

**Exclusive electrodisintegration of  ${}^3\text{He}$  at high  $Q^2$ . I. Generalized eikonal approximation**

M. M. Sargsian and T. V. Abrahamyan

*Department of Physics, Florida International University, Miami, Florida 33199*

M. I. Strikman

*Department of Physics, Pennsylvania State University, University Park, Pennsylvania 16802*

L. L. Frankfurt

*Department of Nuclear Physics, Tel Aviv University, Tel Aviv, Israel 69978*

(Received 21 June 2004; revised manuscript received 15 October 2004; published 29 April 2005)

We develop a theoretical framework for calculation of high- $Q^2$  exclusive electrodisintegration of  $A = 3$  systems. The main result of this work is the calculation of the final state interaction of the struck energetic nucleon with recoil nucleons within the generalized eikonal approximation (GEA), which allows us to account for the finite and relatively large momenta of the bound nucleons in the nucleus. This approach makes it possible to study in a self-consistent way the short-range correlations in nuclei. This is because the GEA does not require a stationary condition for recoil nucleons as does the conventional Glauber approximation. As a result GEA treats explicitly the Fermi motion of recoil nucleons in the nucleus.

DOI: 10.1103/PhysRevC.71.044614

PACS number(s): 25.30.Fj, 11.80.Fv, 25.10.+s, 25.30.Rw

**I. INTRODUCTION**

Advances in experimental studies of high-energy exclusive electro-disintegration reactions of few-nucleon systems [1–4] as well as the multitude of the planned experiments at Jefferson Lab with the upgraded energies of CEBAF [5,6] emphasize strongly the need for systematic theoretical studies of these reactions. Recently, there have been several theoretical works addressing many of the outstanding issues related to the physics of high-energy exclusive breakup of few-nucleon systems [7–16].

The heightened interest in these reactions is based on expectations that the high-resolution power of the energetic probe (virtual photon) and the relative simplicity of the target (consisting of two or three nucleons) will boost considerably our ability to probe the dynamics of bound systems at small space-time separations and allow systematic studies of transition from hadronic to quark-gluon degrees of freedom in nuclear interactions. In many instances, these studies can take advantage of recent progresses in developing the realistic wave functions of few-nucleon systems [17,18].

In this work we are interested particularly in high- $Q^2$  ( $4 \lesssim Q^2 \lesssim 1 \text{ GeV}^2$ ) exclusive  ${}^3\text{He}(e, e'NN)N$  reactions in which one nucleon in the final state can be clearly identified as a knocked-out nucleon that carries practically all of the momentum of the virtual photon. We calculate the scattering amplitude of this reaction within the generalized eikonal approximation (GEA) [7,19,20] in which one starts by expressing the scattering amplitude through the sum of the diagrams corresponding to the  $n$ th order rescattering of the knocked-out nucleon with the residual nucleons in the nucleus. Then we evaluate each diagram based on the effective Feynman diagram rules derived within the GEA [19,20]. The advantage of this approach is that the number of the diagrams contributing to the scattering amplitude is defined

by the finite number of  $NN$  rescatterings that can be evaluated within the eikonal approximation. The manifestly covariant nature of Feynman diagrams allows us to preserve both the relativistic dynamics and the kinematics of the rescattering while identifying the low-momentum nuclear part of the amplitude with the nonrelativistic nuclear wave function. Such an approach allows us to account for the internal motion of residual target nucleons in the rescattering amplitude. (For recent developments see also Ref. [15].) Additionally, the GEA accounts for the transferred longitudinal momentum in the rescattering amplitude, which is important for describing the inelastic processes (or processes with large excitation energies) in which the final state is strongly energy dependent. All these constitute a generalization of the conventional eikonal approximation [21] in which the nucleons in the nucleus are considered as stationary scatterers and only the transverse momentum is transferred in the reinteractions. These features of the GEA are crucial in describing electro-production reactions aimed at the study of short-range nuclear configurations since they are characterized by non-negligible values of bound nucleon momenta and excitation energies. The study of short-range nucleon correlations is the main goal of Part II of this work.

In the present paper we work in the virtual nucleon framework in describing the dynamics of the reaction. In this framework one describes the reaction in the lab frame relating all non-nucleonic degrees of freedom effectively to the off-shellness of the knocked-out (virtual) nucleon in the nucleus. This justifies the use of only nucleonic degrees of freedom in the ground-state wave function of the nucleus. If the probed internal momenta are sufficiently small,  $p^2/m_N^2 \ll 1$ , one can use the nonrelativistic ground-state nuclear wave functions, which are calculated based on realistic  $NN$  interaction potentials. Furthermore, considering only the kinematics of quasi-elastic reactions we neglect the isobar current and meson

exchange contributions. All these impose specific restrictions on the kinematics of the reaction that will be discussed in detail in the present paper.<sup>1</sup>

The paper is organized as follows: In Sec. II the specifics of the considered electro-nuclear reaction and its kinematical requirements are discussed. In Sec. III we derive the scattering amplitude within the GEA by calculating the contributions from single and double rescattering of the knocked-out nucleon off recoil nucleons in the reaction. We also calculate the pair distortion effects due to the interaction of slow residual nucleons in the final state of the reaction. In the last part of Sec. III we discuss the general form of the differential cross section. Section IV summarizes the results of the derivations. The effective Feynman diagram rules of the GEA are given in the Appendix.

## II. REACTION AND KINEMATICS

We are considering the electrodisintegration of  ${}^3\text{He}$  in the reaction

$$e + {}^3\text{He} \rightarrow e' + N_f + N_{r2} + N_{r3}, \quad (1)$$

where  $e$  and  $e'$  are the initial and scattered electrons with four-momenta  $k_e$  and  $k'_e$ , respectively. The  ${}^3\text{He}$  nucleus has a four-momentum  $p_A$ .  $N_f$ ,  $N_{r2}$ , and  $N_{r3}$  correspond to knocked-out and two recoil nucleons with four-momenta  $p_f$ ,  $p_{r2}$ , and  $p_{r3}$ , respectively. We define also the four-momentum of the virtual photon  $q = (q_0, |\mathbf{q}|, 0_\perp) \equiv k_e - k'_e$  with  $Q^2 = -q^2$ . Hereafter the  $z$  direction is chosen parallel to  $\mathbf{q}$  and the scattering plane is the plane of the  $\mathbf{q}$  and  $\mathbf{k}_e$  vectors.

We will investigate the reaction of Eq. (1) in the kinematic region defined as follows:

$$\begin{aligned} \text{(a) } & 4 \geq Q^2 \geq 1 \text{ GeV}^2; \quad \text{(b) } \mathbf{p}_f \approx \mathbf{q}; \\ \text{(c) } & |\mathbf{p}_m|, |\mathbf{p}_{r2}|, |\mathbf{p}_{r3}| \leq 400\text{--}500 \text{ MeV}/c, \end{aligned} \quad (2)$$

where one defines a missing momentum  $\mathbf{p}_m = \mathbf{p}_f - \mathbf{q}$ . The lower limit of Eq. (2a) is what provides a high-momentum transfer in the electrodisintegration. This condition together with Eqs. (2b) and (2c) allows us to identify  $N_f$  as a knocked-out nucleon, whereas  $N_{r2}$  and  $N_{r3}$  could be considered as recoil nucleons that do not interact directly with the virtual photon. The upper limit of Eq. (2a) comes from the condition that the color coherence effects are small and the produced hadronic state represents a single state (i.e., nucleon) rather than a superposition of different hadronic states in the form of the wave packet (see, e.g., Ref. [22]).

