Charge-symmetry-breaking three-nucleon forces

J. L. Friar,[1] G. L. Payne,[2] and U. van Kolck[3]

[1]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
[2]*Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52242*
[3]*Department of Physics, University of Arizona, Tucson, Arizona 85721*
and RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973

Leading-order three-nucleon forces that violate isospin symmetry are calculated in heavy-baryon chiral perturbation theory. The effect of the charge-symmetry-breaking three-nucleon force is investigated in the trinucleon systems using Faddeev calculations. We find that the contribution of this force to the ^{3}He-^{3}H binding-energy difference is given by $\Delta E_{3NF}^{CSB} \simeq 5$ keV.

I. INTRODUCTION

Isospin violation [1–3] has recently been investigated in the context of (heavy-baryon) chiral perturbation theory (χPT). This powerful technique [4,5] casts the symmetries of QCD into effective Lagrangians that are expressed in terms of pions and nucleons, which are the effective degrees of freedom of nuclear physics. These Lagrangian building blocks can then be combined in a systematic way to develop isospin-violating forces. Although most of the forces resulting from this procedure were anticipated and developed using phenomenological methods, new forces have also been found. It is the purpose of this work to complement the earlier work on isospin-violating two-nucleon [6–14] forces by calculating the leading-order three-nucleon isospin-violating force, which breaks charge symmetry. In addition, we estimate the contribution of this force to the ^{3}He-^{3}H binding-energy difference. This is the first such calculation of isospin-violating three-nucleon forces[1] and completes the χPT calculations of isospin violation—both charge-independence breaking (CIB) and charge-symmetry breaking (CSB)—through the first three orders.

Chiral perturbation theory is organized around power counting (dimensional analysis [15]), which allows estimates of the sizes of various mechanisms to be made in terms of the parameters and scales intrinsic to QCD. These scales (using Weinberg power counting [4,6,16]) include the pion decay constant, $f_\pi \sim 93$ MeV, which sets the scale for pion emission or absorption; the pion mass, m_π, which sets the scale for chiral-symmetry breaking; the typical nucleon momentum, $Q \sim m_\pi$ (which also determines the inverse correlation length in nuclei); and the characteristic QCD bound-state scale, $\Lambda \sim m_\rho$, which is appropriate for heavy mesons, nucleon resonances, etc. The latter states are frozen out and do not explicitly appear in χPT, although their effect is present in the counter terms of the effective interactions. The resulting

field theory is a power series in Q/Λ, and the number of implicit powers of $1/\Lambda$ (e.g., n) can be used to label individual terms in the Lagrangian (viz., $\mathcal{L}^{(n)}$). In this way, higher powers denote smaller terms, and this is a critical part of the organizing principle of χPT. We note that power-counting estimates of sizes are typically within a factor of 2–3 of the actual sizes.

The nucleon-nucleon correlation-length scale is not relevant for one-body operators; it occurs once for two-body operators, twice for three-body operators, etc. Thus it is important to incorporate this mechanism into the power counting if we wish to compare mechanisms with differing numbers of interacting nucleons. In addition to adding the indexes "n" for each of the individual Lagrangians (see above) that are used in a given calculation, we must add 2 for each loop and $N - 1$ for an N-body Feynman diagram in order to determine the effective order Δ [16]. Thus three-nucleon forces with an order determined by an index $n - 2$ should be comparable in size in nuclei to one-body operators (such as the kinetic energy) corresponding to an index n, or to a two-body force corresponding to an index $n - 1$. (Notice that this accounting of the relative sizes of few-body forces is different from Weinberg's [4,6] by one order, which reflects the fact that we are counting the nucleon mass M_N as Λ, rather than Λ^2/Q.) This is the underlying reason why N-body forces in nuclei get systematically smaller as N increases (and this makes nuclear physics tractable).

Isospin violation in nuclei arises from three distinct mechanisms. The first is the up-down quark-mass difference, which dominates and makes the neutron heavier than the proton. The second mechanism is hard electromagnetic (EM) interactions at the quark level, which try to make the proton heavier than the neutron. This is also the mechanism that produces most of the pion-mass difference. The final mechanisms are the soft-photon interactions (such as the Coulomb interaction between protons) that dominate isospin violation in nuclei.

