Collective modes in relativistic asymmetric nuclear matter

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Isospin and density waves in asymmetric nuclear matter (ANM) are studied in the framework of relativistic mean-field hadron models. A semiclassical relativistic approach based on the Vlasov equation is applied to the study of infinite asymmetric nuclear matter. The isovector and isoscalar collective modes are determined for a wide range of densities as a function of isospin asymmetry and momentum transfer. The condition for the existence of each type of mode is referred. The instabilities of ANM, related to the liquid-gas phase transition, are investigated.

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I. INTRODUCTION

Relativistic phenomenological models have been extensively used in describing nuclei and nuclear matter nowadays. Besides the good description of both stable and unstable nuclei [1,2], these same models, with conveniently adjusted parameters, are used to describe the properties of neutron stars and supernovae [3,4]. Therefore, it is important to test these models at finite temperature, different densities, and different isospin asymmetries.

The instabilities and phase transitions appearing in asymmetric nuclear matter (ANM) are important quantities in the understanding of the physics underlying isospin distillation, multifragmentation, and fractionation effects. At low nuclear densities, mechanical and chemical modes are coupled in such a way that the instability of the ANM system appears as a mixture of baryon density and concentration fluctuations [5]. The region of instability, determined by the spinodal curve, depends on the model used and shrinks considerably with the increase of the temperature [6]. The study of metastable states, very important from the technological point of view and the appearance of nucleation processes close to the spinodal are also related with the instabilities of the system [7].

The inclusion of the isovector channel is essential to study both isoscalar and isovector colletive modes present in nuclear matter [8,9]. The chemical effect may be detected in the relative participation of protons and neutrons with respect to the initial isospin asymmetry. Examples are the distillation effect at low densities or the pure neutron waves proposed in Ref. [8].

Relevant quantities in the description of the gravitational collapse of a massive star are the equation of state and the description of neutrino transport phenomena, namely, the neutrino mean free path in the medium. It has been shown that the neutrino opacity is affected by nucleon-nucleon interactions due to coherent scattering off density fluctuations [10]. Both single particle and collective contribution have to be taken into account. It is, therefore, important to have a thorough understanding of the collective modes in asymmetric nuclear matter in order to predict the behavior of neutrinos.

In previous works the nuclear and mesonic collective modes have been already studied with the help of different formalisms. In Ref. [11] isospin and density waves in asymmetric nuclear matter have been studied in a nonrelativistic approach and using the Landau-Fermi liquid theory with an effective quasiparticle interaction. In Refs. [12,13] a semiclassical approach to the quantum hadrodynamical model (QHD) was used with the relativistic Vlasov equation, which may be regarded as the semiclassical limit of the mean field theory (MFT). The relativistic Vlasov equation based on QHD was also used to study heavy-ion collisions [14,15], and its predictions are similar to the more involved calculations based on the time-dependent Dirac equations [14]. This means that the use of the relativistic Vlasov equation appears as an alternative way to study relativistic systems. In Ref. [9] a relativistic Hartree calculation was performed and the zero sound, the longitudinal, and the transverse modes were obtained, but only symmetric matter was investigated. In Ref. [8] the asymmetry was also considered and the calculations were done via the linear response equations. The collective modes were considered in a Landau-Fermi liquid formalism within a relativistic mean field theory in Ref. [16], but the nonlinear terms were not included and only symmetric matter and neutron matter were discussed. In Ref. [17] the Landau parameters were calculated in relativistic nonlinear models with different parameter sets. All possible meson self-interactions were taken into account but only the symmetric nuclear matter was considered. Stable and unstable modes were calculated and discussed.

In the present work we are interested in studying the collective modes corresponding to small amplitude oscillations around a stationary state in nuclear matter, extending the formalism used in Ref. [12] in order to consider asymmetric nuclear matter as well. We shall restrict ourselves to the longitudinal modes. Besides the nuclear collective modes we shall also investigate the mesonic modes. For the sake of comparison with the Vlasov formalism, we also show the corresponding Landau parameters obtained in the Landau-Fermi liquid theory, which is an extension for ANM of the calculations performed in Refs. [16,17].

In Sec. II we formulate the Vlasov equation based on QHD. The eigenmodes and the dispersion relation are given in Sec. III. In Sec. IV the Fermi liquid theory is reviewed with the introduction of the nonlinear terms and the ρ meson, necessary for the description of ANM. The numerical results are presented in Sec. V and the conclusions are drawn in Sec. VI.

II. THE VLASOV EQUATION FORMALISM

We consider a system of baryons, with mass M interacting with and through a isoscalar-scalar field ϕ with mass m_s, a isoscalar-vector field V^μ with mass m_v, and an isovector-vector field $\mathbf{b}^\mu$ with mass m_ρ. The Lagrangian density reads

$$\mathcal{L} = \bar{\psi}\left[\gamma_\mu\left(i\partial^\mu - g_v V^\mu - \frac{g_\rho}{2}\boldsymbol{\tau}\cdot\mathbf{b}^\mu\right) - (M - g_s\phi)\right]\psi + \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2\right) - \frac{1}{3!}\kappa\phi^3 - \frac{1}{4!}\lambda\phi^4 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V^\mu - \frac{1}{4}\mathbf{B}_{\mu\nu}\cdot\mathbf{B}^{\mu\nu} + \frac{1}{2}m_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu, \tag{1}$$

where $\Omega_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $\mathbf{B}_{\mu\nu} = \partial_\mu\mathbf{b}_\nu - \partial_\nu\mathbf{b}_\mu - g_\rho(\mathbf{b}_\mu \times \mathbf{b}_\nu)$. The model comprises the following parameters: three coupling constants g_s, g_v, and g_ρ of the mesons to the nucleons, the nucleon mass M, the masses of the mesons m_s, m_v, m_ρ, and the self-interacting coupling constants κ and λ. We have used the set of constants, identified as NL3 taken from Ref. [18]. For this case, the saturation density that we refer as ρ_0 is 0.148 fm^{-3}.

Denoting by

$$f(\mathbf{r},\mathbf{p},t) = \begin{pmatrix} f_p & 0 \\ 0 & f_n \end{pmatrix}$$

the one-body phase-space distribution function in isospin space and by

$$h = \begin{pmatrix} \sqrt{(\mathbf{p}-\boldsymbol{\mathcal{V}}_p)^2 + (M - g_s\phi)^2} + \mathcal{V}_{0p} & 0 \\ 0 & \sqrt{(\mathbf{p}-\boldsymbol{\mathcal{V}}_n)^2 + (M - g_s\phi)^2} + \mathcal{V}_{0n} \end{pmatrix} \tag{2}$$

the one-body Hamiltonian, where

$$\mathcal{V}_{0i} = g_v V_0 + \frac{g_\rho}{2}\tau_i b_0, \quad \boldsymbol{\mathcal{V}}_i = g_v\mathbf{V} + \frac{g_\rho}{2}\tau_i\mathbf{b}, \quad i = p, n,$$

$\tau_i = 1$ (protons) or -1 (neutrons), the energy of the system is

$$E = 2\int\frac{d^3r d^3p}{(2\pi)^3} f(\mathbf{r},\mathbf{p},t)\,h(\mathbf{r},\mathbf{p},t) + \frac{1}{2}\int d^3r\left(\Pi_\phi^2 + \nabla\phi\cdot\nabla\phi + m_s^2\phi^2 + \frac{1}{3!}\kappa\phi^3 + \frac{1}{4!}\lambda\phi^4\right) + \frac{1}{2}\int d^3r\left[\Pi_{V_i}^2 - 2\Pi_{V_i}\partial_i V_0 + \nabla V_i\cdot\nabla V_i - \partial_j V_i\partial_i V_j + m_v^2\left(\mathbf{V}^2 - V_0^2\right)\right] + \frac{1}{2}\int d^3r\left[\Pi_{b_i}^2 - 2\Pi_{b_i}\partial_i b_0 + \nabla b_i\cdot\nabla b_i - \partial_j b_i\partial_i b_j + m_\rho^2\left(\mathbf{b}^2 - b_0^2\right)\right], \tag{3}$$

where Π_ϕ (Π_{V_i}, Π_{b_i}) is the field canonically conjugated to ϕ (V_i, b_i). The ρ-meson self-interacting terms have not been included.

The time evolution of the distribution function is described by the Vlasov equation

$$\frac{\partial f_i}{\partial t} + \{f_i, h_i\} = 0, \quad i = p,\ n, \tag{4}$$

where { , } denote the Poisson brackets. It has been argued in Refs. [19,20] that Eq. (4) expresses the conservation of the number of particles in phase space and is, therefore, covariant. Antiparticles should certainly be taken into account at finite temperature. However, at a given temperature, the dynamics described by the Vlasov equation does not change the number of particles or antiparticles in this semiclassical approach.

