# Energy loss of leading partons in a thermal QCD medium

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We consider bremsstrahlung energy loss for hard partons traversing a quark-gluon plasma (QCP). Accounting correctly for the probabilistic nature of the energy loss, and making a leading-order-accurate treatment of bremsstrahlung, we find that the suppression of the spectrum is nearly flat, with the most suppression at energies $E \sim 30T$ (where $T$ is the QGP temperature), without the need for initial state effects such as shadowing and the Cronin effect. This flat pattern should also be observed at the LHC (Large Hadron Collider) out to an energy of $\sim$30 GeV.

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## I. INTRODUCTION

In highly relativistic heavy-ion collisions, production of hard partons precedes most other processes, simply because the time scale of the production is short, $\tau \sim 1/p_T$. In particular, the production of high-$p_T$ partons precedes the formation of a quark-gluon plasma (QGP). Therefore, the produced hard partons find themselves in an environment far different from a vacuum. The interaction between the parton and the environment influences the final spectrum of high-$p_T$ hadrons in a nontrivial way. In particular, if the QGP is very dense, we expect energy loss, leading to an energy-dependent suppression of the high-$p_T$ spectrum, a phenomenon called "jet quenching." The extent of jet quenching can be used to learn about the QGP [1]. Experimentally, the CERN SPS gave little evidence of jet quenching, but RHIC has seen a rather dramatic suppression of high-$p_T$ pions.

The results from RHIC experiments [2–5] are surprising in many ways. At $p_T \sim 3$ GeV, we already see a substantial suppression. Furthermore, the suppression continues more or less in constant fashion up to the highest $p_T$ measured so far ($\sim$10 GeV). The amount of suppression is also rather substantial; the ratio of high-$p_T$ events to the number expected based on proton-proton data is $R \simeq 1/5$.

Theoretically, it is well established that the main energy loss mechanism of a fast parton is the bremsstrahlung of gluons in the medium. The strength of the bremsstrahlung in the medium depends on a coherence effect called the Landau-Pomeranchuk-Migdal (LPM) effect. The first quantitative treatment of this effect (in QED) was by Migdal [6]; more recently it has been considered in QCD by several authors [7–10].

Jet quenching through bremsstrahlung energy loss has been considered by several authors before [7–9,11–17]. The purpose of this paper is to revisit the energy-loss calculation, particularly emphasizing two points. In many previous treatments, the (path-length-dependent) average energy loss is computed and applied to all hard partons. However, bremsstrahlung does not cause all particles of a given energy, traversing a given amount of material, to have the same energy loss. Rather, bremsstrahlung is a stochastic process, typically dominated by a few emissions; an ensemble of partons with the same initial energy, traversing the same path length of medium, will have a final distribution of energies almost as broad as the mean energy loss, as illustrated in Fig. 1. This is especially important when the initial energy distribution is a rapidly falling function of energy; the final distribution is dominated by the few partons that happen not to lose much energy. This has previously been emphasized by Baier *et al.* [12], who found that it significantly influences the final spectrum (see also [18]). To account for it, we directly evolve the spectrum of the partons as they undergo bremsstrahlung energy loss.

Second, most previous treatments have taken the LPM effect to be a parametrically large suppression. This is formally true whenever the parton and the emitted gluon are very energetic, that is, when $E_{\rm parton}, E_{\rm gluon} \gg T$. However, the LPM suppression (actual rate over Bethe-Heitler rate) is only a factor of $1/2$ at $E_{\rm gluon} = 10T$; for less energetic gluons the LPM suppression is small (and approximations that take it to be large are grossly in error). When the parton spectrum is steeply falling, the most important bremsstrahlung events have $E_{\rm gluon} \ll E_{\rm parton}$; so at realistic parton energies one cannot take the LPM effect to be *parametrically* large. Therefore, we will use the formalism developed by Arnold, Moore, and Yaffe (AMY) [19], which correctly treats the LPM effect [up to $O(g_s)$ corrections] at all energies $E_{\rm gluon} > g_s T$.