Additionally, the condition of Eq. (2c) allows us to consider the nucleons as the basic degrees of freedom in describing the interacting nuclear system. From the technical point of view, this means that in the set of noncovariant diagrams comprising the covariant scattering amplitude, one can neglect

the noncovariant diagrams containing non-nucleonic degrees of freedom (e.g., negative energy projections of the bound nucleon spinors contributing to the vacuum fluctuations in the scattering amplitude). Within this approximation one can reduce the nuclear vertices to the nonrelativistic nuclear wave functions of nuclei [see, e.g., Eq. (A4)]. Note that on several occasions in Ref. [23] we will extend our calculations to the region of missing and recoil momenta  $\geq 500 \text{ MeV}/c$ . We justify such an extension by the expectation that the onset of the relativistic effects in the nuclear wave function should happen rather smoothly. However, in all these cases our results should be considered as qualitative.

## III. SCATTERING AMPLITUDE

Within the one-photon exchange approximation, the amplitude  $M_{fi}$  of reaction (1) can be written as follows (see, e.g., [24]):

$$M_{fi} = -4\pi\alpha \frac{1}{q^2} j_\mu^e \cdot \langle f | J_A^\mu(Q^2) | i \rangle. \quad (3)$$

Here  $\alpha$  is the fine structure constant,  $j_\mu^e = \bar{u}(k'_e)\gamma_\mu u(k_e)$  is the electromagnetic current of the electron, and  $J_A^\mu(Q^2)$  is the operator of the nuclear electromagnetic current. The initial state  $|i\rangle$  is the totally antisymmetric state of  ${}^3\text{He}$ . The final state  $|f\rangle$  also has to be antisymmetric. However, because of the kinematical constraints of Eqs. (2) one can neglect the antisymmetrization between the outgoing fast and slow recoil nucleons. Such an approximation is justified because the diagrams in which an energetic photon will produce the slow hadrons are strongly suppressed.

Here we need to calculate the electromagnetic transition amplitude  $A^\mu$ , defined as

$$A^\mu \equiv \langle f | J_A^\mu(Q^2) | i \rangle. \quad (4)$$

We will perform the calculation within the generalized eikonal approximation [19,20]. In this approach, the interaction of the fast knocked-out nucleon with slow spectator nucleons is calculated based on the eikonal approximation for the corresponding covariant diagrams. The reduction theorem [20] derived for this approximation allows us to reduce an infinite sum of rescattering diagrams to a finite set of covariant Feynman diagrams. In these diagrams, soft  $NN$  reinteractions are described through phenomenological  $NN$  interaction vertices, which can be parameterized using experimental data on the small-angle  $NN$  scattering amplitudes. In its final form this approximation can be formulated through a set of effective Feynman diagram rules (summarized in the Appendix) for calculating the scattering amplitude of the  $e + A \rightarrow e' + N + (A-1)'$  reaction in the given order of the rescattering of the fast knocked-out nucleon off spectator nucleons (for review on GEA, see Ref. [20]). Based on the kinematic constraints of Eq. (2c) we will neglect non-nucleonic degrees of freedom in the ground-state wave function of  ${}^3\text{He}$ . This allows us, in the limit of  $p_m^2/m^2 \ll 1$ , to reduce the covariant nuclear vertices to the nonrelativistic wave function of initial and final nuclear states with nucleonic constituents only. Note that we still account for effects of the order of magnitude  $p_m/m$ . One such term is the flux factor, which is

<sup>1</sup>The relativistic effects can be incorporated self-consistently in GEA using the light-cone formalism (see appendix in Ref. [19]). These and studies of isobar contributions in the reaction dynamics will be addressed in subsequent papers.

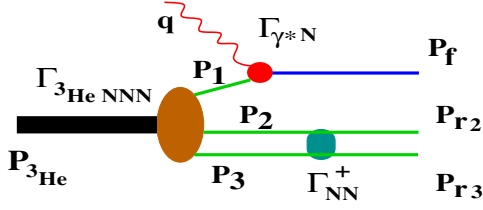


FIG. 1. (Color online) Impulse approximation contribution to the scattering amplitude of the  ${}^3\text{He}(e, e' N_f N_r2) N_r3$  reaction.

proportional to  $1 - p_m^z/m$  and which should be taken into account to preserve the baryon number conservation in the reaction (see, e.g., Ref. [25]). In addition to the flux-factor effects, in the present approach, the initial off-shellness of the struck nucleon renders ambiguity in choosing the proper form of the electromagnetic current of the  $eN$  interaction. This problem is addressed usually by imposing an electromagnetic current conservation relation that allows us to express the off-shell component through the on-shell component of the electromagnetic current (see, e.g., [26]). Note, however, that the ambiguity resulting from the off-shellness is proportional to  $p_m^2/Q^2$ , and for the kinematic range of Eqs. (2), it is a small correction, the discussion of which lies outside the scope of the present paper.

### A. Impulse approximation

The contribution to the electromagnetic transition amplitude  $A^\mu$ , in which the knocked-out nucleon does not interact with other nucleons, corresponds to the impulse approximation (IA). In this case the wave function of the knocked-out nucleon is a plane wave.

The IA term of the scattering amplitude,  $A_0^\mu$ , is described by the Feynman diagram of Fig. 1. Using the diagrammatic rules summarized in the Appendix and identifying knocked-out and two recoil nucleons in the initial state by indexes 1, 2, and 3, respectively, one obtains

$$A_0^\mu = - \int \frac{d^4 p_2}{i(2\pi)^4} \bar{u}(p_{r2}) \bar{u}(p_{r3}) \bar{u}(p_f) \cdot \Gamma_{NN}^+(p_2, p_3) \cdot \Gamma_{\gamma^* N}^\mu \cdot \frac{\hat{p}_3 + m}{p_3^2 - m^2 + i\epsilon} \frac{\hat{p}_2 + m}{p_2^2 - m^2 + i\epsilon} \cdot \frac{\hat{p}_1 + m}{p_1^2 - m^2 + i\epsilon} \cdot \Gamma_{\text{HeNNN}}(p_1, p_2, p_3) \chi^A, \quad (5)$$

where  $\mathbf{p}_1 = \mathbf{p}_m \equiv \mathbf{p}_f - \mathbf{q}$  and  $p_3 = p_A - p_1 - p_2$ . For the kinematic range of Eq. (2c) one can integrate over  $dp_2^0$ , estimating it through the residue at the positive energy pole of the propagator of nucleon 2. This corresponds to a positive energy projection of the virtual nucleon state. Such integration effectively corresponds to the replacement

$$\int \frac{dp_2^0}{p_2^2 - m^2 + i\epsilon} = -\frac{i2\pi}{2E_2} \approx -\frac{i2\pi}{2m}. \quad (6)$$

The condition that the internal momenta of the nucleons remain small ( $\mathbf{p}_{m,2,3}^2 \ll m^2$ ) also allows one to use the closure relation for on-mass shell nucleons to express the numerator of the

bound nucleon propagator as follows:

$$\hat{p} + m = \sum_s u(p, s) \bar{u}(p, s). \quad (7)$$

Using Eqs. (6) and (7) in Eq. (5) one obtains

$$A_0^\mu = \sum_{s_1, s_2, s_3} \int \frac{d^3 p_2}{2m(2\pi)^3} \times \frac{\bar{u}(p_{r2}, s_{r2}) \bar{u}(p_{r3}, s_{r3}) \Gamma_{NN}^+(p_2, p_3) u(p_3, s_3) u(p_2, s_2)}{p_3^2 - m^2 + i\epsilon} \times \bar{u}(p_f, s_f) \Gamma_{\gamma^* N}^\mu u(p_1, s_1) \times \frac{\bar{u}(p_1, s_1) \bar{u}(p_2, s_2) \bar{u}(p_3, s_3) \Gamma_{\text{HeNNN}}(p_1, p_2, p_3) \chi^A}{p_1^2 - m^2 + i\epsilon}. \quad (8)$$