From Hamilton's equations we derive the equations describing the time evolution of the fields ϕ, V^μ and the third component of the ρ-field $b_3^\mu = (b_0, \mathbf{b})$:

$$\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi + m_s^2\phi + \frac{\kappa}{2}\phi^2 + \frac{\lambda}{6}\phi^3 = g_s\rho_s(\mathbf{r},t), \tag{5}$$

$$\frac{\partial^2 V_0}{\partial t^2} - \nabla^2 V_0 + m_v^2 V_0 = g_v j_0(\mathbf{r},t) + \frac{\partial}{\partial t}\left(\frac{\partial V_0}{\partial t} + \nabla\cdot\mathbf{V}\right), \tag{6}$$

$$\frac{\partial^2 V_i}{\partial t^2} - \nabla^2 V_i + m_v^2 V_i = g_v j_i(\mathbf{r},t) + \frac{\partial}{\partial x_i}\left(\frac{\partial V_0}{\partial t} + \nabla\cdot\mathbf{V}\right), \tag{7}$$

$$\frac{\partial^2 b_0}{\partial t^2} - \nabla^2 b_0 + m_\rho^2 b_0 = \frac{g_\rho}{2} j_{3,0}(\mathbf{r},t) + \frac{\partial}{\partial t}\left(\frac{\partial b_0}{\partial t} + \nabla\cdot\mathbf{b}\right), \tag{8}$$

$$\frac{\partial^2 b_i}{\partial t^2} - \nabla^2 b_i + m_\rho^2 b_i = \frac{g_\rho}{2} j_{3,i}(\mathbf{r},t) + \frac{\partial}{\partial x_i}\left(\frac{\partial b_0}{\partial t} + \nabla\cdot\mathbf{b}\right), \tag{9}$$

where the scalar density is

$$\rho_s(\mathbf{r},t) = 2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3} f_i(\mathbf{r},\mathbf{p},t)\frac{M^*}{\epsilon_i}, \tag{10}$$

with M^* denoting the effective baryon mass, $M^* = M - g_s\phi$. The components of the baryonic four-current density are

$$j_0(\mathbf{r},t) = 2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3} f_i(\mathbf{r},\mathbf{p},t) = \rho_p + \rho_n, \tag{11}$$

$$\mathbf{j}(\mathbf{r},t) = 2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3} f_i(\mathbf{r},\mathbf{p},t)\frac{\mathbf{p}-\boldsymbol{\mathcal{V}}_i}{\epsilon_i}, \tag{12}$$

where ρ_p, ρ_n are the proton and neutron densities, and the components of the isovector four-current density are

$$j_{3,0}(\mathbf{r},t) = 2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3} f_i(\mathbf{r},\mathbf{p},t)\tau_i = \rho_p - \rho_n, \tag{13}$$

$$\mathbf{j}_3(\mathbf{r},t) = 2\sum_{i=p,n}\int \frac{d^3p}{(2\pi)^3} f_i(\mathbf{r},\mathbf{p},t)\frac{\mathbf{p}-\boldsymbol{\mathcal{V}}_i}{\epsilon_i}\tau_i, \tag{14}$$

with $\epsilon_i = \sqrt{(\mathbf{p}-\boldsymbol{\mathcal{V}}_i)^2 + M^{*2}}$.

It can be easily seen that the four-current satisfies the continuity equation. From Eq. (2) we can write

$$\mathbf{j}(\mathbf{r},t) = 2\sum_{i=p,n}\int \frac{d^3p}{(2\pi)^3} f_i(\mathbf{r},\mathbf{p},t)\frac{\partial h_i}{\partial \mathbf{p}}$$

and therefore it is straightforward to show that

$$\frac{\partial j_0}{\partial t} + \nabla\cdot\mathbf{j} = 2\sum_{i=p,n}\int \frac{d^3p}{(2\pi)^3}\left(\frac{\partial f_i}{\partial t} + \{f_i, h_i\}\right).$$

Using Eq. (4) it follows that $\partial_\mu j^\mu = 0$. This continuity equation also gives [from Eqs. (6) and (7)] the following relation between the components of the vector field: $(\partial V_0)/(\partial t) + \nabla\cdot\mathbf{V} = 0$. In a similar way we have

$$\frac{\partial j_{3,0}}{\partial t} + \nabla\cdot\mathbf{j}_3 = 2\sum_{i=p,n}\int \frac{d^3p}{(2\pi)^3}\tau_i\left(\frac{\partial f_i}{\partial t} + \{f_i, h_i\}\right).$$

Using Eq. (4) it follows that $\partial_\mu j_3^\mu = 0$ or $\frac{\partial b_0}{\partial t} + \nabla\cdot\mathbf{b} = 0$.

At zero temperature and for particles obeying Fermi-Dirac statistics, the value of the distribution function is either 1 or 0, since the single particle state is either occupied by one particle or empty. The state which minimizes the energy of asymmetric nuclear matter is characterized by the Fermi momenta P_{Fi}, $i = p, n$, and is described by the distribution function

$$f_0(\mathbf{r},\mathbf{p}) = \begin{pmatrix}\Theta\left(P_{Fp}^2 - p^2\right) & 0\\ 0 & \Theta\left(P_{Fn}^2 - p^2\right)\end{pmatrix} \tag{15}$$

and by the constant mesonic fields which obey the following equations: $m_s^2\phi_0 + (\kappa/2)\phi_0^2 + (\lambda/6)\phi_0^3 = g_s\rho_s^{(0)}$, $m_v^2 V_0^{(0)} = g_v j_0^{(0)}$, $V_i^{(0)} = 0$, $m_\rho^2 b_0^{(0)} = (g_\rho/2) j_{3,0}^{(0)}$, $b_i^{(0)} = 0$.

Collective modes in the present approach correspond to small oscillations around the equilibrium state. These small deviations are described by the linearized equations of motion and, therefore, collective modes are given as solutions of the linearized equations of motion. To construct these equations let

$$\begin{aligned} &f = f_0 + \delta f, \quad \phi = \phi_0 + \delta\phi, \quad V_0 = V_0^{(0)} + \delta V_0,\\ &V_i = \delta V_i, \quad b_0 = b_0^{(0)} + \delta b_0, \quad b_i = \delta b_i. \end{aligned} \tag{16}$$

As in Ref. [12] we introduce a generating function

$$S(\mathbf{r},\mathbf{p},t) = \begin{pmatrix} S_p & 0\\ 0 & S_n\end{pmatrix}$$

defined in isospin space such that the variation of the distribution function is

$$\delta f_i = \{S_i, f_{0i}\} = -\{S_i, p^2\}\delta\left(P_{Fi}^2 - p^2\right). \tag{17}$$

In terms of this generating function, the linearized Vlasov equations for δf_i are equivalent to the following time evolution equations:

$$\frac{\partial S_i}{\partial t} + \{S_i, h_{0i}\} = \delta h_i = -g_s\delta\phi\frac{M^*}{\epsilon_0} + \delta\mathcal{V}_{0i} - \frac{\mathbf{p}\cdot\delta\boldsymbol{\mathcal{V}}_i}{\epsilon_0},\quad i = p,n. \tag{18}$$

where $\delta\mathcal{V}_{0i} = g_v\delta V_0 + \tau_i(g_\rho/2)\delta b_0$ and $\delta\boldsymbol{\mathcal{V}}_i = g_v\delta\mathbf{V} + \tau_i(g_\rho/2)\delta\mathbf{b}$, which has to be satisfied only for $p = P_{Fi}$. In Eq. (18)

$$h_{0i} = \sqrt{p^2 + M^{*2}} + \mathcal{V}_{0i}^{(0)} = \epsilon_0 + \mathcal{V}_{0i}^{(0)}. \tag{19}$$

The linearized equations of the fields read

$$\frac{\partial^2\delta\phi}{\partial t^2} - \nabla^2\delta\phi + \left(m_s^2 + \kappa\phi_0 + \frac{\lambda}{2}\phi_0^2\right)\delta\phi = g_s\delta\rho_s, \tag{20}$$

$$\frac{\partial^2\delta V_0}{\partial t^2} - \nabla^2\delta V_0 + m_v^2\delta V_0 = g_v\delta j_0, \tag{21}$$

$$\frac{\partial^2\delta V_i}{\partial t^2} - \nabla^2\delta V_i + m_v^2\delta V_i = g_v\delta j_i, \tag{22}$$

$$\frac{\partial^2\delta b_0}{\partial t^2} - \nabla^2\delta b_0 + m_\rho^2\delta b_0 = \frac{g_\rho}{2}\delta j_{3,0}, \tag{23}$$

$$\frac{\partial^2\delta b_i}{\partial t^2} - \nabla^2\delta b_i + m_\rho^2\delta b_i = \frac{g_\rho}{2}\delta j_{3,i}, \tag{24}$$

with

$$\begin{aligned}
\delta\rho_s &= 2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3}\delta f_i\frac{M^*}{\epsilon_0} - g_s\delta\phi\frac{d\rho_s^0}{dM^*},\\
M^* &= M - g_s\phi_0,\\
\delta j_0 &= 2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3}\delta f_i,\\
\delta\mathbf{j} &= 2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3}\delta f_i\frac{\mathbf{p}}{\epsilon_0}\\
&\quad - 2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3} f_{0i}\left(\frac{\delta\boldsymbol{\mathcal{V}}}{\epsilon_0} - \mathbf{p}\frac{\mathbf{p}\cdot\delta\boldsymbol{\mathcal{V}}}{\epsilon_0^3}\right),\\
\delta j_{3,0} &= 2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3}\tau_i\delta f_i,\\
\delta\mathbf{j}_3 &= 2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3}\tau_i\delta f_i\frac{\mathbf{p}}{\epsilon_0}\\
&\quad - 2\sum_{i=p,n}\int\frac{d^3p}{(2\pi)^3}\tau_i f_{0i}\left(\frac{\delta\boldsymbol{\mathcal{V}}}{\epsilon_0} - \mathbf{p}\frac{\mathbf{p}\cdot\delta\boldsymbol{\mathcal{V}}}{\epsilon_0^3}\right).
\end{aligned}$$

