Ratio of the proton electromagnetic form factors from meson dressing

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The dressed *K*-matrix model, developed previously for low- and intermediate-energy Compton scattering, is generalized to calculate the photon-nucleon vertex with a virtual photon and an off-shell nucleon. This model was used to compute the ratio of the proton electromagnetic form factors. The calculated ratio is in excellent agreement with the ratio measured in polarized electron-proton scattering.

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I. INTRODUCTION

Results of the recent experiments [1] on polarized electron-proton scattering and their apparent disagreement [2] with the electromagnetic form factors (EM FFs) extracted from Rosenbluth cross-section measurements indicate that much remains to be learned about nucleon EM interactions. While a definitive understanding of the nucleon EM FFs is still lacking, several theoretical models [3–8] are able to describe their momentum-transfer dependence consistently with the JLab experiments [1].

Since the ultimate goal is to reach a coherent understanding of all aspects of the nucleon EM interactions, important insights can be gained by describing as many different photon-nucleon reactions as possible in a unified theoretical approach. The dressed *K*-matrix model (DKM) [9–11] has been developed for a description of real Compton scattering in a wide energy region both below and above the pion production threshold. The central element of this model is a nonperturbative nucleon dressing based on the iterative use of dispersion relations, thus adding analyticity constraints to the properties of relativistic covariance, unitarity, crossing symmetry, and gauge invariance of the usual *K*-matrix approaches.

The parameters of the model are not entirely free: the convergence requirement of the dressing procedure imposes constraints on the allowed range of these parameters while introducing an interdependence among them. The thus-constrained parameters were then completely fixed by a fit to Compton cross sections and pion-nucleon (πN) phase shifts at intermediate energies [10]. With all the parameters fully fixed, low-energy observables—such as the nucleon polarizabilities [10], pion-nucleon scattering lengths, and Σ term [11]—were then calculated and shown to agree with experimental data. The main reason for this success is that important analyticity (causality) constraints are implemented in DKM by dressing the γNN, πNN, and $\pi N\Delta$ vertices and the nucleon and Δ propagators with meson loops up to infinite order. The extent to which analyticity is fulfilled can be quantified by comparing the low-energy amplitudes with corresponding sum rules, both evaluated within the same model. Within DKM, such a comparison was made for the Baldin-Lapidus and Gerasimov-Drell-Hearn sum rules vs. the nucleon polarizabilities [12] and for the Adler-Weisberger and Goldberger-Miyazawa-Oehme sum rules vs. the near-threshold πN amplitude [13]. We generally found good agreement between the low-energy and sum-rule evaluations.

In this report we extend the dressing procedure to the case of virtual spacelike photons. The resulting dressed photon-nucleon (γNN) vertex has six invariant functions depending on two invariant variables: the four-momentum squared of the photon and of one of the nucleons, the other nucleon being on-shell. By putting both nucleons on the mass shell we calculate the momentum-transfer dependence of the ratio of the proton EM FFs. We find good agreement of our calculation with the JLab polarization-transfer measurements [1].

The crucial point of this result is that the momentum-transfer dependence of the ratio of the FFs is calculated here with the *same* parameters as used in Refs. [10–13] to describe the Compton and πN amplitudes at low and intermediate energies. This shows that the dynamics of DKM captures features which are important in a wide kinematical range relevant to the photon-nucleon interactions.

II. FORM FACTORS IN THE DRESSED γNN VERTEX

The most general Lorentz-covariant structure of the γNN vertex with an incoming off-shell photon (four-momentum q) and an incoming off-shell nucleon (four-momentum p) can be written as [14]

$$\Gamma^\mu(p,q) = \sum_{r=\pm}\left(\gamma^\mu F_1^r(p^2,q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m}F_2^r(p^2,q^2) + \frac{q^\mu}{m}F_3^r(p^2,q^2)\right)\Lambda_r(\not p), \tag{1}$$

where $\Lambda_\pm(\not p) = (\pm\not p + m)/(2m)$ are the nucleon positive- and negative-energy projection operators,[1] and the six invariant

[1]Throughout the paper we use the conventions and definitions of Ref. [15].

functions $F_i^r(p^2, q^2)$ $(i = \{1, 2, 3\}, r = \pm)$ are called half-off-shell FFs. The outgoing nucleon is on the mass shell, i.e., its four-momentum $p' = p + q$ obeys $p'^2 = m^2$. We consider protons interacting with real or virtual spacelike photons, i.e., $q^2 \leqslant 0$.