In this short paper we will concentrate on the qualitative features of the spectrum, specifically its shape (the $p_T$ dependence of $R$). The goal is to show that the trend seen in the data—that $R$ falls slightly and levels off, but does not rise with $p_T$ at least at accessible momenta—is the trend expected from bremsstrahlung energy loss. In previous literature it has appeared necessary to explain the data by invoking additional many-body effects such as the Cronin effect and shadowing [12,15,16,20] or by invoking strongly energy dependent energy loss [14,17]. In this study, we find that the interaction of a hard parton with a hot and dense medium alone is enough to yield the flat ratio. In particular, we predict that even at the LHC (Large Hadron Collider), the flat suppression pattern seen in RHIC experiments should persist up to $\sim$30 GeV. Since we are only after these qualitative features, we will simplify the treatment somewhat and consider a static thermal medium of quarks and gluons at a constant temperature $T > T_c$. To be more quantitative, we need to take into account the nuclear

 

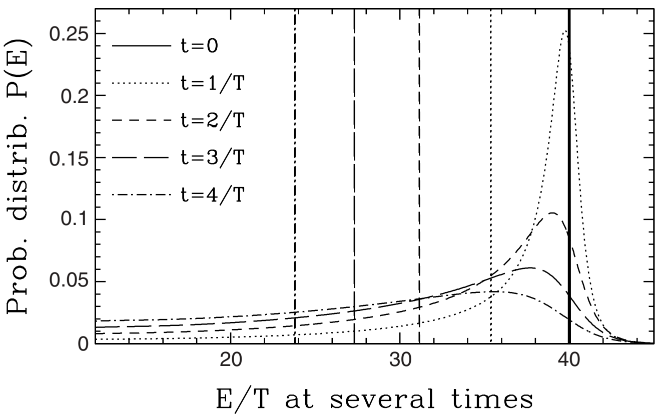


FIG. 1. Time evolution of an initially monoenergetic ensemble of quarks. For comparison, vertical bars show the energy of the quarks if we take the energy loss to be a steady process, determined by Eq. (10).

geometry and the expanding system and the hadronization of the hard partons after their traversal of the QGP. In particular, experiments measure $R$ for hadrons, not partons—though if the suppression of partons is flat, the suppression should remain flat after folding by the fragmentation function. We intend to return to these limitations of our treatment in a subsequent publication [21].

## II. BRIEF DESCRIPTION OF FORMALISM

We consider a small number of high-energy partons traversing a thermalized QGP. The high-energy partons are rare enough that their dominant interactions are with the thermal bath particles. We also work at leading nontrivial order in $\alpha_s$. This is obviously an idealization, but it is hard to see how to do better at present.

A parton traversing the QGP undergoes a series of soft scatterings with other constituents of the medium, with leading-order cross section

$$\sigma_{\rm soft} = C_s g_s^2 \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp)\,. \tag{1}$$

[The group Casimir $C_s$ is $C_f = \frac{4}{3}$ (quarks) or $C_A = 3$ (gluons).] Here $C(\mathbf{q}_\perp)$ is the differential rate to exchange transverse (to the parton) momentum $\mathbf{q}_\perp$. In a hot thermal medium, its value at leading order in $\alpha_s$ is [22]

$$C(\mathbf{q}_\perp) = \frac{m_D^2}{\mathbf{q}_\perp^2\left(\mathbf{q}_\perp^2 + m_D^2\right)}\,, \quad m_D^2 = \frac{g_s^2 T^2}{6}(2N_c + N_f)\,. \tag{2}$$

[The scattering cross section $\sigma_{\rm soft}$ is logarithmically infrared divergent. However, for all physical applications, the effect of very soft scatterings will be suppressed and this divergence is harmless. In our application this occurs because the integrand of Eq. (6) contains cancellations in the $\mathbf{q}_\perp \to 0$ limit.]

