Surface diffuseness anomaly in heavy-ion potentials for large-angle quasielastic scattering

K. Hagino,[1] T. Takehi,[1] A. B. Balantekin,[1,2] and N. Takigawa[1]
[1]*Department of Physics, Tohoku University, Sendai 980-8578, Japan*
[2]*Department of Physics, University of Wisconsin, Madison, Wisconsin 53706*
(Received 12 December 2004; published 29 April 2005)

Recent high precision experimental data for heavy-ion fusion reactions at sub-barrier energies systematically show that a surprisingly large surface diffuseness parameter for a Woods-Saxon potential is required in order to fit the data. We point out that experimental data for quasielastic scattering at backward angles also favor a similar large value of the surface diffuseness parameter. Consequently, a double folding approach with a short-range imaginary potential for the compound nucleus formation fails to reproduce the experimental excitation function of quasielastic scattering for the $^{16}O + {}^{154}Sm$ system at energies around the Coulomb barrier. We also show that the deviation of the ratio of the quasielastic to the Rutherford cross sections from unity at deep sub-barrier energies offers an unambiguous way to determine the value of the surface diffuseness parameter in the nucleus-nucleus potential.

DOI: 10.1103/PhysRevC.71.044612 PACS number(s): 25.70.Bc, 25.70.Jj, 24.10.Eq, 27.70.+q

The nucleus-nucleus potential is the primary ingredient in nuclear reaction calculations. Its nuclear part has often been parametrized as a Woods-Saxon form [1]. Elastic and inelastic scattering are sensitive mainly to the surface region of the nuclear potential, where the Woods-Saxon parametrization has a simple exponential form. This fact has been exploited to study the surface property of nuclear potential. Usually, the best fit to experimental data for scattering is obtained with a diffuseness of around 0.63 fm [1–5]. This value is consistent with a double folding potential [6,7], and it seems to be well accepted [1,8].

In marked contrast, recent high precision experimental data for heavy-ion fusion reactions at energies around the Coulomb barrier suggest that a much larger value of diffuseness, ranging from 0.75 to 1.5 fm, is required to fit the data [6,7,9–12] (see Ref. [13] for a detailed systematic study). The Woods-Saxon potential which fits elastic scattering overestimates fusion cross sections at energies both above and below the Coulomb barrier, having an inconsistent energy dependence with the experimental fusion excitation function. When the height of the Coulomb barrier is fixed, the larger diffuseness parameter leads to the smaller barrier position and the smaller barrier curvature (thus the larger tunneling region). The main effect on the fusion cross sections comes from the barrier position and the tunneling width of the barrier at energies above and below the Coulomb barrier, respectively. A large diffuseness parameter appears to be desirable in both these respects [6]. The reason for the large discrepancies in diffuseness parameters extracted from scattering and fusion analyses, however, is not yet understood.

The purpose of this paper is to discuss the dependence of a quasielastic excitation function at a large scattering angle on the surface diffuseness parameter in a nucleus-nucleus potential. The quasielastic cross section is defined as the sum of the cross sections of elastic, inelastic, and transfer reactions. Its excitation function at backward angles provides complementary information to the fusion process [14–16]. It therefore offers an ideal test ground for the large diffuseness parameter suggested by recent fusion data. This is particularly of interest in connection to the steep falloff phenomena of fusion cross sections at deep sub-barrier energies observed recently in several systems [11,17–19]. This is so because the measurement of quasielastic scattering is experimentally much easier than that of a fusion reaction, especially at deep sub-barrier energies [16]. Contrary to what one might expect, we demonstrate below that the surface diffuseness parameter which fits the experimental data of quasielastic scattering is consistent with the one for fusion, rather than the commonly accepted value for scattering.

As a concrete example, let us consider the $^{16}O + {}^{154}Sm$ reaction. Neglecting the finite excitation energy of the ground state rotational band in the target nucleus ^{154}Sm, the cross sections for fusion and quasielastic scattering are given by [14,16,20,21]