The usual Dirac and Pauli FFs, denoted as $F_D(q^2)$ and $F_P(q^2)$, respectively, are obtained by putting both nucleons on the mass shell in Eq. (1):

$$F_{D,P}(q^2) = \lim_{p^2 \to m^2} F_{1,2}^+(p^2, q^2). \tag{2}$$

When the two nucleons are on-shell, $F_{1,2,3}^-(m^2, q^2)$ do not enter the vertex since $\Lambda_-(\not{p} = m) = 0$; in addition, space-time reflection invariance requires [14] that

$$F_3^+(m^2, q^2) = 0. \tag{3}$$

$F_{D,P}(q^2)$ are related to the Sachs (electric and magnetic) FFs $G_{E,M}(q^2)$ by

$$\begin{aligned} G_E(q^2) &= F_D(q^2) + \frac{q^2}{4m^2} F_P(q^2), \\ G_M(q^2) &= F_D(q^2) + F_P(q^2). \end{aligned} \tag{4}$$

The γNN vertex Eq. (1) is calculated by dressing a bare vertex with an infinite number of meson loops including pions and vector isoscalar mesons. The πNN and $\pi N\Delta$ vertices and the nucleon and Δ propagators, which enter in the loop integrals for the γNN vertex, are also dressed nonperturbatively with pions and ρ and σ mesons. The detailed description of the dressing technique for the γNN vertex (with real photons) can be found in Ref. [9], and for the πNN, $\pi N\Delta$ vertices and the propagators in Ref. [11]. This dressing is part of DKM [10] where it amounts to restoring the principal-value parts of loop contributions to the Compton and πN amplitudes.

In the present work we have generalized the dressing procedure of Ref. [9] from real to virtual spacelike photons in Eq. (1). Thus we have calculated the six invariant functions $F_i^r(p^2, q^2)$ for $-\infty < p^2 < \infty$ and $q^2 \leqslant 0$, from which the momentum-transfer dependence of the EM FFs has been obtained according to Eqs. (2) and (4). For brevity, we will focus only on the important new features related to the off-shellness of the photon.

We write the bare vertex as

$$\gamma^\mu + i\frac{\sigma^{\mu\nu}q_\nu}{2m}\kappa_B + \frac{q^\mu}{m}h_B(q^2). \tag{5}$$

The renormalization consists in choosing the bare constant κ_B such that the dressed vertex with all particles on-shell reproduces the physical anomalous magnetic moment of the proton, i.e., $F_2^+(m^2, 0) = \kappa_p = \mu_p - 1 = 1.79$, and adjusting $h_B(q^2)$ so that the calculated function $F_3^+(p^2, q^2)$ obeys Eq. (3).

III. GAUGE INVARIANCE OF THE MODEL

The Ward-Takahashi identity (WTI) is a consequence of gauge invariance of the theory [16]. It relates the γNN vertex Eq. (1) to the propagator $S(\not{p})$ of the off-shell proton: $q_\mu \Gamma^\mu(p, q) = -S^{-1}(\not{p})$. Thus, the WTI dictates that the half-off-shell FFs in Eq. (1) must obey the relations

$$\begin{aligned} F_1^+(p^2, q^2) = \frac{m}{p^2 - m^2}\Bigg(&\frac{\alpha(p^2)(p^2 + m^2)}{m} - 2\alpha(p^2)\xi(p^2) \\ &+ \frac{q^2}{2m^3}\left[(3m^2 + p^2)F_3^+(p^2, q^2)\right. \\ &\left. + (m^2 - p^2)F_3^-(p^2, q^2)\right]\Bigg), \end{aligned} \tag{6}$$

$$F_1^-(p^2, q^2) = \alpha(p^2) + \frac{q^2}{2m^2}\left[F_3^+(p^2, q^2) - F_3^-(p^2, q^2)\right], \tag{7}$$

where the self-energy functions $\alpha(p^2)$ and $\xi(p^2)$ enter into the dressed nucleon propagator $S(\not{p}) = [\alpha(p^2)(\not{p} - \xi(p^2))]^{-1}$ which was calculated together with the dressed Δ propagator and πNN and $\pi N\Delta$ vertices [11]. The renormalization conditions $S^{-1}(m) = 0$ and $\operatorname{Res} S(m) = 1$ can be explicitly written as

$$\alpha(m^2)(m - \xi(m^2)) = 0, \tag{8}$$

$$\begin{aligned} &2m(m - \xi(m^2))\left.\frac{d\alpha(p^2)}{dp^2}\right|_{p^2=m^2} \\ &\quad - \alpha(m^2)\left(2m\left.\frac{d\xi(p^2)}{dp^2}\right|_{p^2=m^2} - 1\right) = 1. \end{aligned} \tag{9}$$

On expanding Eq. (6) in powers of p^2 around m^2 and using Eqs. (2), (3), (8), and (9), we obtain the WTI for the Dirac FF of the proton as

$$F_D(q^2) = 1 + q^2\left(2\left.\frac{\partial F_3^+(p^2, q^2)}{\partial p^2}\right|_{p^2=m^2} - \frac{F_3^-(m^2, q^2)}{2m^2}\right), \tag{10}$$

yielding the familiar constraint $F_D(q^2 = 0) = 1$ for an on-shell vertex with a real photon. Note that we do not impose Eqs. (6) and (7) "by hand" at any stage of the dressing; nevertheless, the resulting vertex does obey these constraints of the WTI because of the gauge invariance of the model.

