PHYSICAL REVIEW C **71**, 022201 (2005)

Virtual photon asymmetry for confined, interacting Dirac particles with spin symmetry

V. R. Pandharipande*
Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801

M. W. Paris†
Thomas Jefferson National Accelerator Facility, MS12H2, 12000 Jefferson Ave., Newport News, Virginia 23606

I. Sick‡
Departement für Physik und Astronomie, Universität Basel, Switzerland
(Received 22 October 2004; published 17 February 2005)

We study the Bjorken x dependence of the virtual photon spin asymmetry in polarized deep inelastic scattering of electrons from hadrons. We use an exactly solved relativistic potential model of the hadron, treating the constituents as independent massless Dirac particles bound to an infinitely massive force center. The potential is chosen to have spin symmetry and a linear radial dependence with spherical symmetry. The effect of interactions of the struck constituent with the remainder of the target on the longitudinal photon asymmetry is demonstrated. In particular, the small-x suppression of the photon asymmetry observed in polarized deep inelastic scattering from the proton is shown to be a consequence of these interactions. The effect of p-wave components of the Dirac wave function, long known to give an important contribution to the spin of hadrons, is explicitly demonstrated through their interference with the s-wave term.

DOI: 10.1103/PhysRevC.71.022201 PACS number(s): 13.60.Hb, 12.39.Ki, 12.39.Pn

I. INTRODUCTION

The spin dependent structure functions measured in deep inelastic scattering (DIS) of electrons from nucleons have recently been measured to high precision [1–3]. Calculations of the polarized structure functions have been performed in an array of models [4–11].

The spin symmetry [12,13] and the closely related pseudospin symmetry have been exploited to explain approximate degeneracies in nuclear [14] and hadronic [15] spectroscopy. In this communication we calculate the virtual photon longitudinal spin asymmetry [16] using a model with spin symmetry [17]. The present work demonstrates the effect of including interactions among the constituents of the composite hadronic target on physical observables. This exact calculation allows us to make unambiguous statements regarding the effects of interactions within the model and the validity of commonly used approximations.

The model of Ref. [17] is a quantum mechanical model for a single massless Dirac particle confined by a linear potential, assumed valid for all radii. It neglects the effects of $q\bar{q}$ pairs. The gluons are imagined to have been integrated out, resulting in the confining potential via a flux tube. Ignoring radiative gluon corrections should not be an impediment in the calculation of the spin asymmetry, observed in polarized electron scattering experiments to be nearly independent of Q^2. The potential is assumed to be one-half vector plus one-half scalar and therefore enjoys the spin symmetry [15]. At small radii the potential is nearly zero and so, for a given resonance, displays asymptotic freedom.

Previous calculations of inclusive unpolarized and polarized structure functions performed within the cavity approximation to the bag model neglect the role of a confining interaction. The naive bag model assumes that the constituents of the hadron are free inside the bag; confinement only restricts the momentum of the constituents. It is interesting to study the consequences of a confining interaction which takes the constituent off the mass-shell in an exactly solvable model.

The present calculation is similar to the bag model calculations in Refs. [6,7,18,19]. These works treat the struck constituent as a free particle whose state is described by a plane wave. Here we use an exactly solved single particle relativistic potential model of the hadron. The eigenstates of this Hamiltonian, which are four-component Dirac spinors, describe the state of the struck constituent. Our model calculation is exact and includes eigenstates with a maximum excitation energy of about 10 GeV [17].

We compare the exact result to the plane wave impulse approximation (PWIA) and find good agreement for the longitudinal photon asymmetry $A_1^q(\xi)$ to a level of about ten percent. The qualitative agreement of the exact calculation with PWIA allows us to compare this calculation to a bag model PWIA evaluation of the asymmetry. We attribute differences between the asymmetries calculated in the present model and the bag model to interactions.

Moreover, the present work shows the effect that interactions have on the spin structure of hadrons within a relativistic potential formalism. It has long been known that p-wave components, which are necessary in the Dirac description of confined particles, reduce the contribution of valence quarks to the spin of the nucleon [4,18–23]. We emphasize the role

*E-mail address: vrp@uiuc.edu.
†E-mail address: mparis@jlab.org.
‡E-mail address: Ingo.Sick@unibas.ch.

of the (lower component) p-wave terms of the ground state Dirac wave function in determining the x dependence of the $A_1^q(\xi)$. The interference between the dominant s-wave and the p-wave parts of the valence quark Dirac wave function suppresses $A_1^q(\xi)$ at small values of ξ. This novel observation demonstrates unambiguously an effect of treating hadronic constituents as bound Dirac particles. It suggests that interactions among the constituents reduce the contribution of valence quarks to the spin of nucleons.

