New approach for calculating the dressed quark propagator at finite chemical potential

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(Received 2 August 2004; published 26 January 2005)

A new method for obtaining the chemical potential dependence of the dressed quark propagator in the rainbow approximation of the Dyson-Schwinger equation is developed. In the above approximation we prove that the dressed quark propagator at finite chemical potential μ can be written as $\mathcal{G}_0^{-1}[\mu] = i\gamma\cdot\tilde{p}A(\tilde{p}^2) + B(\tilde{p}^2)$ with $\tilde{p}_\mu = (\vec{p}, p_4 + i\mu)$. From this the chemical potential dependence of the "effective" two-quark condensate is evaluated. A comparison with previous results is given.

DOI: 10.1103/PhysRevC.71.015205 PACS number(s): 24.85.+p, 11.10.Wx, 12.39.Ba, 14.20.Dh

The quark propagator at finite chemical potential plays an essential role in the study of chiral symmetry restoration and quark deconfinement. Due to the well-known difficulties to deal directly with finite density QCD, it is interesting to give a general recipe to study the chemical potential dependence of the dressed-quark propagator at nonzero chemical potential in the framework of a suitable nonperturbative QCD model.

Because the global color symmetry model (GCM) [1–3] provides a nonperturbative framework that admits the simultaneous study of dynamical chiral symmetry breaking and confinement, it is expected to be well suited to explore the transition from hadronic matter to QGP [4]. It is the aim of this article to study the chemical potential dependence of the dressed-quark propagator in the framework of GCM, which provides a means of determining the behavior of the chiral and deconfinement order parameters. Up to this end let us start from the Euclidean action of GCM at finite chemical potential μ (in the case of the chiral limit):

$$S_{\rm GCM}[\bar{q}, q; \mu] = \int d^4x \{\bar{q}(x)[\gamma\cdot\partial_x - \mu\gamma_4]q(x)\} + \int d^4x d^4y \left[\frac{g_s^2}{2} j_\mu^a(x) D_{\mu\nu}^{ab}(x-y) j_\nu^b(y)\right], \quad (1)$$

where $j_\mu^a(x) = \bar{q}(x)\gamma_\mu \frac{\lambda_c^a}{2} q(x)$ denotes the color octet vector current and $g_s^2 D_{\mu\nu}^{ab}(x-y)$ is the dressed model gluon propagator in GCM. For convenience, we employ a model ansatz $D_{\mu\nu}^{ab}(x-y) = \delta_{\mu\nu}\delta^{ab} D(x-y)$ for the gluon propagator, which is often referred to as the so-called "Feynman-like" gauge propagator [1,2]. (It should be noted that the above ansatz should be regarded merely as a model form for the gluon two-point function.)

Introducing an auxiliary bilocal field $B^\theta(x, y)$ and applying the standard bosonization procedure the partition function of GCM [1,2]:

$$\mathcal{Z}[\mu] = \int \mathcal{D}\bar{q}\mathcal{D}q e^{-S_{\rm GCM}[\bar{q},q;\mu]} \quad (2)$$

can be rewritten in terms of the bilocal fields $B^\theta(x, y)$ as follows:

$$\mathcal{Z}[\mu] = \int \mathcal{D}B^\theta e^{-S_{\rm eff}[B^\theta;\mu]} \quad (3)$$

with the effective bosonic action as follows:

$$S_{\rm eff}[B^\theta; \mu] = -{\rm Tr}\ln \mathcal{G}^{-1}[B^\theta; \mu] + \int d^4x d^4y \frac{B^\theta(x, y)B^\theta(y, x)}{2g_s^2 D(x-y)} \quad (4)$$

and the quark operator as follows:

$$\mathcal{G}^{-1}[B^\theta; \mu] = [\gamma\cdot\partial_x - \mu\gamma_4]\delta(x-y) + \Lambda^\theta B^\theta(x, y), \quad (5)$$

where the matrices $\Lambda^\theta = D^a \otimes C^b \otimes F^c$ are determined by Fierz transformation in Dirac, color, and flavor spaces of the current-current interaction in Eq. (1) (more detail can be found in Refs. [1,2]).

