α-decay half-lives of the observed superheavy nuclei ($Z = 108-118$)

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A systematic and comprehensive study of the decay half-lives of nuclei appearing in the observed α-decay chains of superheavy elements ($Z = 108-118$) is presented. The calculation proceeds in three steps. First, the relativistic mean-field equations are solved in the axially symmetric deformed oscillator basis to obtain ground-state properties such as binding energies, radii, deformations, and densities. The results are in good agreement with the available experimental systematics, as expected. Next, the calculated densities are used in the double-folding prescription to determine the interaction potentials for the α-daughter systems. Finally, these potentials, along with calculated and experimental Q values, are used in the WKB approximation to estimate the decay half-lives. The calculated half-lives, which sensitively depend on Q values, qualitatively reproduce the experiment.

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The production and study of superheavy elements (SHE) has been the cherished aim of experimentalists since the prediction of the island of stability, around $Z \sim 114$, $N \sim 184$ in the 1960s. It has still not been possible to reach the $N = 184$ closed shell. Earlier experiments had partial success. Cold (hot) fusion with Pb/Bi (actinides: ^{235}U / ^{244}Pu / ^{243}Am) targets and suitable projectiles of ^{64}Ni / ^{70}Zn (^{48}Ca) have been successfully used for the production of superheavy elements $110-113$ ($114-116$). The search for isotopes of these elements and also for elements with higher atomic numbers (Z) is pursued vigorously by a number of laboratories around the world.

On the theoretical front, primarily two kinds of approaches have been used to describe superheavy nuclei: microscopic theories and microscopic-macroscopic models. The relativistic mean-field (RMF) theory [1,2] belongs to the former, and the Möller-Nix [3] or the Muntian [4] models are examples from the latter category. The aim of the earlier RMF studies [5] had been to predict the most stable N and Z combination. In self-consistent models, the occurrence of a spherical proton (neutron) shell closure with given Z (N) may change with varying neutron number N (Z). Such systematic investigations are still being reported [6–8]. A number of RMF investigations for the ground-state properties of nuclei appearing in the observed decay chains of specific superheavy nuclei have also been reported (e.g., [9–11]). The calculated binding energies reproduce Audi-Wapstra systematics [12] rather well. However, most of these use the phenomenological Viola-Seaborg formula [13] for the calculation of the decay half-lives. Our emphasis in this Brief Report is on microscopic calculation of the α-decay half-lives, where the experimental data are available [14–19]. First, the ground-state properties are calculated in the RMF framework. We do not intend to present all the details of the ground-state properties; instead, we list essentials of the emerging systematic features, which are consistent with earlier investigations. The present calculation of the half-lives requires specific ground-state information, namely, binding energies (through the Q values) and density distributions (entering through the α-daughter interaction potentials). Therefore, we present and discuss only the calculated Q values and the decay half-lives.

Relativistic mean-field theories work at the level of nucleons and mesons. The Dirac spinor nucleons interact only through the σ, ω, and ρ mesons and the photon fields. Starting with a suitable interaction Lagrangian [1,2], the Euler-Lagrange variational principle yields the equations of motion. Replacing the field operators by c numbers and imposing symmetry requirements, one finally ends up with a set of coupled nonlinear differential equations:

- the Dirac equation with potential terms involving meson and electromagnetic fields describing the nucleon dynamics and
- a set of Klein-Gordon-type equations with sources involving nucleonic currents and densities, for mesons and the photon.

These equations, known as the RMF equations, are to be solved self-consistently. The pairing correlations, important for open-shell nuclei, are incorporated by using a simple BCS prescription (constant gap) or self-consistently, through the Bogoliubov transformation. The latter leads to the relativistic Hartree-Bogoliubov (RHB) equations [2]. Under the constant-gap approximation, the RHB equations reduce to the usual RMF equations.

The explicit calculations require the following input information:

- the parameters appearing in the Lagrangian and
- the pairing gaps or suitable pairing interaction.

