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Gamow-Teller 1^+ states in ^{208}Bi

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The properties of the Gamow-Teller (GT) 1^+ states in ^{208}Bi have been investigated by using the Pyatov-Salamov method. The GT resonance (GTR) energy, the contribution of the GTR β transition strength to the Ikeda sum rule, and the differential cross sections for the ^{208}Pb$(p,n)^{208}$Bi and ^{208}Pb(^{3}He, t)^{208}Bi charge exchange reactions at different energies have been calculated. Our results show good agreement with experimental values.

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It is well established that the Gamov-Teller resonance (GTR) studies are very important in understanding such basic astrophysical and nuclear processes as the initial step of the hydrogen fusion reaction leading to nucleosynthesis and the electron capture reactions leading to stellar collapse and supernova formation [1]. They are also of great importance to checking the validity of the theories for the double-β-decay process.

The GT resonances, which were theoretically predicted in 1963 in the explanation of the allowed unfavored β-decay hindrance [2], were first experimentally identified in the ^{90}Zr (p,n) reaction at $E(p) = 35$ MeV in 1975 [3]. Such (p,n) charge exchange reactions have been used as an efficient way to extract the GTR in heavy nuclei (^{208}Bi) at intermediate energies [4,5]. Moreover, it has also been shown experimentally that the (^{3}He, t) reaction becomes a very suitable alternative for investigating these spin-isospin excitations when the bombarding energies exceed 100 MeV/nucleon [6]. There have been other attempts to extract these excitations using the ^{208}Pb(^{3}He, t)^{208}Bi reaction at different energies [7–9].

The GTR distribution in ^{208}Bi has also been investigated within the frameworks of different theoretical models. Kuzmin and Soloviev [10] calculated the fragmentation of the GTR in heavy nuclei within the framework of the quasiparticle phonon model. Their results showed that the GTR spreads only up to the excitation energies of around 30 MeV. Colo *et al.* [11] studied the spreading of the GTR and calculated its particle decay width by using the continuum Tamm-Dancoff approximation (TDA) and Hartree-Fock (HF) formalism with several types of the Skyrme interaction. They found that GTR has been located within the energy interval of 18–24 MeV, and the calculated energy for the GTR peak is higher than the experimental value by about 2–4 MeV. The GTR strength exhausts 61–68% of the Ikeda sum rule. Their calculated value for the particle decay width of the GTR (∼3 MeV) is lower than the experimental one (3.8 MeV). The spreading properties of the GTR in ^{208}Bi have also been studied by Dang *et al.* [12] including the two-particle, two-hole (2p2h) configurations with use of a two-body residual interaction in the form of the M3Y effective nucleon-nucleon force and the ground state correlations beyond the random phase approximation (RPA). Their calculations give a main peak for the GTR energy at 16.6 MeV, which is 2.6 MeV lower than the experimental value. The coupling to 2p2h states spreads the GTR up to around 60 MeV, and the GTR strength amounts to 57% of the Ikeda sum rule. The continuum RPA approach has been used in Ref. [13] in the calculations of the GTR distribution in ^{208}Bi. Moukhai *et al.* [13] use the Landau-Migdal parameter (g') in their calculations and obtain a value of 0.76 to describe the experimental energy of the GTR in ^{208}Bi. The GT states in ^{208}Bi have also been investigated by Suzuki and Sagawa [14] using the self-consistent HF and TDA. Their calculated value for the GTR energy is 0.4 MeV lower than the experimental value, and 63.6% of the total strength is concentrated in this state. The GTR energy has been calculated by means of an extended continuum RPA approach, and a value of 16.2 MeV was obtained by taking the Landau-Migdal parameter g' as 0.78 [15]. This is 0.6 MeV higher than the experimental value. The calculated relative strength of the GTR main peak is 69%. Recently, Bender *et al.* [16] investigated the effect of the spin-isospin channel of the Skyrme energy functional on the predictions of the GT distributions in ^{208}Pb nucleus. The calculations done in Ref. [11,14,16] are self-consistent.

The main aim of this work is to apply the Pyatov-Salamov method [17] to the investigation of the properties of the GT 1^+ states in ^{208}Bi in which the effective interaction strength has been determined self-consistently by relating it to the average field. This method has been applied to different kinds of studies [18–24]. In the present study, the GTR properties in spherical nuclei have been studied by imposing the commutativity of the central term in the nuclear part of the shell-model single-particle Hamilton operator with the GT operator. The problem has been treated in the RPA method. The details of the corresponding formalism have been given in Ref. [25].