Of particular interest on account of their physical relevance are the longitudinal modes, with momentum $\mathbf{k}$ and frequency ω, described by the ansatz

$$\begin{pmatrix} S_p(\mathbf{r},\mathbf{p},t)\\ S_n(\mathbf{r},\mathbf{p},t)\\ \delta\phi\\ \delta V_0\\ \delta V_i\\ \delta b_0\\ \delta b_i\end{pmatrix} = \begin{pmatrix}\mathcal{S}_\omega^p(\cos\theta)\\ \mathcal{S}_\omega^n(\cos\theta)\\ \delta\phi_\omega\\ \delta V_\omega^0\\ \delta V_\omega^i\\ \delta b_\omega^0\\ \delta b_\omega^i\end{pmatrix} \mathrm{e}^{i(\omega t - \mathbf{k}\cdot\mathbf{r})},$$

where θ is the angle between $\mathbf{p}$ and $\mathbf{k}$. For these modes, we get $\delta V_\omega^x = \delta V_\omega^y = \delta b_\omega^x = \delta b_\omega^y = 0$. Calling $\delta V_\omega^z = \delta V_\omega$ and $\delta b_\omega^z = \delta b_\omega$, the equations of motion become

$$i(\omega - \omega_{0i}x)\mathcal{S}_\omega^i(x) = -g_s \frac{M^*}{\epsilon_{Fi}}\delta\phi_\omega + \delta\mathcal{V}_\omega^{0i} - \frac{P_{Fi}}{\epsilon_{Fi}} x\delta\mathcal{V}_\omega^i\,, \quad i = p, n, \tag{25}$$

$$\left(\omega^2 - k^2 - m_{s,\text{eff}}^2\right)\delta\phi_\omega = -\frac{2ig_s}{(2\pi)^2}\sum_{i=p,n}\omega_{0i}P_{Fi}M^* \times \int_{-1}^1 dx\, x\mathcal{S}_\omega^i(x), \tag{26}$$

$$\left(\omega^2 - k^2 - m_v^2\right)\delta V_\omega^0 = -\frac{2ig_v}{(2\pi)^2}\sum_{i=p,n}\omega_{0i}P_{Fi}\epsilon_{Fi}\int_{-1}^1 dx\, x\mathcal{S}_\omega^i(x), \tag{27}$$

$$\left(\omega^2 - k^2 - m_v^2 - g_v^2\left(\Omega_p^2 + \Omega_n^2\right)\right)\delta V_\omega - g_v\frac{g_\rho}{2}\left(\Omega_p^2 - \Omega_n^2\right)\delta b_\omega = -\frac{2ig_v}{(2\pi)^2}\sum_{i=p,n}\omega_{0i}P_{Fi}^2\int_{-1}^1 dx\, x^2\mathcal{S}_\omega^i(x), \tag{28}$$

$$\left(\omega^2 - k^2 - m_\rho^2\right)\delta b_\omega^0 = -\frac{2ig_\rho}{2(2\pi)^2}\sum_{i=p,n}\tau_i\omega_{0i}P_{Fi}\epsilon_{Fi} \times \int_{-1}^1 dx\, x\mathcal{S}_\omega^i(x), \tag{29}$$

$$\left(\omega^2 - k^2 - m_\rho^2 - \left(\frac{g_\rho}{2}\right)^2\left(\Omega_p^2 + \Omega_n^2\right)\right)\delta b_\omega - g_v\frac{g_\rho}{2}\left(\Omega_p^2 - \Omega_n^2\right)\delta V_\omega = -\frac{2ig_\rho}{2(2\pi)^2}\sum_{i=p,n}\tau_i\omega_{0i}P_{Fi}^2 \times \int_{-1}^1 dx x^2\mathcal{S}_\omega^i(x), \tag{30}$$

where $\omega_{0i} = kP_{Fi}/\epsilon_{Fi}$, $\epsilon_{Fi} = \sqrt{P_{Fi}^2 + M^{*2}}$,

$$m_{s,\text{eff}}^2 = m_s^2 + \kappa\phi_0 + \frac{\lambda}{2}\phi_0^2 + g_s^2\frac{d\rho_s^0}{dM^*}, \tag{31}$$

and

$$\Omega_i^2 = \frac{2}{(2\pi)^3}\int d^3p f_{0i}\left(\frac{1}{\epsilon_0} - \frac{p^2\cos^2\theta}{\epsilon_0^3}\right) = \frac{\rho_i}{\epsilon_{Fi}}.$$

If we integrate x times Eq. (25) from -1 to 1 we get

$$\omega\int_{-1}^1 dx\, x\mathcal{S}_\omega^i(x) - \omega_{0i}\int_{-1}^1 dx\, x^2\mathcal{S}_\omega^i(x) = i\frac{2}{3}\frac{P_{Fi}}{\epsilon_{Fi}}\delta\mathcal{V}^i.$$

Using the above relations in Eqs. (27), (28) and (29), (30) we obtain

$$\omega\delta V_\omega^0 = k\delta V_\omega; \qquad \omega\delta b_\omega^0 = k\delta b_\omega, \tag{32}$$

which are equivalent to the continuity equations. Substituting the relations obtained in Eq. (32) into Eqs. (28) and (30), they become respectively identical to Eqs. (27) and (29).

III. SOLUTIONS FOR THE EIGENMODES AND THE DISPERSION RELATION

The solutions of Eqs. (25)–(30) form a complete set of eigenmodes which may be used to construct a general solution for an arbitrary longitudinal perturbation [21–23]. Substituting the set of equations (26), (27), (29), (32) into Eq. (25) we get

$$A_{\omega p}\left[1 - \left(-C_s^{\text{pp}} + C_v^{\text{pp}} + C_\rho^{\text{pp}}\right)L(s_p)\right] - A_{\omega n}\left(-C_s^{\text{pn}} + C_v^{\text{pn}} - C_\rho^{\text{pn}}\right)L(s_p) = 0, \tag{33}$$

$$-A_{\omega p}\left(-C_s^{\text{np}} + C_v^{\text{np}} - C_\rho^{\text{np}}\right)L(s_n) + A_{\omega n}\left[1 - \left(-C_s^{\text{nn}} + C_v^{\text{nn}} + C_\rho^{\text{nn}}\right)L(s_n)\right] = 0, \tag{34}$$

with $A_{\omega i} = \int_{-1}^1 xS_{\omega i}(x)\,dx$ and $L(s_i), s_i = \omega/\omega_{oi}$, related with the Lindhard function Φ defined in

$$L(s_i) = 2\Phi(s_i) = 2 - s_i\ln\left(\frac{s_i+1}{s_i-1}\right). \tag{35}$$

We also have

$$C_s^{ij} = \frac{1}{2\pi^2}\frac{M^{*2}g_s^2}{\omega^2 - \omega_s^2}\frac{1}{P_{Fi}}P_{Fj}V_{Fj},$$
$$C_v^{ij} = \frac{1}{2\pi^2}\frac{g_v^2}{\omega^2 - \omega_v^2}\left(1 - \frac{\omega^2}{k^2}\right)\frac{P_{Fj}^2}{V_{Fi}},$$
$$C_\rho^{ij} = \frac{1}{2\pi^2}\frac{g_\rho^2}{4\left(\omega^2 - \omega_\rho^2\right)}\left(1 - \frac{\omega^2}{k^2}\right)\frac{P_{Fj}^2}{V_{Fi}},$$

where

$$V_{F_i} = \frac{P_{F_i}}{\epsilon_{F_i}} = \omega_{0i}/k \tag{36}$$

is the nucleon Fermi velocity and

$$\omega_s^2 = k^2 + m_{s,\text{eff}}^2, \quad \omega_v^2 = k^2 + m_v^2, \quad \omega_\rho^2 = k^2 + m_\rho^2. \tag{37}$$

From Eqs. (33) and (34) we get the following dispersion relation:

$$1 + L(s_p)F^{\text{pp}} + L(s_n)F^{\text{nn}} + L(s_p)L(s_n) \times \left(F^{\text{pp}}F^{\text{nn}} - F^{\text{pn}}F^{\text{np}}\right) = 0 \tag{38}$$

with $F^{ij} = (C_s^{ij} - C_v^{ij} - \tau_i\tau_j C_\rho^{ij})$. The density fluctuations are given by

$$\delta\rho_i = \frac{3}{2}\frac{k}{P_{Fi}}\rho_{0i}A_{\omega i}.$$

Is it important to stress that the inclusion of the ρ mesons is necessary to study isovector modes even for symmetric nuclear matter.

As mentioned in the introduction, the instabilities of the system are a subject of interest. At low densities, corresponding to a negative value of the compressibility, the system presents unstable modes characterized by an imaginary frequency. In order to obtain these modes, one has to replace s by $i\beta$ in Eq. (35). In this case, the Lindhard functions become

$$L(i\beta) = 1 - \beta\tan^{-1}(1/\beta). \tag{39}$$

IV. THE LANDAU-FERMI LIQUID THEORY

Next we outline the main expressions for the calculation of the collective modes within a Landau-Fermi liquid theory. In this approximation the meson fields are kept constant and equal to their values in equilibrium, i.e., a variation of the source will not give rise to a fluctuation of the meson fields. This is a good approximation for small momentum transfers. The following expressions differ from Ref. [16] in some cases because we have introduced the nonlinear terms in the QHD Lagrangian density. For the nonlinear Walecka model the only Landau parameters which are different from zero are

$$f_0^{ii'} = \frac{g_v^2}{m_v^2} + \frac{g_\rho^2}{4m_\rho^2}\tau_i\tau_{i'} - \frac{g_s^2}{m_{s,\mathrm{eff}}^2}\frac{M^{*2}}{\epsilon_{F_i}\epsilon_{F_{i'}}},$$

$$\begin{aligned} f_1^{ii'} &= -\frac{g_v^2}{m_v^2}\frac{P_{F_i}}{\epsilon_{F_i}}\frac{P_{F_{i'}}}{\epsilon_{F_{i'}}}\left(1-\frac{\frac{g_\rho^2}{4m_\rho^2}\tau_{i'}B}{1+\frac{g_\rho^2}{4m_\rho^2}A}\right) \\ &\quad\times\left(1+\frac{g_v^2}{m_v^2}A-\frac{\frac{g_v^2}{m_v^2}\frac{g_\rho^2}{4m_\rho^2}B^2}{1+\frac{g_\rho^2}{4m_\rho^2}A}\right)^{-1} \\ &\quad-\frac{g_\rho^2}{4m_\rho^2}\tau_i\tau_{i'}\frac{P_{F_i}}{\epsilon_{F_i}}\frac{P_{F_{i'}}}{\epsilon_{F_{i'}}}\left(1-\frac{\frac{g_v^2}{m_v^2}\tau_{i'}B}{1+\frac{g_v^2}{m_v^2}A}\right) \\ &\quad\times\left(1+\frac{g_\rho^2}{4m_\rho^2}A-\frac{\frac{g_v^2}{m_v^2}\frac{g_\rho^2}{4m_\rho^2}B^2}{1+\frac{g_v^2}{m_v^2}A}\right)^{-1}, \end{aligned} \tag{40}$$

where we have defined $A = \Omega_p^2 + \Omega_n^2$ and $B = \Omega_p^2 - \Omega_n^2$.

