Intensity interferometry of thermal photons from relativistic heavy-ion collisions

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Intensity interferometry of thermal photons having transverse momenta $k_T \approx 0.1-2.0$ GeV produced in relativistic collision of heavy nuclei is studied. It is seen to provide an accurate information about the temporal and spatial structure of the interacting system. The source dimensions, and their k_T dependence revealed by the photon interferometry, display a richness not seen in pion interferometry. We attribute this to the difference in the source functions, the fact that photons come out from every stage of the collision and from every point in the system, and the fact that the rate of production of photons is different for the quark-gluon plasma, which dominates the early hot stage and the hadronic matter that populates the last phase of the collision dynamics. The usefulness of this procedure is demonstrated by an application to collision of lead nuclei at the CERN SPS. Prediction for the transverse momentum dependence of the sizes for SPS, RHIC, and LHC energies are given.

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I. INTRODUCTION

The search for quark hadron phase transition and the investigation of the properties of quark gluon plasma stand among the most challenging as well as rewarding pursuits of high energy nuclear physics today. The observations of jet-quenching [1,2] and the elliptic flow [3,4], and the success of the partonic recombination [5] as a model for hadronization in recent experiments at the Relativistic Heavy Ion Collider at Brookhaven, presage the shift of the focus to even more interesting questions about how the plasma is formed and how it evolves.

It is expected that the quantum statistical interference between identical particles emitted from the relativistic heavy ion collisions will provide valuable inputs for these investigations. The pion intensity interferometry experiments have, however, thrown up some startling questions, which need to be answered before we can address these issues. The hydrodynamics models that have traditionally provided a quantitative description to data on particle spectra and elliptic flow fail to explain the near identity of the so-called outward and sideward radii seen in these experiments and invariably lead to outward radii that are up to a factor of 2 larger than the sideward radii [6].

An investigation of the structure and dimensions of the source in terms of photon interferometry should provide a more direct and valuable insight into these questions. The advantages of using photons for such studies are well known [7]; they interact only weakly with the system after their production and are free from such distorting effects as the rescattering [8] and Coulomb interactions, which have dogged these investigations using identical hadrons. Photons are also emitted at every stage of the collision dynamics and from every point in the system—and not only from the freeze-out surface. We note though that some studies have postulated that pions may be emitted not only from the surface as normally assumed; they also escape the system continuously from all points in the system [9] with some escape probability. A large probability for this will, however, be hard to justify, as the pions do interact strongly with the hadronic medium. Still this possibility, along with the modifications introduced due to final state interactions for hadrons, necessitates a great deal of further study before the radii obtained from the intensity interferometry of hadrons are used to get the spatial and temporal extent of the source in these collisions.

Theoretical investigations of the nature of the photon correlation function for relativistic heavy collisions, where a quark-gluon plasma may be formed have been carried out by several authors [10,11]. An experimental measurement is a far more difficult proposition though, because of the huge background of decay photons and a considerably smaller production of direct photons. Thus, so far only one experiment at cyclotron energies [12] (where direct photons mostly originate from bremsstrahlung of protons against neutrons) and a measurement by the WA98 collaboration [13] (where photons originate from hadronic reactions and possibly quark matter) for the central collision of lead nuclei at CERN SPS have been reported.

It is expected that such experiments will have a higher possibility of success at RHIC and LHC energies. First, the larger initial temperatures expected there will lead to a larger production of direct photons. Second, a large suppression of high-momentum pions because of the onset of jet quenching will lead to a reduction in the background from the decay photons. Furthermore, a large production of photons having high k_T is expected from the preequilibrium stage of the collision [14,15], which can be reliably crafted using the parton cascade model [16], and it has been shown to have a very distinct distribution of the source [17].

The early theoretical investigations [10,11] of photon interferometry were also aimed at isolating photons coming from the quark matter from those coming from the hadronic matter. These studies concentrated on photons having high transverse momenta ($k_T \gg 1$ GeV), which are more likely to be emitted from the early hot and dense phase of the plasma.

In the present work we focus our attention on the photons having low and intermediate $k_T \approx 100$ MeV–2 GeV. The photons at the lower end of the k_T range considered would have a dominant contribution from the late stages of the hadronic phase. This should reveal a source that is strongly affected by the radial flow. The higher end of the k_T should have a leading contribution from the early hot and dense stage of the plasma, when the flow is still small and the temperature is large.

We also incorporate several improvements over the early exploratory works [10]. First, we use the complete leading order results for the production of photons from the quark matter [18], which has important contributions from the bremsstrahlung, and annihilation of off-shell quarks [19]. Second, for hadronic reactions we use the state-of-the-art results from Turbide *et al*. [20] along with inclusion of hadronic form factors at the vortexes, incorporating strange mesons. The dominant hadronic bremsstrahlung process ($\pi\pi \to \pi\pi\gamma$) is included [21] for low k_T photons and double counting via $\pi\pi \to \rho\gamma$ and $\rho \to \pi\pi\gamma$ avoided by limiting the contributions of the latter two processes to $E_\gamma > 500$ MeV. This should be adequate until a more detailed and complete calculation is available. Finally, we use a much richer equation of state for the hadronic matter, with the inclusion of all the particles in the particle data book, having $M < 2.5$ GeV. Results of our model calculations are compared with the recent data obtained by the WA98 experiment and predictions given for RHIC and LHC energies.

We shall see that the competition between the cooling because of expansion and the blue shift of the photon spectra because of transverse expansion gives rise to a unique k_T dependence of the outward and sideward correlation radii, which evolves rapidly as the initial temperature of the system changes, as we go from SPS to RHIC to LHC.

We first discuss the tools necessary for the analysis of the intensity interferometry and briefly discuss the initial state and the rate of production of photons. Next we check whether the description used by us provides a reasonable description of the single photon spectra measured at CERN SPS. The results for some typical momenta of the photons and transverse momentum dependence of correlation radii are discussed next. Finally we give our conclusions.

II. FORMULATION

A. The correlation function

The spin-averaged intensity correlation between two photons with momenta $\mathbf{k_1}$ and $\mathbf{k_2}$, emitted from a completely chaotic source, is given by the following:

$$C(\mathbf{q},\mathbf{K}) = 1 + \frac{1}{2}\frac{\left|\int d^4x\, S(x,\mathbf{K})e^{ix\cdot q}\right|^2}{\int d^4x\, S(x,\mathbf{k_1})\int d^4x\, S(x,\mathbf{k_2})}, \tag{1}$$

where $S(x,\mathbf{k})$ is the space-time emission function and

$$\mathbf{q} = \mathbf{k_1} - \mathbf{k_2}, \quad \mathbf{K} = (\mathbf{k_1} + \mathbf{k_2})/2. \tag{2}$$

In a hydrodynamics calculation, the space-time emission function S is replaced by the rate of production of photon $EdN/d^4x d^3k$, in the hadronic or the quark matter, as discussed earlier.

The results for the correlation function $C(\mathbf{q},\mathbf{K})$ are best discussed in terms of the so-called outward, sideward, and longitudinal momentum differences. Thus for the four-momentum k_i^μ of the ith photon, we have the following:

$$k_i^\mu = (k_{iT}\ \cosh\ y_i, \mathbf{k_i}) \tag{3}$$

with

$$\mathbf{k_i} = (k_{iT}\ \cos\psi_i, k_{iT}\sin\psi_i, k_{iT}\ \sinh\ y_i), \tag{4}$$

where k_T is the transverse momentum, y is the rapidity, and ψ is the azimuthal angle. Defining the difference and the average of the transverse momenta,

$$\mathbf{q_T} = \mathbf{k_{1T}} - \mathbf{k_{2T}}, \mathbf{K_T} = (\mathbf{k_{1T}} + \mathbf{k_{2T}})/2\,, \tag{5}$$

we can write [22] the following:

$$\begin{aligned} q_{\rm long} &= k_{1z} - k_{2z} \\ &= k_{1T}\sinh y_1 - k_{2T}\sinh y_2 \end{aligned} \tag{6}$$

$$\begin{aligned} q_{\rm out} &= \frac{\mathbf{q_T}\cdot\mathbf{K_T}}{K_T} \\ &= \frac{\left(k_{1T}^2 - k_{2T}^2\right)}{\sqrt{k_{1T}^2 + k_{2T}^2 + 2k_{1T}k_{2T}\cos(\psi_1-\psi_2)}} \end{aligned} \tag{7}$$

$$\begin{aligned} q_{\rm side} &= \left|\mathbf{q_T} - q_{\rm out}\frac{\mathbf{K_T}}{K_T}\right| \\ &= \frac{2k_{1T}k_{2T}\sqrt{1-\cos^2(\psi_1-\psi_2)}}{\sqrt{k_{1T}^2 + k_{2T}^2 + 2k_{1T}k_{2T}\cos(\psi_1-\psi_2)}}. \end{aligned} \tag{8}$$

The corresponding radii are obtained by approximating the following:

$$\begin{aligned} &C(q_{\rm out}, q_{\rm side}, q_{\rm long}) \\ &\quad = 1 + \tfrac{1}{2}\exp\left[-\left(q_{\rm out}^2R_{\rm out}^2 + q_{\rm side}^2R_{\rm side}^2 + q_{\rm long}^2R_{\rm long}^2\right)/2\right]. \end{aligned} \tag{9}$$

Note that for this choice of parametrization for the correlation function $R_i^2 = 1/\langle q_i^2\rangle$, where $i =$ out, side, and long, and the average is performed over the distribution $(C-1)$.