We employ here the most widely used Lagrangian parameter set, NL3 [20]. The pairing gaps, in principle, are to be calculated from the experimental odd-even mass differences. However, in the absence of experimental odd-even mass differences (as in the present case), one generally uses some phenomenological prescription for the pairing gaps (see, for example, [11]). For a better description of pairing, one often uses the finite-range Gogny-D1S [21,22] interaction, which

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is known to have the right pairing content. Here, the gaps are determined so as to reproduce the individual proton and neutron RHB pairing energies obtained with the Gogny-D1S interaction. It has been shown [23] that this choice of pairing gaps yields reliable and satisfactory results for the ground-state properties.

We now present and discuss briefly the calculated ground-state properties. The calculated binding energies are found to be in good agreement with the corresponding Audi-Wapstra systematics [12]; the maximum departure is a few MeV (<0.25%). The rms matter radii are found to vary monotonically. The calculated quadrupole deformation parameters (β) are found to be close to the corresponding values of Möller and Nix [3]. Most of the nuclei considered here turn out to be of prolate shape, except for $^{287,288}115$, with $\beta \approx 0$. However, for some of the isotopes of the elements with $Z \geqslant 110$, superdeformed ground-state solutions are obtained. Such solutions are reported by Ren [24] and also by Sharma *et al*. [25]. However, these superdeformed solutions may disappear if higher multipolar constraints are imposed. This is supported by the recently reported microscopic-macroscopic analysis [26]. With these brief comments on the ground-state properties, we now present and discuss the Q values, which are required in the subsequent calculation of the α-decay half-lives.

The Q value of a parent nucleus against α decay is just the difference between the binding energy of the parent nucleus and the sum of binding energies of the α and daughter nuclei. The calculated Q values (denoted Cal.) for the superheavy nuclei against α decay are presented in Fig. 1. The corresponding values for the hypothetical α-decay chains of $^{294,295}118$ nuclei are also shown in the same figure. The so-called experimental Q values for the decay chains of $^{294,295}118$ have been obtained from the Audi-Wapstra systematics [12]. The calculations are found to be in good qualitative agreement with the corresponding experimental values [14–19]. At a finer level, differences do exist at some places, the maximum departure being of the order of 1 MeV. However, for the $Z = 115$ chains, the maximum departure is found to be about 2 MeV. The kind of agreement obtained here is gratifying in view of the fact that the Q value is the difference between two large numbers. A small error even in one of them could alter the Q values considerably. This agreement reflects a very precise and delicate cancellation.

The results obtained by using the other successful Lagrangian parameter sets (e.g., NL1 [1,27], NL-SH [28], and NL-SV1 [29]) exhibit identical systematics. Therefore, the conclusions drawn here will generally remain valid.

The present calculation of the decay half-lives also requires the α-daughter interaction potentials. These are obtained using the double-folding prescription. The double-folding (DF) model [30] yields the nucleus-nucleus interaction potential, using the density distribution of the constituent nuclei and nucleon-nucleon interaction. The calculated deformed densities are first expanded in terms of multipoles (Y_L^0). The $L = 0$ projected and renormalized (spherical) densities are then used to obtain the DF potential. Here, we use the density-dependent version of the M3Y interaction ($\mathcal{V}$). The density dependence