As in Sec. II, the collective modes are obtained when small perturbations around the equilibrium density are introduced. Let $n_i(\mathbf{r}, t)$ be the distribution function at time t:

$$n_i(\mathbf{r},t) = n_i^{(0)} + \delta n_i(\mathbf{r},t), \quad i = p, n,$$

where $n_i^{(0)} = f_{0i}(\mathbf{r},\mathbf{p})$, Eq. (15), and

$$\delta n_i(\mathbf{r},t) = \delta n_{\mathbf{p}_i}(\mathbf{r},t) = \delta n_{\mathbf{p}_i}(\mathbf{k},\omega)\,\mathrm{e}^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}.$$

Assuming that

$$\delta n_{\mathbf{p}_i}(\mathbf{k},\omega) = u_i(\theta)\delta\big(P_{F_i} - p_i\big),$$

with

$$u_i(\theta) = \sum_l u_l^{(i)}\mathrm{P}_l(\cos\theta), \tag{41}$$

and θ the angle between $\mathbf{p}$ and $\mathbf{k}$ one can show that the collective modes come from the solution of a system formed by four equations, i.e.,

$$\frac{u_l^{(i)}}{2l+1} = -\sum_{i',l'}\frac{F_{l'}^{ii'}}{2l'+1}D_{ll'}(s_i)u_{l'}^{(i')}, \tag{42}$$

where

$$F_l^{ii'} = \left(\frac{P_{F_{i'}}}{P_{F_i}}\right)^2 N_{F_i} f_l^{ii'}.$$

As usual, the density of states in the Fermi surface is given by $N_{F_i} = P_{F_i}\epsilon_{F_i}/\pi^2$.

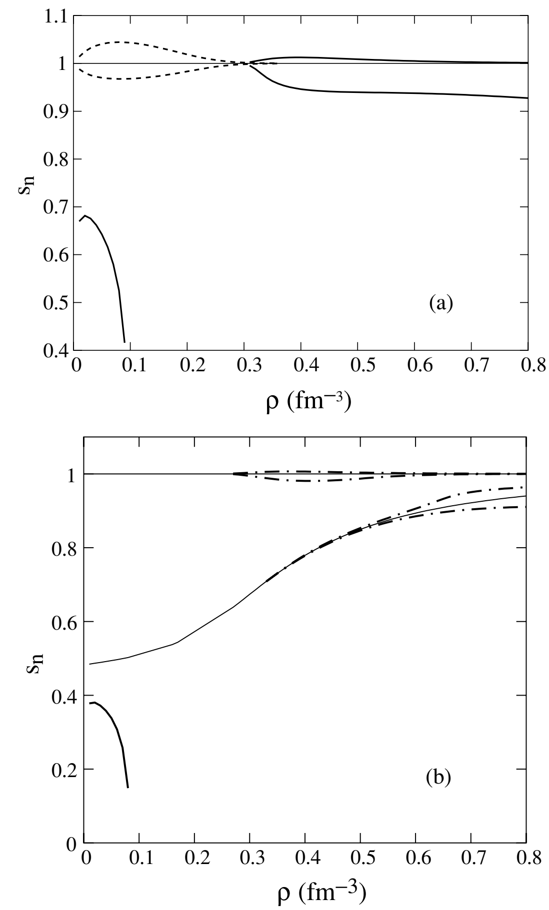

FIG. 1. Nuclear collective modes $s_n = v_s/V_{Fn}$ for $k = 10$ MeV and (a) $y_p = 0.5$ (b) $y_p = 0.1$. Solid lines represent isoscalar modes, dashed lines stand for isovector modes, and dot-dashed lines modes with mixed character.

We introduce the D functions, related with $L(s_i)$ given in Eq. (35), defined as

$$\begin{aligned} \Phi(s_i) \equiv D_{00}(s_i) &= \frac{L(s_i)}{2}, \\ D_{01}(s_i) = D_{10}(s_i) &= s_i\Phi(s_i), \\ D_{11}(s_i) &= s_i^2\Phi(s_i) + \frac{1}{3}. \end{aligned}$$

Once the above equations are substituted into Eq. (42), the system of equations for the nucleon i reads

$$\begin{aligned} u_0^{(p)} &= \frac{1}{3s_p}\left(1+\frac{1}{3}F_1^{\mathrm{pp}}\right)u_1^{(p)} + \frac{1}{9s_p}F_1^{\mathrm{pn}}u_1^{(n)} \\ &= G_{\mathrm{pp}}u_1^{(p)} + G_{\mathrm{pn}}u_1^{(n)} \\ u_0^{(n)} &= \frac{1}{9s_n}F_1^{\mathrm{np}}u_1^{(p)} + \frac{1}{3s_n}\left(1+\frac{1}{3}F_1^{\mathrm{nn}}\right)u_1^{(n)} \\ &= G_{\mathrm{np}}u_1^{(p)} + G_{\mathrm{nn}}u_1^{(n)}, \end{aligned} \tag{43}$$

and after being rearranged, the following system is obtained:

$$\begin{aligned} C_{\mathrm{pp}}u_1^{(p)} + C_{\mathrm{pn}}u_1^{(n)} &= 0, \\ C_{\mathrm{np}}u_1^{(p)} + C_{\mathrm{nn}}u_1^{(n)} &= 0, \end{aligned} \tag{44}$$

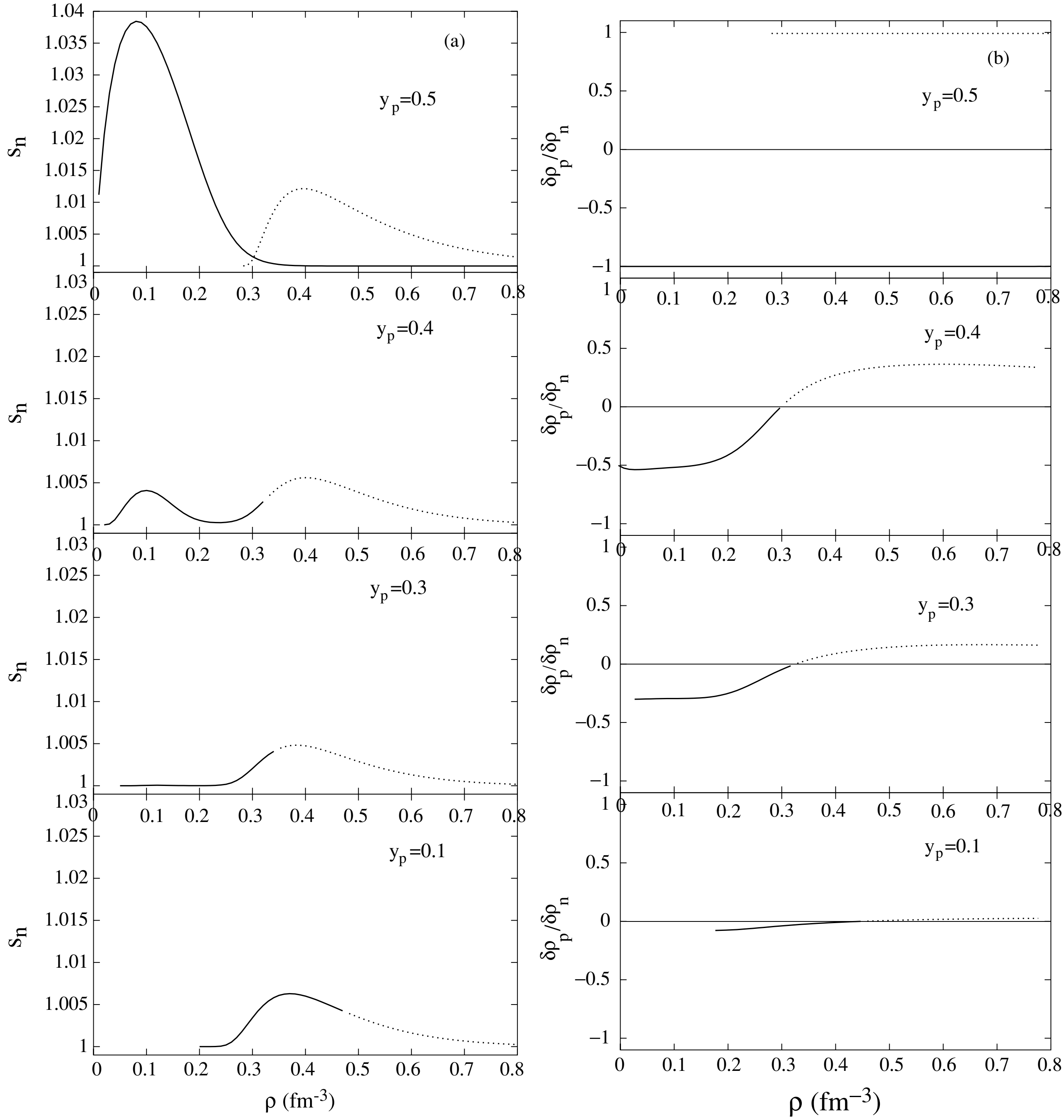

FIG. 2. (a) Collective modes and (b) ratio between the proton and neutron amplitudes obtained for $k = 200$ MeV in terms of the baryonic density for symmetric nuclear matter (top figure) with proton fraction decreasing up to very large asymmetric matter (bottom figure). Solid lines represent isovector modes and dashed lines stand for isoscalar modes.