Direct comparison [10,11] of the sizes of the EM and quark-mass terms demonstrates that the EM terms (which contain a factor of α, the fine-structure constant) are roughly the same size as quark-mass terms that are formally three orders smaller in the power counting. We adjust our power counting accordingly and adopt the convenient mnemonic of adding 3 to the order of the EM-induced isospin-violating

---

[1]While this manuscript was being written, we learned of a similar investigation—using, however, a different choice of fields—by Epelbaum, Meißner and Palomar [37], where there is no attempt to calculate binding energies.

Lagrangian when comparing sizes with quark-mass-induced mechanisms [6]. Henceforth our power counting for any EM-induced interactions will contain this additional factor of 3.

Our prior work on isospin-violating nucleon-nucleon forces (both CSB and CIB) in the context of (heavy-baryon) chiral perturbation theory with the Δ-isobar integrated out [6–11] identified the leading mechanisms for isospin violation in nuclei. In Sec. II we discuss their impact on the ^{3}He-^{3}H binding-energy difference. In Sec. III we calculate the leading isospin-violating three-nucleon force and evaluate its contribution to the ^{3}He-^{3}H binding-energy difference. We then conclude.

II. VARIOUS CSB MECHANISMS

We have shown [6,7,10,11] that the following ten mechanisms are expected to contribute dominantly to CSB in nuclei:
- The mass difference of the proton and neutron, $\delta M_N = m_p - m_n < 0$
- The CSB nuclear kinetic energy
- The Coulomb potential between protons
- The Breit-interaction $((v/c)^2)$ corrections to the Coulomb potential
- The CSB one-pion-exchange potential (OPEP)
- The CSB short-range nuclear potential
- The CSB two-pion-exchange potential incorporating the nucleon-mass difference
- The Class IV CSB interactions (antisymmetric in isospin coordinates and with a spin-orbit-type spin-space dependence)
- An OPEP with Class IV isospin structure that vanishes in the two-nucleon center-of-mass, but not in a three-nucleon system
- A two-pion-exchange three-nucleon force proportional to the quark-mass-difference contribution, $\delta M_N^{\rm qm}$, to the nucleon-mass difference

We briefly discuss each of them in turn in the context of the ^{3}He-^{3}H binding-energy difference.

The first four mechanisms are fairly well known. They include the two largest mechanisms, and we start our discussion with them.

The mass (rest-energy) difference of the nucleons, δM_N, contributes to the χPT Lagrangian at order $n = 1$. From a nuclear-physics perspective it makes an uninteresting contribution to the mass difference of ^{3}He and ^{3}H and is conventionally removed, leaving only a binding-energy difference. Although ^{3}H is heavier than ^{3}He, this removal leads to ^{3}He being less bound than ^{3}H by 764 keV, which is the target for all CSB calculations in the three-nucleon systems. The nucleon-mass difference nevertheless plays a nontrivial role in intermediate states where two protons are converted to two neutrons (or vice versa) by exchanging pions. That effect was recently treated in a systematic fashion [11] by removing the δM_N mass-difference term from the χPT Lagrangian. This removes δM_N from asymptotic states and nuclear energies, but its effect in intermediate states is compensated by the addition of new terms in the Lagrangian that must be incorporated in any calculations. The resulting scheme is much simpler to use than older techniques, and we use it below.

The kinetic-energy difference between two protons and two neutrons caused by their different masses corresponds to $n = 3$ in power counting. In the trinucleon systems this mechanism leads to a robust 14-keV contribution [17–22] to the binding-energy difference of ^{3}He and ^{3}H.

The Coulomb potential between two protons is the dominant CSB interaction in nuclei. According to the way we bookkeep EM interactions, this is an effect one order down compared to the leading, isospin-conserving nucleon-nucleon force, so it is effectively an $n = 1$, or $\Delta = 2$, term. This contribution has a nominal size in terms of scales given by $E_C \sim \alpha Q \sim 1$ MeV, where α is the fine-structure constant. In the trinucleon systems it has been well studied over several decades and leads to a robust and dominant 648-keV contribution to the 764-keV trinucleon binding-energy difference [23].