In the mean-field approximation, the fields $B^\theta(x, y)$ are substituted simply by their vacuum value $B_0^\theta(x, y)$, which is defined as $\frac{\delta S_{\rm eff}}{\delta B}|_{B_0} = 0$ and is given by the following:

$$B_0^\theta[\mu](x, y) = g_s^2 D(x-y)\, {\rm tr}[\Lambda^\theta \mathcal{G}_0[\mu](x, y)], \quad (6)$$

where the notation *tr* includes trace over the Dirac, color, and flavor indices and $\mathcal{G}_0^{-1}[\mu](x, y)$ denotes the inverse propagator with the self-energy $\Sigma_0[\mu](x, y) = \Lambda^\theta B_0^\theta[\mu](x, y)$ at the finite chemical potential μ. Employing the stationary condition Eq. (6), and reversing the Fierz transformation, we have the following:

$$\Sigma_0[\mu](x, y) = \frac{4}{3} g_s^2 D(x-y)\gamma_\nu \mathcal{G}_0[\mu](x, y)\gamma_\nu. \quad (7)$$

In this case, Eq. (5) reduces to the following:

$$\begin{aligned}\mathcal{G}_0^{-1}[\mu](x, y) &= [\gamma\cdot\partial_x - \mu\gamma_4]\delta(x-y) + \Lambda^\theta B_0^\theta[\mu](x, y) \\ &= [\gamma\cdot\partial_x - \mu\gamma_4]\delta(x-y) \\ &\quad + \frac{4}{3} g_s^2 D(x-y)\gamma_\nu \mathcal{G}_0[\mu](x, y)\gamma_\nu. \quad (8)\end{aligned}$$

Fourier transforming Eq. (8) leads to the momentum space form of $\mathcal{G}_0^{-1}[\mu](p)$ as follows:

$$\mathcal{G}_0^{-1}[\mu](p) = i\gamma\cdot p - \mu\gamma_4 + \frac{4}{3}\int\frac{d^4q}{(2\pi)^4}g_s^2 \times D(p-q)\gamma_\nu\mathcal{G}_0[\mu](q)\gamma_\nu. \quad (9)$$

Here we want to stress that Eq. (9) (commonly referred to as the "rainbow" approximation), which employs the bare quark-gluon vertex and solves the Dyson-Schwinger equation for the dressed quark propagator $\mathcal{G}_0[\mu](p)$ at finite chemical potential with a given chemical potential independent gluon propagator $g_s^2D(p)$ as input, is our starting point for studying the dressed quark propagator at finite chemical potential in the framework of GCM. Our main conclusion [see Eq. (33) below] derived form Eq. (9) is valid only for the GCM model.

It should be noted that both $B_0^\theta[\mu](x,y)$ and $\mathcal{G}_0^{-1}[\mu](x,y)$ depend on the chemical potential μ. When the chemical potential μ is switched off, $\mathcal{G}_0[\mu]$ and $\Sigma_0[\mu]$ go into the usual dressed quark propagator $G\equiv\mathcal{G}_0[\mu=0]$ and self-energy, which satisfy the following:

$$\begin{aligned}\Sigma(p) &= \int d^4x e^{ip\cdot x}\left[\Lambda^\theta B_0^\theta(x)\right]\\ &= \frac{4}{3}\int\frac{d^4q}{(2\pi)^4}g_s^2D(p-q)\gamma_\nu G(q)\gamma_\nu\\ &\equiv i\gamma\cdot p[A(p^2)-1]+B(p^2)\end{aligned} \quad (10)$$

and

$$G^{-1}(p) = i\gamma\cdot p + \frac{4}{3}\int\frac{d^4q}{(2\pi)^4}g_s^2D(p-q)\gamma_\nu G(q)\gamma_\nu, \quad (11)$$

where the self-energy functions $A(p^2)$ and $B(p^2)$ are determined by the rainbow Dyson-Schwinger equation (DSE) as follows:

$$\begin{aligned}[A(p^2)-1]p^2 &= \frac{8}{3}\int\frac{d^4q}{(2\pi)^4}g_s^2D(p-q)\times\frac{A(q^2)p\cdot q}{q^2A^2(q^2)+B^2(q^2)},\\ B(p^2) &= \frac{16}{3}\int\frac{d^4q}{(2\pi)^4}g_s^2D(p-q)\times\frac{B(q^2)}{q^2A^2(q^2)+B^2(q^2)}.\end{aligned} \quad (12)$$

Here we want to stress that the $B(p^2)$ in Eq. (12) has two qualitatively distinct solutions. The "Nambu-Goldstone" solution, for which

$$B(p^2)\neq 0, \quad (13)$$

describes a phase in which (a) chiral symmetry is dynamically broken, because one has a nonzero quark mass function, and (b) the dressed quarks are confined, because the propagator described by these functions does not have a Lehmann representation. The alternative "Wigner" solution, for which

$$B(p^2)\equiv 0, \quad (14)$$

describes a phase in which chiral symmetry is not broken and the dressed quarks are not confined.