These frequent soft scatterings can induce collinear splitting (bremsstrahlung) of the parton. The time scale over which the parton and emitted gluon overlap, in the absence of other scatterings, is $\tau \sim xp/\mathbf{p}_\perp^2 \sim xp/g^2T^2$, with $x$ the momentum fraction of the gluon and $p_\perp$ the momentum component of the gluon perpendicular to the original parton. When $\sigma_{\rm soft}\tau$ is large, additional collisions typically occur while the parton and gluon are still coherent; this can frustrate the original emission.

This problem has been treated in the QCD context by BDMPS [8] and by Zakharov [10]. AMY have reanalyzed it with almost the same conclusions [19]; we outline the physics and summarize their results. The probability of emission of a gluon of momentum $k$ is schematically

$$\int dk_\perp \left| \langle p-k; k| \int_t J_\mu^a G^{\mu a}_{\rm hard}(t)\,|p\rangle \right|^2 , \tag{3}$$

with the effects of soft collisions implicitly included in the time evolution. The time for the $J \cdot G$ insertion in the amplitude and its conjugate differ; to get the emission rate we must integrate over the difference of these times,

$$\frac{d\Gamma}{dkdt} \sim \int dt' \langle p| J_\mu^a G_a^\mu(t')|p-k;k\rangle\langle p-k;k|J_\nu^b G_b^\nu(0)|p\rangle. \tag{4}$$

The problem is then to evolve $|p-k;k\rangle\langle p|$ between time 0 and time $t'$. Its evolution equation is similar to a Boltzmann equation, but with an extra phase-accumulating term because the states $|p\rangle$ and $|p-k;k\rangle$ have different energies. Performing the time integration (taking the medium to be uniform on the scale of the formation time), we obtain the complete expression for the bremsstrahlung rate:

$$\frac{d\Gamma(p,k)}{dkdt} = \frac{C_s g_s^2}{16\pi p^7}\frac{1}{1\pm e^{-k/T}}\frac{1}{1\pm e^{-(p-k)/T}} \times \left\{ \begin{array}{ll} \frac{1+(1-x)^2}{x^3(1-x)^2} & q \to qg \\ N_{\rm f}\frac{x^2+(1-x)^2}{x^2(1-x)^2} & g \to q\bar{q} \\ \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g \to gg \end{array} \right\} \times \int \frac{d^2\mathbf{h}}{(2\pi)^2} 2\mathbf{h}\cdot {\rm Re}\,\mathbf{F}(\mathbf{h},p,k)\,, \tag{5}$$

where $x \equiv k/p$ is the momentum fraction in the gluon (or either quark, for the case $g \to q\bar{q}$). The factors $1/(1 \pm e^{-k/T})$ are Bose stimulation or Pauli blocking factors for the final states, with $-$ for bosons and $+$ for fermions. The vector $\mathbf{h} \equiv \mathbf{p} \times \mathbf{k}$ determines how noncollinear the final state is; we treat it as parametrically $O(gT^2)$ and therefore small compared to $\mathbf{p}\cdot\mathbf{k}$. Therefore it can be taken as a two-dimensional vector in the transverse space. The vector $\mathbf{F}(\mathbf{h}, p, k)$ is the solution of an integral equation describing how $|p-k;k\rangle\langle p|$ evolves with time, owing to collisions and the energy difference of the two states:

$$\begin{aligned} 2\mathbf{h} = {} & i\delta E(\mathbf{h},p,k)\mathbf{F}(\mathbf{h}) + g^2\int\frac{d^2\mathbf{q}_\perp}{(2\pi)^2}C(\mathbf{q}_\perp) \\ & \times \{(C_s - C_A/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}-k\,\mathbf{q}_\perp)] \\ & + (C_A/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}+p\,\mathbf{q}_\perp)] \\ & + (C_A/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}-(p-k)\,\mathbf{q}_\perp)]\}, \end{aligned} \tag{6}$$

$$\delta E(\mathbf{h},p,k) = \frac{\mathbf{h}^2}{2pk(p-k)} + \frac{m_k^2}{2k} + \frac{m_{p-k}^2}{2(p-k)} - \frac{m_p^2}{2p}. \tag{7}$$