Using Eq. (A4) and introducing the electromagnetic nucleon current

$$j^\mu(p_f, s_f; p_m, s_1) = \bar{u}(p_f, s_f) \Gamma_{\gamma^* N}^\mu u(p_m, s_1), \quad (9)$$

for  $A_0^\mu$ , one arrives at

$$A_0^\mu = \sqrt{2E_{r2} 2E_{r3}} (2\pi)^3 \sum_{s_1, s_2, s_3, t_2, t_3} \int d^3 p_2 \times \Psi_{NN}^\dagger(p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3})(p_2, s_2, t_2; p_3, s_3, t_3) \times j_{t_1}^\mu(p_m + q, s_f; p_m, s_1) \times \Psi_A^{s_A}(p_m, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3), \quad (10)$$

where  $s_A, s_1, s_2, s_3, s_f, s_{r2}, s_{r3}$  describe the spin projections of the  ${}^3\text{He}$  nucleus, the initial nucleons, and the final nucleons, respectively. We represent the isospin projections of nucleons by  $t_1, t_2, t_3, t_f, t_{r2}, t_{r3}$  and use these indexes to identify the protons and neutrons. In Eq. (10),  $\Psi_A^{s_A}$  is the ground-state wave function of the  ${}^3\text{He}$  nucleus with polarization vector  $\mathbf{s}_A$ , and  $\Psi_{NN}$  represents the bound or continuum  $NN$  wave function. One can simplify Eq. (10) further using the fact that  $\Psi_{NN}$  is a function only of the relative three-momenta of spectator nucleons and the spins. This allows us to replace the  $d^3 p_2$  integration by  $d^3 p_{23}$ , which yields<sup>2</sup>

$$A_0^\mu = \sqrt{2E_{r2} 2E_{r3}} (2\pi)^3 \sum_{s_1, s_2, s_3, t_2, t_3} \int d^3 p_{23} \times \Psi_{NN}^\dagger(p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3})(p_{23}, s_2, t_2; s_3, t_3) \times j_{t_1}^\mu(p_m + q, s_f; p_m, s_1) \times \Psi_A^{s_A} \left( p_m, s_1, t_1; -\frac{p_m}{2} + p_{23}, s_2, t_2; -\frac{p_m}{2} - p_{23}, s_3, t_3 \right). \quad (11)$$

For the case of the reaction of Eq. (1),  $\Psi_{NN}$  is a continuum  $NN$  wave function, which can be represented through the solution

<sup>2</sup>To do this one can introduce  $d^3 p_3 \delta^3(p_{r2} + p_{r3} - p_2 - p_3)$  in Eq. (10), then replace  $d^3 p_2 d^3 p_3$  by  $d^3 p_{23} d^3 p_{\text{cm}23}$ , with  $\mathbf{p}_{23} = \frac{\mathbf{p}_2 - \mathbf{p}_3}{2}$  and  $\mathbf{P}_{\text{cm}23} = \mathbf{p}_2 + \mathbf{p}_3$ , and integrate out the  $\delta$  function through  $d^3 p_{\text{cm}23}$ .

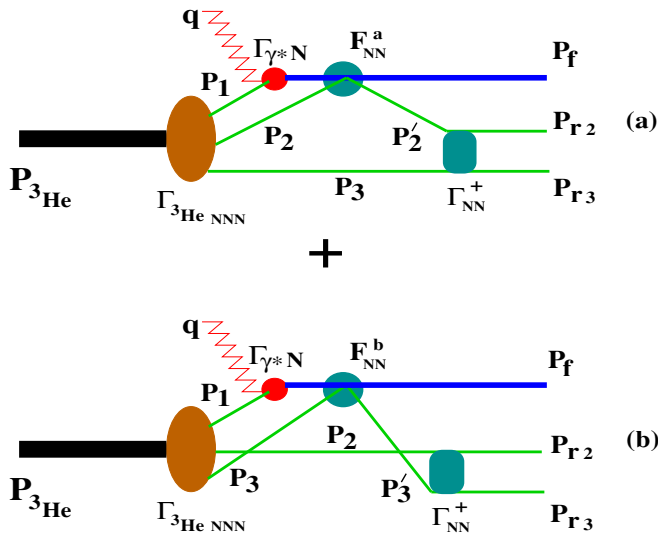


FIG. 2. (Color online) Single rescattering contribution to the scattering amplitude of the  ${}^3\text{He}(e, e'N_f N_{r2})N_{r3}$  reaction.

of Lippmann-Schwinger equation as follows:

$$\Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(\mathbf{p}_{23}, s_2, t_2; s_3, t_3) = \delta^3(\mathbf{p}_{23} - \mathbf{p}_{r23}) + \frac{1}{2\pi^2} \frac{\langle s_{r2}, t_{r2}; s_{r3}, t_{r3} | f_{NN}^{\text{off shell}}(\mathbf{p}_{r23}, \mathbf{p}_{23}) | s_2, t_2; s_3, t_3 \rangle}{\mathbf{p}_{23}^2 - \mathbf{p}_{r23}^2 - i\epsilon}, \quad (12)$$

where  $\mathbf{p}_{23} = \frac{\mathbf{p}_2 - \mathbf{p}_3}{2}$ ,  $\mathbf{p}_{r23} = \frac{\mathbf{p}_{r2} - \mathbf{p}_{r3}}{2}$ , and  $f_{NN}^{\text{off shell}}$  is a half-off-shell nonrelativistic amplitude of  $NN$  scattering (see, e.g., Ref. [27]). Terms on the right-hand side of Eq. (12) characterize two distinctive dynamics of production of the recoil  $NN$  state. If only the first term of Eq. (12) is kept in Eq. (11), this will correspond to the approximation in which all three final nucleons propagate as plane waves. Hereafter we will refer

to this approximation as a plane wave impulse approximation (PWIA). The second term in Eq. (12) describes a reinteraction between the pair of slow nucleons, which distorts the plane wave of the outgoing recoil nucleons. Following Ref. [28] we will refer to this term as a pair distortion contribution.