Our method described above is applied for the first time to calculations for the ^{208}Bi nucleus, a representative of double closed-shell nuclei. The dependence of the energies of the excited GT 1^+ states in ^{208}Bi and the β transition strength from the ground state of the ^{208}Pb isotope to these 1^+ states on the parameters of the average field potential have been investigated. In addition, the differential cross sections for the ^{208}Pb$(p,n)^{208}$Bi and ^{208}Pb(^{3}He, t)^{208}Bi charge exchange reactions at $E(p) = 120$ MeV and $E(^3\text{He}) = 200$

TABLE I. The calculated and experimental energy spectra of the single-neutron-hole states in ^{208}Pb.

States	$3p_{1/2}$	$2f_{5/2}$	$3p_{3/2}$	$1i_{13/2}$	$2f_{7/2}$	$1h_{9/2}$
$-\varepsilon_{j_n}$(cal.)	7.12	8.03	8.17	9.07	10.70	11.04
$-\varepsilon_{j_n}$(exp.) [7]	7.37	7.94	8.27	9.00	9.71	10.78

and 450 MeV have been calculated. These results have been compared with the corresponding experimental values [26–28]. In the calculations, the Woods-Saxon potential with the Chepurnov parametrization ($V_0 = 53.3$ MeV, $\eta = 0.63$, $a = 0.63$ fm, $\xi_{ls} = 0.263$ fm^2) [29] was used. The basis used in our calculation contains all neutron-proton transitions which change the radial quantum number n by $\triangle n = 0, 1, 2, 3$. The single-particle Ikeda sum rule is fulfilled with $\sim$1.5% accuracy. The shell-model parameters chosen in our calculations allow one to satisfactorily describe the single-particle spectrum of the ^{208}Pb nucleus. In Table I, the calculated energy spectra ε_{j_n} of the single-neutron-hole states are given and compared with the experimental values. All the excitation energies (ω'_{GTR} and ω'_{k_F}) of the Gamow-Teller 1^+ states in all the figures and tables have been calculated from the ground state of the ^{208}Bi nucleus.

The dependence of the GTR energy in ^{208}Bi and the contribution of its β transition strength to the Ikeda sum rule on the isovector parameter η are shown in Figs. 1(a) and 1(b), respectively. In these figures, the region between the dashed lines corresponds to the range of the experimental values [8]. As seen in Fig. 1(a), the GTR energy decreases as the isovector parameter increases. The reason for this can be attributed to the fact that the energies of the proton and neutron states which are far away from each other because of the Coulomb potential now come closer. The contribution of the GTR β transition strength to the Ikeda sum rule also shows a decreasing tendency with the increase in the isovector parameter [Fig. 1(b)]. This stems from the different shift rates of the 1^+ excited states. As known, the spectrum of these 1^+ excited states is composed of three energy regions: the low-energy region, the GTR region, and the pigmy isovector spin monopole resonance (IVSMR) region. We can now examine the influence of the η parameter on the spectrum of these regions. When we change the η value from 0.5 to 0.7, the GTR state shifts toward the lower energy values by an amount of $\sim$0.8–1.0 MeV, while the energy shift in the other two states is in the same direction as the GTR state by an amount of 3–5 MeV. This means that the IVSMR and GTR states come closer to each other although the energy difference between the low-energy and GTR states has increased. As a result of this closeness, some part of the GTR β transition strength has been transmitted to the IVSMR states. For example, the numerical values at $\eta = 0.5$ for the contribution mentioned above at different energy regions are 5.17%, 87.13%, and 7.70%, respectively. The following values for the same contribution at $\eta = 0.7$ are obtained: 2.42%, 66.73%, and 30.85%.

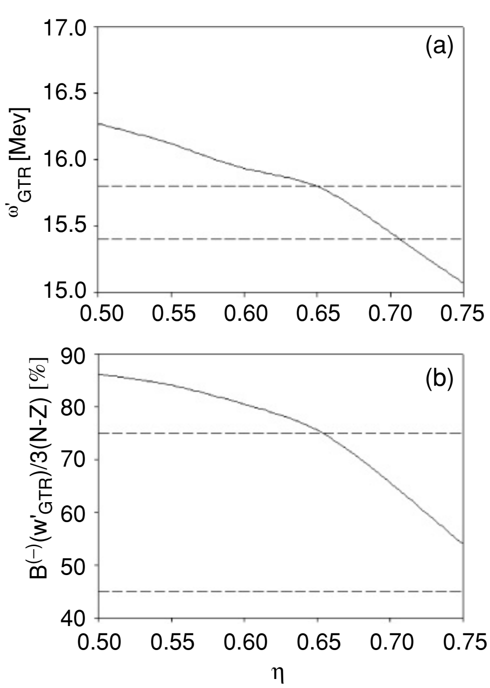

FIG. 1. The dependence of (a) the GTR energy and (b) the contribution of its β transition strength to the Ikeda sum rule on the isovector parameter.