Here $m^2$ are the medium-induced thermal masses, equal to $m_D^2/2$ for a gluon and $C_f g_s^2T^2/4 = g_s^2T^2/3$ for a quark. For the case of $g \to q\bar{q}$, the $(C_s - C_A/2)$ term is the one with $\mathbf{F}(\mathbf{h} - p\,\mathbf{q}_\perp)$ rather than $\mathbf{F}(\mathbf{h} - k\,\mathbf{q}_\perp)$.

The treatment of BDMPS is equivalent [cf. Eq. (20) in Ref. [9]] except that it uses $(q^2+m_D^2)^2$ in the denominator of Eq. (2) and drops the mass terms in Eq. (7). These errors are not numerically significant. They also typically solve Eq. (6) in a large-$h$ approximation, valid for large $p/T$ and large $k/T$ but unreliable for $k \leqslant 10T$.

Next, we use these expressions to evolve the hard gluon distribution $P_g(p, t = 0)$ and the hard quark plus antiquark distribution $P_{q\bar{q}}(p, t = 0)$ with time, as they traverse the medium. The joint equations for $P_{q\bar{q}}$ and $P_g$ are

$$\begin{aligned}
\frac{dP_{q\bar{q}}(p)}{dt} &= \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,k)}{dkdt} - P_{q\bar{q}}(p)\frac{d\Gamma^q_{qg}(p,k)}{dkdt} \\
&\quad + 2P_g(p+k)\frac{d\Gamma^g_{q\bar{q}}(p+k,k)}{dkdt}, \\
\frac{dP_g(p)}{dt} &= \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,p)}{dpdt} + P_g(p+k) \\
&\quad \times \frac{d\Gamma^g_{gg}(p+k,k)}{dkdt} - P_g(p)\left(\frac{d\Gamma^g_{q\bar{q}}(p,k)}{dkdt}\right. \\
&\quad \left. + \frac{d\Gamma^g_{gg}(p,k)}{dkdt}\Theta(2k-p)\right),
\end{aligned} \tag{8}$$

where the $k$ integrals run from $-\infty$ to $\infty$. The integration range with $k < 0$ represents absorption of thermal gluons from the QGP; the range with $k > p$ represents annihilation against an antiquark from the QGP, of energy $(k-p)$. In writing Eq. (8), we used $d\Gamma^g_{gg}(p,k) = d\Gamma^g_{gg}(p, p-k)$ and similarly for $g \to q\bar{q}$; the $\Theta$ function in the loss term for $g \to gg$ prevents double counting of final states. Since bremsstrahlung energy loss involves only small $O(g_sT/p)$ changes to the directions of particles, these equations can be used for the momentum distributions in any particular direction. For a single initial hard particle, they can be viewed as Fokker-Planck equations for the evolution of the probability distribution of the particle energy and of accompanying gluons.

Our treatment implicitly assumes that the formation time is parametrically short compared to the time in the medium. This is formally justified if one considers the calculation as an expansion in $\alpha_s$. This is because the formation time is parametrically $\sim 1/(\alpha_s T)$, whereas the mean time between bremsstrahlung emissions is $\sim 1/(\alpha_s^2 T)$. Therefore, either the formation time is shorter than the time in the medium, or the mean number of emissions is $\ll 1$, in which case energy loss is not important. Whether the bremsstrahlung rate and the formation time are well separated in practice is another question; but it is related to the question of whether, at the realistic value of $\alpha_s$, any perturbative treatment has any validity. The strength of our approach is that it is systematic in treating all effects at leading order in $\alpha_s$.