II. MODEL CALCULATION

We consider the calculation of the virtual photon spin asymmetry in DIS of a charged leptonic probe from a hadronic target within the model of Ref. [17]. The model Hamiltonian is chosen as

$$H = \boldsymbol{\alpha} \cdot \mathbf{p} + \frac{1+\beta}{2}\sqrt{\sigma} r, \tag{1}$$

where $\boldsymbol{\alpha}$ and β are Dirac matrices in the standard representation [24]. It describes a massless Dirac particle in a linear confining well. The half-vector plus half-scalar structure of the confining potential is chosen for its spin symmetry [15] wherein spin-orbit doublets are degenerate. Relatively small spin-orbit splittings seen in meson spectra motivate this choice. Computations are simple with this choice since the lower components of the wave function are not coupled by the potential. The value of the string tension $\sqrt{\sigma}$ is assumed to be 1 GeV/fm, as indicated by the slopes of baryon Regge trajectories. In Ref. [17] all the eigenstates of this model were obtained exactly for excitation energies up to ~12 GeV. The ground-state energy E_0 for this string tension is 840 MeV. The model may be viewed as a heavy-light meson, such as $\bar{t}u$, in the limit that the antiquark mass goes to infinity. However, it retains only the confining part of the $\bar{t}u$ interaction modeled by a flux tube.

The model neglects gluon and sea-quark contributions to DIS as well as the quantum chromodynamics evolution. However, the observed ratio of the spin structure function $g_1(x)$ to $F_1(x)$, the unpolarized structure function, is relatively independent of Q^2 [16]. Our objective is to calculate the x dependence of this ratio for the contribution of valence quarks to DIS, and we hope that the model is useful in this context. The utility of our potential model is limited, however, and we note that it has known shortcomings. It cannot, for example, reproduce the observed ratio of g_1^p/g_A [25].

The virtual photon asymmetry is defined as [16]

$$A_1 = \frac{\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}}{\sigma_{\frac{1}{2}} + \sigma_{\frac{3}{2}}}, \tag{2}$$

with $\sigma_{1/2}$ and $\sigma_{3/2}$ the helicity cross sections for the target angular momentum antiparallel and parallel to the photon helicity, respectively. We may calculate the inclusive virtual photon helicity cross sections in the rest frame of the target as

$$\sigma_{\frac{1}{2}} = \sigma_M \sum_I \left|\langle I|\alpha_+ e^{i|\mathbf{q}|z}\left|0, -\tfrac{1}{2}\right\rangle\right|^2 \delta(E_I - E_0 - \nu) \tag{3}$$

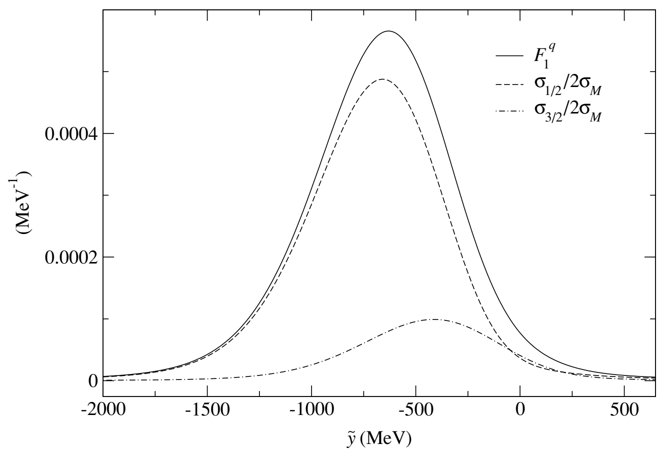

FIG. 1. Virtual photon helicity cross section of a confined massless quark, modulo twice the Mott cross section, as a function of $\tilde{y}$. The dashed ($\sigma_{1/2}$) and dashed-dotted ($\sigma_{3/2}$) curves sum to the unpolarized structure function (solid curve).