$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\rm fus}(E;\theta_T) \tag{1}$$

and

$$\sigma_{\rm qel}(E,\theta) = \int_0^1 d(\cos\theta_T)\sigma_{\rm el}(E,\theta;\theta_T), \tag{2}$$

respectively, in the isocentrifugal approximation, where one neglects the angular momentum transfer in the centrifugal potential [16,22]. θ and θ_T are the scattering angle and the orientation angle of the deformed target with respect to the projectile direction, respectively. $\sigma_{\rm fus}(E;\theta_T)$ and $\sigma_{\rm el}(E,\theta;\theta_T)$ are the fusion and the elastic cross sections for the angle dependent potential $V(r,\theta_T)$ given by

$$V(r,\theta_T) = V_N(r,\theta_T) + V_C(r,\theta_T), \tag{3}$$

$$V_N(r,\theta_T) = \frac{-V_0}{1+\exp[(r-R-R_T\sum_\lambda \beta_\lambda Y_{\lambda 0}(\theta_T))/a]}, \tag{4}$$

$$V_C(r,\theta_T) = \frac{Z_P Z_T e^2}{r} + \sum_\lambda \left(\beta_\lambda + \frac{2}{7}\sqrt{\frac{5}{\pi}}\beta_2^2\delta_{\lambda,2}\right) \times \frac{3Z_P Z_T e^2}{2\lambda+1}\frac{R_T^\lambda}{r^{\lambda+1}}Y_{\lambda 0}(\theta_T). \tag{5}$$

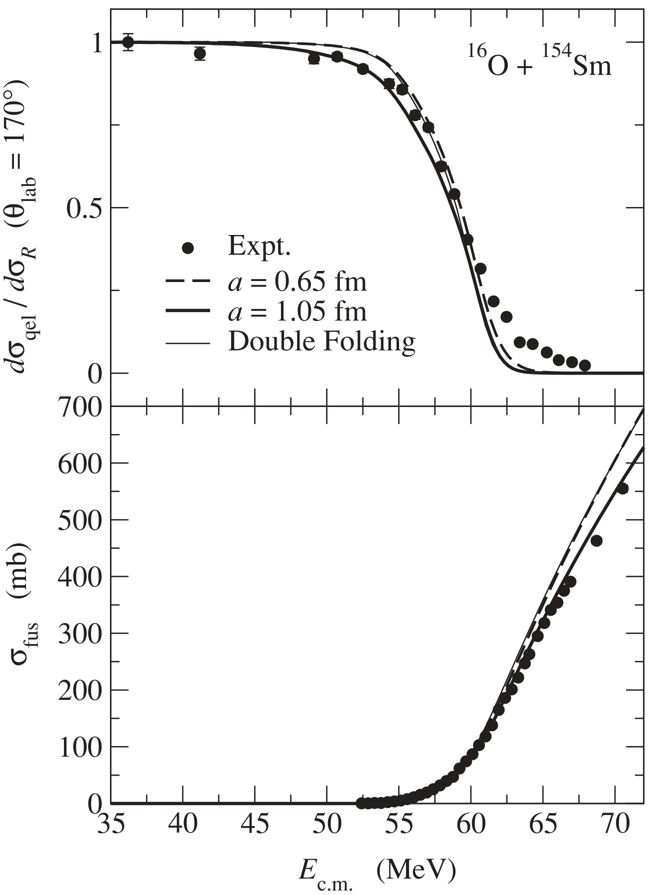

FIG. 1. The ratio of the quasielastic to the Rutherford cross sections at $\theta_{\rm lab} = 170^\circ$ (upper panel) and the fusion cross section (lower panel) for the ^{16}O + ^{154}Sm reaction. The solid line is obtained with the orientation-integrated formula with $\beta_2 = 0.306$ and $\beta_4 = 0.05$ by using the Woods-Saxon potential with the surface diffuseness parameter a of 1.05 fm; the dashed line is obtained with an a of 0.65 fm. The result of the double folding potential with the density-dependent M3Y interaction is denoted by the thin solid line. The experimental data are taken from Refs. [9,15].

Figure 1 compares the experimental data for the quasielastic (given as the ratio to the Rutherford cross section; the upper panel) and the fusion (the lower panel) cross sections with calculated cross sections obtained with different values for the surface diffuseness parameter in the Woods-Saxon potential. The experimental data are taken from Refs. [9,15], where the quasielastic cross sections were measured at 170° in the laboratory frame. The solid and dashed lines are obtained with a Woods-Saxon potential with $a = 1.05$ and 0.65 fm, respectively. The depth and the radius parameters of the potentials are $V_0 = 165$ MeV and $R = 0.95 \times (A_P^{1/3} + A_T^{1/3})$ fm for the former, and $V_0 = 220$ MeV and $R = 1.1 \times (A_P^{1/3} + A_T^{1/3})$ fm for the latter. The deformation parameters are taken to be $\beta_2 = 0.306$ and $\beta_4 = 0.05$ with $R_T = 1.06 \times A_T^{1/3}$ fm. As is usually done, we use a short-range imaginary potential with $W = 50$ MeV, $a_w = 0.4$ fm, and $r_w = 1.0$ fm in order to simulate the compound nucleus formation, and we assume that the potential parameters are independent of energy.[1] The absorption cross sections are thus identified with the fusion cross sections in the present calculations. The calculated fusion cross sections are insensitive to the parameters of the imaginary part of the potential as long as it is strong enough and well localized inside the Coulomb barrier.