### B. Single rescattering amplitude

The diagrams in Fig. 2 describe the process in which the knocked-out (fast) nucleon rescatters off one of the spectator nucleons. Using the diagrammatic rules from the Appendix for the amplitude corresponding to the diagram of Fig. 2(a) one obtains

$$A_{1a}^{\mu} = - \int \frac{d^4 p_2}{i(2\pi)^4} \frac{d^4 p_3}{i(2\pi)^4} \bar{u}(p_{r3}) \bar{u}(p_{r2}) \bar{u}(p_f) \times \frac{\Gamma_{NN}^+(p_2', p_3)(\hat{p}_2' + m) F_{NN}^a(p_2' - p_2)(\hat{p}_1 + \hat{q} + m)}{p_2'^2 - m^2 + i\epsilon} \frac{1}{(p_1 + q)^2 - m^2 + i\epsilon} \cdot \Gamma_{\gamma^* N}^{\mu} \cdot \frac{\hat{p}_3 + m}{p_3^2 - m^2 + i\epsilon} \frac{\hat{p}_2 + m}{p_2^2 - m^2 + i\epsilon} \cdot \frac{\hat{p}_1 + m}{p_1^2 - m^2 + i\epsilon} \cdot \Gamma_{3\text{He}NNN}(p_1, p_2, p_3) \chi^A. \quad (13)$$

Using the same arguments as in the previous subsection we evaluate the  $d^0 p_2$  and  $d^0 p_3$  integrals through the residues at positive energy poles in the recoil nucleon propagators. This yields a replacement of  $\int [d_{2,3}^0 / (p_{2,3}^2 - m^2 + i\epsilon)] \approx -i \frac{2\pi}{m}$  and reduces the covariant Feynman diagram of Eq. (13) to the time-ordered noncovariant diagram in which nucleon 1 is first struck by a virtual photon and then rescatters off the spectator nucleon, 2. The rescattered nucleon 2 subsequently combines with nucleon 3 into the  $NN$  continuum (or bound) state. Now, we can use Eq. (7) for the intermediate nucleons. Furthermore, relating the nuclear vertex functions to the nuclear wave functions according to Eq. (A4) and the  $NN$  rescattering vertex functions to the  $NN$  scattering amplitude according to Eq. (A1), one obtains

$$A_{1a}^{\mu} = -F \sum_{s_1', s_2', s_1, s_2, s_3} \sum_{t_1, t_2', t_2, t_3} \frac{1}{2m} \int \frac{d^3 p_2}{(2\pi)^3} d^3 p_3 \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p_2', s_2', t_2'; p_3, s_3, t_3) \times \frac{\sqrt{s_2^{NN}(s_2^{NN} - 4m^2)} f_{NN}(p_2', s_2', t_2', p_f, s_f, t_f; |p_2, s_2, t_2, p_1 + q, s_1, t_1)}{(p_1 + q)^2 - m^2 + i\epsilon} \times J_{t_1}^{\mu}(p_1 + q, s_1'; p_1, s_1) \cdot \Psi_A^{s_A}(p_1, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3), \quad (14)$$

where  $F = \sqrt{2E_{r2} 2E_{r3}} (2\pi)^3$ ,  $p_1 = p_A - p_2 - p_3$ ,  $p_2' = p_{r2} + p_{r3} - p_3$ , and  $s_2^{NN} = (p_1 + q + p_2)^2$ .

Now we analyze the propagator of the knocked-out nucleon:

$$(q + p_1)^2 - m^2 + i\epsilon = 2q \left[ \frac{2mq_0 - Q^2}{2q} - p_{1z} + i\epsilon \right]. \quad (15)$$

The factor  $(2mq_0 - Q^2)/2q$  is fixed by external (measurable) kinematical variables such as  $Q^2$ ,  $q_0$ ,  $\mathbf{p}_f$ , and  $\mathbf{p}_{r2}$ . Using the condition of the quasi-elastic scattering for reaction (1),

$(q + p_A - (p_{r2} + p_{r3}))^2 = m^2$ , one obtains

$$\frac{2mq_0 - Q^2}{2q} = p_{mz} + \frac{q_0}{q} (T_{r2} + T_{r3} + |\epsilon_A|) + \frac{m^2 - \tilde{m}^2}{2q} \approx p_{mz} + \Delta^0, \quad (16)$$

where  $p_{mz} = p_{fz} - q$ ,  $T_{r2}$  and  $T_{r3}$  are the kinetic energies of recoil nucleons,  $|\epsilon_A|$  is the modulus of the binding energy of the target, and  $\tilde{m}^2 = [p_A - (p_{r2} + p_{r3})]^2$ . In the last step in Eq. (16) we neglected the  $(m^2 - \tilde{m}^2)/(2q)$  term, since for the

fixed decay kinematics it vanishes with an increase of  $q$ . We have also denoted

$$\Delta^0 = \frac{q_0}{q}(T_{r2} + T_{r3} + |\epsilon_A|), \quad (17)$$

which defines the effective longitudinal momentum transferred in the  $NN$  rescattering. Note that  $\Delta^0$ , which is absent in the conventional eikonal approximation, is important at large values of recoil nucleon energies  $T_{r2}, T_{r3} \sim m$ . As will be shown in Part II of this paper [23] the kinematics with the large values of recoil nucleon energies are most relevant for accessing the short-range correlations in nuclei. Substituting Eqs. (15) and (16) into Eq. (14) for the scattering amplitude described by Fig. 2, one obtains

$$\begin{aligned} A_{1a}^\mu &= -\frac{F}{2} \sum_{s_{1'}, s_{2'}, s_1, s_2, s_3} \sum_{t_1, t_2, t_3} \int \frac{d^3 p_2}{(2\pi)^3} d^3 p_3 \\ &\times \Psi_{NN}^\dagger(p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3})(p_2', s_2', t_2'; p_3, s_3, t_3) \\ &\times \frac{\sqrt{s_2^{NN}(s_2^{NN} - 4m^2)}}{2qm} \\ &\times \frac{f_{NN}(p_2', s_2', t_2'; p_f, s_f, t_f | p_2, s_2, t_2; p_1 + q, s_{1'}, t_1)}{p_{mz} + \Delta^0 - p_{1z} + i\epsilon} \\ &\times j_{t_1}^\mu(p_1 + q, s_{1'}; p_1, s_1) \\ &\times \Psi_A^{s_A}(p_1, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3). \end{aligned} \quad (18)$$

Two important features of soft  $NN$  scattering allow us to simplify Eq. (18). One is that in the high-energy regime the soft, low- $t$   $NN$  amplitude, which dominates Eq. (18), conserves the helicities of nucleons. Starting at  $Q^2 \geq 1 \text{ GeV}^2$  the approximation of helicity conservation is accurate on the level of 2–3% [29] (for recent analysis see Ref. [30]). It is an even smaller factor for the observables of reaction (1) in which no polarization degrees of freedom are fixed. The second feature is that the soft amplitude is a function of the transverse component of the transferred momentum,  $(p_2' - p_2)_\perp$ , only. Since  $\hat{z} \parallel \mathbf{q}$ , the helicity-conserving argument implies the conservation of the polarization projections of the interacting nucleons in the  $z$  direction. These simplifications yield the equation

$$\begin{aligned} f_{NN}(p_2', s_2', t_2', p_f, s_f, t_f; | p_2, s_2, t_2, p_1 + q, s_{1'}, t_1) \\ = f_{NN}^{t_2', t_f | t_2, t_1}(p_{2\perp}' - p_{2\perp}) \delta^{s_f, s_{1'}} \delta^{s_2', s_2}. \end{aligned} \quad (19)$$

Using this relation and defining the transferred momentum in  $f_{NN}$  as  $k \equiv p_2' - p_2 = p_1 - p_m$  we can rewrite Eq. (18) as follows:

$$\begin{aligned} A_{1a}^\mu &= -\frac{F}{2} \sum_{s_1, s_2, s_3} \sum_{t_1, t_2, t_3} \int \frac{d^3 k}{(2\pi)^3} d^3 p_3 \\ &\times \Psi_{NN}^\dagger(p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3})(p_2', s_2, t_2; p_3, s_3, t_3) \\ &\times \frac{\chi_1(s_2^{NN}) f_{NN}^{t_2', t_f | t_2, t_1}(k_\perp)}{\Delta^0 - k_z + i\epsilon} \\ &\cdot j_{t_1}^\mu(p_1 + q, s_f; p_1, s_1) \\ &\cdot \Psi_A^{s_A}(p_m + k, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3), \end{aligned} \quad (20)$$

where  $\chi_1(s_2^{NN}) = \sqrt{s_2^{NN}(s_2^{NN} - 4m^2)}/2qm$ .