A one-dimensional analysis of the correlation function C is sometimes performed in terms of the invariant momentum difference as follows:

$$C\left(\sqrt{q_{\rm inv}^2}\right) = 1 + \tfrac{1}{2}\exp\left[-q_{\rm inv}^2R_{\rm inv}^2/2\right], \tag{10}$$

where

$$\begin{aligned} q_{\rm inv} &= \sqrt{-\left(k_1^\mu - k_2^\mu\right)^2} = \sqrt{-q_0^2 + \mathbf{q}^2} \\ &= \sqrt{2k_{1T}k_{2T}\left[\cosh(y_1-y_2) - \cos(\psi_1-\psi_2)\right]}. \end{aligned} \tag{11}$$

However, the significance of the corresponding radius has no clear meaning [23], and it is also not useful for comparing results obtained using different particles. The variable $\mathbf{q}^2 + q_0^2 = q_{\rm inv}^2 + 2q_0^2$ is also occasionally used for a one-dimensional analysis [24], and one writes

$$C\left(\sqrt{\mathbf{q}^2 + q_0^2}\right) = 1 + \tfrac{1}{2}\exp\left[-\left(\mathbf{q}^2 + q_0^2\right)R^2/2\right]. \tag{12}$$

Note also that $\mathbf{q}^2 = q_{\rm out}^2 + q_{\rm side}^2 + q_{\rm long}^2$.

Here, it is useful to realize that in actual studies of photon interferometry the correlation function C would be parametrized as follows:

$$C(q_{\rm out}, q_{\rm side}, q_{\rm long}) = 1 + \frac{1}{2} \times \lambda \times \exp\left[-\left(q_{\rm out}^2 R_{\rm out}^2 + q_{\rm side}^2 R_{\rm side}^2 + q_{\rm long}^2 R_{\rm long}^2\right)/2\right], \tag{13}$$

where λ is a measure of single (S) versus decay photons (D), under the assumption that the source is completely chaotic. Obviously the latter do not contribute to the correlation (the lifetime of π^0 is $\sim 10^{-16}$ seconds) and thus

$$C(q_{\rm out} = 0, q_{\rm side} = 0, q_{\rm long} = 0) = 1 + \frac{1}{2}\lambda \tag{14}$$

so that

$$\lambda = \frac{S^2}{(S+D)^2}. \tag{15}$$

and therefore using the measured values of the total photon yield $(S + D)$ at the momentum $\mathbf{K}$, we can get the yield of single photons from the intercept of C on the y axis (Peressounko [11]). This can provide a check on the results on single photon obtained by, say, a subtraction of photons from decay of pions and η's.

It is important to realize that this procedure is not directly applicable to the case where only a one-dimensional analysis in terms of $q_{\rm inv}$ [Eq. (10)] is performed. The photon pairs having transverse momenta in some bin around K_T, rapidity difference within Δy, and azimuthal angle difference within $\Delta\psi$ will lead to a range of outward, sideward, and longitudinal momentum differences that will decide the actual correlation function C [Eq. (9)]. Even a given value of $q_{\rm inv}$ will admit a range of these momentum differences, and thus the value of one-dimensional C will be obtained by taking an average over the corresponding results. This will lead to an effective λ that will not reduce to unity, even if there were no decay photons [25]. This "effective" λ, as well as the $R_{\rm inv}$, will depend on K_T, ΔK_T, Δy, and $\Delta\psi$ and the correlation radii, $R_{\rm out}$, $R_{\rm side}$, and $R_{\rm long}$. Thus the direct photon yield can be measured only by assigning a value to these radii and becomes model dependent. Of course, $R_{\rm inv}$ can be assigned, though its meaning remains unclear and it remains model dependent.

A simple instance should exemplify this issue. Consider a case where $y_1 = y_2 = 0$ and $\psi_1 = \psi_2$ so that $q_{\rm side} = q_{\rm long} \equiv 0$ [see Eqs. (7) and (8)]. In this case $q_{\rm out} = (k_{1T} - k_{2T})$, but $q_{\rm inv} \equiv 0$. Thus the "true" correlation function [Eq. (9)] will be decided by the value of the outward momentum difference and the outward correlation radius, while naively one would expect the one-dimensional correlation function [Eq. (10)] to reduce to 1.5. Thus even for a case when there are no decay photons, we would need a correction factor to account for the range of values admitted by the "true" correlation function.

The procedure may, however, still work for the one-dimensional analysis in terms of the variable $\sqrt{q_0^2 + \mathbf{q}^2}$ [Eq. (12)], as this vanishes only when all three momentum differences, $q_{\rm out}$, $q_{\rm side}$, and $q_{\rm long}$ also vanish simultaneously. The meaning of the corresponding radius and its relation to the outward, sideward, and longitudinal correlation radii remains unclear though.

While we are discussing the merits of the one-dimensional analysis in terms of the invariant momentum difference, $q_{\rm inv}$, we recall that for typical cases, the correlation function C differs substantially from unity for momentum differences $\leqslant$0.2 GeV. For studies using single photons this limit has to be less than m_{π^0} to stay clear of the π^0 peak. If k_{iT} are large, choosing Δy and $\Delta\psi$ as small, we can get very small values of the sideward, outward, and longitudinal momentum differences. However, as $q_{\rm inv} \propto k_T$, small values for $q_{\rm inv}$ can be obtained only if k_{iT} are quite small. For larger value of k_{iT}, the necessary Δy and $\Delta\psi$ bins would be too small to admit meaningful statistics. Thus, it is no wonder that both the photon interferometry experiments reported in the literature so far [12,13] use the one-dimensional analysis in terms of $q_{\rm inv}$ and utilize photons having very low k_T. We hope that a much larger statistics expected at RHIC and LHC will help us get over this problem by providing results for the full three-dimensional correlation function.

B. Single photons

We very briefly recall the treatment for the space-time evolution of the system and the production mechanism of the photons in these calculations.

For the first set of calculations, we consider central collision of lead nuclei, corresponding to the conditions realized at the CERN SPS. We further assume that a thermally and chemically equilibrated quark-gluon plasma is produced in such collisions at the initial time τ_0 and use the assumption of isentropic expansion to estimate the initial temperature T_0. Thus we have the following:

$$\frac{2\pi^4}{45\zeta(3)} \frac{1}{A_T} \frac{dN}{dy} = 4aT_0^3\tau_0, \tag{16}$$

where A_T is the transverse area of the system, dN/dy is the particle rapidity density, and $a = 42.25\pi^2/90$ for a plasma of massless quarks (u, d, and s) and gluons. The number of flavors for this purpose is taken as $\approx$2.5 to account for the mass of the strange quarks. We further assume a rapid thermalization of the plasma, constrained by the uncertainty relation, $\tau_0 = 1/3T_0$. The initial energy density is taken as proportional to the "wounded-nucleon" distribution, appropriate for SPS energies.

The quark-hadron phase transition is assumed to take place at 180 MeV and the freeze-out at 100 MeV. The relevant hydrodynamic equations are solved under the assumption of boost-invariant longitudinal and azimuthally symmetric transverse expansion using the procedure discussed earlier [26] and integration performed over the history of evolution. This procedure is known to give an accurate description of hadronic spectra [27].

We have already indicated the sources for evaluation of rates of photon production from quark and hadronic matter.