The dependence of the same quantities on the spin-orbit parameter ξ_{ls} has been presented in Figs. 2(a) and 2(b). As expected, the energy of the GTR state goes to the higher values as the spin-orbit parameter increases. However, the contribution of the GTR β transition strength to the Ikeda sum rule decreases with the increase of this parameter. This decrease occurs because the spin-orbit parameter affects the GT 1^+ states at different energy regions in a different way. For example, when the value of the spin-orbit parameter changes from 0.20 to 0.32 fm^2, the 1^+ states in the pigmy IVSMR region move down to the lower energy values by an amount of $\sim$1.5–2.0 MeV whereas those in the low-energy and GTR region go up to the higher energy values by $\sim$0.5–1.5 and 0.5 MeV, respectively. This means that both states in the low-energy and pigmy IVSMR region approach the GTR state. As a result of this closeness, the states in these two regions take some part of the β transition strength from the GTR state. The numerical values obtained for the contribution of the GTR β transition strength to the Ikeda sum rule at $\xi_{ls} = 0.20$ fm^2 are 1.13%, 84.30%, and 14.48%; while at $\xi_{ls} = 0.32$ fm^2, they are 4.07%, 65.90%, and 30%, respectively.

From all the figures presented above, we see that the calculated values obtained by our Pyatov-Salamov method [17] for different quantities at the well-known values [29] of the average field potential parameter are not very far from the related experimental values. Then it can be said that these standard parameter values will be suitable for our calculations.

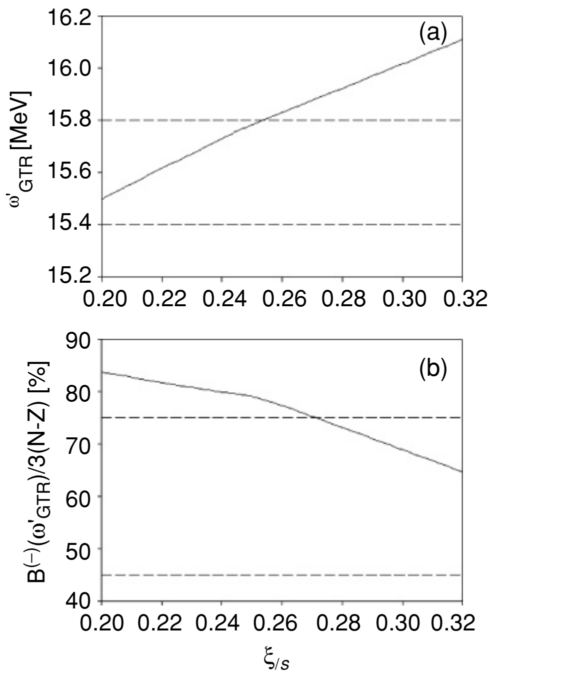

FIG. 2. The dependence of the same quantities in Fig. 1 on the spin-orbit parameter.

Our next calculation results were obtained by using these parameter values.

The distribution of the GT β transition strength in ^{208}Bi is given in Fig. 3. This distribution can be divided into three energy regions: low-energy region ($\omega'_{k_F} \leqslant 9$ MeV), the GTR region, and the pigmy IVSMR region (19 MeV $\leqslant \omega'_{k_F} \leqslant$ 24 MeV). There are six states with $B^{(-)}_{\omega'_{k_F}} > 0.1$ in the low-energy region as mentioned in Ref. [27] which exhaust 2.5% of the Ikeda sum rule. The 1^+ excited states in this region are composed of the proton-particle–neutron-hole transitions with $\Delta n = 0$, and these transitions are weakly collectivized. In Table II, the calculated values of the differential cross section for the ^{208}Pb(^{3}He,t)^{208}Bi charge exchange reaction

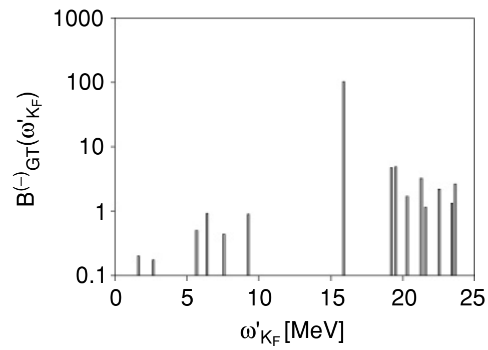

FIG. 3. GT β transition strength distribution in ^{208}Bi.

at $E(^3\text{He}) = 200$ MeV and $\theta \approx 0$ are compared with the corresponding experimental values. As seen from this table, both the energies and differential cross-section values of the first two excited 1^+ states agree with the experimental values [7], while they differ from the experimental values for the remaining states. The 1^+ states having a considerable β transition strength in the low-energy region are in the energy ranges of 1.6–3.0 and 5.68–9.27 MeV. The energies of these 1^+ states agree with the ones obtained in experiments in Ref. [27], whereas there is a disagreement for the β transition strength.