## III. NUMERICAL EVALUATIONS

We evaluate Eq. (5) by the impact parameter space method of [23], on a grid of points $p, k$; Eq. (8) is then solved on this momentum space grid by a second-order Runge-Kutta algorithm. The errors are quadratic in the momentum discretization provided proper care is taken with the small-$k$ behavior of $d\Gamma/dkdt$. Discretization errors are under control and numerical costs are modest.

To have a realistic $p_T$ spectrum for both peripheral and central collisions, we need a realistic initial $p_T$ spectrum. Here we will use a parametrization taken from Ref. [13],

$$\left.\frac{dN}{d^2p_T}\right|_{\mathrm{init}} \approx \frac{C}{\left(p_0^2 + p_T^2\right)^5}, \tag{9}$$

as the initial $p_T$ spectrum of hard partons. Reference [13] fitted this to $\sqrt{s} = 200\,\mathrm{GeV}$ $p\bar{p}$ data and obtained $p_0 \sim 1.75$ GeV (consistent with the initial spectrum in Ref. [12]).

The $p_T$ spectrum used here is that for quarks. The gluon spectrum may be different; however, for traversal of a large amount of medium, it is the quark spectrum that is most important. This is because the gluon's frequency of bremsstrahlung emissions is at least twice that of the quarks, mostly because of the different color Casimir in Eq. (5). When a thick medium is traversed, this means that the gluon suppression is more than the square of the quark suppression; and if the suppression of quarks is large, the suppression of gluons will be almost complete. (When we fold in a realistic nuclear geometry, the quarks will be less dominant because of surface emission.)

It is also expected that at ultra-high $p_T$, the exponent in Eq. (9) will change. The suppression pattern for those ultra-high $p_T$ particles should reflect such a change (cf. Fig. 3). However, as Eq. (9) is rather good for observable energies at RHIC, it should be safe to assume its validity for our calculation.

## IV. RESULTS AND DISCUSSION

Consider first the evolution of a sample of quarks, all of momentum $p$ at $t = 0$. Since bremsstrahlung is a statistical process, the range of momenta broadens and the mean value falls. We show such an evolution in Fig. 1. The unit time step in the figure was $16/g^4T$, which is $\simeq 1/T$ for $\alpha_s = 1/3$. The figure shows clearly that the momentum distribution broadens as fast as the mean value falls. For comparison, we also plotted the particle energy we would obtain if we took the energy-loss process to be a steady $dE/dt$ energy loss, determined by

$$\frac{dE}{dt} = \int dk \frac{d\Gamma^q_{qg}(p,k)}{dkdt}, \tag{10}$$

in which case all the particles would have the same energy at any given time. We see that this is a bad description of the real energy distribution.

In Fig. 2, we show the ratio of the final and the initial quark spectra calculated in two ways. The solid and dashed curves are the result of directly solving Eq. (8), and the dotted and dash-dot curves are obtained by evolving particle energies according to Eq. (10). As the quarks move with almost the speed of light, we need not distinguish the time and the length. With a plasma temperature of about 0.4 GeV, the energy range shown corresponds to about 6–20 GeV, with a time range of 0.5–1.5 fm/$c$.

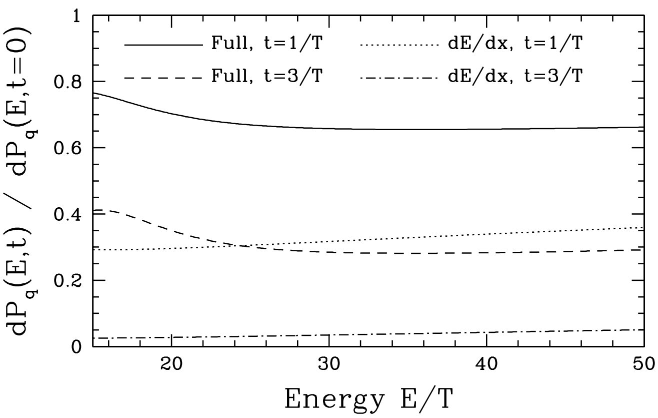


FIG. 2. The ratios of the final and the initial momentum spectra for quarks. The solid and dashed curves are calculated by solving Eq. (8) directly. The dotted and dash-dot curves are calculated by first calculating $dE/dt$.