Figure 1 clearly shows that the experimental data favor the internuclear potential with the larger value of the diffuseness parameter, $a = 1.05$ fm, both for fusion and quasielastic scattering. We have checked that the fit to the experimental data with the potential with $a = 0.65$ fm does not improve even if we vary the depth and the radius parameters of the potential as well as the deformation parameters. The discrepancy between the experimental data and the theoretical curve for the quasielastic excitation function around $E = 65$ MeV is due to the transfer process [15], which is not included in the present calculations.

For a single channel problem, the ratio of the elastic to the Rutherford cross sections at backward angles is given by [16,25]

$$\frac{d\sigma_{\rm el}}{d\sigma_R}(E,\theta) \sim 1 + \frac{V_N(r_c)}{ka}\frac{\sqrt{2a\pi k\eta}}{E}, \qquad (6)$$

at energies well below the Coulomb barrier, where the tunneling probability is exponentially small (see Ref. [16] for a more general formula which is also valid at higher energies). This formula is obtained with the semiclassical perturbation theory by assuming that the nuclear potential $V_N(r)$ is proportional to $\exp(-r/a)$ around the distance of closest approach, that is, $r_c = (\eta + \sqrt{\eta^2 + \lambda_c^2})/k$, where η is the Sommerferd parameter and $\lambda_c = \eta\cot(\theta/2)$. The deviation of the ratio of the cross sections at sub-barrier energies from unity is therefore sensitive only to the surface property of nuclear potential, and it provides a relatively model-independent way to study the effect of the surface diffuseness parameter. In order to demonstrate that the surface diffuseness is indeed more influential than the channel coupling effect on quasielastic scattering at low energies, Fig. 2 shows the effect of deformation of the target nucleus on the quasielastic cross sections. We find that the effect is negligible at deep sub-barrier energies, and the role played by the surface diffuseness parameter is indeed identified unambiguously. In the interpretation of the channel coupling effects based on the barrier distribution picture [9,26,27], this is a natural consequence of the fact that the reflection probability is almost unity for all the distributed barriers at deep sub-barrier energies. In terms of the dynamical polarization potential (DPP), channel couplings induce both the real and imaginary

[1]In another model of fusion by Udagawa *et al.* [23,24], a relatively long-range imaginary potential is assumed. To satisfy the dispersion relation, the real part then has a strong energy dependence. Although this approach has been as successful as the approach with a short-range imaginary potential, we choose the latter approach, i.e., the energy-independent approach in the present paper, assuming that the imaginary potential around the barrier position has been taken into account by explicitly considering the couplings to the rotational states (or the deformation) of the target nucleus.

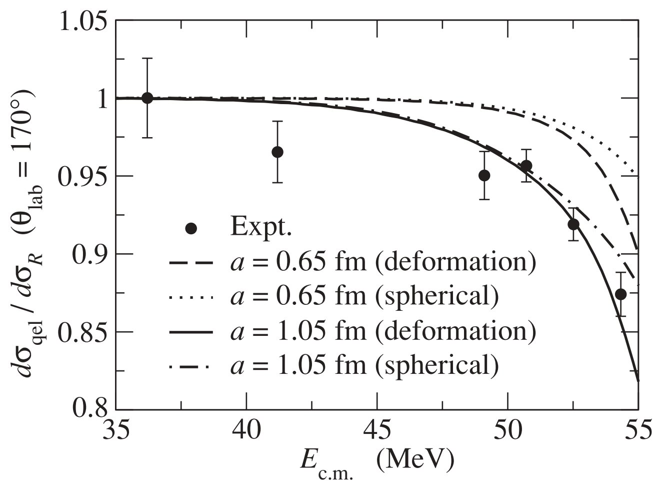

FIG. 2. Effects of the deformation of the target nucleus on the quasielastic scattering for the $^{16}O + ^{154}Sm$ reaction. The meaning of the solid and dashed lines is the same as in Fig. 1. The dot-dashed and the dotted lines are obtained by assuming a spherical target for a Woods-Saxon potential with a surface diffuseness parameter a of 1.05 and 0.65 fm, respectively.

polarization potentials. These two effects cancel each other in quasielastic cross sections at deep sub-barrier energies.