$$\sigma_{\frac{3}{2}} = \sigma_M \sum_I \left|\langle I|\alpha_+ e^{i|\mathbf{q}|z}\left|0, +\tfrac{1}{2}\right\rangle\right|^2 \delta(E_I - E_0 - \nu), \tag{4}$$

where $|\mathbf{q}|$ and ν are the momentum and energy transferred to the target, σ_M is the Mott cross section; and we assume that the virtual photon is in the $\hat{\mathbf{z}}$ direction. The ground states $|0, j_z = \pm 1/2\rangle$ have the total angular momentum projection $j_z = \pm 1/2$. The operator α_+ corresponds to a virtual photon with positive helicity, and $|I\rangle$ are eigenstates of the Hamiltonian H [Eq. (1)] with energies E_I.

The calculation of the virtual photon helicity cross sections proceeds in this model, without approximations, exactly as the calculation of the unpolarized structure functions described in Ref. [17]. When $|\mathbf{q}|$ is large, the σ/σ_M depend only on $\tilde{y} = |\mathbf{q}| - \nu$. Figure 1 shows the calculated $\sigma_{1/2}/(2\sigma_M)$ and $\sigma_{3/2}/(2\sigma_M)$ plotted as a function of the scaling variable $\tilde{y}$, and their sum

$$F_1^q(\tilde{y}) = \frac{1}{2\sigma_M}(\sigma_{1/2} + \sigma_{3/2}), \tag{5}$$

the unpolarized structure function. The conventionally defined Bjorken and Nachtmann scaling variables are related to $\tilde{y}$ by [26]

$$x(Q^2 \to \infty) = \xi = -\frac{\tilde{y}}{M_T}, \tag{6}$$

where M_T is the target mass. Thus small (large) negative $\tilde{y}$ correspond to small (large) x. We note that the $\sigma_{1/2}(\tilde{y})$ and $\sigma_{3/2}(\tilde{y})$ are not proportional, which implies that the A_1^q of a confined relativistic quark has a large $\tilde{y}$ or equivalently x dependence.

III. RESULTS

The ground state $|0, j_z\rangle$ of the confined quark has wave function

$$\Psi_{0,j_z}(\mathbf{r}) = \begin{pmatrix} f_0(r)\mathcal{Y}^0_{1/2,j_z}(\hat{\mathbf{r}}) \\ i g_0(r)\mathcal{Y}^1_{1/2,j_z}(\hat{\mathbf{r}}) \end{pmatrix}, \tag{7}$$

RAPID COMMUNICATIONS

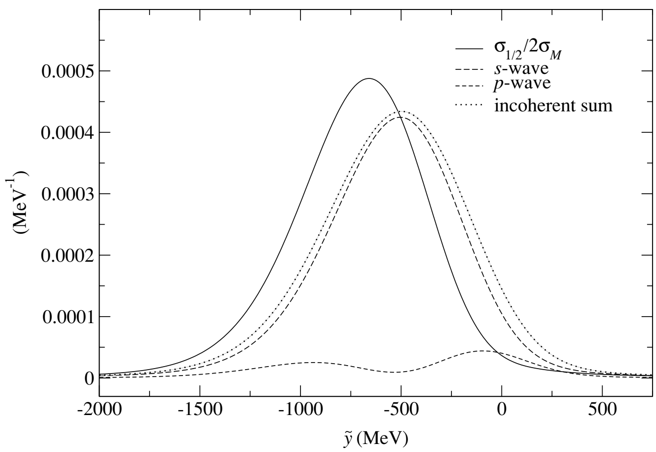

FIG. 2. Interference effects in $j_z = -\frac{1}{2}$ $(\sigma_{1/2})$ structure function. The dashed lines give the contributions of the s and p waves alone; the dotted line shows their incoherent sum; and the solid line is the exact result.