Let us now study the chemical potential dependence of the dressed quark propagator. It is clear that the free inverse quark propagator at finite chemical potential is obviously analytic in μ and can be obtained from the free inverse quark propagator at zero μ by the substitution $p_4\to p_4+i\mu$. In the case of real QCD, we expect that the full inverse quark propagator at finite chemical potential is also analytic in μ, at least for small μ. This is supported by lattice study of finite-density QCD. In fact, in the lattice treatment of finite-density QCD, it is generally believed that physical quantities are analytic in the neighborhood of $\mu=0$ and two kinds of methods, that is, the Taylor expansion in powers of μ and analytic continuation from simulations at imaginary μ are adopted [5–7]. Therefore, we think it is interesting to assume the analyticity property of the full inverse quark propagator at finite chemical potential in a continuum nonperturbative QCD model such as GCM and study its physical consequences. Under this assumption, one can expand $\mathcal{G}_0^{-1}[\mu](p)$ in powers of μ as follows:

$$\begin{aligned}\mathcal{G}_0^{-1}[\mu] &= \mathcal{G}_0^{-1}[\mu]\big|_{\mu=0} + \left.\frac{\partial\mathcal{G}_0^{-1}[\mu]}{\partial\mu}\right|_{\mu=0}\mu + \frac{1}{2!}\left.\frac{\partial^2\mathcal{G}_0^{-1}[\mu]}{\partial\mu^2}\right|_{\mu=0}\mu^2+\cdots + \frac{1}{n!}\left.\frac{\partial^n\mathcal{G}_0^{-1}[\mu]}{\partial\mu^n}\right|_{\mu=0}\mu^n+\cdots\\ &= G^{-1}+\Gamma^{(1)}\mu+\frac{1}{2!}\Gamma^{(2)}\mu^2+\cdots + \frac{1}{n!}\Gamma^{(n)}\mu^n+\cdots,\end{aligned} \quad (15)$$

with $\Gamma^{(1)}(p,0)$, $\Gamma^{(2)}(p,0)$, and $\Gamma^{(n)}(p,0)$

$$\Gamma^{(1)}(p,0) = \left.\frac{\partial\mathcal{G}_0^{-1}[\mu](p)}{\partial\mu}\right|_{\mu=0}, \quad (16)$$

$$\Gamma^{(2)}(p,0) = \left.\frac{\partial^2\mathcal{G}_0^{-1}[\mu](p)}{\partial\mu^2}\right|_{\mu=0}, \quad (17)$$

$$\Gamma^{(n)}(p,0) = \left.\frac{\partial^n\mathcal{G}_0^{-1}[\mu](p)}{\partial\mu^n}\right|_{\mu=0}. \quad (18)$$

It should be noted that Eq. (15) is valid only within the radius of convergence of μ expansion. In addition, we stress here that the model gluon propagator [see Eq. (9)] has no explicit μ dependence, whereas the actual gluon propagator should be μ dependent due to quark loop insertions. As such it may be inadequate at large values of μ, particularly near any critical chemical potential.

Applying the differential operation $\partial/\partial\mu$ on both sides of Eq. (9), we obtain the following:

$$\frac{\partial\mathcal{G}_0^{-1}[\mu](p)}{\partial\mu} = -\gamma_4 + \frac{4}{3}\int\frac{d^4q}{(2\pi)^4}g_s^2D(p-q)\gamma_\nu \times\frac{\partial\mathcal{G}_0[\mu](q)}{\partial\mu}\gamma_\nu$$

$$= -\gamma_4 - \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g_s^2 D(p-q)\gamma_\nu \mathcal{G}_0[\mu](q) \times \frac{\partial \mathcal{G}_0^{-1}[\mu](q)}{\partial\mu}\mathcal{G}_0[\mu](q)\gamma_\nu, \tag{19}$$

where we have made use of the following identity:

$$\frac{\partial \mathcal{G}_0[\mu](q)}{\partial\mu} = -\mathcal{G}_0[\mu](q)\frac{\partial \mathcal{G}_0^{-1}[\mu](q)}{\partial\mu}\mathcal{G}_0[\mu](q). \tag{20}$$