As a first step we plot the rate of production of single photons due to all the reactions among mesons considered in Ref. [20] (see Fig. 1) at $T = 180$ MeV. We see that the inclusion of the form factor considerably reduces the rate for production of photons having higher energies, leaving the results for the lowest energy photons unaltered. We also give the results for

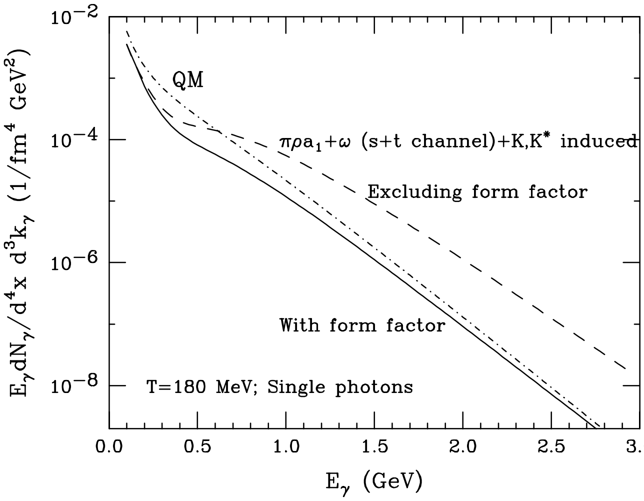

FIG. 1. The rate of emission of single photons from hadronic reactions considered in Ref. [20] at $T = 180$ MeV with the inclusion (solid curve) of hadronic vortex form factors.

the rates from quark matter at the same temperature. It should be remembered that photon intensity interferometry should be less sensitive to the specific details of the rates of production from quark and hadronic matter but should depend on their relative strengths.

To uniquely establish the importance of the mandatory requirement of including the form factors discussed in Ref. [20], we plot the sum of the production of thermal photons from the quark matter and the hadronic matter in a collision of lead nuclei at the CERN SPS, with the initial conditions discussed earlier (see Fig. 2). The data obtained by the WA98 experiments [13,28] and the production from the quark matter are also shown for a comparison. We immediately note that the exclusion of the form factors will considerably overestimate the production of single photons at higher k_T, as

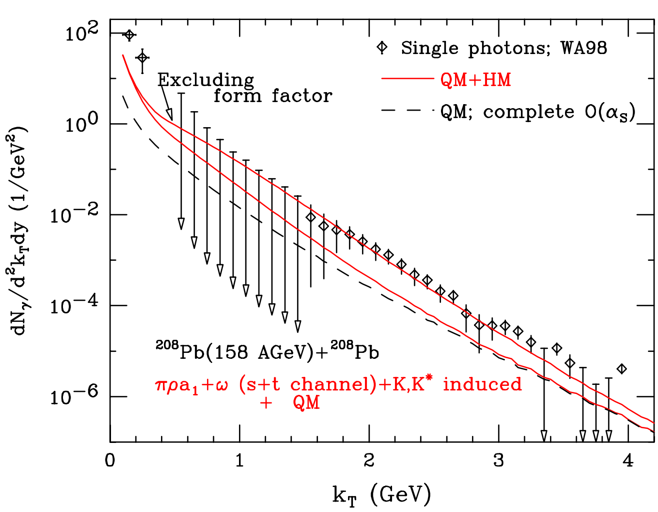

FIG. 2. (Color online) The production of thermal photons from the quark matter plus hadronic matter with and without the inclusion of the hadronic form factors in central collision of lead nuclei at CERN energies.

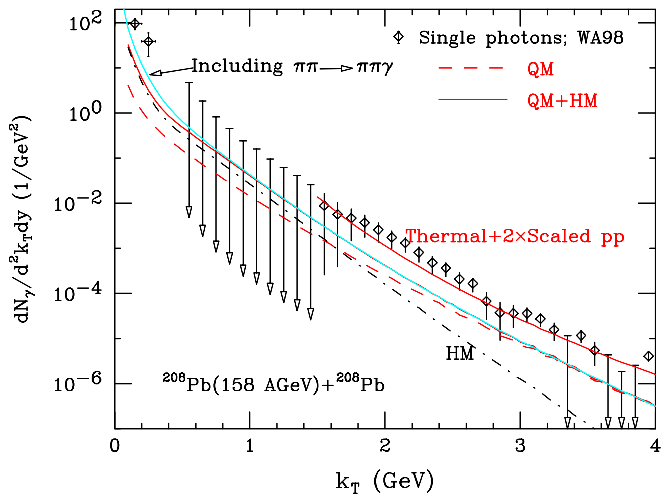

FIG. 3. (Color online) The production of single photons in collision of lead nuclei at CERN.

it is well established from several studies that up to half of the single photons measured by the WA98 experiment have their origin in the prompt-QCD process, which should be added to the thermal photons to get a quantitative description of the production of the single photons. We further note that the calculations underestimate the data by a factor of 7–9 at the lowest k_T reported recently [13] (see discussion later).

Finally, in Fig. 3 we show our results for the production of single photons, by summing the thermal contributions (with the inclusion of form factors while estimating the hadronic rates) and the prompt-QCD contribution. For the latter we use a parametrization of all the *pp* data [29] and scale it for collision of lead nuclei. We account for the intrinsic k_T of the partons by multiplying this scaled *pp* contribution by a factor of 2. We see that the sum of the thermal and prompt contributions provides a satisfactory explanation of the single photon spectrum at large k_T in the present model.

We also note that although an inclusion of the pionic bremsstrahlung improves the description of the data at the lower k_T marginally, the theoretical calculations are still well below the estimated (see later) experimental results. We can think of two possible reasons for this shortfall. One possibility is that there may be additional reactions, for example, involving baryons, which may contribute at low k_T. The other, and possibly more plausible, reason could be the neglect of the pionic chemical potential [33] in the present work, which can arise toward the end of hadronic phase [34]. Let us elaborate on this aspect. Recall that the pion-chemical potential (μ_π) may reach a value of about 60 MeV in collision of lead nuclei at SPS energies [35] when T drops to 100 MeV. This will imply an enhancement in the rate of production of photons from the hadronic matter by approximately a factor of $\exp(\mu_\pi/T)$ for every pion in the entrance channel and also some increase due to Bose enhancement factor in the exit channel, if there is a pion there. These aspects are under investigation. We may add, however, that the inclusion of these corrections should not drastically alter the following results, as the hadronic phase contributions already dominate the yield of photons at low k_T considerably.

The calculations at RHIC and LHC energies are performed in an analogous manner, taking dN/dy at $y = 0$ as 1260 and 5625 [32], respectively, for central collisions of gold (at RHIC) and lead (at LHC) nuclei. This procedure has been discussed repeatedly in the literature; the only difference we have in the present work is that we use the state-of-the-art results for the rates and a more complete equation of state for the hadronic matter.

C. Photon intensity interferometry at SPS energies

As a first step, let us investigate the differences in the patterns of the intensity interferometry of photons having high and low k_T. Thus we perform calculations by choosing k_{iT}, ψ_i, and y_i of photons such that two of the three momentum differences, $q_{\rm out}$, $q_{\rm side}$, and $q_{\rm long}$, vanish in turn. This theoretical construction helps us obtain the corresponding correlation functions.

We show typical results for $k_{1T} = 0.15$, 1.05, and 1.95 GeV in Fig. 4. We see a very interesting feature in the outward correlation function: as the transverse momentum of the photon increases, we see a clear emergence of two sources, one that has a smaller correlation radius (see larger $q_{\rm out}$) and another that has a larger correlation radius. This aspect has remained a recurring theme in results for photon intensity interferometry [10] and has its origin in the emissions from quark and the hadronic phases respectively, which have vastly differing source dimensions. It is interesting that this feature noted in early exploratory studies has survived the vast improvements in the rate calculations and the dynamics of evolution. The emergence of the two-source structure at larger k_T is facilitated by the decreasing contribution of the hadronic phase there. This feature is not expected for pion intensity correlations, as they leave the system mostly at the time of freeze-out only.

The sideward correlations show a decreasing radial dimension as k_T increases, as expected for a transversely expanding source, and look Gaussian in nature. The longitudinal correlation function also shows a similar behavior, as far as the variation of the correlation radius is concerned; however, it is definitely not Gaussian in form. Thus neither does the function [Eq. (9)] describe these variations satisfactorily, nor does the procedure of taking $R_i = 1/\tilde{q}_i$, where $\tilde{q}_i$ denotes the momentum difference where $C - 1$ has dropped by a factor of $1/e$ compared to its value at $q_i = 0$, do full justice to these distributions.