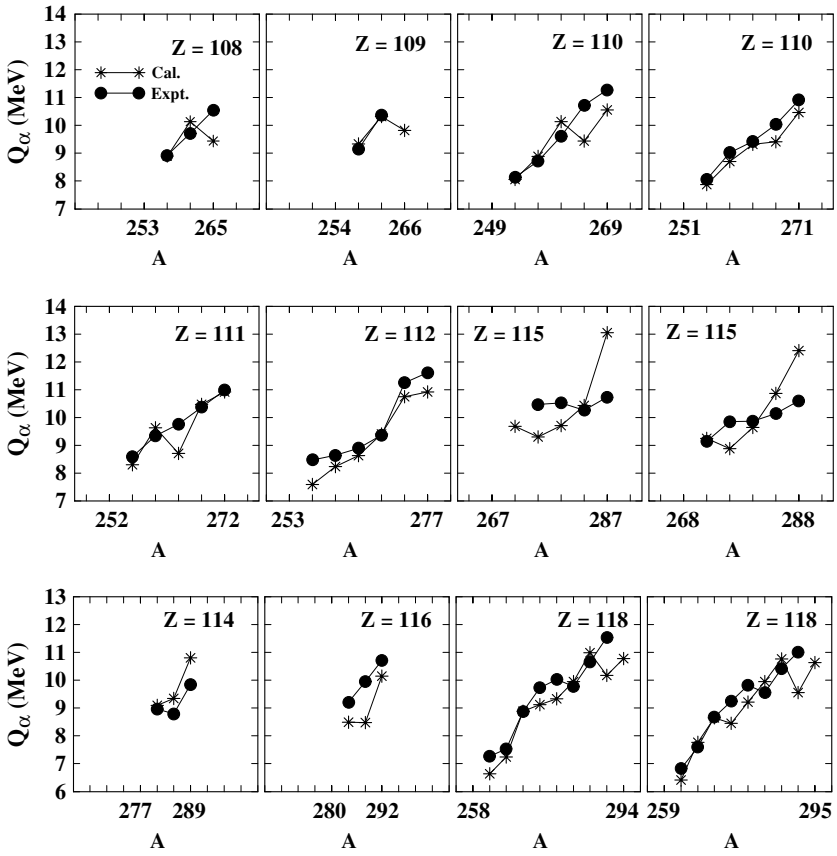

FIG. 1. Calculated and experimental Q values against α decay. The corresponding experimental values [14–19] are also shown, where available.

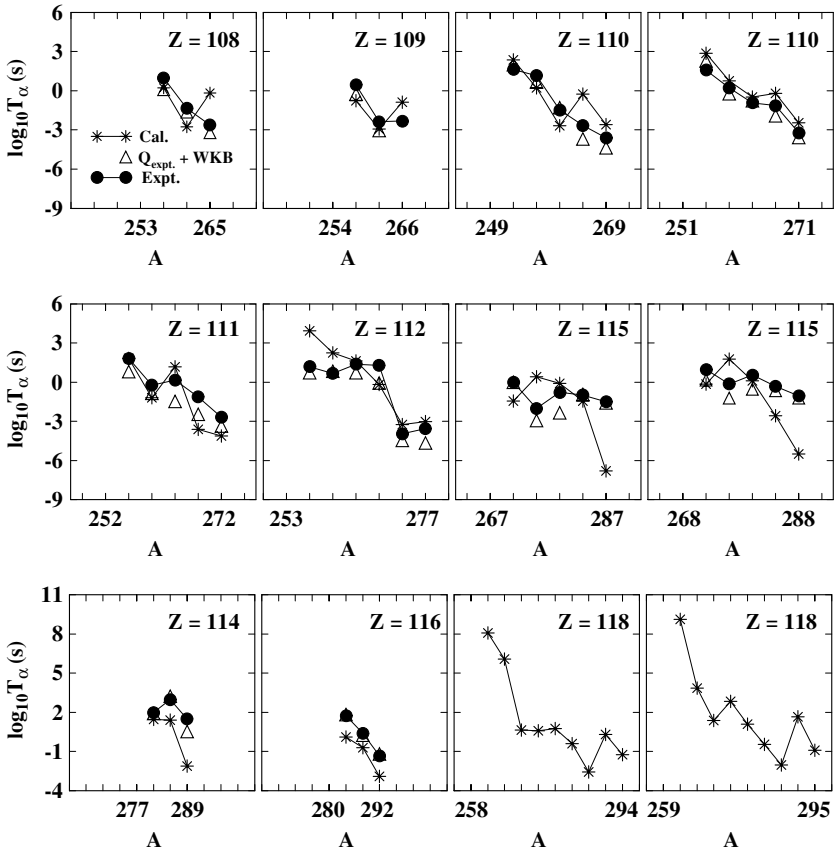

FIG. 2. Decay half-lives against α decay of superheavy nuclei. See text for details.