The strongest energy dependence of the cross section ratio comes from the exponential factor, $\exp(-r_c/a)$, in the nuclear potential $V_N(r_c)$. The larger value of diffuseness parameter results in the stronger energy dependence, and thus the larger deviation of the ratio from unity. The measured quasielastic cross sections at energies between 35 and 55 MeV are clearly inconsistent with $a = 0.65$ fm. As in sub-barrier fusion reactions, a larger diffuseness parameter seems to be required in order to fit the experimental data.

For completeness of our study, we next examine the performance of a double folding potential [28–30] for the sub-barrier reactions. In order to construct a nucleus-nucleus potential with the double folding procedure, we assume a deformed Fermi function for the (intrinsic) target density,

$$\rho_T(\boldsymbol{r}) = \frac{\rho_0}{1 + \exp\left[(r - R - R\sum_\lambda \beta_\lambda Y_{\lambda 0}(\hat{\boldsymbol{r}}))/a_d\right]}. \qquad (7)$$

We use the same parameters as in Ref. [31], including the β_2 and β_4 deformations. We numerically expand Eq. (7) into multipoles up to $L = 6$, and construct the double folding potential for each multipole component, leading to an orientation-dependent potential which corresponds to Eq. (3). We use the same (spherical) density for ^{16}O as in Ref. [32]. For an effective nucleon-nucleon interaction, we use the density-dependent Michigan three-range Yukawa (DDM3Y) interaction [33], together with the zero-range approximation for the exchange contribution (see Ref. [29] for the parameters). We introduce an overall scaling factor to the nuclear potential so that the barrier height is the same as that of the Woods-Saxon potentials. The cross sections computed with a double folding potential thus obtained are denoted by the thin solid line in Fig. 1. Those are similar to the results of the Woods-Saxon potential with a diffuseness parameter of $a = 0.65$ fm. In particular, compared with the experimental data, the double folding potential leads to a much weaker falloff of quasielastic cross sections at energies well below the Coulomb barrier. Evidently, the double folding model does not provide a good representation for both the quasielastic scattering and the fusion reaction at sub-barrier energies, at least when it is used with a short-range imaginary potential.

In summary, we have studied the sensitivity of large-angle quasielastic scattering to the surface diffuseness parameter in the nucleus-nucleus potential. To this end, we assumed that the imaginary potential is well localized inside the Coulomb barrier in order to simulate the compound nucleus formation. We have argued that the deviation of the ratio of quasielastic to Rutherford cross sections from unity at deep sub-barrier energies is sensitive mainly to the surface property of nuclear potential, and thus it provides a useful way to determine the value of the surface diffuseness parameter. Using this fact, we have shown that the experimental excitation function for quasielastic scattering at energies around the Coulomb barrier can be reproduced only when a much larger diffuseness parameter is used in a Woods-Saxon potential than the commonly accepted value, that is, around 0.63 fm. This finding is consistent with a recent observation in heavy-ion sub-barrier fusion reactions. It would be helpful to perform other quasielastic measurements at deep sub-barrier energies, so that a systematic study of the diffuseness parameter for the scattering process is possible.

We have also discussed the applicability of a double folding potential in quasielastic scattering. We have shown that the cross sections obtained with the double folding potential are similar to the ones obtained with a Woods-Saxon potential whose surface diffuseness parameter is around 0.65 fm. Consequently, the double folding potential with a short-range imaginary potential does not reproduce the experimental excitation function for large-angle quasielastic scattering around the Coulomb barrier. This may appear rather surprising, given that a double folding approach has enjoyed success in reproducing an angular distribution for elastic and inelastic scattering in many systems. In order to reconcile this apparent contradiction, a more careful investigation, e.g., for the energy dependence of a double folding potential due to the exchange contribution, would be necessary. Detailed comparison between the approach with a short-range imaginary potential employed in this paper and the one by Udagawa *et al.* [23,24] with a long-range imaginary potential is also important. We will report these in a separate paper.

This work was supported by the Grant-in-Aid for Scientific Research, Contract No. 16740139, from the Japanese Ministry of Education, Culture, Sports, Science, and Technology; the U.S. National Science Foundation Grant No. PHY-0244384; and the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation. A. B. B. gratefully acknowledges the 21st Century Center of Excellence Program "Exploring New Science by Bridging Particle-Matter Hierarchy" at Tohoku University for financial support, and thanks the Nuclear Theory Group at Tohoku University for their hospitality.