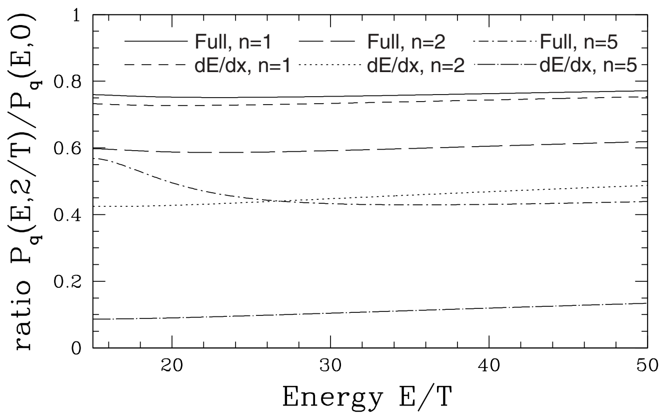


FIG. 3. The ratios of the final and the initial momentum spectra for quarks. The solid, long-dashed, and dash-dotted curves are calculated by solving Eq. (8) directly. The short-dashed, dotted, and long-dash dotted curves are calculated by first calculating $dE/dt$. The evolution time is $2/T$. The integer $n$ corresponds to having initial spectrum $f_0 \propto 1/(p_0^2 + p^2)^n$.

The times are chosen to illustrate the evolution of the spectrum. Obviously, $t = 0.5$ fm is too short to yield a large enough $R$ to explain the data. For our adopted value of $T$, the time of 1.5 fm almost gives enough suppression. This may sound too short for the RHIC context, but this is because we took a static plasma at a high temperature and did not yet take into account the nuclear geometry. The goal of this paper is not so much to predict the size of $R$, which would require a realistic hydro treatment, but to show how the flat pattern of the $R_{AA}$ may arise from energy loss in a hot QGP.

From Fig. 2, it is obvious that the $dE/dt$ method is not a good approximation. When particles with a steeply falling energy spectrum lose energy, the final spectrum is typically dominated by those few particles that happened not to lose much energy. To give an extreme example, consider a spectrum falling as $p^{-10}$, and compare the effect of the two energy-loss mechanisms. In the first, half the particles lose all their energy and half are unaffected; in the second, all particles lose exactly half their energy. In the former case, the spectrum is reduced by a factor of 1/2; in the latter case, it is reduced by $(1/2)^{10} \sim 1/1000$. Yet the mean energy loss of the two hypothetical processes is the same. The real case is somewhere in between. Almost all particles lose energy, but a few particles happen to suffer fairly small energy loss, whereas others suffer large energy loss. This causes less suppression of the spectrum than if the energy-loss process were uniform.

This effect becomes more important as the spectrum of initial particle energies becomes steeper. We illustrate this in Fig. 3, which shows the ratio $P_{q\bar{q}}(E, t = 2/T)/P_{q\bar{q}}(E, t = 0)$ for initial distributions of the form $C/(p_0^2 + p^2)^n$. For small values of $n$, the $dE/dt$ method is not that bad. However, as $n$ grows, the result of the $dE/dt$ calculation deviates more and more from the correct one.