Small EM contributions of relativistic order contained in the Breit interaction (viz., the interaction between nucleon magnetic moments and between the currents associated with moving protons) plus a smaller vacuum-polarization force appear two orders down ($n = 3$ or $\Delta = 4$ in our power counting). In terms of scales, the relativistic contributions behave like $E_B \sim \alpha Q^3/\Lambda^2 \sim 25$ keV. Indeed, they lead to a robust [17–21] 28 keV.

The effect of CSB on two-nucleon potentials is subsumed by the next four mechanisms on the list above, of which three are Class III and one is Class IV. We discuss them separately.

Charge-symmetry breaking in the pion-nucleon coupling constants can lead to a CSB OPEP that has nominal size $n = 2$, which corresponds to $\Delta = 3$. Only an upper limit of size 50 keV (with unknown sign) constrains this mechanism [10]. A conventional short-range interaction of undetermined strength corresponding to size $\Delta = 3$ (and a nominal size of roughly 50 keV) is also present [7]. The last of the three Class III mechanisms is the recently calculated two-pion-exchange potential that incorporates various aspects of the nucleon-mass difference. It is of nominal order $n = 3$ or $\Delta = 4$ [10]. Each of these three mechanisms contributes to the difference between the pp force (with the EM interaction removed) and the nn force. At present the only experimental information on this difference is contained in the scattering-length difference. The resulting $a_{nn} - a_{pp}$ scattering-length difference [2,3,7] of $-1.5(5)$ fm is then attributed to CSB in the three forces discussed above, which cannot be further disentangled at the present time. (Of course, in principle these mechanisms could be separated thanks to their different ranges, provided that the nucleon-nucleon data are accurate enough.) This difference then produces a contribution to the ^{3}He-^{3}H binding-energy difference of approximately 65(22) keV, a number that also appears to be robust [20,21,24].

The Class IV two-nucleon CSB OPEP [2,3,11] has a nominal $n = 2$ or $\Delta = 3$ size, which is suppressed by nearly an order of magnitude by nature's fine tuning of δM_N to its physical value [11]. This type of force is further suppressed in the trinucleon bound states because S-wave components of the wave function do not contribute to a spin-orbit force. Although this force and a short-range force of order $n = 4$ are formally part of the CSB two-nucleon potential, they make a negligible contribution to the trinucleon binding-energy difference.

The last two mechanisms are three-body effects.

The recent study [11] of Class IV CSB forces found a peculiar two-body force that vanishes in the center-of-mass of two nucleons, but does not vanish in a system of more than two nucleons. Although nominally of order $n = 2$ (or $\Delta = 3$), this force should be much smaller than that for three reasons. The first reason is that this two-body force is constrained by kinematics to vanish in the center-of-mass of those two nucleons. In addition, this force is proportional to δM_N, which results from the cancellation of the separate quark-mass and EM contributions and has been fine-tuned by nature to a rather small value (a factor of 5 smaller than the nominal value of the power-counting estimate for the quark-mass part δM_N^{qm} of that mass difference, viz., -7 MeV). The final suppression is caused by approximate SU(4) symmetry in the few-nucleon systems. The spin-isospin dependence of the force is antisymmetric under the interchange of those coordinates for the two nucleons, caused by a $(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z$ type of isospin dependence. The dominant component of the trinucleon wave function ($\sim 90\%$) is the S-state [an SU(4) classification], which is completely antisymmetric under that interchange. These symmetry considerations cause the diagonal S-state matrix element of the force to vanish. The net result of these suppressions is that this force should be much smaller than its nominal order indicates (i.e., $\Delta = 3$) and is therefore very unlikely to be significant. For these reasons we will ignore this contribution in this work.

The remaining force is a three-nucleon force of nominal order $n = 1$ or $\Delta = 3$ that originates in the chiral-symmetry-breaking properties of the quark-mass difference. It has never before been calculated, and we turn to it in Sec. III.

III. CSB THREE-NUCLEON FORCES

In this section we examine isospin-violating three-nucleon forces within heavy-baryon chiral perturbation theory. Isospin-conserving three-nucleon forces have been derived within this approach in Refs. [25,26]. We follow the same method here. In particular, we ignore terms that cancel against recoil in the iteration of the two-nucleon potential.