In view of the above, we have numerically evaluated

$$R_i^2 = 1/\langle q_i^2\rangle; \quad i = \text{out, side, long}, \tag{17}$$

where

$$\langle q_i^2\rangle = \frac{\int dq_i\, q_i^2\,(C-1)}{\int dq_i\,(C-1)} \tag{18}$$

and plotted it as a function of k_{1T} in Fig. 5.

The k_T variation of the $R_{\rm out}$ reveals a very rich structure. To fully appreciate the observations, we recall that the source of photons has a dependence $\sim T^\nu \exp(-E/T)$, where $\nu \geqslant 2$ and E is the energy of the photons in the rest frame of the fluid. This is very distinct from the corresponding dependence for pions, which varies as $\sim \exp(-E/T_f)$, where T_f is the freeze-out temperature and contributions accrue from locations along

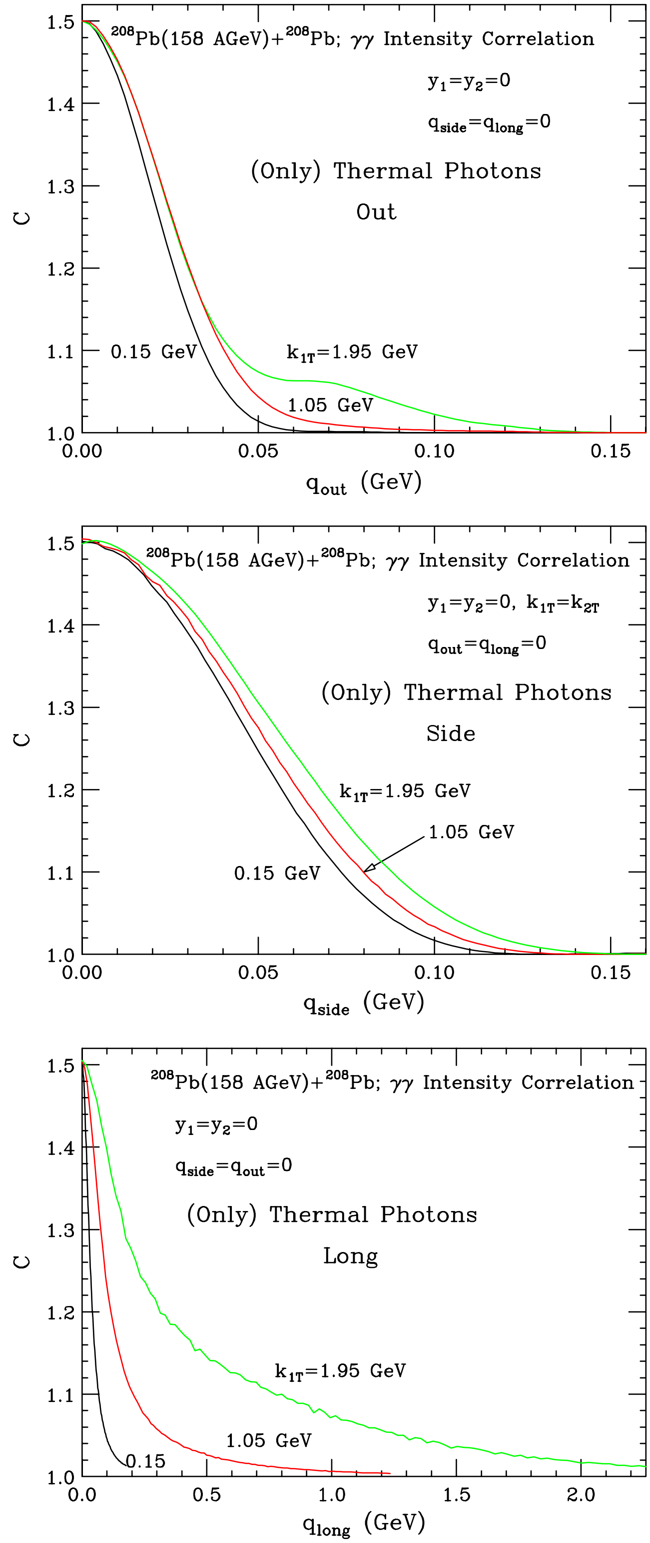

FIG. 4. (Color online) The outward, sideward, and longitudinal correlation function for direct photons produced in central collision of lead nuclei at CERN.

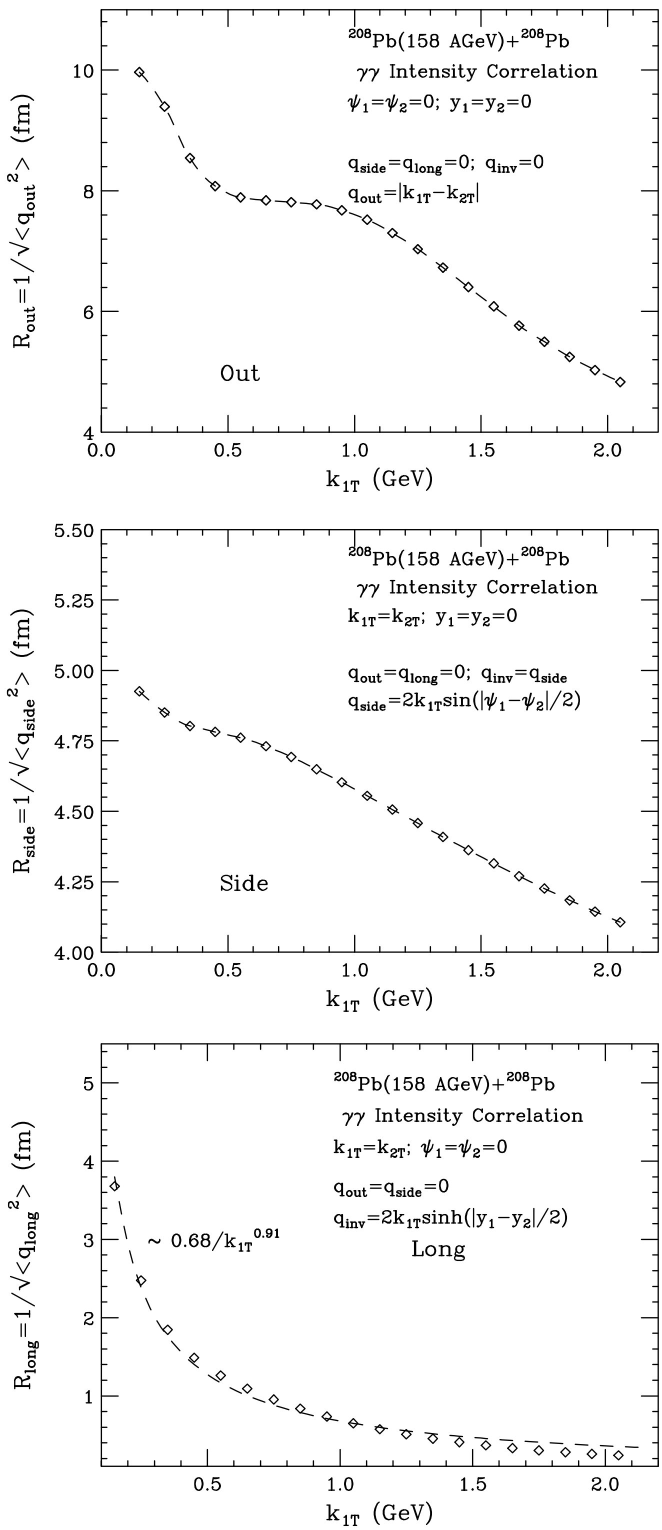

FIG. 5. The transverse-momentum dependence of outward, sideward, and longitudinal radii for photons from central collision of lead nuclei at CERN.