IV. RESULTS OF THE CALCULATION

The ratio of the calculated proton EM FFs is shown in Fig. 1 as a function of the photon momentum squared, $Q^2 \equiv -q^2 \geqslant 0$, together with the recent JLab measurements [1].

We found that in order to obtain accurate agreement with experiment over the wide range of momenta-transferred Q^2, the coupling of the vector isoscalar particle to the nucleon has to be different from the ωNN coupling given in Ref. [10]. While the part of the vertex proportional to γ_μ is comparable to that of the ωNN vertex (instead of $g_\omega = 12$ as in [10], now we use $g_{\text{vect}} = 8$), the part proportional to $\sigma_{\mu\nu}q^\nu$ has to be quite

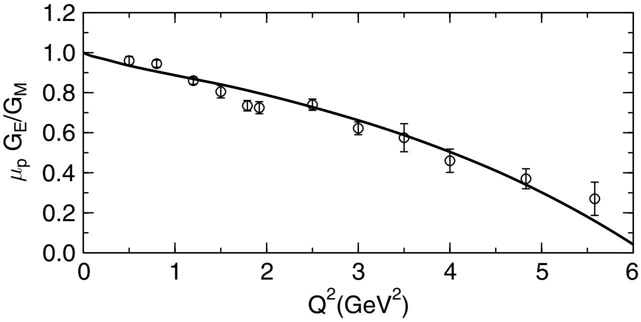

FIG. 1. Momentum-transfer dependence of the ratio of the proton EM FFs Eq. (4) ($Q^2 = -q^2$). The present calculation is compared with the experimental results from polarized electron-proton scattering [1].

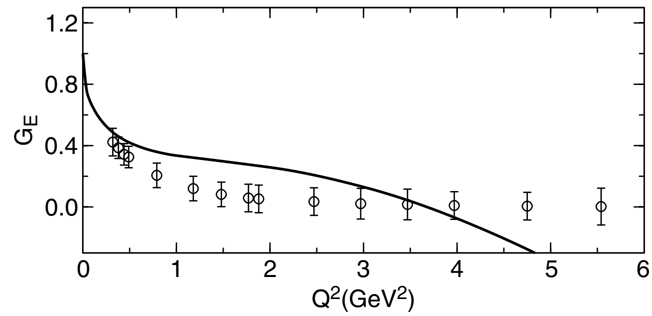

FIG. 2. Calculated electric FF of the proton is compared with the data analysis of Ref. [17].

large (we need $\kappa_{\text{vect}} = 60$ instead of $\kappa_\omega = -0.8$).[2] The other coupling constants are the *same* as in [10], i.e., as fixed in the description of the πN and Compton scattering processes. It is important that even without the contribution of the vector isoscalar particle, we find the slope of the ratio of the FFs to be negative at $Q^2 = 0$, although in that case it is too steep. Including the vector particle in the dressing makes the slope less steep. A negative slope of the ratio implies that the electric mean-square radius of the proton is larger that the magnetic mean-square radius normalized to the total magnetic moment; i.e., if

$$\left.\frac{d}{dQ^2}\frac{G_E(Q^2)}{G_M(Q^2)}\right|_{Q^2=0} < 0, \tag{11}$$

then taking into account that $G_E(Q^2 = 0) = 1$, $G_M(Q^2 = 0) = \mu_p$, and using the usual definitions of the mean-square radii

$$\langle r^2_{E,M}\rangle = -6\left.\frac{dG_{E,M}(Q^2)}{dQ^2}\right|_{Q^2=0}, \tag{12}$$

we find

$$\langle r^2_E\rangle > \frac{\langle r^2_M\rangle}{\mu_p}. \tag{13}$$

Note that the traditional one-photon exchange analyses of the Rosenbluth cross sections would yield $\mu_p\, G_E(Q^2)/G_M(Q^2) = \text{const} \approx 1$ (see, e.g., Ref. [2]), which would imply equal electric and (normalized) magnetic radii, in sharp contrast to Eq. (13). We would also like to point out that the relation in Eq. (13) is consistent with the well-known fact that the electric polarizability of the proton, $\alpha_E \approx 12 \times 10^{-4}\ \text{fm}^3$, is larger than its magnetic polarizability, $\beta_M \approx 2 \times 10^{-4}\ \text{fm}^3$, that is,

$$\alpha_E > \beta_M. \tag{14}$$

Thus, one might speculate that the proton behaves as a "larger" and hence more "deformable" object in the presence of an electric field than in the presence of a magnetic field. Since the mean-square radii are extracted from the momentum-transfer dependence of the FFs while the polarizabilities come from Compton scattering, the dressing procedure of DKM could provide a theoretical framework for understanding this (possibly fortuitous) similarity between Eqs. (13) and (14).