The field redefinition that we employed in Ref. [11] eliminated the nucleon-mass difference in the free Lagrangian at the cost of additional effective interactions proportional to powers of that mass difference. Only Lagrangian terms that had explicit time derivatives generated additional terms. Incorporating the results of that field redefinition through orders $n = 0$ and $n = 1$ in the Lagrangian (including short-range two- and three-body terms) plus several other terms from Ref. [6] leads to the following terms that contribute to isospin-violating three-nucleon forces at orders $n = 1$ (CSB) and $n = 2$ (CIB), plus omitted terms that would contribute only to higher orders:

$$\mathcal{L}_{\mathrm{iv}} = \frac{\delta M_N^{\mathrm{qm}}}{4 f_\pi^2} N^\dagger \left[\boldsymbol{\tau} \cdot \boldsymbol{\pi} \pi_3 + ((\boldsymbol{\tau} \times \boldsymbol{\pi}) \times \boldsymbol{\pi})_3 \right] N$$
$$- \frac{1}{2} \left(\delta m_\pi^2 - \delta M_N^2 \right) \left(\boldsymbol{\pi}^2 - \pi_3^2 \right)$$
$$+ \frac{\tilde{c}_2 \delta M_N^2 + \bar{\beta}_1 / 4}{f_\pi^2} N^\dagger (\boldsymbol{\pi}^2 - \pi_3^2) N + \cdots. \quad (1)$$
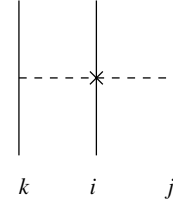

FIG. 1. Leading isospin-violating three-nucleon force W_i, which is charge-symmetry breaking. A solid (dashed) line represents a nucleon (pion), and the cross represents the interaction generated by the quark-mass difference component of the nucleon-mass difference [first term in Eq. (1)].

All these terms originate in the interactions that break isospin in QCD. The quark-mass-difference term in the QCD Lagrangian and short-distance photon exchange between quarks both have definite transformation properties under chiral symmetry. They generate effective hadronic interactions that break isospin in the same way. These include pion-nucleon interactions that are fixed by the quark-mass (δM_N^{qm}) and electromagnetic ($\delta M_N^{\mathrm{em}} \simeq \delta M_N - \delta M_N^{\mathrm{qm}}$) contributions to the nucleon-mass difference. Details can be found in Refs. [6,7,10]. After a field redefinition [11], we arrive at Eq. (1). The first of these terms breaks charge symmetry, and the remaining two break charge independence. We focus here on the first term, which is the largest of all ($n = 1$). We discuss corrections at the end of this section.

Using the lowest-order isospin-conserving Lagrangian

$$\mathcal{L}^{(0)} = \frac{1}{2} \left[\dot{\boldsymbol{\pi}}^2 - (\vec{\nabla} \boldsymbol{\pi})^2 - m_\pi^2 \boldsymbol{\pi}^2 \right] + N^\dagger \left[i \partial_0 - \frac{1}{4 f_\pi^2} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \dot{\boldsymbol{\pi}}) \right] N$$
$$+ \frac{g_A}{2 f_\pi} N^\dagger \vec{\sigma} \cdot \vec{\nabla} (\boldsymbol{\tau} \cdot \boldsymbol{\pi}) N + \cdots, \quad (2)$$

a simple calculation along the lines of Ref. [26] leads to the following three-nucleon force corresponding to $\Delta = 3$. We define the total three-nucleon force W as

$$W = W_1 + W_2 + W_3, \quad (3)$$

where the subscript refers to the number of the nucleon that emits both pions (the other two nucleons each absorbs one of those pions), as shown in Fig. 1. The expressions W_i are symmetric under the interchange of nucleons j and k. We then find that

$$W_1^{\mathrm{CSB}} = -\frac{\delta M_N^{\mathrm{qm}} g_A^2 m_\pi^2}{8 f_\pi^4 (4\pi)^2} (\vec{\sigma}_2 \cdot \hat{x}_{12} \, Y'(|\vec{x}_{12}|) \, \vec{\sigma}_3 \cdot \hat{x}_{13} \, Y'(|\vec{x}_{13}|))$$
$$\times \left(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \tau_3^{(3)} + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \tau_2^{(3)} - \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \tau_1^{(3)} \right), \quad (4)$$

where $\tau_i^{(k)}$ is the kth component of the isospin operator for nucleon i, $\vec{\sigma}_i$ is the spin operator for nucleon i, $\vec{x}_{ij}$ is the vector from nucleon j to nucleon i,

$$Y(x) = \exp(-m_\pi x)/(m_\pi x),$$

$g_A \cong 1.25$ is the axial-vector constant, and δM_N^{qm} is the currently unknown quark-mass portion of the nucleon-mass difference.