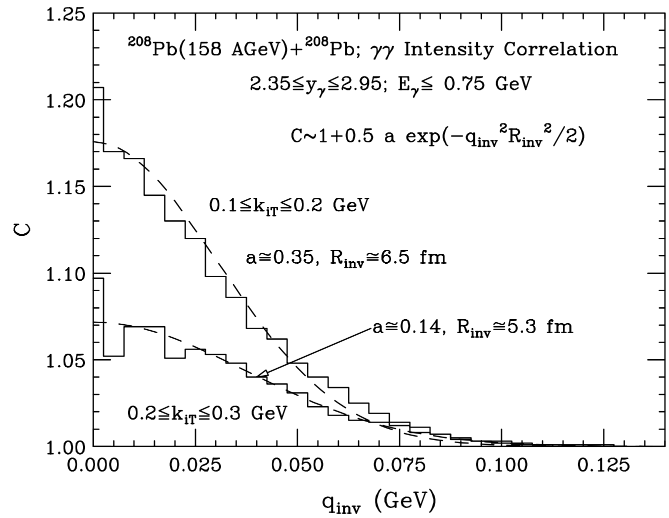

FIG. 6. The one dimensional correlation function for the kinematic window used in WA98 experiment [13], assuming a fully source and emitting only single photons. Central collision of lead nuclei is considered.

the freeze-out surface. The "reclining chair" behavior seen in this variation is an outcome of competition between a high temperature (limited to small radii and early times) and a large transverse flow velocity (predominant at large radii and late times) in producing photons having moderate k_T. Thus the large radial flow ensures that photons having a large k_T can also be produced at later times, thus enhancing the so-called duration of the source measured as $R_{\text{out}} - R_{\text{side}}$. It is shown that the the "seat" of this "reclining chair" gets narrower for RHIC energies and vanishes at LHC energies, as the increasing radial flow rapidly cools the system. A hint of this behavior is also present in the momentum dependence of the sideward correlation radius, which decreases almost linearly with k_T. The momentum dependence of the longitudinal correlation radius is completely different from the $1/\sqrt{m_T}$ expected for pions [30]. We see that R_{long} for photons is almost inversely proportional to the transverse momentum. We believe that these differences do arise because of the difference in the source function for photons and pions.

D. One-dimensional analysis and comparison to WA98 results

Let us now look at our results for the one-dimensional analysis of the correlation function in terms of the invariant momentum difference q_{inv} corresponding to the transverse momentum and rapidity window used in the WA98 experiment [13]. Our results with all the kinematic cuts are shown in Fig. 6. To simulate the probabilistic selection of photons, we first generated a sufficiently large number of photons according to the thermal distribution calculated by us earlier, in the transverse-momentum window of $k_T \epsilon$ [0.10, 2.5] GeV, and (randomly) distributed them uniformly over the azimuthal directions and the rapidity window corresponding to the experiment. Next we sampled pairs so that their average transverse momentum K_T was in the appropriate window. The correlation function was then calculated using the expression Eq. (1). The results were then averaged by binning in q_{inv}.

We see that our results are described to a reasonable accuracy by the following form:

$$C = 1 + 0.5\,a\,\exp\left[-q_{\text{inv}}^2 R_{\text{inv}}^2/2\right], \qquad (19)$$

where $R_{\rm inv} \approx 6.5$ fm for $0.10 \leqslant k_T \leqslant 0.20$ GeV and 5.3 fm for the transverse-momentum window $0.20 \leqslant k_T \leqslant 0.30$ GeV. To compare our results with the numbers quoted the WA98 experiment [13] we need to multiply their results with $\sqrt{2}$ as the fits discussed in that work do not include the factor of 2 in the exponential used in the present work [Eq. (10)]. Thus the corresponding experimental results for the $R_{\rm inv}$ obtained by the WA98 experiment are 8.34 ± 1.7 fm and 8.63 ± 2.0 fm, respectively. Although our predictions are in reasonable agreement with the "experimental" findings (within the errors), the experimental results are on the larger side. Before passing judgment on our results, recall that that the values quoted by the WA98 experiment are obtained by assigning values of 7.85 and 7.25 fm for the $R_{\rm side}$, 8.4 and 7.7 fm for the $R_{\rm long}$, and 8.5 fm for the $R_{\rm out}$ based on the values obtained for pion interferometry [31].

Our results for the $R_{\rm side}$ are 4.93 and 4.85 fm, for the $R_{\rm long}$ are 3.7 and 2.5 fm, and for the $R_{\rm out}$ are 9.97 and 9.39 fm, respectively, at $k_T = 0.15$ and 0.25 GeV respectively (see Fig. 5). Thus we feel that the results for the $R_{\rm inv}$ as well as the results for the single photons at low k_T quoted by the WA98 experiment (based on the value of the parameter a in the [Eq. (19) above] are model dependent and uncertain to the extent that they use correlation radii determined from pion interferometry. We also see that at least the sideward and the longitudinal radii for photon correlations are much smaller than the corresponding values for the pion interferometry.

A full three-dimensional determination of the correlation function will go a long way in getting reliable results and constraining the theoretical calculations discussed here.

E. Results for RHIC and LHC energies

We now present results for correlation functions for thermal photons at RHIC and LHC energies (see Figs. 7–10).

Looking at Fig. 7 for central collisions of gold nuclei at RHIC energies, we note that the two-source aspect in the outward correlation function becomes more clear, because of increased contributions from the (smaller but stronger) quark matter. Of course, it is known [17] that we shall have a large contribution of preequilibrium photons at larger k_T and this trend should continue at LHC energies. This will further enhance the contribution of the smaller-sized source, and the structure seen here will disappear.

The transverse-momentum dependence of the source sizes (Fig. 8) is similar in nature to what we see at SPS energies, though the seat of the reclining chair seen in the outward size gets narrower, as one would expect for a more rapidly expanding source. The longitudinal correlation length is again seen to decrease rapidly with the increase in the transverse momentum.

These trends continue at LHC energies (Fig. 9), and the transverse-momentum dependence of the source sizes (Fig. 10) becomes very pronounced. The so-called seat in the outward correlation vanishes completely, and all the radii decrease roughly as $1/k_T^{\alpha}$, with $\alpha \approx 0.27$ for the outward correlation, about 0.14 for the sideward correlation, and about 0.94 for the longitudinal correlation. We note that at none of the

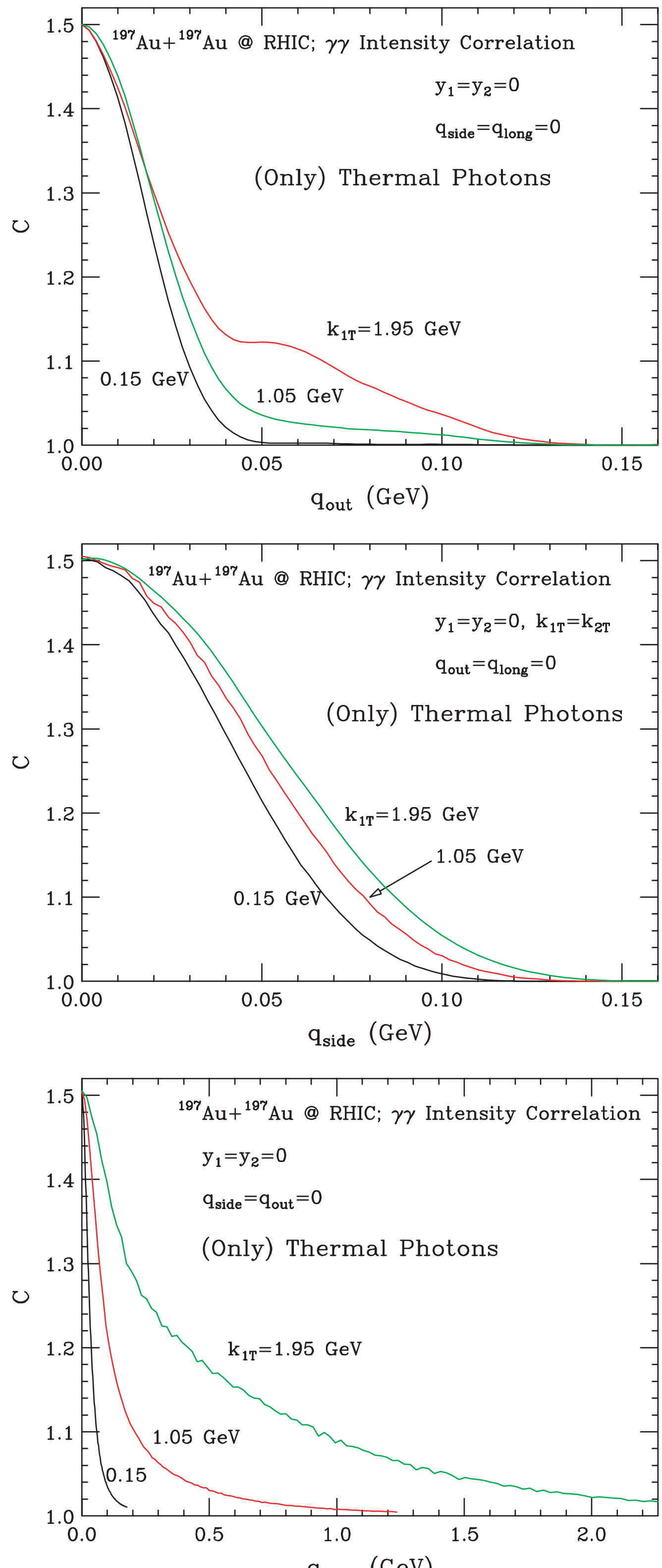

FIG. 7. (Color online) The outward, sideward, and longitudinal correlation function for thermal photons produced in central collision of gold nuclei at BNL RHIC.