where

$$C_{\rm pp} = \frac{1}{3s_p}\left[\left(1+\frac{1}{3}F_1^{\rm pp}\right) + \left(F_0^{\rm pp}\left(1+\frac{1}{3}F_1^{\rm pp}\right) + \frac{1}{3}\frac{s_p}{s_n}F_0^{\rm pn}F_1^{\rm np} + s_p^2F_1^{\rm pp}\right)\Phi_p\right],$$

$$C_{\rm pn} = \frac{1}{3s_p}\left[\frac{F_1^{\rm pn}}{3} + \left(\frac{s_p}{s_n}F_0^{\rm pn}\left(1+\frac{1}{3}F_1^{\rm nn}\right) + \frac{1}{3}F_0^{\rm pp}F_1^{\rm pn} + s_p^2F_1^{\rm pn}\right)\Phi_p\right],$$

$$C_{\rm np} = \frac{1}{3s_n}\left[\frac{F_1^{\rm np}}{3} + \left(\frac{s_n}{s_p}F_0^{\rm np}\left(1+\frac{1}{3}F_1^{\rm pp}\right) + \frac{1}{3}F_0^{\rm nn}F_1^{\rm np} + s_n^2F_1^{\rm np}\right)\Phi_n\right],$$

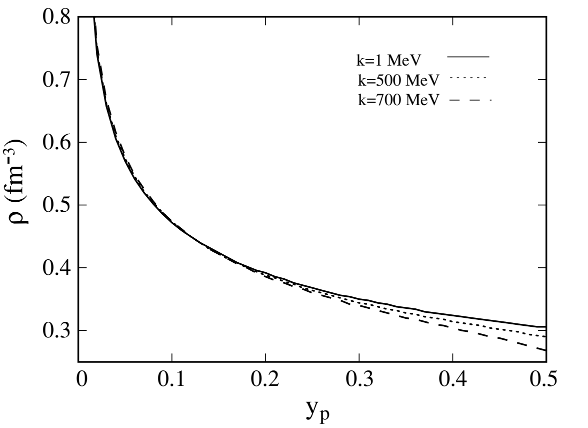

FIG. 3. Density dependence of the crossing of isoscalar and isovector modes as function of y_p for different values of k.

$$C_{\rm nn} = \frac{1}{3s_n}\left[\left(1+\frac{1}{3}F_1^{\rm nn}\right) + \left(F_0^{\rm nn}\left(1+\frac{1}{3}F_1^{\rm nn}\right) + \frac{1}{3}\frac{s_n}{s_p}F_0^{\rm np}F_1^{\rm pn} + s_n^2F_1^{\rm nn}\right)\Phi_n\right]. \tag{45}$$

The solutions of the system (44) are obtained imposing the condition:

$$C_{\rm pp}C_{\rm nn} - C_{\rm pn}C_{\rm np} = 0,$$

the only variables being s_i, $(i = p, n)$, which are related to the zero sound velocity, $s_i = v_s/V_{Fi}$, usually defined as $v_s = \omega/k$.

In order to distinguish between the isoscalar and isovector collective modes, we calculate the ratio between the angle integrated proton (u_p) and neutron (u_n) amplitudes from Eqs. (41), (43), (44), (45): these quantities are referred to as $\delta\rho_p/\delta\rho_n$ in Ref. [8].

In the sequel we consider that, when u_p/u_n is positive ($\delta\rho_p/\delta\rho_n > 0$), the corresponding modes are isoscalar while when this ratio is negative ($\delta\rho_p/\delta\rho_n < 0$), isovector modes arise [24]. These definitions coincide with the fact that when protons and neutrons move in phase the modes are isoscalar. When they move out of phase, the modes are isovector.

V. NUMERICAL RESULTS AND DISCUSSIONS

From the solutions of the dispersion relation given in Eq. (38), we have obtained nuclear and mesonic collective modes which we plot and discuss in this section. At low momenta transfer k the two sectors have quite different energy scales. From Eqs. (26)–(30) it is seen that the coupling of the mesonic sector to the nucleonic is suppressed by a factor k/m_i, (being m_i the mass of meson $i = \sigma, \omega, \rho$). We first discuss the nucleonic sector.

In Fig. 1 we show the longitudinal isoscalar and isovector modes for (a) symmetric nuclear matter, $y_p = \rho_p/\rho = 0.5$, and (b) $y_p = 0.1$ for the momentum transfer equal to $k =$ 10 MeV. Only the modes which lie above the Fermi velocity of neutrons are collective modes which do not suffer Landau damping. In Fig. 1(a) there are a couple of isovector modes between 0 and 0.35 fm^{-3}, one above the neutron Fermi velocity and one Landau damped below this velocity. In a similar way there are a couple of isoscalar modes above 0.3 fm^{-3}: only the one that lies above the neutron Fermi velocity is not damped. There is a fifth isoscalar mode at low densities and below the neutron Fermi velocity which comes alone. This is a sign of instability. In fact this mode appears in the region of spinodal instability. This same mode is also present in Fig. 1(b) at low densities and only disappears for almost pure neutron matter. In Fig. 1(b) we also represent the Fermi velocity of neutrons (solid thin line at $s_n = 1$) and protons (solid thin line). Above and below each of these two lines there are two modes. From these four modes the only one that is not Landau damped lies above the neutron Fermi velocity. In the sequel we discuss the stable (not damped) collective modes, the mesonic modes, and comment the appearance of unstable modes in the region of spinodal instability.

In order to study the dependence of the collective modes on the isospin asymmetry, we show in Fig. 2, for $k =$ 200 MeV and decreasing proton fractions, the sound velocity of the collective modes and their corresponding ratio of proton

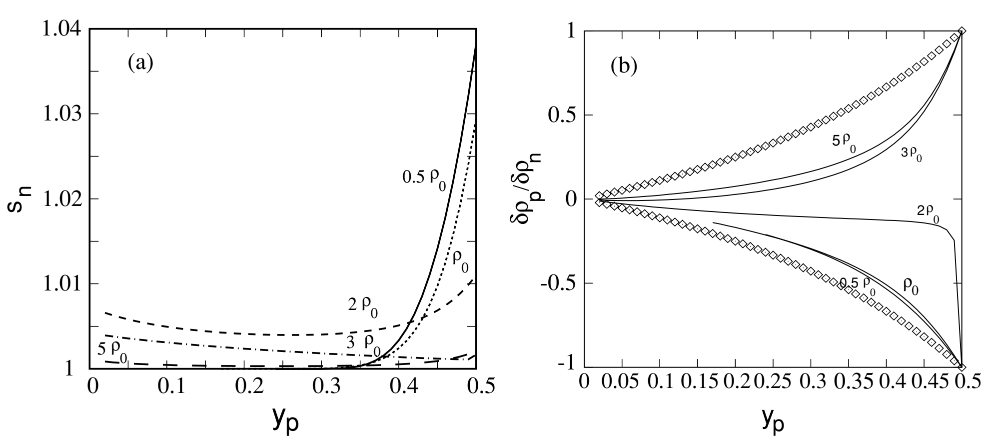

FIG. 4. (a) Collective modes in function of the isospin asymmetry for different densities and (b) corresponding proton to neutron transition density ratio. In this case $k = 200$ MeV. The diamonds represent $\pm\rho_p/\rho_n$.

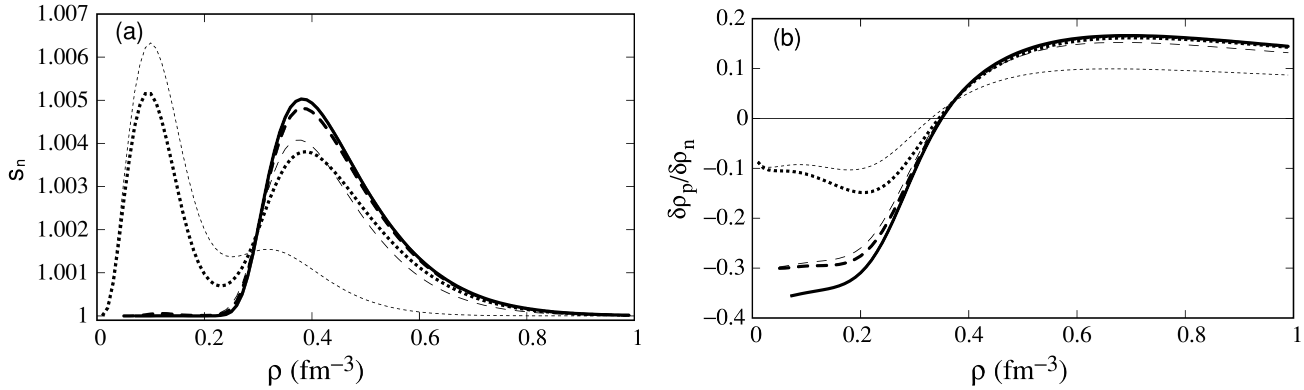

FIG. 5. (a) Collective modes in function of density for $y_p = 0.3$ and different k values and (b) their corresponding proton to neutron transition density ratios. The dotted lines stand for $k = 500$ MeV, the dashed lines stand for $k = 200$ MeV, and the solid lines for $k = 1$ MeV. The thin lines represent the corresponding static quantities.

to neutron transition densities. Comparing both sides of the figure one sees that for symmetric matter the isoscalar and isovector modes are completely separated. For $y_p < 0.5$ only one mode exists and it changes character from isovector to isoscalar at the crossing density, already referred to in Ref. [8]. The crossing density increases with decreasing proton fraction as can be clearly seen in Fig. 3.