The three-nucleon force in Eqs. (3) and (4) is charge-symmetry breaking. It is remarkable that this force appears

TABLE I. Contributions to the ^{3}He-^{3}H binding-energy difference in keV. The Coulomb interaction and associated (relativistic) Breit-interaction corrections dominate, and the CSB kinetic-energy difference (E_K), the sum of the short-range two-body CSB force mechanisms, and the three-nucleon CSB force (calculated here for the first time) all make significant contributions. (In the three-nucleon force, we used $\delta M_N^{\rm qm} = -2.5$ MeV for illustration.) "Theory" labels the sum of these mechanisms.

Coulomb	Breit	(E_k)	Two-body	Three-body	Theory	Experiment
648	28	14	65(22)	5	760(22)	764

formally at the same order as the leading isospin-conserving three-nucleon force [25,26] in the theory without an explicit Δ isobar. That is, the factors of Q and Λ are the same in both isospin-conserving and CSB three-nucleon forces, although, of course, the CSB force is smaller by a factor of $\varepsilon = (m_d - m_u)/(m_d + m_u) \sim 1/3$.

CSB is thus relatively large among three-nucleon effects, an unusual phenomenon. It is agreed that three-nucleon effects (those not fixed by two-nucleon data) provide about 1 MeV of the three-nucleon binding energies. The leading three-nucleon forces contain relatively large subleading interactions (due to effects of the Δ isobar), perhaps by a factor of 3 or so. Combining this with a factor of ε, we could expect the CSB force to contribute as much as 100 keV to the three-nucleon binding-energy difference. Indeed, using the replacement $f_0^2 = (g_A m_\pi/2f_\pi)^2/4\pi \cong 0.075$ (which is strictly valid only if the Goldberger-Treiman [28] relation is exact; that is, to lowest order), Eq. (4) can be written as $-2\delta M_N^{\rm qm} f_0^4/g_A^2$ times a dimensionless function of coordinates (in units of $1/m_\pi$), spins, and isospins. Assuming that the matrix elements of this function give numbers of order 1 and that $\delta M_N^{\rm qm}$ has its naive-dimensional-analysis value of -7 MeV, we arrive at 50 keV as an estimate for the size of the CSB three-nucleon force. This is significant, but obviously it could differ from the actual value by a factor of a few. The two sources of uncertainty in the size of this force are the values of $\delta M_N^{\rm qm}$ and of the dimensionless function above.

In order to better estimate the size of this CSB three-nucleon force, we have implemented it in our Faddeev codes. The cutoff parameter in the TM' force [26,27] was adjusted slightly to produce the correct binding energy for ^{3}H when used in conjunction with the AV18 two-nucleon force [18]. In any numerical calculation it is necessary to regulate the Yukawa function, $Y(x)$ in Eq. (4), and this was done in a way that is consistent with the Tucson-Melbourne force [27]. Perturbation theory was then used to calculate the binding-energy difference of ^{3}He and ^{3}H. We find

$$E_{\rm 3NF}^{\rm CSB} = 2 \times \left[-\frac{\delta M^{\rm qm}}{\rm MeV} \right] \text{ keV}, \qquad (5)$$

which is about a factor of 3 smaller than our estimate. Any other set of realistic two- and three-nucleon forces should give similar results.

As mentioned above, the actual value of $\delta M_N^{\rm qm}$ is uncertain. It has been suggested [29,30] that it could be extracted from pion-production experiments [31,32], but it is unclear if this

can be achieved in the near future. It is likely to be smaller by a factor of a few than the naive estimate, so -7 MeV is to be viewed as an overestimate. Using -2.5 MeV for $\delta M_N^{\rm qm}$, the contribution of our three-nucleon force is listed in Table I together with all the other significant contributions to the ^{3}He-^{3}H binding-energy difference that we have discussed.