Our calculations determined that there is only one highly collectivized GTR state at $\omega'_{\text{GTR}} = 15.897$ MeV. It is obvious that the main contribution to the GTR state comes from the transitions occurring from the states having the neutron excess. Moreover, the calculated value for the energy of the GTR state is very close to the corresponding experimental value (15.6 ± 0.2 MeV) [8]. Note that among the self-consistent calculations, our calculated value for the GTR energy is the closest one to the experimental value. Our calculations show that the β transition strength coming from this GTR state at $\omega'_{\text{GTR}} = 15.897$ MeV exhausts 79.66% of the Ikeda sum rule. This value is very close to the upper limit of the experimental value $(60 \pm 15)\%$ [8], and it is larger than the corresponding value obtained in Refs. [11,12,14,15].

TABLE II. The calculated and experimental values for the characteristic quantities of the GT 1^+ states in ^{208}Bi. The energies are given in MeV.

Our calculations		Ref. [7]		Ref. [27]	
ω'_{k_F}	$\frac{d\sigma}{d\Omega}$ (mb/sr)	$\omega^{\exp'}_{k_F}$	$(\frac{d\sigma}{d\Omega})^{\exp}$ (mb/sr)	$\omega^{\exp'}_{k_F}$	$\frac{I_{\text{GT}}}{I_{\text{GTR}}}(\%)$
1.652	0.490	1.803 ± 0.025	0.302 ± 0.015	1.803	2.7
2.686	0.423	3.174 ± 0.025	0.204 ± 0.014	3.2	0.7
3.582	0.018	–	–	–	–
3.846	0.033	–	–	–	–
4.379	0.207	3.863 ± 0.025	0.194 ± 0.013	4.1	3.0
4.702	0.068	4.043 ± 0.025	0.173 ± 0.013	4.7	1.8
5.076	0.019	4.621 ± 0.025	0.350 ± 0.018	–	–
5.683	1.213	–	–	5.9	3.2
6.414	2.204	–	–	–	–
7.589	1.046	–	–	8.0	7.0
9.270	2.138	–	–	9.8	12.0

The differential cross section for the ^{208}Pb(^{3}He, t)^{208}Bi and ^{208}Pb(p, n)^{208}Bi charge exchange reactions occurring from the excitation of the GTR state has been calculated at $E(^3\text{He}) = 450$ MeV and $E(p) = 200$ MeV, respectively. In the calculations, the value of $J_{\sigma\tau} = 172$ MeV/fm^3 [26] was used for the volume integral. The calculated values for the differential cross section of the above charge exchange reactions are 249.98 and 26.34 mb/sr, respectively. The corresponding experimental values for these reactions are 163 ± 33 [8] and 41 ± 12 mb/sr [28]. On the other hand, using the GT sum rules [2], the differential cross section for the ^{208}Pb(^{3}He, t)^{208}Bi at $E(^3\text{He}) = 450$ MeV was calculated in Ref. [26], and a value of 335 mb/sr was obtained. It is obvious that the value for the differential cross section of the ^{208}Pb(^{3}He, t)^{208}Bi at $E(^3\text{He}) = 450$ MeV calculated by our present method is closer to the experimental data than that calculated in Ref. [26].

Our calculations also show the presence of the pigmy IVSMR region which is distributed over the energy range of $\sim$19–24 MeV, and the pigmy IVSMR β transition strength exhausts 17.84% of the Ikeda sum rule (see Fig. 3). The proton-particle–neutron-hole multiplets with $\Delta n \neq 0$ form the general structure of the 1^+ states in this region. Recall that our calculated value for the contribution of the pigmy IVSMR β transition strength to the Ikeda sum rule shows a good agreement with the results of other theoretical calculations [15,30].

Based on the Pyatov-Salamov method [17], the properties of the GT 1^+ states in the ^{208}Bi isotope have been investigated by providing the commutativity of the central term in the nuclear part of the shell-model single-particle Hamilton operator with the GT operator. The use of this method makes our formalism free of the GT interaction strength parameter. The GT 1^+ states have been treated within the framework of RPA where the ground state correlations are included. Our analysis for the dependence of the properties of the GTR state on the average field potential parameters indicates that the energy of the GTR state shifts down to the lower energy values with the increase of the isovector parameter while it moves up to higher energy values as the spin-orbit parameter increases. It has also been established that the contribution of the GTR β transition strength to the Ikeda sum rule decreases with an increase in all these parameters. Moreover, our analysis has shown that the standard parameter values for the average field potential taken from Ref. [29] could be suitable for our calculations. The results obtained by the present method for the GTR energy, the contribution of the GTR β transition strength to the Ikeda sum rule, and the differential cross section of the ^{208}Pb(p, n)^{208}Bi and ^{208}Pb(^{3}He, t)^{208}Bi charge exchange reactions at different energies show a good agreement with the experimental values.

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