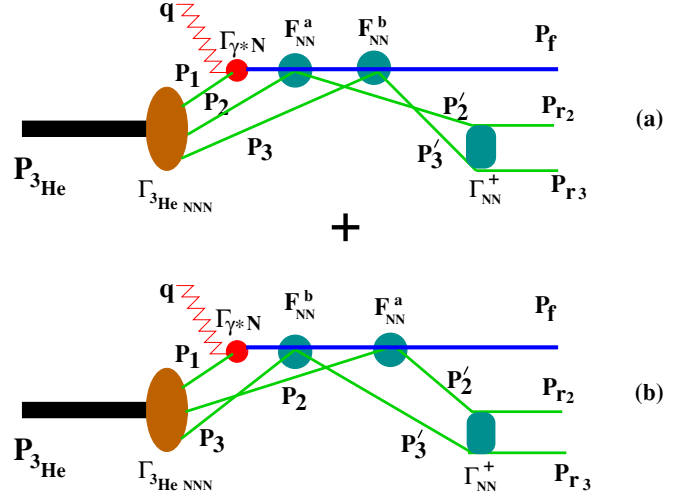


FIG. 3. (Color online) Double rescattering contribution to the scattering amplitude of the  ${}^3\text{He}(e, e'N_f N_{r2})N_{r3}$  reaction.

The contribution of the second diagram in Fig. 2 can be calculated by interchanging the momenta of 2 and 3 nucleons in Eq. (20). Doing this and changing the integration variables  $d^3 p_3$  to  $d^3 p_{23}$  (similar to what was done in Sec. IIIA) for complete single rescattering amplitude one obtains

$$\begin{aligned} A_1^\mu &= A_{1a}^\mu + A_{1b}^\mu = -\frac{F}{2} \sum_{s_1, s_2, s_3} \sum_{t_2, t_3} \sum_{t_1, t_2, t_3} \\ &\times \int \frac{d^3 k d^3 p_{23}}{(2\pi)^3} \Psi_{NN}^\dagger(p_{r23}, s_{r2}, t_{r2}; s_3, t_3) \\ &\cdot j_{t_1}^\mu(p_1 + q, s_f; p_1, s_1) \left[ \frac{\chi_1(s_2^{NN}) f_{NN}^{t_f, t_2' | t_1, t_2}(k_\perp) \delta^{t_3, t_3}}{\Delta^0 - k_z + i\epsilon} \right. \\ &\times \Psi_A^{s_A} \left( p_m + k, s_1, t_1; -\frac{p_m}{2} + p_{23} - k, s_2, t_2; -\frac{p_m}{2} \right. \\ &\left. \left. - p_{23}; s_3, t_3 \right) + \frac{\chi_1(s_3^{NN}) f_{NN}^{t_f, t_3' | t_1, t_3}(k_\perp) \delta^{t_2', t_2}}{\Delta^0 - k_z + i\epsilon} \right. \\ &\left. \times \Psi_A^{s_A} \left( p_m + k, s_1, t_1; -\frac{p_m}{2} + p_{23}, s_2, t_2; -\frac{p_m}{2} \right. \right. \\ &\left. \left. - p_{23} - k; s_3, t_3 \right) \right]. \end{aligned} \quad (21)$$

### C. Double rescattering amplitude

Next we discuss the double rescattering contribution, in which the knocked-out nucleon rescatters off both spectator nucleons in the nucleus (Fig. 3). Using the diagram rules summarized in the Appendix, one obtains

$$\begin{aligned} A_{2a}^\mu &= -\int \frac{d^4 p_3'}{i(2\pi)^4} \frac{d^4 p_2}{i(2\pi)^4} \frac{d^4 p_3}{i(2\pi)^4} \bar{u}(p_{r3}) \bar{u}(p_{r2}) \bar{u}(p_f) \\ &\times \frac{\Gamma_{NN}^+(p_2', p_3')(\hat{p}_2' + m)(\hat{p}_3' + m)}{(p_2'^2 - m^2 + i\epsilon)(p_3'^2 - m^2 + i\epsilon)} \end{aligned}$$

$$\begin{aligned}
& \times \frac{F_{NN}^b(p'_3 - p_3)(\hat{p}_1 + \hat{q} + \hat{p}_2 - \hat{p}'_2 + m)}{(p_1 + q + p_2 - p'_2)^2 - m^2 + i\epsilon} \\
& \times \frac{F_{NN}^a(p'_2 - p_2)(\hat{p}_1 + \hat{q} + m)}{(p_1 + q)^2 - m^2 + i\epsilon} \cdot \Gamma_{\gamma^* N}^\mu \cdot \frac{\hat{p}_3 + m}{p_3^2 - m^2 + i\epsilon} \\
& \times \frac{\hat{p}_2 + m}{p_2^2 - m^2 + i\epsilon} \cdot \frac{\hat{p}_1 + m}{p_1^2 - m^2 + i\epsilon} \cdot \Gamma_{\text{HeNNN}}^\mu(p_1, p_2, p_3) \chi^A, \quad (22)
\end{aligned}$$

where  $p_2$  and  $p_3$  are the momenta of the spectator nucleons before rescattering;  $p_1 = p_A - p_2 - p_3$ .

Using the same approximation we used for the IA and single rescattering amplitudes we estimate the integrals over  $d^0 p_{3',2}$  through the positive energy poles of the nucleon propagators with momenta  $p'_3$ ,  $p_3$  and  $p_2$ , respectively. These integrations result in the replacement of

$$\int \frac{d^0 p_j}{2\pi i (p_j^2 - m^2 + i\epsilon)} \rightarrow -\frac{1}{2E_j} \approx -\frac{1}{2m}, \quad (j = 2, 3, 3').$$

Applying the closure condition of Eq. (7) and using the reduction [Eq. (A4)] of nuclear vertices to the nonrelativistic nuclear wave functions (both in the ground state and in the continuum) as well as applying the relations of Eqs. (9) and (A1), for  $A_{2a}^\mu$  one obtains

$$\begin{aligned}
A_{2a}^\mu &= \frac{F}{(2m)^2} \sum_{s_1, s_2, s_3} \sum_{t_1, t_2, t_3, t_{1'}, t_{2'}, t_{3'}} \int \frac{d^3 p'_3}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} d^3 p_3 \\
& \times \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p'_2, s_2, t_{2'}; p'_3, s_3, t_{3'}) \\
& \times \frac{\sqrt{s_{b3}^{NN} (s_{b3}^{NN} - 4m^2)} f_{NN}^{t_{3'}, t_f | t_3, t_{1'}}(p'_{3\perp} - p_{3\perp})}{(p_1 + q + p_2 - p'_2)^2 - m^2 + i\epsilon} \\
& \times \frac{\sqrt{s_{a2}^{NN} (s_{a2}^{NN} - 4m^2)} f_{NN}^{t_{2'}, t_{1'} | t_2, t_1}(p'_{2\perp} - p_{2\perp})}{(p_1 + q)^2 - m^2 + i\epsilon} \\
& \times j_{t_1}^\mu(p_1 + q, s_f; p_1, s_1) \\
& \cdot \Psi_A^s(p_1, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3), \quad (23)
\end{aligned}$$

where  $s_{b3}^{NN}$  and  $s_{a2}^{NN}$  are total invariant energies of nucleons coupling at the vertices  $F_{NN}^b$  and  $F_{NN}^a$  respectively.