The three-nucleon results are in agreement with experiment when the error bar associated with the strong-interaction CSB strength is taken into account. This conclusion is also consistent with the CSB results extracted in Ref. [33] for $A = 6$–10.

There are, of course, other isospin-violating three-nucleon forces, but they are higher order in our power counting and thus should be smaller. Some are generated by subleading interactions, such as those depicted in Fig. 2. The second term in Eq. (1) reflects the additional amount that should be added to the charged-pion mass (squared) in all pion propagators in isospin-conserving three-nucleon forces, such as the TM' force [26], which comes from the subleading isospin-conserving Lagrangian $\mathcal{L}^{(1)}$. The third term in Eq. (1) is an isospin-violating contribution to the nucleon σ-term, often called c_1. It modifies the three-nucleon force that is generated by charged-pion exchanges in that interaction. Both of these modifications break, of course, charge independence, but not charge symmetry. These forces are transparently easy to implement, and we refrain from writing explicit forms. They correspond to $n = 2$ or $\Delta = 4$.

At the same subleading order there are also soft-EM forces, where a pion and a photon are in the air at the same time, as in Fig. 3. While in flight between two nucleons, a charged pion can exchange a photon with the third nucleon. The photon couples either (i) to the charge of the nucleon and the energy of the pion, which in the nuclear environment is Q^2/M_N or (ii) through the momentum of the pion and the
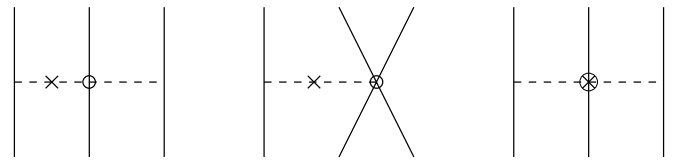

FIG. 2. Subleading isospin-violating three-nucleon forces from subleading interactions. The cross represents the pion-mass difference [second term in Eq. (1)], and the circled cross stands for the subleading isospin-violating seagull [third term in Eq. (1)]; a circle represents an interaction from the subleading isospin-conserving Lagrangian.
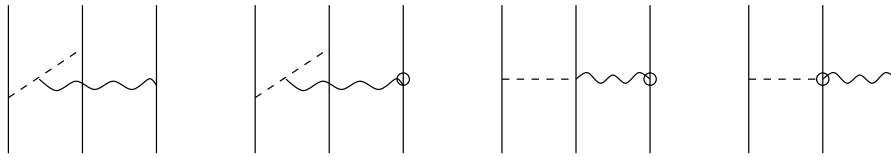
FIG. 3. Subleading isospin-violating three-nucleon forces from pion-photon exchange. A wavy line represents a photon, and a circle represents an interaction from the gauging of the subleading isospin-conserving Lagrangian.

magnetic moment of the nucleon, which is a $1/M_N$ effect contained in the subleading Lagrangian $\mathcal{L}^{(1)}$. In addition, there can be simultaneous emission of a photon and a charged pion by one nucleon followed by their absorption on two other nucleons. This can happen when the photon couples (i) to the pion-nucleon vertex through the gauging of the axial-vector coupling [third term in Eq. (2)], and to the nucleon magnetic moment or (ii) to the nucleon charge, and to the pion-nucleon vertex through the gauging of the relativistic correction to the pion-nucleon coupling contained in the subleading Lagrangian $\mathcal{L}^{(1)}$. These mechanisms are formally $n = 2$ in power counting (i.e., $-1 + 3$) and are suppressed by one power of Q/M_N compared to the CSB three-nucleon force we calculated above. These three-nucleon forces therefore also correspond to $\Delta = 4$. They break both charge independence and charge symmetry. Notice that they are entirely determined by gauge and Galilean invariance in terms of known parameters (the axial-vector coupling of the pion, the pion charge, the nucleon charge and magnetic moment, and the pion and nucleon masses). Effects from integrated-out resonances (most importantly the Δ isobar) only appear one order further up. A subset of these EM effects has been calculated before [34–36].