Even though the ratio of the calculated FFs agrees with experiment very well as shown in Fig. 1, the model fails in describing $G_E(Q^2)$ and $G_M(Q^2)$ separately. This is demonstrated in Fig. 2 for the electric FF. A possible remedy could be to use an appropriate FF in the bare γNN vertex, which would introduce additional cutoff parameters. We have not done so in the present calculation since our aim is to keep the same model parameters as fixed in describing Compton and πN scattering. In this way we impose a stringent dynamical constraint on the model.

The dependence of the calculated γNN vertex on the momentum squared p^2 of the off-shell proton is illustrated in Fig. 3, where we show the p^2 dependence of the ratio of generalized (half-off-shell) Sachs FFs, which we define as

$$\begin{aligned} G_E^{\text{gen}}(p^2, q^2) &= F_1^+(p^2, q^2) + \frac{q^2}{4m^2}F_2^+(p^2, q^2), \\ G_M^{\text{gen}}(p^2, q^2) &= F_1^+(p^2, q^2) + F_2^+(p^2, q^2). \end{aligned} \tag{15}$$

The usual Sachs FFs (4) are obtained from (15) by taking the limit $p^2 \to m^2$ in accordance with Eq. (2).

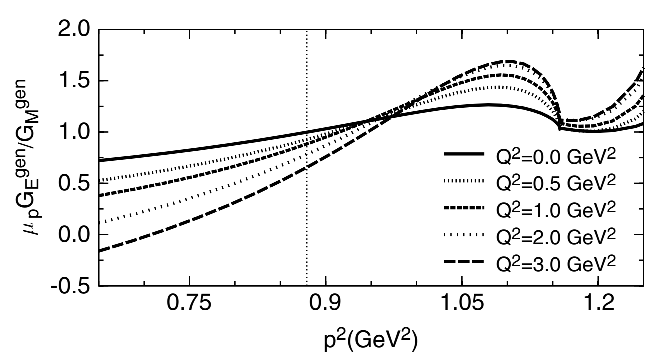

FIG. 3. Ratio of the generalized Sachs form factors (15) as a function of the momentum squared p^2 of the off-shell proton, for various fixed values of the photon momentum squared $Q^2 = -q^2$. The vertical dotted line corresponds to both nucleons being on-shell with $p^2 = p'^2 = m^2$. The cusp at $p^2 \approx 1.16\ \text{GeV}^2$ indicates the opening of the pion production threshold.

[2]A large additional vector particle contribution is also needed in other approaches in the literature, e.g., in a soliton model of Ref. [4].

Although the half-off-shell FFs are not measurable by themselves [18], they can be an important part of various models for physical processes. For example, a two-photon exchange contribution to electron-proton scattering contains γNN vertices with an off-shell nucleon leg. So far the dependence of these vertices on the nucleon off-shell momentum has not been taken into account in the existing approaches [19]. Our calculation shown in Fig. 3 suggests that this dependence may be significant. It should be borne in mind, however, that in view of the representation dependence of Green's functions [20], the calculation of such off-shell vertices should be consistent with the model for the physical processes to which they are applied.

V. CONCLUSIONS

In this report we extended the nonperturbative dressing procedure of the dressed K-matrix model (DKM) to calculate a γNN vertex with an off-shell nucleon and an off-shell photon. The principal motivation for this work is to study the nucleon electromagnetic interactions in a dynamical approach that describes in a unified manner two distinct types of reactions: those where essential contributions come from an off-shell nucleon (as in Compton scattering) and those where one probes the dependence on the momentum transferred by an off-shell photon (as in form factors extracted from electron-proton scattering experiments). Having previously applied DKM to Compton and πN scattering, in this report we focused on the momentum-transfer dependence. We developed the dressing procedure to calculate the ratio of the proton electromagnetic form factors and found good agreement of our results with the recent JLab measurements.

The main limitation of the present version of DKM is the difficulty in describing the momentum-transfer dependence of the individual form factors as accurately as their ratio. To resolve this problem, additional dynamical contents—with attendant new parameters—might be required in the model. Rather than pursuing this direction in detail, we studied to what extent one can describe the dependence of the nucleon EM interactions on both the nucleon and photon variables using the *same* dynamical approach with a few predetermined parameters. Our approach is based on essential symmetry constraints, including relativistic and gauge invariance, unitarity, crossing, and causality, which are required in order to correlate in one model the very different kinematical regions explored in Compton and πN scattering and in the proton form factors.

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