Let us now consider the denominators of the knocked-out nucleon in the intermediate states. For  $(p_1 + q)^2 - m^2 + i\epsilon$ , similar to the case of single rescattering, one obtains

$$(p_1 + q)^2 - m^2 = 2q(\Delta^0 + p_{mz} - p_{1z} + i\epsilon), \quad (24)$$

where  $\Delta^0$  is defined according to Eq. (17).

For the denominator,  $(p_1 + q + p_2 - p'_2)^2 - m^2 + i\epsilon$  in Eq. (23), using energy-momentum conservation in the reaction (1) we obtain

$$\begin{aligned}
& (p_1 + q + p_2 - p'_2)^2 - m^2 + i\epsilon = (p_f + p'_3 - p_3)^2 - m^2 + i\epsilon \\
& = 2p_{fz} \left[ \frac{E_f}{p_{fz}} (E'_3 - E_3) - (p'_{3z} - p_{3z}) - \frac{p_{f\perp}}{p_{fz}} (p'_{3\perp} - p_{3\perp}) + i\epsilon \right] \\
& = 2p_{fz} [\Delta_3 - (p'_{3z} - p_{3z}) + i\epsilon]. \quad (25)
\end{aligned}$$

where  $\Delta_3 = \frac{E_f}{p_{fz}} (E'_3 - E_3) - \frac{p_{f\perp}}{p_{fz}} (p'_{3\perp} - p_{3\perp})$ . Substituting Eqs. (24) and (25) into Eq. (22) for  $A_{2a}^\mu$  one obtains

$$\begin{aligned}
A_{2a}^\mu &= \frac{F}{4} \sum_{s_1, s_2, s_3} \sum_{t_1, t_2, t_3, t_{1'}, t_{2'}, t_{3'}} \int \frac{d^3 p'_3}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} d^3 p_3 \\
& \times \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p'_2, s_2, t_{2'}; p'_3, s_3, t_{3'}) \\
& \times \frac{\chi_2(s_{b3}^{NN}) f_{NN}^{t_{3'}, t_f | t_3, t_{1'}}(p'_{3\perp} - p_{3\perp})}{\Delta_3 + p'_{3z} - p_{3z} + i\epsilon} \\
& \times \frac{\chi_1(s_{a2}^{NN}) f_{NN}^{t_{2'}, t_{1'} | t_2, t_1}(p'_{2\perp} - p_{2\perp})}{\Delta^0 + p_{mz} - p_{1z} + i\epsilon} \\
& \times j_{t_1}^\mu(p_1 + q, s_f; p_1, s_1) \\
& \cdot \Psi_A^s(p_1, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3), \quad (26)
\end{aligned}$$

where  $\chi_1(s_{a2}^{NN}) = \sqrt{s_{a2}^{NN} (s_{a2}^{NN} - 4m^2)} / 2qm$  with  $s_{a2}^{NN} = (p_1 + q + p_2)^2$  and  $\chi_2(s_{b3}^{NN}) = \sqrt{s_{b3}^{NN} (s_{b3}^{NN} - 4m^2)} / 2p_{fz}m$  with  $s_{b3}^{NN} = (p_1 + q + p_2 - p'_2 + p_3)^2$ .

To complete the calculation of the double rescattering amplitude one should also calculate the amplitude corresponding to the diagram of Fig. 3(b). This amplitude is obtained by interchanging momenta of nucleons 2 and 3 in Eq. (26). Furthermore, it is more convenient to express the integrand of the double rescattering amplitude through the momentum transfers in the  $NN$  rescattering amplitude  $k_2 = p'_2 - p_2$  and  $k_3 = p'_3 - p_3$ . Using these variables, and changing the  $d^3 p'_3$  integration to  $d^3 p'_{23}$  (similar to what was done in Sec. III A), one obtains for the complete double rescattering amplitude

$$\begin{aligned}
A_2^\mu &= A_{2a}^\mu + A_{2b}^\mu = \frac{F}{4} \sum_{s_1, s_2, s_3} \sum_{t_1, t_2, t_3, t_{1'}, t_{2'}, t_{3'}} \\
& \times \int d^3 p'_{23} \frac{d^3 k_3}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p'_{23}, s_2, t_{2'}; s_3, t_3) \\
& \times \left[ \frac{\chi_2(s_{b3}^{NN}) f_{NN}^{t_{3'}, t_f | t_3, t_{1'}}(k_{3\perp}) \chi_1(s_{a2}^{NN}) f_{NN}^{t_{2'}, t_{1'} | t_2, t_1}(k_{2\perp})}{\Delta_3 - k_{3z} + i\epsilon} \frac{\chi_1(s_{a2}^{NN}) f_{NN}^{t_{2'}, t_{1'} | t_2, t_1}(k_{2\perp})}{\Delta^0 - k_{2z} - k_{3z} + i\epsilon} \right. \\
& \left. + \frac{\chi_2(s_{b2}^{NN}) f_{NN}^{t_{2'}, t_f | t_2, t_{1'}}(k_{2\perp}) \chi_1(s_{a3}^{NN}) f_{NN}^{t_{3'}, t_{1'} | t_3, t_1}(k_{3\perp})}{\Delta_2 - k_{2z} + i\epsilon} \frac{\chi_1(s_{a3}^{NN}) f_{NN}^{t_{3'}, t_{1'} | t_3, t_1}(k_{3\perp})}{\Delta^0 - k_{2z} - k_{3z} + i\epsilon} \right] \\
& \times j_{t_1}^\mu(p_m + k_2 + k_3 + q, s_f; p_m + k_2 + k_3, s_1) \\
& \times \Psi_A^s \left( p_m + k_3 + k_2, s_1, t_1; -\frac{P_m}{2} - k_2 + p'_{23}, s_2, t_2; \right. \\
& \left. -\frac{P_m}{2} - k_3 - p'_{23}, s_3, t_3 \right). \quad (27)
\end{aligned}$$

#### D. Differential Cross Section

The calculated amplitudes in Sec. III allow us to evaluate numerous observables (both polarized and unpolarized) for high- $Q^2$  quasi-elastic electroproduction from a  $^3\text{He}$  target. The differential cross section of reaction (1) is given by

$$\begin{aligned}
d^{12}\sigma &= \frac{1}{4j_A} (2\pi)^4 \delta^4(k_e + P_A - k'_e - p_f - p_{r2} - p_{r3}) \\
&\times \sum_{\text{nucleons}} |M_{fi}|^2 \frac{d^3k'_e}{(2\pi)^3 2E'_e} \frac{d^3p_f}{(2\pi)^3 2E_f} \\
&\times \frac{d^3p_{r2}}{(2\pi)^3 2E_{r2}} \frac{d^3p_{r3}}{(2\pi)^3 2E_{r3}}, \quad (28)
\end{aligned}$$

where  $j_A = \sqrt{(k_e P_A)^2 - m_e^2 M_A^2}$ . Here we sum over the nucleons knocked out by the virtual photon.