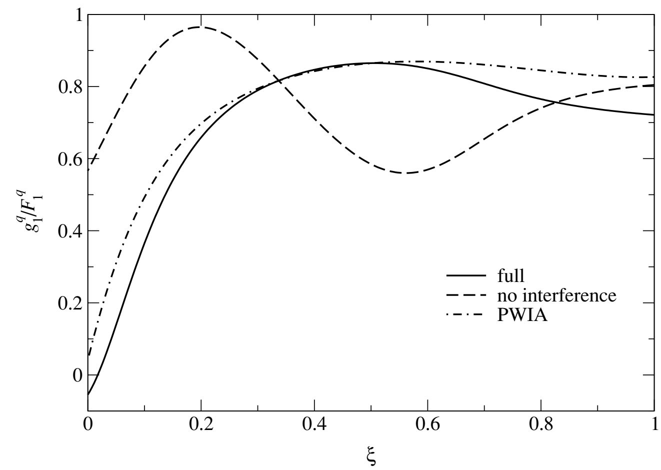

FIG. 3. The g_1^q/F_1^q for a single massless quark confined by a flux tube, as a function of the Nachtmann $\xi = (|\mathbf{q}| - \nu)/M_T$ with and without interference terms (see text) and compared to PWIA. The curves are valid for $\xi \lesssim 0.8$.

where $f_0(r)$ and $g_0(r)$ are the radial functions for the s and p waves, respectively, and $\mathcal{Y}^{\ell}_{j,jz}$ are the spin-angle functions obtained by coupling spin-1/2 and orbital angular momentum ℓ to $j = 1/2$.

The interference in the DIS between the s and p waves contributes significantly to the $\tilde{y}$ dependence of the $\sigma_{1/2}$ helicity cross section, A_1^q and F_1^q. The effect of interference is shown in Fig. 2, where we compare the polarized cross section $\sigma_{1/2}$ including interference terms (solid curve) with the polarized cross section neglecting interference terms (dotted curve). Also shown are the polarized cross sections obtained after keeping only the s or p waves in the $j_z = -1/2$ target. We note that the interference shifts $\sigma_{1/2}$ to more negative $\tilde{y}$ corresponding to larger values of ξ. Only the p waves contribute to $\sigma_{3/2}$, shown in Fig.1.

The virtual photon asymmetry is given in terms of the spin dependent structure functions g_1 and g_2 [16] by

$$A_1 = \frac{g_1 - \gamma^2 g_2}{F_1} \approx \frac{g_1}{F_1}, \tag{8}$$

where $\gamma^2 = 4M_T^2 x^2/Q^2$, in the scaling regime $Q^2 \to \infty$. As mentioned above, the observed A_1 of the proton, A_1^p, is nearly independent of Q^2 and is used to extract values of g_1^p/F_1^p [16].

Using the structure functions given in Fig. 1 we can easily calculate the virtual photon asymmetry A_1^q, or equivalently the ratio g_1^q/F_1^q, for a single confined quark, as a function of $\tilde{y}$. In order to compare it with the data on protons, we have to convert it to a function of ξ by providing a mass scale M_T (see Eq. (6)). Our model target has infinite mass associated with the center of the confining potential. However, that mass is not relevant since only the confined quark contributes to DIS. We use $M_T = 2.5$ GeV $\sim 3E_0$, where E_0 is the energy of a single confined quark in the ground state. With this choice the $F_1^q(\xi)$ becomes small at $\xi \sim 0.8$ as in the proton. The fact that the model target has infinite mass means that response can be nonzero, in principle, at arbitrarily large values of $\xi > 0$. In fact, the calculated structure functions shown in Figs.1 and 2 are very close to zero at $\tilde{y} = -2000$ MeV corresponding to $\xi \approx 0.8$. Nevertheless, the present model should not be used for values of $\xi \gtrsim 0.8$.

The solid curve in Fig. 3 shows the $A_1^q(\xi)$, or equivalently, $g_1^q(\xi)/F_1^q(\xi)$ of a confined quark. The calculated ratio goes to zero at small ξ, and this behavior is independent of the chosen value of M_T. The dip at $\xi = 0$ is due to the shift of $\sigma_{1/2}$ to larger values of ξ, produced by the interference effect shown in Fig. 2. When the interference terms are omitted we obtain the dashed curve in Fig. 3, which has $g_1^q/F_1^q \sim 0.6$ at $\xi = 0$.

Alternatively, we could have chosen the string tension $\sqrt{\sigma}$ such that $3E_0 = M_N$, the nucleon mass. However, since $\sqrt{\sigma}$ provides the only mass scale in the Hamiltonian H [Eq. (1)], $A_1^q(\xi)$ is independent of this choice.