Setting $\mu = 0$ in Eq. (19), we obtain the following integral equation satisfied by $\Gamma^{(1)}$

$$\Gamma^{(1)}(p,0) = -\gamma_4 - \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g_s^2 D(p-q) \times \gamma_\nu G(q)\Gamma^{(1)}(q,0)G(q)\gamma_\nu. \tag{21}$$

Similarly, applying the differential operation $\partial/\partial\mu$ on both sides of Eq. (19) successively (n-1)($n \geqslant 2$) times and subsequently setting $\mu = 0$, we obtain the following:

$$\Gamma^{(n)}(p,0) = \left.\frac{\partial^n \mathcal{G}_0^{-1}[\mu](p)}{\partial\mu^n}\right|_{\mu=0} = -\frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g_s^2 D(p-q)\gamma_\nu \left\{\frac{\partial^{n-1}}{\partial\mu^{n-1}} \times \left[\mathcal{G}_0[\mu](q)\frac{\partial \mathcal{G}_0^{-1}[\mu](q)}{\partial\mu}\mathcal{G}_0[\mu](q)\right]\right\}\Bigg|_{\mu=0}\gamma_\nu. \tag{22}$$

To write the integral equation satisfied by $\Gamma^n(p,0)$, one should perform the differentiation operation in the expression as follows:

$$\frac{\partial^{n-1}}{\partial\mu^{n-1}}\left[\mathcal{G}_0[\mu](q)\frac{\partial \mathcal{G}_0^{-1}[\mu](q)}{\partial\mu}\mathcal{G}_0[\mu](q)\right]. \tag{23}$$

For example, when $n = 2$, we have the following:

$$\frac{\partial}{\partial\mu}\left[\mathcal{G}_0[\mu](q)\frac{\partial \mathcal{G}_0^{-1}[\mu](q)}{\partial\mu}\mathcal{G}_0[\mu](q)\right] = \mathcal{G}_0[\mu](q)\left\{\frac{\partial^2 \mathcal{G}_0^{-1}[\mu](q)}{\partial\mu^2} - 2\frac{\partial \mathcal{G}_0^{-1}[\mu](q)}{\partial\mu}\mathcal{G}_0[\mu](q) \times \frac{\partial \mathcal{G}_0^{-1}[\mu](q)}{\partial\mu}\right\}\mathcal{G}_0[\mu](q). \tag{24}$$

Putting $n = 2$ in Eq. (22) and substituting Eq. (24) into Eq. (22), we have the following integral equation for $\Gamma^{(2)}(p,0)$:

$$\Gamma^{(2)}(p,0) = -\frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g_s^2 D(p-q)\gamma_\mu G(q)\Gamma^{(2)}(q,0) \times G(q)\gamma_\mu + \frac{8}{3}\int \frac{d^4q}{(2\pi)^4} g_s^2 D(p-q)\gamma_\mu \times G(q)\Gamma^{(1)}(q,0)G(q)\Gamma^{(1)}(k,0)G(q)\gamma_\mu. \tag{25}$$

Generally speaking there is no definite relation between $\Gamma^{(n)}(p,0)(n \geqslant 1)$ and the general vertex obtained by differentiating the fermion propagator (the Ward identity). However, as shown below, based on the rainbow approximation of Dyson-Schwinger [Eq. (9)] and the assumption that the full inverse quark propagator is analytic in the neighborhood of $\mu = 0$, one can find a simple relation between $\Gamma^{(n)}(p,0)$ and the general vertex. For $n = 1$ this relation reads $\Gamma^{(1)}(p,0) = \frac{\partial G^{-1}(p)}{\partial(-ip_4)}$ and we prove this specific case first.