Also note that the energy dependence of the suppression factor is different from the $dE/dt$ prediction; at $p/T \sim 15$ the suppression is actually becoming larger as $p$ increases, though eventually this behavior turns over. This $p$ dependence is similar to what is observed experimentally in the ratio plot of $AA$ and $pp$ high-$p_T$ spectra. As noticed before [20,24] the previous LPM effect calculations seem to be incompatible with the current RHIC data essentially because these calculations show rising ratios whereas the data show a flat or slightly decreasing ratio. What we show here is that this discrepancy does not mean that the basic energy-loss mechanism (LPM suppressed bremsstrahlung) is wrong. Instead, the failure of most previous approximations is largely due to making the $dE/dt$ approximation. Part of the explanation also lies in the treatment of bremsstrahlung made here, in which the LPM effect is not taken as parametrically large, and absorption as well as radiation is allowed.

## V. CONCLUSION

In this work, we have demonstrated that LPM-suppressed bremsstrahlung alone can in fact explain the qualitative features of the high-$p_T$ experimental data. This is because of two features of our treatment. First, we determine the

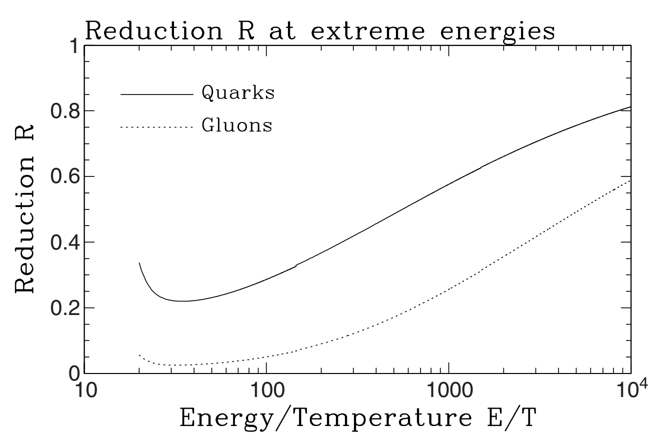


FIG. 4. The ratios of the final and the initial momentum spectra for quarks and gluons up to the energy of $E = 10^4\,T$. The parameters used are $\alpha_s = 0.315$ and $T = 400$ MeV.

distribution of final energies an energetic parton can end up with, rather than using the mean. This is important when the initial spectrum is steeply falling. Second, we do not assume that the LPM effect is *parametrically* large from the beginning but use a treatment that handles the transition between Bethe-Heitler and LPM correctly. Since this transition occurs at emitted gluon energies $\sim 10T$, such a treatment is necessary. We find that the ratio $R$ of the data to the *pp*-based expectations at first falls with energy, reaches a minimum around $30T$, and then rises slowly thereafter. In previous studies, explaining the flat suppression pattern required additional nuclear effects such as shadowing or a strongly energy-dependent energy loss, which was motivated solely by phenomenology.

We have checked that $R$ approaches 1 at very large $p_T$. However, the trend becomes visible only after $p_T$ of a few hundred $T$ (see Fig. 4). For the LHC, with a reasonable estimate of $T \simeq 0.5$ GeV, this indicates that even at $p_T = 25$ GeV, the suppression pattern will be nearly flat. This contrasts with the previous finding of Gyulassy and Vitev [16], who claim that the Cronin effect and nuclear shadowing are essential in describing the flat $R$ from RHIC but that these effects are small at the LHC, which will therefore exhibit rising $R$ at large $p_T$.

To turn our observations into a quantitative prediction for the $p_T$ spectrum will require a proper inclusion of the nuclear geometry, of the expansion and cooling of the QGP medium, and of hadronization. In general, the $p_T$-dependent suppression of the partons and of the hadrons are different, because the partonic function is folded by the fragmentation functions to get the hadronic spectrum. However, if the suppression of the partonic spectrum is flat, then the suppression of the hadronic spectrum must also be flat, since a constant multiplicative shift before the folding will factor out as the same multiplicative shift in the result. This is why we do not expect hadronization to modify the flatness of the suppression. Calculation of the ratio including the above effects has now been carried out and will be reported in a future publication [21].

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