[1] R. A. Broglia and A. Winther, *Heavy Ion Reactions*, in Frontiers in Physics Lecture Note Series (Addison-Wesley, Redwood City, CA, 1991), Vol. 84.
[2] P. R. Christensen and A. Winther, Phys. Lett. **B65**, 19 (1976).
[3] M. Lozano and G. Madurga, Nucl. Phys. **A334**, 349 (1980).
[4] L. C. Chamon, D. Pereira, E. S. Rossi, C. P. Silva, H. Dias, L. Losano, and C. A. P. Ceneviva, Nucl. Phys. **A597**, 253 (1996).
[5] C. P. Silva *et al.*, Nucl. Phys. **A679**, 287 (2001).
[6] K. Hagino, M. Dasgupta, I. I. Gontchar, D. J. Hinde, C. R. Morton, and J. O. Newton, in *Proceedings of the Fourth Italy-Japan Symposium on Heavy-Ion Physics, Tokyo,* edited by K. Yoshida *et al.* (World Scientific, Singapore, 2002), pp. 87–98; nucl-th/0110065.
[7] I. I. Gonttchar, D. J. Hinde, M. Dasgupta, and J. O. Newton, Phys. Rev. C **69**, 024610 (2004).
[8] H. Esbensen and B. B. Back, Phys. Rev. C **54**, 3109 (1996).
[9] J. R. Leigh *et al.*, Phys. Rev. C **52**, 3151 (1995).
[10] J. O. Newton, C. R. Morton, M. Dasgupta, J. R. Leigh, J. C. Mein, D. J. Hinde, H. Timmers, and K. Hagino, Phys. Rev. C **64**, 064608 (2001).
[11] K. Hagino, N. Rowley, and M. Dasgupta, Phys. Rev. C **67**, 054603 (2003).
[12] M. Dasgupta, D. J. Hinde, J. O. Newton, and K. Hagino, Prog. Theor. Phys. Suppl. **154**, 209 (2004).
[13] J. O. Newton, R. D. Butt, M. Dasgupta, D. J. Hinde, I. I. Gontchar, C. R. Morton, and K. Hagino, Phys. Lett. **B586**, 219 (2004); Phys. Rev. C **70**, 024605 (2004).
[14] M. V. Andres, N. Rowley, and M. A. Nagarajan, Phys. Lett. **B202**, 292 (1988).
[15] H. Timmers, J. R. Leigh, M. Dasgupta, D. J. Hinde, R. C. Lemmon, J. C. Mein, C. R. Morton, J. O. Newton, and N. Rowley, Nucl. Phys. **A584**, 190 (1995).
[16] K. Hagino and N. Rowley, Phys. Rev. C **69**, 054610 (2004).
[17] C. L. Jiang *et al.*, Phys. Rev. Lett. **89**, 052701 (2002).
[18] C. L. Jiang, H. Esbensen, B. B. Back, R. V. F. Janssens, and K. E. Rehm, Phys. Rev. C **69**, 014604 (2004).
[19] C. L. Jiang *et al.*, Phys. Rev. Lett. **93**, 012701 (2004).
[20] T. Rumin, K. Hagino, and N. Takigawa, Phys. Rev. C **63**, 044603 (2001).
[21] C. Y. Wong, Phys. Rev. Lett. **31**, 766 (1973).
[22] K. Hagino, N. Rowley, and A. T. Kruppa, Comput. Phys. Commun. **123**, 143 (1999).
[23] T. Udagawa, B. T. Kim, and T. Tamura, Phys. Rev. C **32**, 124 (1985).
[24] T. Izumoto, T. Udagawa, and B. T. Kim, Phys. Rev. C **51**, 761 (1995).
[25] S. Landowne and H. H. Wolter, Nucl. Phys. **A351**, 171 (1981).
[26] N. Rowley, G. R. Satchler, and P. H. Stelson, Phys. Lett. **B254**, 25 (1991).
[27] M. Dasgupta, D. J. Hinde, N. Rowley, and A. M. Stefanini, Annu. Rev. Nucl. Part. Sci. **48**, 401 (1998).
[28] G. R. Satchler and W. G. Love, Phys. Rep. **55**, 183 (1979).
[29] M. E. Brandan and G. R. Satchler, Phys. Rep. **285**, 143 (1997).
[30] D. T. Khoa and G. R. Satchler, Nucl. Phys. **A668**, 3 (2000).
[31] T. Cooper, W. Bertozzi, J. Heisemberg, S. Kowalski, W. Turchinetz, C. Williamson, L. Cardman, S. Fivozinsky, J. Lightbody Jr., and S. Penner, Phys. Rev. C **13**, 1083 (1976).
[32] M. El-Azab Farid and G. R. Satchler, Nucl. Phys. **A438**, 525 (1985).
[33] A. M. Kobos, B. A. Brown, R. Lindsay, and G. R. Satchler, Nucl. Phys. **A425**, 205 (1984).