energies, the longitudinal correlation function resembles a Gaussian, as normally assumed in parametrization. This is

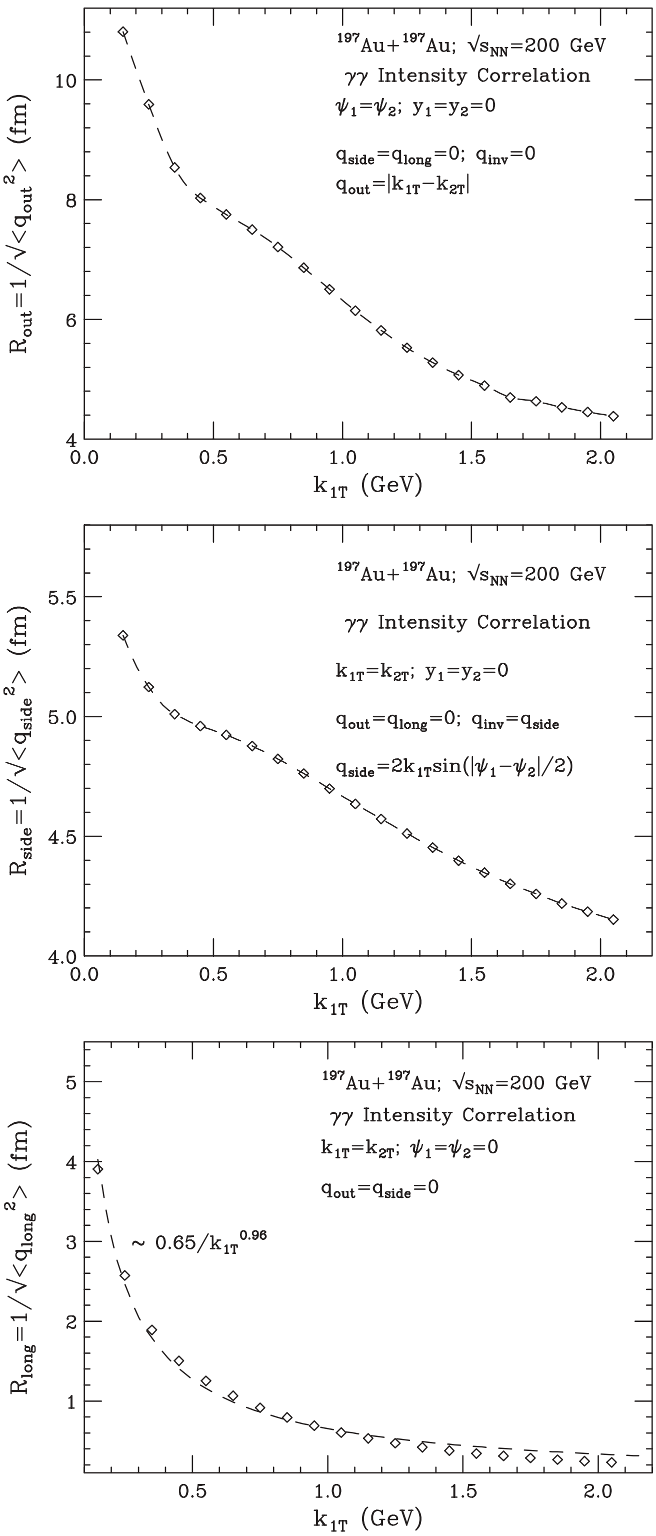

FIG. 8. The transverse-momentum dependence of outward, sideward, and longitudinal radii for thermal photons from central collision of gold nuclei at BNL RHIC.

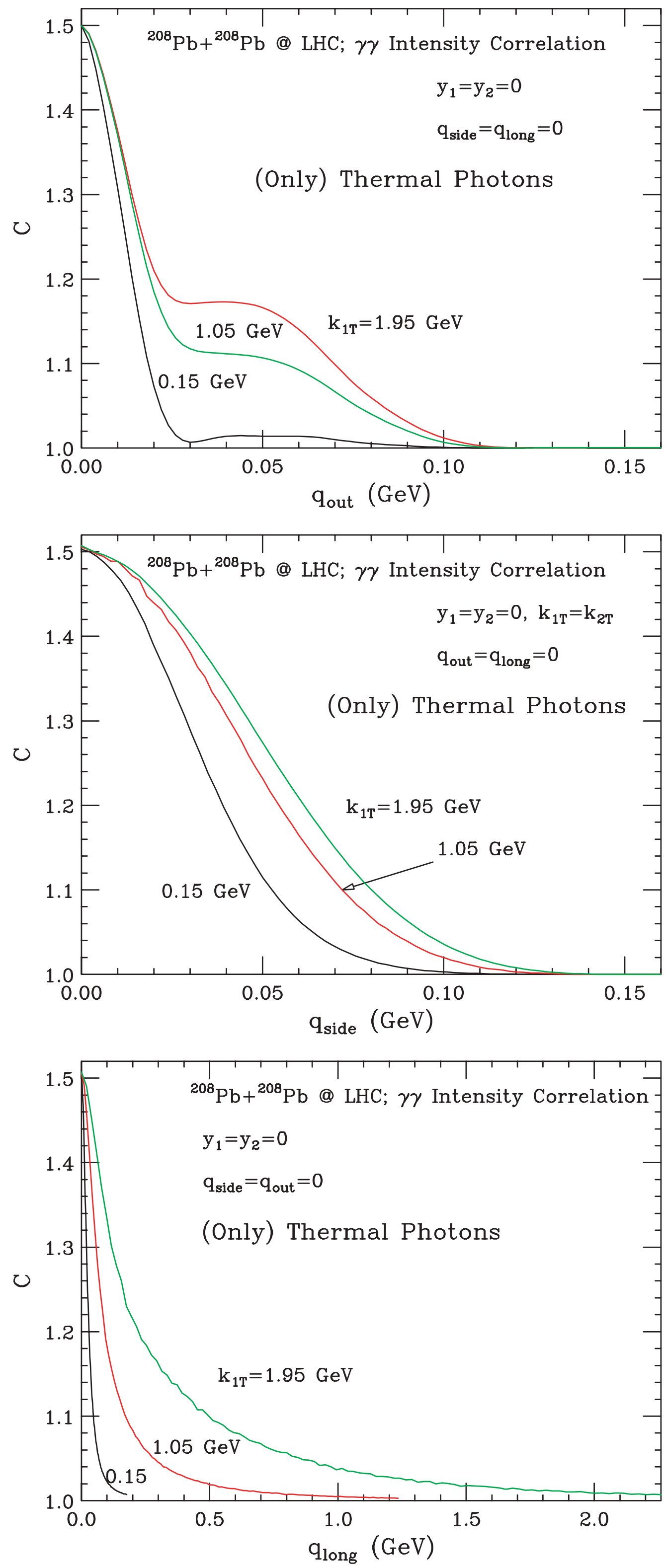

FIG. 9. (Color online) The outward, sideward, and longitudinal correlation function for thermal photons produced in central collision of lead nuclei at CERN LHC.

perhaps related to the emission of the photons from all the points in a longitudinally expanding system. In fact, a similar shape is seen for the longitudinal correlation function for pions "escaping" from the system [9] and for pions produced from partonic cascades [36].