In order to understand this behavior, we use Eqs. (33) and (34) to write the ratio $\delta\rho_p/\delta\rho_n$ as

$$\delta\rho_p/\delta\rho_n = -\frac{P_{F_n}\rho_p}{P_{F_p}\rho_n}\cdot\frac{F^{\rm pn}L(s_p)}{1+F^{\rm pp}L(s_p)}. \qquad (46)$$

Since in the above expression the denominator is always positive and the Lindhard function $L(s)$ is negative for $s > 1$, we immediately conclude that the crossing density is obtained by calculating the baryon density for which $F^{\rm pn}$ vanishes. For low momentum transfer ($k < 100$ MeV), $F^{\rm pn}$ is given by

$$F^{\rm pn} = -\frac{1}{2\pi^2}\frac{P_{F_n}V_{F_n}}{P_{F_p}}\left[\frac{M^{*2}g_s^2}{m_{s,\rm eff}^2} - \left(\frac{g_v^2}{m_v^2} - \frac{g_\rho^2}{4m_\rho^2}\right)\right.$$
$$\left.\times\left(1-\frac{\omega^2}{k^2}\right)\left(\frac{P_{F_n}P_{F_p}}{V_{F_n}V_{F_p}}\right)\right]. \qquad (47)$$

The character of the collective mode is determined by the signal of $F^{\rm pn}$, i.e., if $F^{\rm pn}$ is positive (negative) the mode has an isoscalar-(isovector)like character. So, the behavior of the collective mode depends on the balance between the scalar and the vector terms. For low baryon density the scalar term is dominant, and therefore $F^{\rm pn}$ is negative, hence the corresponding collective mode has an isovector like character. For densities larger than the crossing density, the nucleon effective

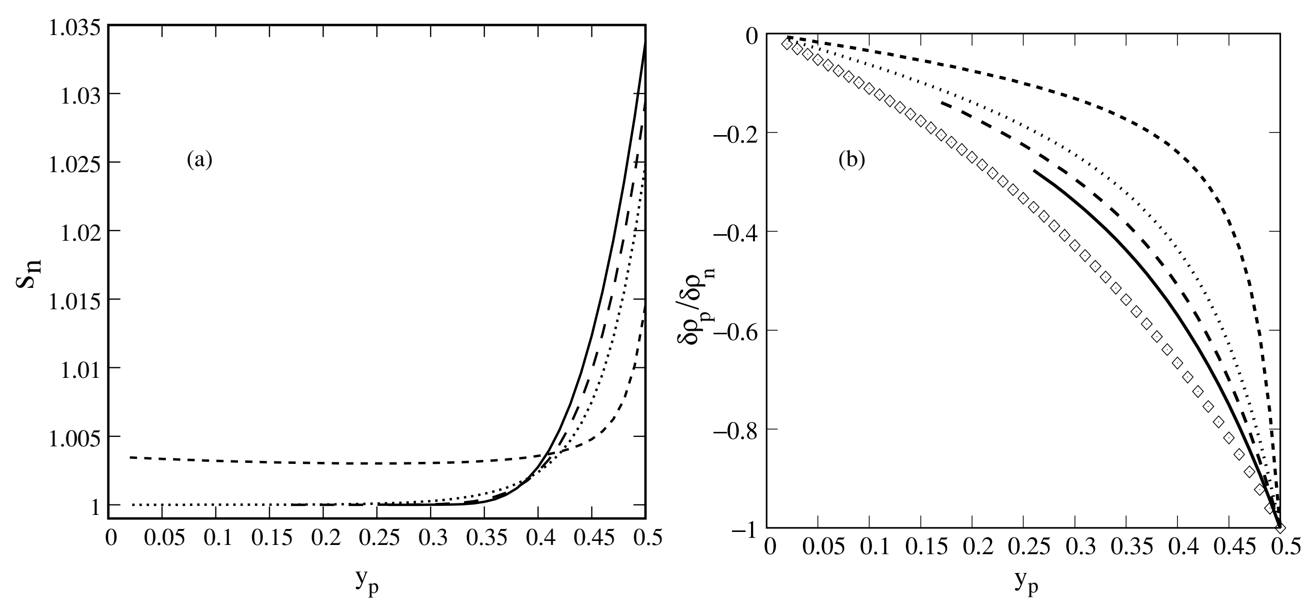

FIG. 6. (a) Isovector modes in function of the proton fraction for different k values and (b) their corresponding proton to neutron transition density ratio. From right top to bottom in figure (a) and from bottom to top in figure (b), the curves are drawn for $k = 1$ MeV (solid line), $k = 200$ MeV (dashed line), $k = 300$ MeV (dotted line), and $k = 500$ MeV (short-dashed line). The diamonds represent ρ_p/ρ_n.

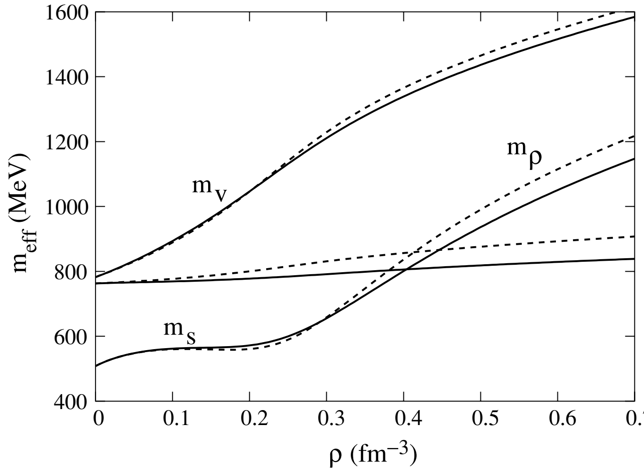

FIG. 7. Mesonic effective masses versus the density for symmetric nuclear matter (short-dashed line) and a system with high isospin asymmetry, i.e., $y_p = 0.1$ (solid line).

mass $M^\star$ goes to zero and the effective scalar meson mass $m_{s,\rm eff}$ increases (see Fig. 7 and discussion below). This makes the second term of Eq. (47) dominant, $F^{\rm pn}$ becomes positive, and the corresponding mode becomes isoscalar. The isospin asymmetry dependence of the modes can be understood noticing that, for fixed baryon density, the nucleon ($M^\star$) and meson ($m_{s,\rm eff}$) effective masses have opposite behavior with increasing asymmetry (decreasing y_p). While $M^\star$ increases with the isospin asymmetry conversely $m_{s,\rm eff}$ decreases, so the ratio $M^{\star 2}/m_{s,\rm eff}{}^2$ becomes always larger if the isospin asymmetry increases. Then for large isospin asymmetries, it is necessary for a larger baryon density to compensate the scalar term and make $F^{\rm pn}$ change sign. Therefore, we conclude that the larger the isospin asymmetry the larger the crossing density as seen in Fig. 3.

From Fig. 2 we also notice that as isospin asymmetry increases the collective modes start to appear at higher and higher densities and the sound velocity of the mode at lower densities decreases with the increase of isospin asymmetry and disappears for $y_p < 0.3$. The appearance of the collective mode at higher and higher densities can be understood from the dispersion relation given in Eq. (38). Using similar arguments as above, one can easily show that all coefficients F^{ij} have a similar behavior, i.e., they are negative for low density and for larger isospin asymmetries they change sign at higher densities. The numerical analysis of the term, $F^{\rm pp}F^{\rm nn} - F^{\rm pn}F^{\rm np}$, appearing in Eq. (38), shows that this term is negative at low densities, with a modulus smaller than 0.5, which decreases when the density grows and is almost independent of the isospin asymmetry. So, for low density the dispersion relation has no solution since the last term is greater than 1 and the other terms are positive. Its behavior is essentially determined by the linear terms in the Lindhard function, namely by $F^{\rm pp}$ and $F^{\rm nn}$. We conclude that for larger isospin asymmetries the dispersion relation has solution only for larger densities as seen in Fig. 2.

In Fig. 4 we display the collective modes in function of the isospin asymmetry for different densities and corresponding ratio of proton to neutron density fluctuations for $k = 200$ MeV. These figures summarize Fig. 2: for the two lower densities, $0.5\rho_0$ and ρ_0, the mode, with an isovector character, disappears at $y_p \sim 0.3$; the higher values of the density, $3\rho_0$ and $5\rho_0$ belong to the second branch of the collective mode represented in Fig. 2, mainly with isoscalar character; the value $2\rho_0$ is at the borderline of the two branchs of the collective mode, still with an isovector like character but with a very small value for the ratio of proton to neutron transition densities. A similar behavior was discussed in Ref. [8].

In Fig. 5 we study the momentum transfer dependence of the collective modes and retardation effects. The lower density branch with isovector character is more sensitive to the wavelength of the excitation. This effect is also seen from Fig. 5(b): as the momentum transfer increases the proton to neutron transition density ratio decreases in magnitude for the isovector modes and remains almost unaffected for the isoscalar ones. Notice also that the maxima decrease with the increase of the momentum transfer, in the isoscalar branch and the opposite occurs in the isovector branch. Within the Landau-Fermi liquid approximation [8] the longitudinal waves do not depend on the k and ω separately, but just on their ratio. The Landau-Fermi liquid theory results coincide with the curves obtained at low momentum transfers ($k = 1$ MeV in Fig. 5).