Let us compare the exact calculation of $A_1^q(\xi)$ to that obtained in the plane wave impulse approximation (PWIA). We replace the final state $\langle I|$ in Eqs. (3) and (4) with a positive energy plane wave $\langle u_{\mathbf{k}+\mathbf{q},s}|$ with momentum $\mathbf{k}+\mathbf{q}$ and spin projection s and replace the energy of the struck constituent by that of a free particle: $E_I \to |\mathbf{k}+\mathbf{q}| + \langle V\rangle_0$; here $\langle V\rangle_0$ is the expectation value of the potential in the ground state, chosen to reproduce the first moment of the exact result. We may simplify the resulting expression for $\Delta\sigma = \sigma_{1/2} - \sigma_{3/2}$ in PWIA using the Dirac equation,

$$f_0'(r) = E_0 g_0(r) \tag{9}$$

$$g_0'(r) + \frac{2}{r} g_0(r) = -[E_0 - \sqrt{\sigma} r] f_0(r), \tag{10}$$

for the ground state. Note the simplicity of Eq. (9) owing to the form of the Dirac structure of the potential in Eq. (1) required by spin symmetry. In PWIA we obtain

$$\Delta\sigma = \frac{\sigma_M}{2}\left\{\left[\frac{1}{2}\left(1 + \frac{4\tilde{y}^2}{E_0^2}\right)\right]\int_0^\infty dk_\perp k_\perp |\tilde{f}_0(\bar{k})|^2 - \frac{1}{E_0^2}\int_0^\infty dk_\perp k_\perp \bar{k}^2 |\tilde{f}_0(\bar{k})|^2\right\}, \tag{11}$$

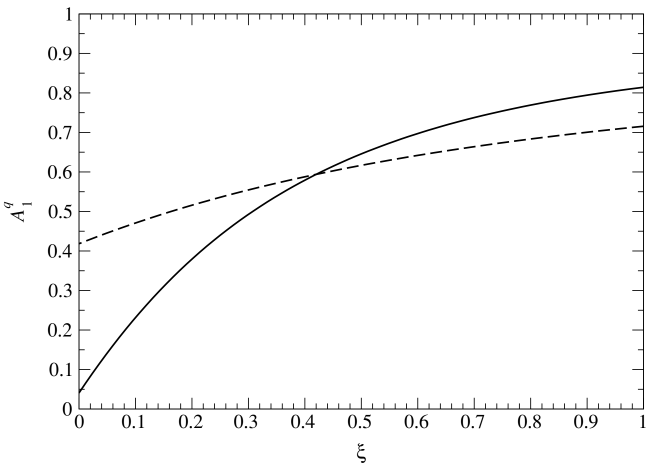

FIG. 4. The A_1^q in the linear confinement model (solid curve) and in the cavity approximation to the bag model (dashed curve) versus the dimensionless variable ξ described in the text.

where $\bar{k} = \sqrt{k_\perp^2 + (E_0/2 + \tilde{y})^2}$ and we have used the fact that $\langle V \rangle_0 = E_0/2$ [17]. The PWIA for $A_1^q(\xi)$ is shown as a dot-dash curve in Fig. 3. The final state interactions (FSI) do not change the qualitative behavior of $A_1^q(\xi)$ in the present model; however, they contribute to its suppression at small ξ.

We also compare the results of the present model with those of the cavity approximation to the bag model [23,27]. We neglect FSI in the bag model and compare the results for A_1^q within PWIA. The bag model wave function has the same form as Eq. (7) with the radial functions $f_0(r) = n_0 j_0(pr)$ and $g_0(r) = -n_0 j_1(pr)$, where n_0 is a normalization factor and j_ℓ are the spherical Bessel functions. The value of p, the momentum of the confined constituent in a cavity of radius R, is fixed by a boundary condition and has the numerical value $pR \approx 2.04$. We choose the cavity radius $R = 0.65$ fm, the rms radius of the ground state in the present model. The results are shown in Fig. 4, where the asymmetries for the linear confining model and the bag model are plotted versus $\xi = -\tilde{y}/E_0$, where E_0 is the ground state energy of the constituent and takes on the value 0.84 GeV in the linear confinement model and 0.62 GeV in the bag model. The photon point $\xi = 0$ is, of course, scale invariant and independent of whatever mass scale one uses to obtain a dimensionless ξ.