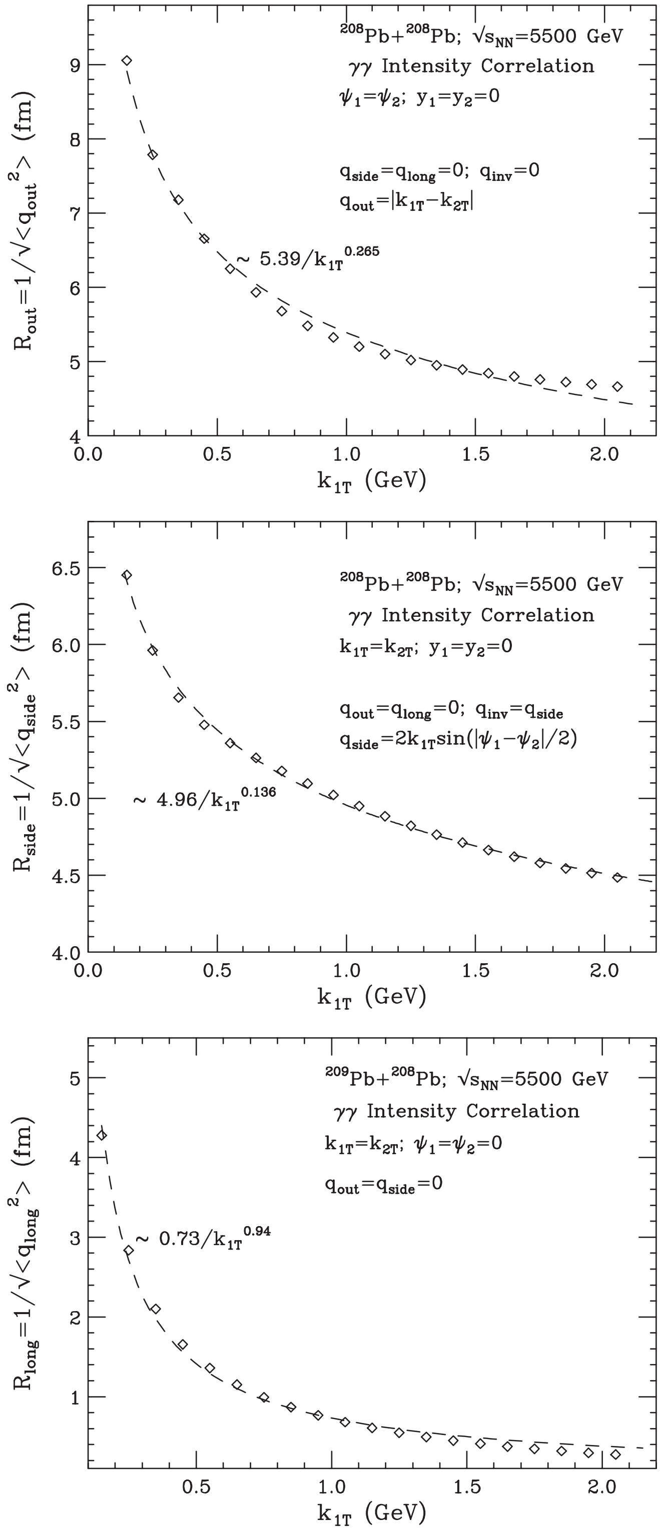

FIG. 10. The transverse-momentum dependence of outward, sideward, and longitudinal radii for thermal photons from central collision of lead nuclei at CERN LHC.

F. The lifetime of the source

The difference of the outward and sideward correlation radii is often associated with the lifetime of the source. To see the evolution of this parameter with the transverse momentum of the thermal photons we plot their difference as well as their ratios in Fig. 11 for the three systems studied in the present work. We see the "reclining chair" behavior for the SPS energies, and a more rapid decrease as we go from RHIC to LHC energies. Several interesting results emerge.

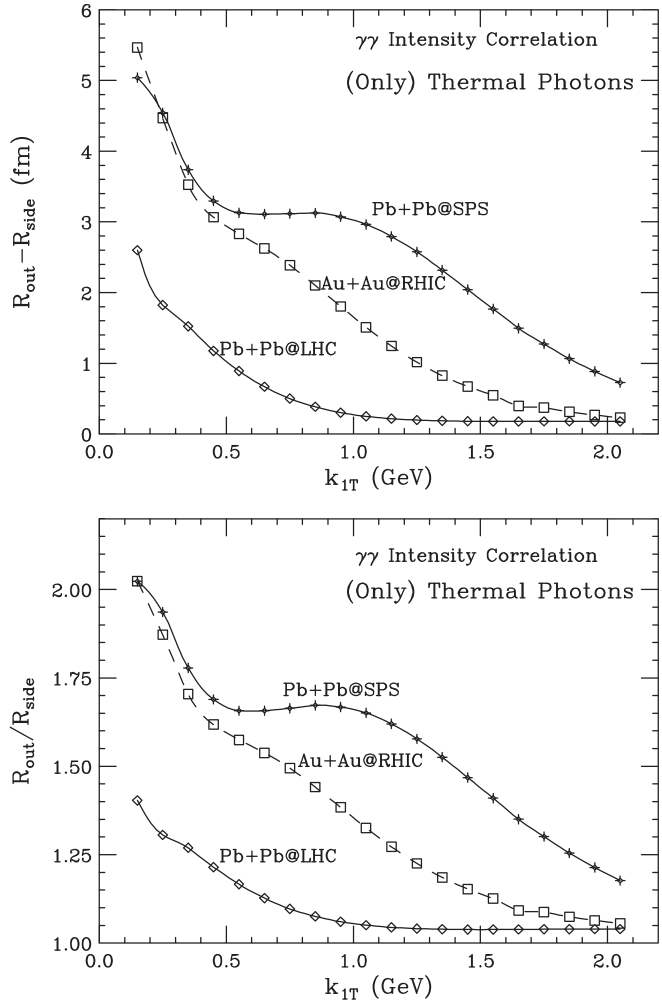

FIG. 11. The difference of outward and sideward correlation radii for thermal photons at CERN SPS, BNL RHIC, and CERN LHC.

First the difference ($R_{out} - R_{side}$) is nearly unaltered as we go from SPS to RHIC for photons having transverse momenta less than 0.5 GeV. We have already noted that the 'seat' of variation is caused by the competition between cooling due to expansion and the blue shift of the momenta due to the transverse flow. The more rapid decrease of the difference arises due to a more rapid cooling and an increased flow as the initial temperature rises. The same trend is seen for the ratios. An experimental confirmation or a refutation of these predictions will be very valuable.

III. DISCUSSION AND SUMMARY

Before summarizing, it may be of use to discuss some aspects that we have overlooked in the present work.

It should be of interest to examine the role of the reactions of the type $q + \bar{q} \to \gamma + \gamma$ or $q + g \to q + \gamma + \gamma$, for the intensity interferometry of photons. First, the quark matter contribution to single photons is already marginal for low transverse momenta, which is the subject of our focus here. Then we note that such photon pairs ("diphotons") will be produced at "a point" and not at different locations like a pair which comes from, say, $q + \bar{q} \to g + \gamma$ and $q + g \to q + \gamma + \gamma$. Moreover, their transverse-momentum difference is $\mathbf{q_T} \approx \mathbf{2k_T}$, where k_T is the transverse momentum of either photon. For calculations that we can treat perturbatively, this difference is at least an order of magnitude larger than the values of $\mathbf{q_T}$ where the correlation function has a significant value. Of course one could include them as a source of single photons. However, they are suppressed by a factor α_s/α compared to the $q + \bar{q} \to g + \gamma$, for example.

Let us also discuss the role of direct (QCD) photons to the k_T region under consideration here and for which we have experimental data at SPS energies from the WA98 experiment. In addition to the results given here, calculations have been reported by Dumitru *et al*. [37], where large values for intrinsic momenta of partons are used to exhaust the WA98 data beyond $k_T \approx 3$ GeV. The calculations exhaust only about 50% of the single photon yield at 2 GeV (as in the present work) and less than 20% of the yield at 1 GeV. Turbide *et al*. [20] have extrapolated the pQCD results down to zero transverse momenta (!) and find that the thermal photon yield is up to 4 orders of magnitude larger than the direct photon yield at lower transverse momenta. Parton cascade model calculations for SPS energies [14] show the dominance of quark-matter contributions beyond 3 GeV (including photons that could be termed thermal, from the quark matter) but a much smaller production from them at lower k_T. These considerations should convince us that indeed thermal photons dominate in the region of transverse momenta considered here.

We may also recall here the results of two works that address similar issues. Peressounko [11] has used hydrodynamics calculations with rates for the production of photons from quark matter and hadronic matter, along with the photons from the decay of pions to get λ, as well as the correlation radii for direct photons. Although the R_{out} and the R_{long} vs. k_T behavior estimated by him are similar to ours, his work shows a very unusual result: the R_{side} actually increases, and substantially, with increase in k_T. Considering that photons having large transverse momenta are produced very early in the collision when the transverse expansion has not yet set in, this is hard to understand. Similarly, the decrease in the values of the correlation radii, as k_T decreases below 0.5 GeV at SPS energies is also difficult to fathom. Other differences could be attributed to the more complete rates for photon production used in the present work.