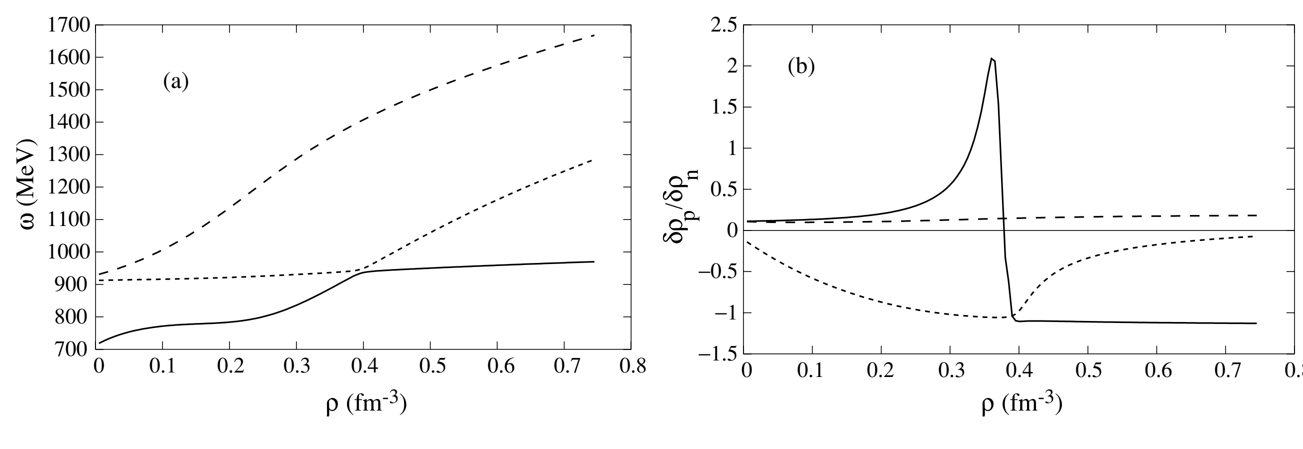

FIG. 8. (a) Mesonic modes for $y_p = 0.1$ and $k = 500$ MeV and (b) their corresponding proton to neutron transition density ratios.

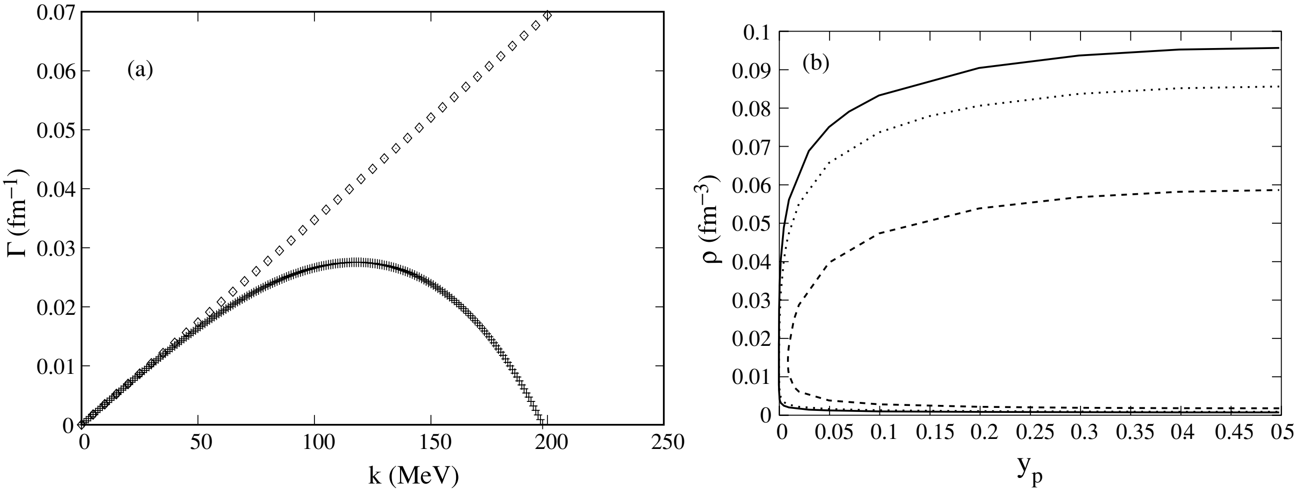

FIG. 9. Unstable sound modes: (a) growth rate Γ as a function of the wave number k for $y_p = 0.5$ and $\rho = 0.4\rho_0$ obtained with the Landau (diamonds) and the Vlasov (solid line) theories. (b) Spinodal curves for $k = 10$ MeV (solid line), $k = 100$ MeV (dotted line), and $k = 200$ MeV (dashed line).

In order to evaluate the importance of the retardation effects in the Vlasov formalism we have included in Figs. 5(a) and (b) the corresponding curves in the static limit. For $k = 1$ MeV there is essentially no effect. For $k = 200$ and 500 MeV there are finite effects on the sound velocity but smaller than 1%. More important effects are felt on the lowest density at which the collective mode occurs, namely at $\rho = 0.066, 0.047, 0.011$ fm^{-3}, respectively, for $k = 1, 200, 500$ MeV/c, or on the crossing density. At $k = 500$ MeV, the crossing density is reduced by 5% and at larger momentum the effects are larger as can be seen in Fig. 3.

In Fig. 6 we analyze in greater detail the momentum dependence of the collective modes. In particular we show both the sound velocity and the proton-neutron transition density ratio for the isovectorlike branch at the saturation density versus proton fraction, and for different momentum transfers. For small asymmetries, $y_p > 0.4$, the sound velocity reduces with an increase of k and a decrease of y_p. For $y_p < 0.4$ the behavior is similar for all cases with $k < 300$ MeV, the sound velocity approaches the neutron Fermi velocity. The only exception to this behavior is the $k = 500$ MeV curve, where the sound velocity approaches the value 1.003 v_{Fn}. However, although the dependence of v_s/v_{Fn} with k is small, the dependence of $\delta\rho_p/\delta\rho_n$ is not negligible. In Fig. 6(b) it is seen that these isovector modes exist as k increases at larger asymmetries and a higher k value implies a smaller proton to neutron density transition ratio, that is the fraction of neutrons participating in the mode is larger.

In Fig. 7 we plot, for $y_p = 0.5$ and 0.1, the effective masses of the mesons given by Eq. (31) and the following equations:

$$m^{\pm}_{v\rho,\mathrm{eff}} = \sqrt{\lambda_\pm},$$
$$\lambda_\pm = \frac{1}{2}\left[\mathrm{Tr}\,(M_{v\rho}) \pm \sqrt{\mathrm{Tr}\,(M_{v\rho})^2 - 4\,\mathrm{Det}\,(M_{v\rho})}\right], \quad (48)$$

with

$$M_{v\rho} = \begin{bmatrix} m_v^2 + g_v^2(\Omega_p^2 + \Omega_n^2) & g_v \frac{g_\rho}{2}(\Omega_p^2 - \Omega_n^2) \\ g_v \frac{g_\rho}{2}(\Omega_p^2 - \Omega_n^2) & m_\rho^2 + \left(\frac{g_\rho}{2}\right)^2(\Omega_p^2 + \Omega_n^2) \end{bmatrix},$$

In the parametrization we have used $m_s = 508.194$ MeV, $m_v = 782.501$ MeV, and $m_\rho = 763.501$ MeV, which are the values of the modes for zero density. The increase of the mesonic modes with density has already been predicted in Ref. [25] and is common to all approaches which only take into account the Fermi sea effects. Effects coming from the polarization of the Dirac sea were shown to be more important than Fermi sea effects, and to affect the meson masses in the opposite way, namely, giving rise to a reduction of the meson masses in the medium [26]. This behavior is related to the chiral symmetry restoration and will be measured in experiments with the spectrometer HADES at GSI.

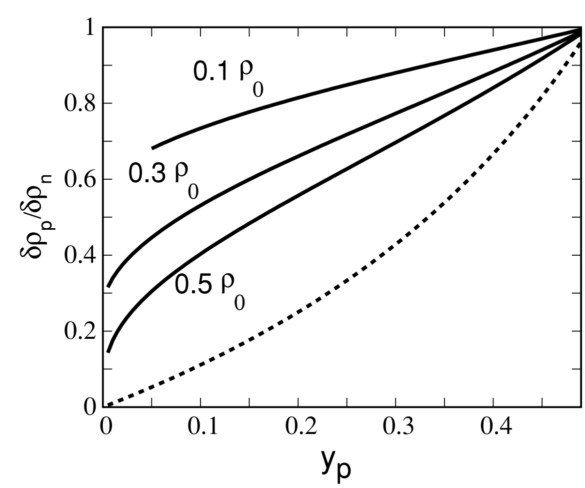

FIG. 10. Density transition ratio as a function of the isospin asymmetry for different densities. The dashed line represents the ratio ρ_p/ρ_n.

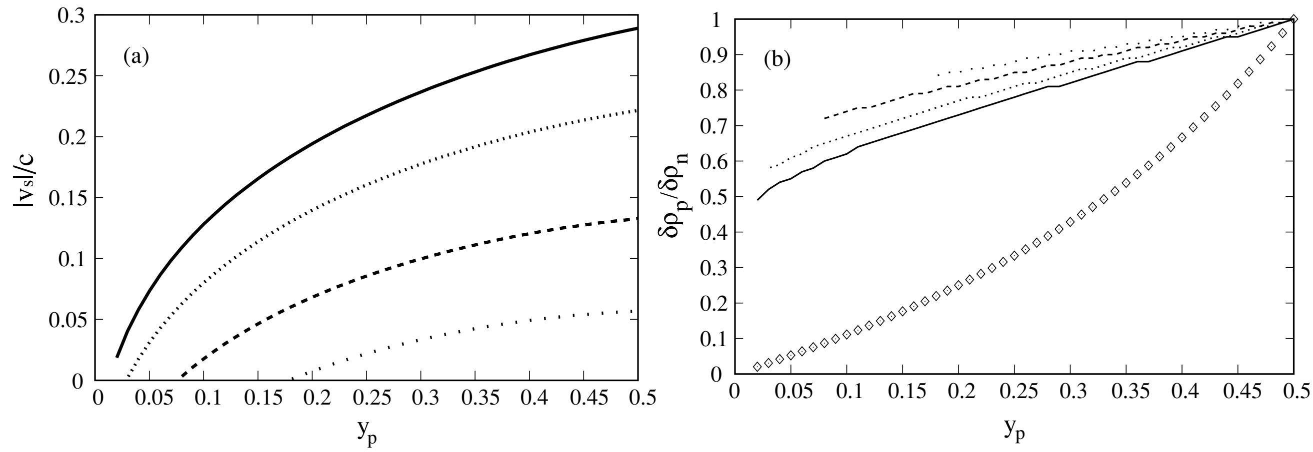

FIG. 11. Unstable modes for $\rho = \rho_0/2$: (a) the modulus of the sound velocity, $|v_s|$, and (b) the proton to neutron transition density ratio as a function of the proton fraction for $k = 1$ MeV (solid line), $k = 100$ MeV (dotted line), $k = 150$ MeV (short-dashed line), and $k = 180$ MeV (long spaced dotted line). In figure(b) the diamonds show the ratio ρ_p/ρ_n.