The experimentally observed suppression of the spin asymmetry in the proton at small values of Bjorken x (or ξ) is seen in the linear confinement model but not in the bag model. Note that although the bag model has similar interference terms of the upper (s-wave) and lower (p-wave) terms of the wave function, these terms do not lead to a suppression at small values of ξ of $A_1^q(\xi)$. The present model demonstrates that the p waves give rise to a dynamical suppression of the helicity distribution at small x when interactions are taken into account. It is, of course, possible to obtain the suppression at small ξ in the bag model by taking into account spin and flavor dependent quark interactions [28]. We obtain this suppression naturally as a consequence of the Dirac character of the interacting constituent.

IV. CONCLUSION

In conclusion, the present work suggests that the ξ dependence of $A_1^p(\xi)$ is a consequence of the interactions among the relativistic fermionic constituents. The p waves in bound quark wave functions interfere with the dominant s waves to suppress $A_1^q(\xi)$ at small ξ when the flux tube model for confinement is used. Although these interference terms are also present in the bag model they do not lead to a suppression of A_1^q at small ξ.

Our model is certainly too simple; it approximates the problem of three interacting quarks by a relativistic one-quark problem. Nevertheless p waves occur very naturally in the wave functions of spin-half relativistic particles, and their effect will presumably exist in more refined treatments of spin asymmetries.

ACKNOWLEDGMENTS

This work has been supported by the U.S. National Science Foundation via Grant PHY-00-98353, and by the U.S. Department of Energy under Contract W-7405-ENG-36 and the Schweizerische Nationalfonds.

[1] P. L. Anthony *et al.* (E155), Phys. Lett. **B553**, 18 (2003), hep-ex/0204028.
[2] A. Airapetian *et al.* (HERMES), Phys. Rev. Lett. **90**, 092002 (2003), hep-ex/0209018.
[3] X. Zheng *et al.* (Jefferson Lab Hall A), Phys. Rev. Lett. **92**, 012004 (2004), nucl-ex/0308011.
[4] R. L. Jaffe and A. Manohar, Nucl. Phys. **B337**, 509 (1990).
[5] R. L. Jaffe and X.-D. Ji, Phys. Rev. D **43**, 724 (1991).
[6] A. W. Schreiber, A. I. Signal, and A. W. Thomas, Phys. Rev. D **44**, 2653 (1991).
[7] X. Song and J. S. McCarthy, Phys. Rev. D **49**, 3169 (1994), hep-ph/9401356.
[8] H. Weigel, L. P. Gamberg, and H. Reinhardt, Phys. Rev. D **55**, 6910 (1997), hep-ph/9609226.
[9] F. M. Steffens, K. Tsushima, A. W. Thomas, and K. Saito, Phys. Lett. **B447**, 233 (1999), nucl-th/9810018.
[10] N. Barik and R. N. Mishra, Phys. Rev. D **61**, 014002 (2000).
[11] M. Gorchtein, D. Drechsel, M. M. Giannini, E. Santopinto, and L. Tiator, Phys. Rev. C **70**, 055202 (2004).
[12] G. B. Smith and L. J. Tassie, Ann. Phys. **65**, 352 (1971).
[13] J. S. Bell and H. Ruegg, Nucl. Phys. **B98**, 151 (1975).
[14] J. N. Ginocchio, Phys. Rev. Lett. **78**, 436 (1997), nucl-th/9611044.
[15] P. R. Page, T. Goldman, and J. N. Ginocchio, Phys. Rev. Lett. **86**, 204 (2001), hep-ph/0002094.
[16] B. Filippone and X. Ji, Adv. Nucl. Phys. **26**, 1 (2001).
[17] M. W. Paris, Phys. Rev. C **68**, 025201 (2003), nucl-th/0305020.
[18] R. L. Jaffe, Phys. Rev. D **11**, 1953 (1975).
[19] R. L. Jaffe and X. Ji, Nucl. Phys. **B375**, 527 (1992).
[20] N. N. Bogoliubov, Ann. Inst. Henri Poincaré, Sect. A **8**, 163 (1968).
[21] J. Kuti and V. F. Weisskopf, Phys. Rev. D **4**, 3418 (1971).

[22] F. E. Close, Nucl. Phys. **B80**, 269 (1974).
[23] A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D **10**, 2599 (1974).
[24] J. Bjorken and S. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
[25] A. Abbas, J. Phys. G **16**, L21 (1990).
[26] O. Benhar, V. R. Pandharipande, and I. Sick, Phys. Lett. **B489**, 131 (2000).
[27] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D **9**, 3471 (1974).
[28] F. E. Close and A. W. Thomas, Phys. Lett. **B212**, 227 (1988).