We expect that these uncalculated parts of the CSB force corresponding to higher orders in the power counting contribute only a few keV or less, which is roughly the level of uncertainty in the EM corrections discussed above.

IV. CONCLUSION

After discussing various charge-symmetry breaking mechanisms in nuclei, we derived the leading isospin-breaking three-nucleon force in chiral perturbation theory without an explicit Δ isobar, given by Eqs. (3) and (4). This force is charge-symmetry breaking and appears formally at the same order as the leading isospin-conserving three-nucleon force. CSB could thus be a relatively large three-nucleon effect. Its strength depends on δM_N^{qm}, the contribution from the quark-mass difference to the nucleon-mass difference. We therefore can directly tie QCD to a three-nucleon effect. Unfortunately, the actual value of δM_N^{qm} has not yet been determined in a model-independent way from low-energy data, nor from lattice QCD.

We have also, for the first time, calculated the contribution of this force to the ^{3}He-^{3}H binding-energy difference, given by Eq. (5). Taking $\delta M_N^{\mathrm{qm}} = -2.5$ MeV for illustration, we find that 5 keV can be attributed to this force. This value has the same sign as the observed difference and is somewhat smaller in magnitude than expected from naive dimensional analysis. As a consequence, it does not upset the agreement between theory and experiment when the uncertainty in two-body effects is accounted for.

[1] E. M. Henley, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969), p. 15; E. M. Henley and G. A. Miller, in *Mesons and Nuclei*, edited by M. Rho and G. E. Brown (North-Holland, Amsterdam, 1979), Vol. I, p. 405.

[2] G. A. Miller, B. M. K. Nefkens, and I. Šlaus, Phys. Rep. **194**, 1 (1990); G. A. Miller and W. T. H. van Oers, in *Symmetries and Fundamental Interactions in Nuclei*, edited by W. Haxton and E. M. Henley (World Scientific, Singapore, 1995).

[3] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1990); G. A. Miller, Nucl. Phys. **A578**, 345 (1990); Chin. J. Phys. **32**, 1075 (1994); S. A. Coon, nucl-th/9903033, invited talk given at XIII$^{\mathrm{th}}$ International Seminar on High Energy Physics Problems (ISHEPP 13), "Relativistic Nuclear Physics and Quantum Chromodynamics," Joint Institute for Nuclear Research, Dubna, Russia, September, 1996. These are all fairly recent reviews of charge symmetry.

[4] S. Weinberg, Physica **96A**, 327 (1979); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B251**, 288 (1990); **B295**, 114 (1992).

[5] P. F. Bedaque and U. van Kolck, Ann. Rev. Nucl. Part. Sci. **52** (2002). This is a comprehensive current review of the application of χPT to nuclear physics.

[6] U. van Kolck, Ph. D. thesis, University of Texas, 1993; Few-Body Syst. Suppl. **9**, 444 (1995).

[7] U. van Kolck, J. L. Friar, and T. Goldman, Phys. Lett. **B371**, 169 (1996).

[8] U. van Kolck, M. C. M. Rentmeester, J. L. Friar, T. Goldman, and J. J. de Swart, Phys. Rev. Lett. **80**, 4386 (1998).

[9] J. L. Friar and U. van Kolck, Phys. Rev. C **60**, 34006 (1999).

[10] J. L. Friar, U. van Kolck, G. L. Payne, and S. A. Coon, Phys. Rev. C **68**, 024003 (2003).

[11] J. L. Friar, U. van Kolck, M. C. M. Rentmeester, and R. G. E. Timmermans, Phys. Rev. C **70**, 044001 (2004).

[12] E. Epelbaum and U.-G. Meißner, Phys. Lett. **B461**, 287 (1999). This work was organized using a method that did not require explicitly calculating a potential.

[13] M. Walzl, U.-G. Meißner, and E. Epelbaum, Nucl. Phys. **A693**, 663 (2001).

[14] J. A. Niskanen, Phys. Rev. C **65**, 037001 (2002).

[15] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); H. Georgi, Phys. Lett. **B298**, 187 (1993).

[16] J. L. Friar, Few-Body Syst. Suppl. **22**, 161 (1997).