The dependence of mesonic modes on the isospin asymmetry of the system is small. The solutions of the dispersion relation (38) corresponding to the mesonic modes are essentially given by $\sqrt{k^2 + m^2_{i,\text{eff}}}$ where $i = s, v\rho+, v\rho-$. However, it is clear from Fig. 8(a) that at the crossing of the σ- and ρ-meson masses there is a level repulsion and the modes exchange character. This behavior is clearly seen in Fig. 8(b) where we have plotted the ratio of the density fluctuations corresponding to these modes, for $k = 500$ MeV.

The unstable modes are discussed next. The growth rate $\Gamma = \text{Im}(\omega)$ is plotted in Fig. 9(a) for $\rho = 0.4\rho_0$ as a function of the wave number k. The maximum growth rate, corresponding to the most amplified modes, occurs $\sim 0.6\, k_{\text{max}}$. For comparison, the growth rate in the framework of the Landau-Fermi theory is also shown. In this case the behavior is linear and not realistic. It coincides with the results obtained from the Vlasov equation formalism for small wave numbers, as expected. For higher k values the Vlasov formalism yields a reasonable growth rate which results from the inclusion of the mesonic fluctuations.

An important point with respect to the unstable modes is that the finite range interaction effects are automatically taken into account within the present formalism. In the framework of nonrelativistic effective models finite range forces have to be considered [5].

In Fig. 9(b) we plot the spinodal curves for different wave numbers k. A larger k reduces the spinodal region, a fact also observed with the increase of the temperature in Ref. [6]. In fact the collective mode is characterized by a wavelength which gives rise to density distortions with the same wavelength. This originates higher and lower densities that take the nuclear matter away from the density unstable region.

In Fig. 10 we plot the ratio $\delta\rho_p/\delta\rho_n$ as a function of isospin asymmetry for different densities. We also show the ratio ρ_p/ρ_n. It is clear that the fluctuations involve a larger fraction of protons than the fraction present in the unperturbed nuclear matter. This is the distillation effect discussed in Ref. [8]. In the region of validity of the Landau-Fermi theory, it was shown in Ref. [8] that the proton oscillations are relatively larger than neutron oscillations and this behavior leads to a more symmetric liquid phase and consequently a more neutron rich gas phase. From Fig. 10 it is seen that the smaller the density, the greater the distillation effect, which is also more efficient at intermediate isospin asymmetries.

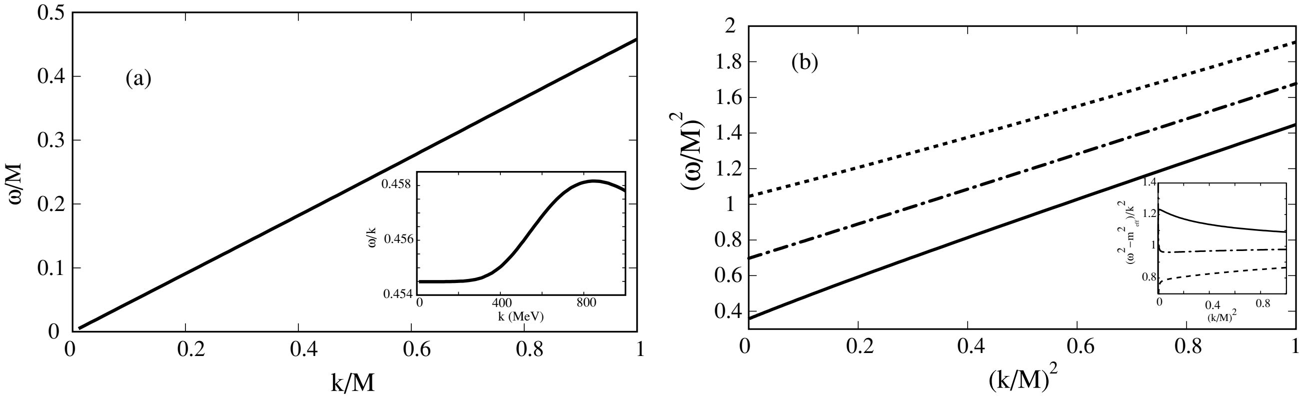

FIG. 12. Dispersion relation for $\rho = \rho_0$ and (a) nucleonic mode; (b) mesonic modes.

Figures 11(a) and (b) show the isospin asymmetry dependence of $|v_s|$, the modulus of the sound velocity, for $\rho = 0.5\rho_0$ and different values of the wave number. An increase of the wave number reduces $|v_s|$, and, in particular, for larger values of k, this quantity goes to zero at larger values of y_p, smaller isospin asymmetries, as expected from the discussion of Fig. 9(b). It is interesting to see in Fig. 11(b) that an increasing wave number k corresponds to a proton-neutron transition density ratio which increases with respect to the ρ_p/ρ_n ratio for a given y_p. For larger values of k the fluctuations tend to restore faster the proton-neutron symmetry of the liquid phase leaving a more neutron rich gas phase, i.e., a larger k value will increase the distillation effect in fragmentation.

Before we conclude, and in order to better understand the momentum dependence of nucleonic and mesonic modes, we plot in Figs. 12(a) and (b), for $\rho = \rho_0$ and $y_p = 0.3$, respectively, the nucleonic mode with $s_n > 1$ in a plot ω versus k and the mesonic modes in a plot ω^2 versus k^2. The insert in Fig. 12(a) shows the ratio ω/k versus k, and that the relation between ω and k is pratically linear. In Fig. 12(b) the insert is a plot of $(\omega^2 - m_{i,\text{eff}}^2)/k^2$ as a function of k. The mesonic modes are essentially described by a dispersion relation of the form $\omega^2 = m_{i,\text{eff}}^2 + a_i\, k^2$, where a_i is a constant close to 1, different for each mode. Similar results have been discussed in Ref. [12].

VI. CONCLUSIONS

In the present work we have applied the formalism of the relativistic Vlasov equation, which is an extension of the Landau-Fermi liquid theory, in the sense that mesonic fluctuations are also taken into account. Collective modes are then calculated. The main consequence of using the Vlasov formalism is the appearance of mesonic collective modes in addition to the usual nuclear modes. Moreover, we could also observe the dependence of the collective modes on the momentum transfer while in the Landau-Fermi liquid theory only very low momentum transfers are possible.

We have concluded that for symmetric nuclear matter isoscalar and isovector modes come out completely separated but, as asymmetry arises, only one mode appears and it changes character from isovector to isoscalar at the crossing density, which increases with decreasing proton fraction, i.e., the larger the isospin asymmetry, the larger the crossing density. This effect is due to the competitive contributions of the scalar meson and the vector mesons. For smaller densities the contribution of the scalar meson is larger giving rise to an isovectorlike mode. On the other hand at high densities this contribution is smaller due to the reduction of the nucleon effective mass and an increase of the scalar meson effective mass. We have shown that the behavior of the dispersion relation, Eq. (38), is essentially determined by the linear terms in the Lindhard function, namely by the quantities F^{pp} and F^{nn}, and for large isospin asymmetries it has a solution only for large densities. It was shown that the larger the density and the momentum transfer, the smaller the fraction of protons that participate in the isovector fluctuations. On the other hand, the isoscalar fluctuations at high density do not depend on the momentum transfer.

We have also studied the arising mesonic modes. In the medium the mesons increase their masses due to Fermi sea effects. However, it is expected that Dirac sea polarization will act in the opposite way, giving rise to a net reduction of the meson masses. In the present approach these effects have not been taken into account. However, using the model proposed in Ref. [27] where the meson masses decrease as the density increases, this effect could be included within the same formalism. The dependence of the mesonic modes on the isospin asymmetry of the system is small. We have noticed that at the crossing of the σ- and ρ-meson masses there is a level repulsion and the modes exchange character.

We have studied the unstable modes appearing at low densities associated with the liquid-gas phase transition. It was shown that the fluctuations tend to restore the isospin symmetry. This effect, known as the distillation effect is more efficient the larger the momentum transfer, the smaller the density, and at intermediate isospin asymmetry. For these modes, the dependence of the growth rate on the momentum transfer within the framework of the Landau-Fermi theory, although unrealistic, coincides for low momentum transfer with the results obtained within the Vlasov formalism. The reasonable behavior obtained from the Vlasov formalism is due to the inclusion of the mesonic fluctuations.

The use of different parametrizations of the nonlinear Walecka model and of relativistic models with density dependent couplings on the calculation of the collective modes may provide different results. This work is under investigation. In the future we also intend to include temperature effects and verify their importance on the collective stable and unstable modes.

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APPENDIX A: USEFUL INTEGRALS

$$\int_{-1}^{1} \frac{x}{x_0 - x}\mathrm{d}x = -\left(2 + x_0 \log\frac{x_0-1}{x_0+1}\right) = -L(x_0), \quad x_0 > 1,$$

$$\int_{-1}^{1} \frac{x^2}{x_0 - x}\mathrm{d}x = x_0\left(2 + x_0 \log\frac{x_0-1}{x_0+1}\right) = x_0 L(x_0), \quad x_0 > 1,$$

$$S(x) = \frac{A + Bx}{\frac{w}{w_0} - x}, \qquad \int_{-1}^{1} x S(x)\,\mathrm{d}x = -\left[A + B\left(\frac{w}{w_0}\right)\right] L\left(\frac{w}{w_0}\right), \quad x_0 > 1,$$

$$P\int_{-1}^{1} \frac{x}{x_0 - x}\mathrm{d}x = -\left(2 + x_0 \log\frac{1-x_0}{1+x_0}\right) = -L(x_0), \quad x_0 < 1.$$

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