The work of Renk [11] is interesting in that it uses a fireball description to model the collision at SPS and RHIC energies. The parameters of the fireball are adjusted to reproduce the pion spectra as well as the HBT radii for pions. The differences seen in the present work then arise because of the explicit appearance of a mixed phase here, during which the speed of sound is zero and acceleration of the expansion is stalled. Results of an investigation using different equations of state, including one that does not admit a mixed phase, will be published shortly.

In brief, the sideward, outward, and longitudinal correlation functions for intensity interferometry of thermal photons at SPS, RHIC, and LHC energies are calculated. The transverse-momentum dependence of these radii are very different from the corresponding results for pions, which are expected to decrease as $1/\sqrt{m_T}$ for all the components. The longitudinal correlations for the three energies are quite similar and may be indicative of boost invariance of the flow assumed in the work. The ratio $R_{\mathrm{out}}/R_{\mathrm{side}}$ at LHC decreases rapidly and approaches unity, because of an increase in R_{side}, because of expansion and decrease in R_{out}, which in turn is because of rapid cooling.

As the results are free from distortions because of final state interactions and uncertainties about the production vertexes, or the production mechanism of the particles under investigation, a confirmation or refutation of the findings will be very valuable.

It is also pointed out that the one-dimensional analysis in terms of the variable Q_{inv} may have only a limited use.

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[1] X. N. Wang, Phys. Rev. C **63**, 054902 (2001); M. Gyulassy, I. Vitev, and X. N. Wang, Phys. Rev. Lett. **86**, 2537 (2001).

[2] PHENIX Collaboration, K. Adox *et al*., Phys. Rev. Lett. **88**, 022301 (2002); STAR Collaboration, J. Adams *et al*., *ibid.* **91**, 172302 (2003).

[3] PHENIX Collaboration, S. S. Adler *et al*., Phys. Rev. Lett. **91**, 182301 (2003); STAR Collaboration, C. Adler *et al*., *ibid.* **90**, 032301 (2003); **89**, 132301 (2002); **87**, 182301 (2001).

[4] P. Huovinen, P. F. Kolb, U. Heinz, P. V. Ruuskanen, and S. A. Voloshin, Phys. Lett. **B503**, 58 (2001); D. Teaney, J. Lauret, and E. V. Shuryak, nucl-th/0110037.

[5] R. J. Fries, B. Müller, C. Nonaka, and S. A. Bass, Phys. Rev. Lett. **90**, 202303 (2003); V. Greco, C. M. Ko, and P. Levai, *ibid.* **90**, 202302 (2003); D. Molnar and S. A. Voloshin, *ibid.* **91**, 092301 (2003); C. Nonaka, R. J. Fries, and S. A. Bass, Phys. Lett. **B583**, 73 (2004).

[6] See, e.g., U. W. Heinz and P. F. Kolb, hep-ph/0204061.

[7] S. Raha and B. Sinha, Int. J. Mod. Phys. A **6**, 517 (1991).

[8] J. Kapusta and Y. Li, J. Phys. G: Nucl. Part. Phys. **30**, S1069 (2004).

[9] F. Grassi, Y. Hama, and T. Kodama, Phys. Lett. **B355**, 9 (1995), O. Socolowski Jr., F. Grassi, Y. Hama, and T. Kodama, hep-ph/0405181.

[10] D. K. Srivastava and J. I. Kapusta, Phys. Lett. **B307**, 1 (1993); Phys. Rev. C **48**, 1335 (1993); D. K. Srivastava, Phys. Rev. D **49**, 4523 (1994); D. K. Srivastava and C. Gale, Phys. Lett. **B319**,

407 (1994); D. K. Srivastava and J. I. Kapusta, Phys. Rev. C **50**, 505 (1994).

[11] A. Timmermann, M. Plümer, L. Razumov, and R. M. Weiner, Phys. Rev. C **50**, 3060 (1994); J. Pisut, N. Pisutova, and B. Tomasik, Phys. Lett. **B345**, 553 (1995); **B353**, 606(E) (1995); C. Slotta and U. W. Heinz, *ibid.* **B391**, 469 (1997); D. Peressounko, Phys. Rev. C **67**, 014905 (2003); J. Alam, B. Mohanty, P. Roy, S. Sarkar, and B. Sinha, *ibid.* **67**, 054902 (2003); T. Renk, hep-ph/040812.

[12] M. Marques *et al*., Phys. Rev. Lett. **73**, 34 (1995); Phys. Lett. **B349**, 30 (1995); Phys. Rep. **284**, 91 (1997).

[13] M. M. Aggarwal *et al*., WA98 Collaboration, Phys. Rev. Lett. **93**, 022301 (2004).

[14] S. A. Bass, B. Müller, and D. K. Srivastava, Phys. Rev. Lett. **90**, 082301 (2003); Phys. Rev. C **66**, 061902(R) (2002).

[15] R. J. Fries, B. Müller, and D. K. Srivastava, Phys. Rev. Lett. **90**, 132301 (2003).

[16] S. A. Bass, B. Müller, and D. K. Srivastava, Phys. Lett. **B551**, 277 (2003); Nucl. Phys. **A715**, 813 (2003); J. Phys. G **30**, S1283 (2004).

[17] S. A. Bass, B. Müller, and D. K. Srivastava, Phys. Rev. Lett. **93**, 162301 (2004).

[18] P. Arnold, G. D. Moore, and L. G. Yaffe, J. High Energy Phys. **112**, 009 (2001).

[19] P. Aurenche, F. Gelis, H. Zaraket, and R. Kobes, Phys. Rev. D **58**, 085003 (1998).

[20] S. Turbide, R. Rapp, and C. Gale, Phys. Rev. C **69**, 014903 (2004).

[21] P. K. Roy, S. Sarkar, D. K. Srivastava, and B. Sinha, Phys. Rev. C **53**, 2364 (1996); D. Pal and D. Mukhopadhyay, *ibid.* **67**, 057001 (2003).

[22] Yu. M. Sinyukov, V. A. Averchenkov, and B. Lörstad, Z. Phys. C **49**, 417 (1991).

[23] L. Ahle *et al*., E802 Collaboration, Nucl. Phys. **A610**, 213c (1996).

[24] H. Beker *et al.* NA44 Collaboration, Nucl. Phys. **A566**, 115c (1994).

[25] D. K. Srivastava (unpublished); WA98 Collaboration, D. Peressounko, J. Phys. G **30**, S1065 (2004).

[26] Jan-e. Alam, D. K. Srivastava, B. Sinha, and D. N. Basu, Phys. Rev. D **48**, 1117 (1993); J. Cleymans, K. Redlich, and D. K. Srivastava, Phys. Rev. C **55**, 1431 (1997); D. K. Srivastava, Eur. Phys. J. C **10**, 487 (1999); **20**, 399(E) (2001).

[27] D. K. Srivastava and B. Sinha, Phys. Rev. C **64**, 034902 (2001); D. K. Srivastava, *ibid.* **64**, 064901 (2001).

[28] WA98 Collaboration, M. M. Aggarwal *et al*., Phys. Rev. Lett. **85**, 3595 (2000).

[29] D. K. Srivastava, Eur. Phys. J. C **22**, 129 (2001).

[30] T. Csörgö, Phys. Lett. **B347**, 354 (1995).

[31] WA98 Collaboration, M. M. Aggarwal *et al*., Phys. Rev. C **67**, 014906 (2003).

[32] J. Kapusta, L. McLerran, and D. K. Srivastava, Phys. Lett. **B283**, 145 (1992).

[33] P. Koch, Phys. Lett. **B288**, 187 (1992).

[34] S. Gavin and P. V. Ruuskanen, Phys. Lett. **B262**, 326 (1991).

[35] B. Tomasik, U. A. Wiedemann, and U. W. Heinz, Heavy Ion Phys. **17**, 105 (2003).

[36] K. Geiger, J. Ellis, U. Heinz, and U. A. Wiedermann, Phys. Rev. D **61**, 054002 (2000). The author thanks U. Heinz for bringing this fact to his notice.

[37] A Dumitru, L. Frankfurt, L. Gerland, H. Stöcker, and M. Strikman, Phys. Rev. C **64**, 054909 (2001).