[17] R. A. Brandenburg, S. A. Coon, and P. U. Sauer, Nucl. Phys. **A294**, 305 (1978).

[18] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).

[19] Y. Wu, S. Ishikawa, and T. Sasakawa, Phys. Rev. Lett. **64**, 1875 (1990); **66 (E)**, 242 (1991).

[20] S. C. Pieper, V. R. Pandharipande, R. B. Wiringa, and J. Carlson, Phys. Rev. C **64**, 014001 (2001) [see Table XI]; S. C. Pieper and R. B. Wiringa, Ann. Rev. Nucl. Part. Sci. **51**, 53 (2001).

[21] A. Nogga, A. Kievsky, H. Kamada, W. Glöckle, L. E. Marcucci, S. Rosati, and M. Viviani, Phys. Rev. C **67**, 034004 (2003).

[22] J. L. Friar, B. F. Gibson, and G. L. Payne, Phys. Rev. C **42**, 1211 (1990).

[23] J. L. Friar, B. F. Gibson, and G. L. Payne, Phys. Rev. C **35**, 1502 (1987). The finite size of the protons reduces the Coulomb energy by about 30 keV, and this size was not accurately known in 1987. Modification of the finite size would lead to a small change in the Coulomb energy, but this would have to be compensated by a corresponding change in the short-range pp force in order to maintain the fit to the pp scattering data. It is believed that such modifications would lead to no significant problems in our understanding of CSB.

[24] R. Machleidt and H. Müther, Phys. Rev. C **63**, 034005 (2001); G. Q. Li and R. Machleidt, *ibid*. **58**, 1393 (1998).

[25] U. van Kolck, Phys. Rev. C **49**, 2932 (1994).

[26] J. L. Friar, D. Hüber, and U. van Kolck, Phys. Rev. C **59**, 53 (1999). This paper treats a variety of three-nucleon forces and includes a χPT derivation of an improved Tucson-Melbourne force, originally given in Ref. [27].

[27] S. A. Coon, M. D. Scadron, P. C. McNamee, B. R. Barrett, D. W. E. Blatt, and B. H. J. McKellar, Nucl. Phys. **A317**, 242 (1979).

[28] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958); Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960).

[29] U. van Kolck, J. A. Niskanen, and G. A. Miller, Phys. Lett. **B493**, 65 (2000).

[30] A. Gårdestig, C. J. Horowitz, A. C. Fonseca, C. Hanhart, G. A. Miller, J. A. Niskanen, A. Nogga, and U. van Kolck, Phys. Rev. C **69**, 044606 (2004).

[31] A. K. Opper *et al.*, Phys. Rev. Lett. **91**, 212302 (2003).

[32] E. J. Stephenson *et al.*, Phys. Rev. Lett. **91**, 142302 (2003).

[33] R. B. Wiringa, Prog. Theor. Phys. Supp. **146**, 403 (2002). Some of the numbers in Table II have changed and are in much better agreement with experiment: see S. C. Pieper, K. Varga, and R. B. Wiringa, Phys. Rev. C **66**, 044310 (2002).

[34] M. D. Shuster, Phys. Lett. **B61**, 231 (1976).

[35] S.-N. Yang, Phys. Rev. C **19**, 1114 (1979); J. Phys. G **9**, L115 (1983).

[36] C. Yalçin and E. Akşahin, Nucl. Phys. **A360**, 223 (1981); C. Yalçin, E. Akşahin, and A. Kolat, Phys. Lett. **B109**, 347 (1982).

[37] E. Epelbaum, U.-G. Meißner, and J. E. Palomar, nucl-th/0407037. Our results for the three-body force we focus on here, proportional to δM_N^{qm} [Eq. (4)], did not agree with the corresponding results in the first version of Ref. [37], but do agree with a version that appeared after our paper. Reference [37] includes also explicit three-body forces proportional to δM_N. The form that a three-body force takes is often dependent on the form one *chooses* for the two-body force. We have verified that the extra three-body forces in Ref. [37] are contained in the off-shell behavior of the peculiar two-body force that we discussed in Ref. [11]. For the reasons given in Sec. II we expect these forces to be small and we neglected them here. We thank E. Epelbaum and U.-G. Meißner for illuminating correspondence on the subject.