is expected to simulate medium effects and Pauli blocking. Exchange effects are taken into account through a δ-function pseudo-potential. The α-daughter potential ($V_{\alpha D}$) within the DF prescription reads

$$V_{\alpha D}(\vec{\mathcal{R}}) = \int \rho_\alpha(\vec{r}_\alpha)\rho_D(\vec{r}_D)\mathcal{V}(\vec{r}_\alpha - \vec{r}_D + \vec{\mathcal{R}})d^3\vec{r}_\alpha d^3\vec{r}_D, \quad (1)$$

where $\mathcal{R}$ is the separation between the centers of α and the daughter; $\vec{r}_\alpha$ and $\vec{r}_D$ are the integration variables for α and daughter nuclei, respectively. An explicit expression for $\mathcal{V}$ may be found in [9]. The Coulomb potential (V_C) is also obtained by the DF prescription. These calculated potentials are used in the WKB approximation to estimate the transmission probability and hence the half-lives. The half-life of a parent nucleus is given by

$$T_{1/2} = \frac{\ln(2)}{\nu_o}(1 + e^K), \quad (2)$$

where the action integral K appearing in Eq. (2) reads

$$K = \frac{2}{\hbar}\int_{\mathcal{R}_a}^{\mathcal{R}_b}\{2\mu(V_{\alpha D}(\mathcal{R}) + V_C(\mathcal{R}) - Q)\}^{1/2}d\mathcal{R}; \quad (3)$$

$\mathcal{R}_a$ and $\mathcal{R}_b$ are the classical turning points, deduced by requiring that the integrand in Eq. (3) vanishes. In Eq. (2) ν_o is the conventional assault frequency obtained from the energy E [$=Q/(1 + M_\alpha/M_D)$, with M_α and M_D being the α and daughter masses, respectively], corrected for the recoil, through

$$\nu_o = \left(\frac{1}{2R}\sqrt{\frac{2E}{M_\alpha}}\right), \quad (4)$$

where $R = 1.2A^{1/3}$ is the nuclear radius parameter of the parent nucleus and A is the mass number.

The calculated and the corresponding experimental half-lives for superheavy nuclei against α decay are presented in Fig. 2. The results obtained with the calculated Q values are denoted Cal. The calculated half-lives are found to be in qualitative agreement with the corresponding experimental values. It is further observed that the half-lives obtained by using the calculated Q values, though trending similar to the experimental data, differ from them quantitatively at some places. This reflects hypersensitivity of the half-lives on Q values. A small change ($\sim$0.2 MeV) in Q values alters the decay half-lives by *at least* an order of magnitude. Therefore, for a reliable and accurate estimation of the half-lives, the calculated Q values should be accurate within a few tens of keV. It is to be stressed that the present prescription for obtaining the decay half-lives is parameter free. Similar results obtained with experimental Q values [14–19] are denoted by $Q_{\text{expt.}}$ + WKB in Fig. 2. Using the experimental Q values further improves the agreement. This indicates that the microscopic nucleus-nucleus potential obtained in the DF model is reliable; therefore, it can be used with confidence in reaction calculations (e.g., as the real part of the optical potential).

In summary, a systematic study of α-decay half-lives of superheavy nuclei has been carried out. The ground-state properties of the relevant nuclei are calculated within the framework of the RMF theory. The binding energies and deformation parameters are found to be in accord with the available corresponding systematics. The calculated Q values are found to be in reasonably good agreement with the experiment. The maximum departure is of the order of 2 MeV in some cases. The half-lives obtained by using these Q values, though qualitatively similar to those from the experiment, do differ significantly at some places. This emphasizes the sensitive dependence of the half-lives on Q values. Use of experimental Q values, however, brings the half-lives closer to the experimental values, thereby implying that the calculated double-folding potential is reliable and can be used with confidence in other reaction studies as the real part of the optical potential.

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