, the backbending is observed only for the cases where density in the core is such that it facilitates a large quark phase region inside the star. For example, although the range of rest mass $1.35M_\odot < M_0 < 1.48M_\odot$ lies in the supermassive domain, no backbending has been found for masses within this range. Higher mass ($1.6M_\odot$ and $1.7M_\odot$) sequences correspond to larger I and Ω. Because of the redistribution of matter, as mentioned, the $dI/d\Omega > 0$ part corresponding to quark core (as seen for masses $1.48M_\odot - 1.5M_\odot$) does not appear.

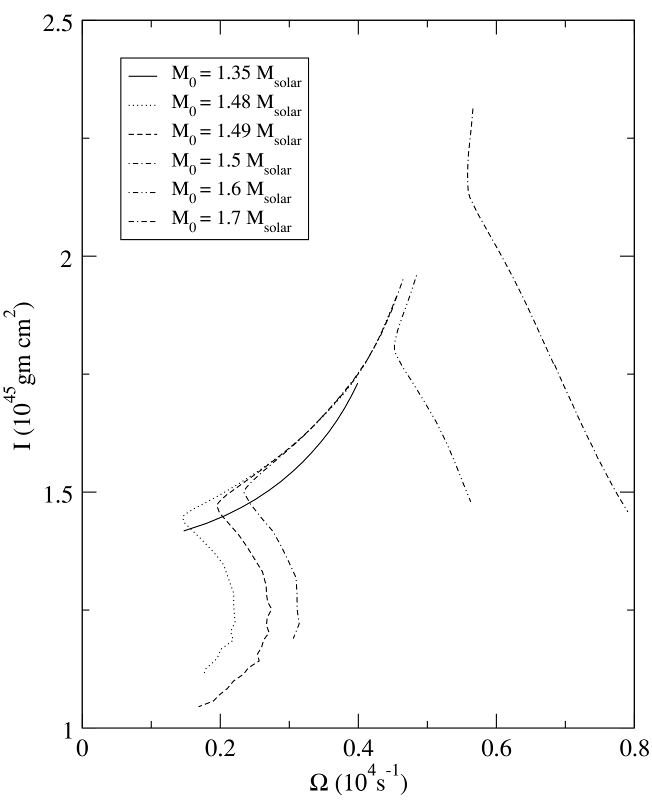

FIG. 4. I as a function of Ω for different rest masses.

In Fig. 5 the angular momentum is plotted as a function of energy density for different rest masses. We have plotted these curves only for the supermassive sequences. Figure 4 shows that initially angular momentum (J) decreases with ϵ_c followed by an unstable region where J increases with ϵ_c. With further increase in ϵ_c, J again decreases, indicating the existence of twin stars.

In Fig. 6 the braking index has been plotted against Ω. It was pointed out by Glendenning *et al.* [4,16] that as the variation of I with Ω shows two distinct regions of stability (Fig. 4), the $dI/d\Omega$ passes through two singularities. This in turn causes the dimensionless braking index given by

$$\nu(\Omega) = 3 - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega} \tag{6}$$

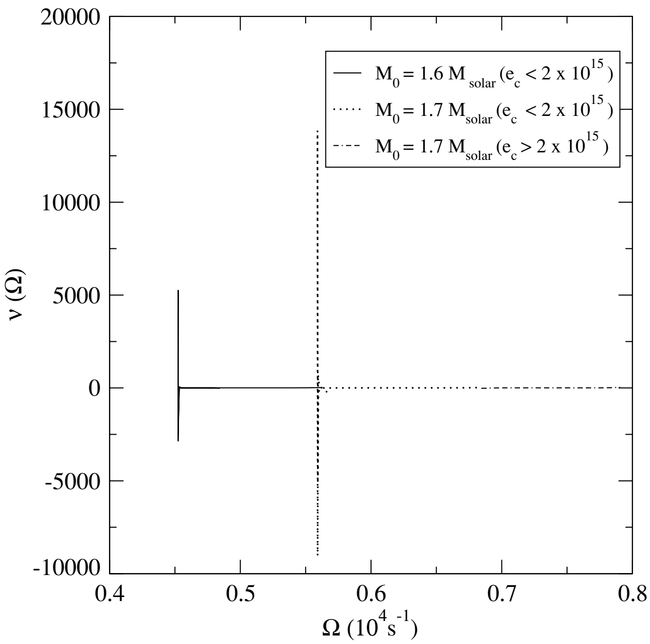

FIG. 6. Braking index as a function of Ω.

to diverge. As this quantity is measurable, this divergence can be taken as a signature of the phase transition. In the present work, the braking index is found to diverge for supermassive sequences. However, the braking index depends very strongly on the extent of the mixed phase. We have plotted this result for three cases in Fig. 6. For the rest mass sequence $M_0 = 1.6M_\odot$ we have varied the central energy density from 10^{15} to $2 \times 10^{15}\,\mathrm{gm/cm}^3$. The same variation has been done for the rest mass sequence $M_0 = 1.7M_\odot$. In the second case the extent (or the spread) of the mixed phase is more than that for the first case, and hence we find that the braking index diverges more. We have also studied the rest mass sequence $M_0 = 1.7M_\odot$ with the central energy density varying from 2×10^{15} to $3 \times 10^{15}\,\mathrm{gm/cm}^3$. Looking at the EOS, one can easily see that throughout the sequence the core of the star lies in the pure quark phase. For this sequence the braking index assumes the canonical value and there is no divergence.

To summarize, we have studied the effect of deconfinement transition in a rotating neutron star using the NLZ model and the MIT bag model for the hadronic and quark sectors, respectively. The third family of stars (i.e., the twin star solution) is expected to occur because of substantial change in the sound speed across the phase boundary. In the present model, the twin star solution is obtained only for bag pressure around 180 MeV. Moreover, with the increase in angular velocity Ω, the mass as well as ϵ_c of the star increases. This implies that the twin star solution that occurs for higher central density vanishes beyond a certain Ω. At this point, most of the star, in the present study is found to be made up of quark matter. So the faster rotating stars have larger domain of mixed phase and do not exhibit the third family of stars.

The jump of the braking index $\nu(\Omega)$ as well as the magnitude of the jump can be a signature of phase tranisiton and the extent of quark core [17]. In Ref. [5], Spyrou *et al.* have observed small oscillations of the braking index in the normal sequence. However, we do not find such oscillation in the normal sequence. This may be because in the central region we have only a mixed phase, whereas they have a quark core in the central region. This is obviously ascribable to the difference in the EOS.

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