The factor  $1/4$  comes from the averaging over the initial polarizations of the electron and  ${}^3\text{He}$ . Since one of the recoil nucleons is not observed, one eliminates this degree of freedom by integrating over  $d^3p_{r3}$ . Thus the integrated differential cross section is

$$\begin{aligned}
d^9\sigma &= \frac{1}{4j_A} (2\pi)^4 \delta(E_e + M_A - E'_e - E_f - E_{r2} - E_{r3}) \\
&\times \sum_{\text{nucleons}} |M_{fi}|^2 \frac{d^3k'_e}{(2\pi)^3 2E'_e} \frac{d^3p_f}{(2\pi)^3 2E_f} \\
&\times \frac{d^3p_{r2}}{(2\pi)^3 2E_{r2}} \frac{1}{(2\pi)^3 2E_{r3}}, \quad (29)
\end{aligned}$$

where  $\mathbf{p}_{r3} = \mathbf{k}_e - \mathbf{k}'_e - \mathbf{p}_f - \mathbf{p}_{r2}$ . In Eqs. (28) and (29) the transition matrix,  $M_{fi}$ , represents the convolution of the electron and nuclear currents, in which the nuclear current represents the sum of the IA and single and double rescattering amplitudes,

$$M_{fi} = -4\pi\alpha \frac{1}{q^2} j_\mu^e \cdot (A_0^\mu + A_1^\mu + A_2^\mu), \quad (30)$$

where  $A_0$ ,  $A_1$ , and  $A_2$  are defined in Eqs. (11), (21), and (27), respectively.

#### IV. SUMMARY

We developed a theoretical framework for calculation of high- $Q^2$  exclusive electro-disintegration of the  $A = 3$  system. The main feature of our approach is the calculation of final state interactions of the struck energetic nucleon with the recoil nucleons within the generalized eikonal approximation, which allows us to account for the finite and relatively large momenta of the recoil nucleons. An important advantage of this approach is that we can now self-consistently study short-range correlations in nuclei since the GEA does not require the recoil nucleons to be stationary as does the conventional Glauber approximation.

To describe the residual interaction between two recoil nucleons, we use a scattering representation of the two-nucleon continuum state wave function. This allows us to evaluate the latter through the  $NN$  scattering amplitudes in the low- to intermediate-energy region.

In the second part of our work [23] we discuss the numerical calculations based on the formulas derived in this paper. In numerical calculations, as an input we use the calculation of ground-state wave functions of  ${}^3\text{He}$  based on realistic  $NN$  interaction potentials as well as including models that account for the three-nucleon forces [18]. The numerical estimates of the interaction between recoil nucleons—referred to as

pair distortion—is implemented through the parameterizations of low- to medium-energy  $NN$  scattering amplitudes provided by the SAID group [30]. For high-energy small-angle  $NN$  scattering we use the parameterization of the form of Eq. (A2). In numerical calculations [23] we are interested mainly in studies of short-range two- and three nucleon correlations for which GEA provides an appropriate theoretical framework.

#### ACKNOWLEDGMENTS

We thank Ted Rogers for careful reading of the manuscript and for many valuable comments. This work is supported by DOE grants under contract DE-FG02-01ER-41172 and DE-FG02-93ER40771 and by the Israel–USA Binational Science Foundation Grant. M.M.S. gratefully acknowledges a contract from Jefferson Lab under which this work was done. The Thomas Jefferson National Accelerator Facility (Jefferson Lab) is operated by the Southeastern Universities Research Association (SURA) under DOE Contract DE-AC05-84ER40150.

#### APPENDIX: FEYNMAN DIAGRAM RULES FOR THE SCATTERING AMPLITUDE IN GEA

Within the GEA the general  $eA$  scattering amplitude of Fig. 4 can be calculated based on effective Feynman diagram rules formulated as follows [19,20]:

- We assign the vertex functions  $\Gamma_A(p_1, \dots, p_A)$  to describe the transition of “nucleus  $A$ ” to “ $A$  nucleons” with momenta  $\{p_n\}$ . The vertex function  $\Gamma_{A-1}^\dagger(p'_2, \dots, p_A)$  describes the transition of “ $(A - 1)$  nucleons” with momenta  $\{p'_n\}$  to “ $(A - 1)$  nucleon final state”.
- For the  $\gamma^*N$  interaction we assign the vertex  $\Gamma_{\gamma^*N}^h$ .
- For each  $NN$  interaction we assign the vertex function  $F_k^{NN}(p_{k+1}, p'_{k+1})$ . This vertex function is related to the amplitude of  $NN$  scattering as follows:

$$\bar{u}(p_3)\bar{u}(p_4)F^{NN}u(p_1)u(p_2) = \sqrt{s(s - 4m^2)}f^{NN}(p_3, p_1), \quad (A1)$$

where  $s$  is the total invariant energy of two interacting nucleons with momenta  $p_1$  and  $p_2$  and

$$f^{NN} = \sigma_{\text{tot}}^{NN} (i + \alpha^{NN}) e^{-\frac{B^{NN}}{2}(p_3 - p_1)_\perp^2}, \quad (A2)$$

where  $\sigma_{\text{tot}}^{NN}$ ,  $\alpha^{NN}$ , and  $B^{NN}$  are known experimentally from  $NN$  scattering data. The vertex functions are accompanied by a  $\delta$ -function of energy-momentum conservation.

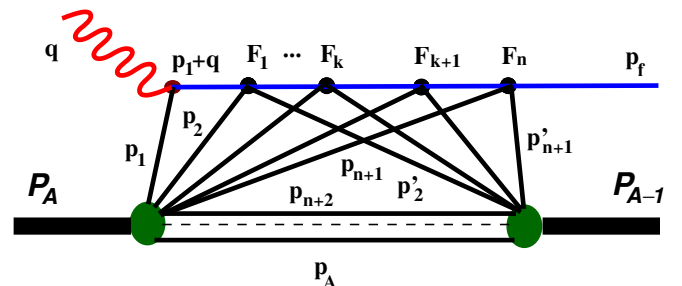


FIG. 4. (Color online)  $n$ -fold  $A(e, e'N)A - 1$  scattering diagram.

- For each intermediate nucleon with four momentum  $p$  we assign propagator  $D(p)^{-1} = -(\hat{p} - m + i\epsilon)^{-1}$ . Following Ref. [31] we choose the minus sign for the nucleon propagators to simplify the calculation of the overall sign of the scattering amplitude.
- The factor  $n!(A - n - 1)!$  accounts for the combinatorics of  $n$  rescatterings and  $(A - n - 1)$  spectator nucleons.
- For each closed contour one gets the factor  $\frac{1}{i(2\pi)^4}$  with no additional sign.

Using the defined rules for the scattering amplitude of Fig. 4 one obtains

$$\begin{aligned}
A_{A,A-1}^{(n)}(q, p_f) &= \sum_h \frac{1}{n!(A - n - 1)!} \prod_{i=1}^A \prod_{j=2}^A \\
&\times \int d^4 p_i d^4 p'_j \frac{1}{[i(2\pi)^4]^{A-2+n}} \delta^4 \left( \sum_{i=1}^A p_i - \mathcal{P}_A \right) \\
&\times \delta^4 \left( \sum_{j=2}^A p'_j - \mathcal{P}_{A-1} \right) \prod_{m=n+2}^A \delta^4(p_m - p'_m) \\
&\times \frac{\bar{u}(p_f) \chi_{A-1}^\dagger \Gamma_{A-1}^\dagger(p'_2, \dots, p'_{n+1}, p_{n+2}, \dots, p_A)}{D(p'_2) \dots D(p'_{n+1})} \\
&\times \frac{f_n^{NN}(p_{n+2}, p'_{n+2}) \dots f_1^{NN}(p_2, p'_2)}{D(l_1) \dots D(l_k) \dots D(l_{n-1})} \\
&\times \frac{\Gamma_{\gamma^* N}^h(Q^2)}{D(p_1 + q)} \frac{\Gamma_A(p_1, \dots, p_A) \chi_A}{D(p_1) D(p_2) \dots D(p_{n+1}) D(p_{n+2}) \dots D(p_A)}, \tag{A3}
\end{aligned}$$

where, for the sake of simplicity, we neglect the spin-dependent indexes. Here,  $\mathcal{P}_A$  and  $\mathcal{P}_{A-1}$  are the four-momenta of the target nucleus and final  $(A - 1)$  system,  $p_i$  and  $p'_i$  are nucleon momenta in the nucleus  $A$  and residual  $(A - 1)$  system, respectively, and  $l_k = q + p_1 + \sum_{i=2}^k (p_i - p'_i)$ . The intermediate spectator states in the diagram of Fig. 4 are expressed in terms of nucleons but not nuclear fragments because, in the

high-energy limit, the closure over various nuclear excitations in the intermediate state is used [19,20]. The sum of  $\sum_h$  in Eq. (A3) goes over virtual photon interactions with different nucleons, in which  $\Gamma_{\gamma^* N}^h(Q^2)$  describes the electromagnetic interaction.

The vertex function  $\Gamma_A$  describes a transition of nucleus  $A$  to the  $A$ -nucleon state, whereas the function  $\Gamma_{A-1}^\dagger$  describes the transition of the  $A - 1$  intermediate nucleons to a final continuum or bound  $A - 1$  nucleon state. Functions  $\chi_A$  and  $\chi_{A-1}$  describe the spin state of the  $A$  and  $A - 1$  systems respectively.

If one considers the kinematic conditions [similar to Eqs. (2)] in which the internal momenta of nucleons are restricted and the only relevant degrees of freedom are nucleons, one can evaluate the intermediate-state nucleon propagators through the poles corresponding to the positive energy solutions. As a result the covariant amplitude will be reduced to a set of time-ordered noncovariant diagrams that allows us to establish the correspondence between the nuclear vertex functions and the nuclear wave functions. In this limit the momentum-space wave function is defined through the vertex function as follows [31,32]:

$$\begin{aligned}
\psi_A(p_1, p_2, \dots, p_A) &= \frac{1}{(\sqrt{(2\pi)^3 2m})^{A-1}} \\
&\times \frac{\bar{u}(p_1) \bar{u}(p_2) \dots \bar{u}(p_A) \Gamma_A(p_1, p_2, \dots, p_A) \chi^A}{p_1^2 - m^2}, \tag{A4}
\end{aligned}$$

normalized as

$$\int |\psi_A(p_1, p_2, \dots, p_A)|^2 \delta^3 \left( \sum p_i - p_A \right) \prod_{i=1}^A d^3 p_i = N, \tag{A5}$$

where  $N = A$  for bound states and  $N = \prod_{i=1}^A \delta^3(p_i - p'_i)$  for  $A$  body continuum state. Note that to apply the relativistic normalization for the spinors ( $\bar{u}u = 2m$ ) the  $\sqrt{(2\pi)^3 2m}^{-1}$  factor should be associated with the plane-wave single-nucleon wave function.

- 
- [1] R. A. Niyazov and L. B. Weinstein (CLAS Collaboration), Phys. Rev. Lett. **92**, 052303 (2004).
- [2] W. Boeglin, M. Jones, A. Klein, J. Mitchell, P. Ulmer, and E. Voutier (spokespersons), Jefferson Lab Proposal E01-020, 2001 (unpublished).
- [3] S. E. Kuhn and K. A. Griffioen (spokespersons), Jefferson Lab Proposal E94-102, 1994 (unpublished).
- [4] K. Sh. Egiyan, K. A. Griffioen, and M. I. Strikman (spokespersons) Jefferson Lab Proposal E94-019, 1994 (unpublished).
- [5] L. Cardman *et al.* (ed.), The Science Driving the 12 GeV Upgrade of CEBAF, *Jefferson Lab Report*, Feb. 2001.
- [6] M. M. Sargsian *et al.*, J. Phys. G **29**, R1 (2003).
- [7] L. L. Frankfurt, W. R. Greenberg, G. A. Miller, M. M. Sargsian, and M. I. Strikman, Z. Phys. A **352**, 97 (1995).
- [8] S. Jeschonnek, Phys. Rev. C **63**, 034609 (2001).
- [9] C. Ciofi degli Atti, L. P. Kaptari, and D. Treleani, Phys. Rev. C **63**, 044601 (2001).
- [10] J. Adam, F. Gross, S. Jeschonnek, P. Ulmer, and J. W. Van Orden, Phys. Rev. C **66**, 044003 (2002).
- [11] J. Ryckebusch, D. Debruyne, P. Lava, S. Janssen, B. Van Overmeire, and T. Van Cauteren, Nucl. Phys. **A728**, 226 (2003).
- [12] C. Ciofi degli Atti and L. P. Kaptari, Phys. Rev. C **66**, 044004 (2002).
- [13] M. A. Braun, C. Ciofi degli Atti, and L. P. Kaptari, arXiv:nucl-th/0303048.
- [14] M. A. Braun, C. Ciofi degli Atti, L. P. Kaptari, and H. Morita, arXiv:nucl-th/0308069.
- [15] C. Ciofi degli Atti and L. P. Kaptari, arXiv:nucl-th/0407024.
- [16] J. M. Laget, nucl-th/0407072.
- [17] R. Machleidt, Phys. Rev. C **63**, 024001 (2001).
- [18] A. Nogga, A. Kievsky, H. Kamada, W. Gloeckle, L. E. Marcucci, S. Rosati, and M. Viviani, Phys. Rev. C **67**, 034004 (2003).



- [19] L. L. Frankfurt, M. M. Sargsian and M. I. Strikman, Phys. Rev. C **56**, 1124 (1997).
- [20] M. M. Sargsian, Int. J. Mod. Phys. E **10**, 405 (2001).
- [21] R. J. Glauber, Phys. Rev. **100**, 242 (1955); *Lectures in Theoretical Physics*, edited by W. Brittain and L. G. Dunham (Interscience, New York, 1959), Vol. 1.
- [22] L. L. Frankfurt, J. A. Miller, and M. I. Strikman, Annu. Rev. Nucl. Part. Sci. **44**, 501 (1994).
- [23] M. M. Sargsian, T. V. Abrahamyan, M. I. Strikman, and L. L. Frankfurt, Phys. Rev. C **71**, 044615 (2005).
- [24] R. P. Feynman, *Photon-Hadron Interactions* (Addison-Wesley Longman, Reading, MA/Palo Alto/London, 1998).
- [25] L. L. Frankfurt and M. I. Strikman, Phys. Lett. **B64**, 433 (1976); **76**, 333 (1978).
- [26] T. De Forest, Nucl. Phys. **A392**, 232 (1983).
- [27] G. Brown and A. D. Jackson, *The Nucleon-Nucleon Interaction* (North-Holland, Amsterdam, 1976).
- [28] J. M. Laget, J. Phys. G **14**, 1445 (1988).
- [29] G. D. Alkhazov, S. L. Belostotsky, and A. A. Vorobev, Phys. Rep. **42**, 89 (1978).
- [30] R. A. Arndt, I. I. Strakovsky, and R. L. Workman, Phys. Rev. C **62**, 034005 (2000).
- [31] V. N. Gribov, Sov. Phys. JETP, **30**, 709 (1970).
- [32] L. Bertocchi, Nuovo Cimento A **11**, 45 (1972).