Applying the differential operation $\partial/\partial(-ip_4)$ on both sides of Eq. (11), we obtain the following:

$$\frac{\partial G^{-1}(p)}{\partial(-ip_4)} = -\gamma_4 + \frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\frac{\partial}{\partial(-ip_4)}\left[g_s^2 D(p-q)\right] \times \gamma_\nu G(q)\gamma_\nu$$
$$= -\gamma_4 + \frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\left\{\frac{\partial}{\partial(iq_4)}\left[g_s^2 D(p-q)\right]\right\} \times\gamma_\nu G(q)\gamma_\nu$$
$$= -\gamma_4 + \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g_s^2 D(p-q)\gamma_\nu \times\frac{\partial G(q)}{\partial(-iq_4)}\gamma_\nu, \tag{26}$$

where we have made use of integration by parts. By means of the following identity:

$$\frac{\partial G(q)}{\partial(-iq_4)} = -G(q)\frac{\partial G^{-1}(q)}{\partial(-iq_4)}G(q),$$

Eq. (26) can be rewritten as follows:

$$\frac{\partial G^{-1}(p)}{\partial(-ip_4)} = -\gamma_4 - \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g_s^2 D(p-q)\gamma_\nu G(q) \times \frac{\partial G^{-1}(q)}{\partial(-iq_4)}G(q)\gamma_\nu, \tag{27}$$

Comparing Eq. (21) with Eq. (27), it is easy to see that $\Gamma^{(1)}(p,0)$ and $\partial G^{-1}(p)/\partial(-ip_4)$ satisfy the same equation. This shows the following:

$$\Gamma^{(1)}(p,0) \equiv \frac{\partial G^{-1}(p)}{\partial(-ip_4)}.$$

We recognize that this relation is the so-called vector "Ward identity" [8,9].

Applying the differential operation $\partial/\partial(-ip_4)$ on both sides of Eq. (27) successively (n-1)($n \geqslant 2$) times, we obtain the following:

$$\frac{\partial^n G^{-1}(p)}{\partial(-ip_4)^n} = -\frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g_s^2 D(p-q)\gamma_\nu \frac{\partial^{n-1}}{\partial(-iq_4)^{n-1}} \times \left[G(q)\frac{\partial G^{-1}(q)}{\partial(-iq_4)}G(q)\right]\gamma_\nu. \tag{28}$$

Similarly, to get the integral equation satisfied by $\partial^n G^{-1}(p)/\partial(-ip_4)^n$, one should perform the differentiation operation in the following expression:

$$\frac{\partial^{n-1}}{\partial(-iq_4)^{n-1}}\left[G(q)\frac{\partial G^{-1}(q)}{\partial(-iq_4)}G(q)\right]. \tag{29}$$

For example, when $n = 2$, we have the following:

$$\frac{\partial}{\partial(-iq_4)}\left[G(q)\frac{\partial G^{-1}(q)}{\partial(-iq_4)}G(q)\right] = G(q)\left\{\frac{\partial^2 G^{-1}(q)}{\partial(-iq_4)^2} - 2\frac{\partial G^{-1}(q)}{\partial(-iq_4)}G(q)\frac{\partial G^{-1}(q)}{\partial(-iq_4)}\right\}G(q). \tag{30}$$

Putting $n = 2$ in Eq. (28) and substituting Eq. (30) into Eq. (28), we have the following:

$$\frac{\partial^2 G^{-1}(p)}{\partial(-ip_4)^2} = -\frac{4}{3}\int\frac{d^4q}{(2\pi)^4}g_s^2 D(p-q)\gamma_\nu G(q)\frac{\partial^2 G^{-1}(q)}{\partial(-iq_4)^2} \times G(q)\gamma_\nu + \frac{8}{3}\int\frac{d^4q}{(2\pi)^4}g_s^2 D(p-q)\gamma_\nu \times G(q)\frac{\partial G^{-1}(q)}{\partial(-iq_4)}G(q)\frac{\partial G^{-1}(q)}{\partial(-iq_4)}G(q)\gamma_\nu. \tag{31}$$

Comparing Eq. (31) with Eq. (25), it is easy to see that $\Gamma^{(2)}(p,0)$ and $\partial^2 G^{-1}(p)/\partial(-ip_4)^2$ satisfy the same integral equation [we have already proved $\Gamma^{(1)}(p,0) = \partial G^{(-1)}(p)/\partial(-ip_4)$]. This shows the following:

$$\Gamma^{(2)}(p,0) \equiv \frac{\partial^2 G^{-1}(p)}{\partial(-ip_4)^2}.$$

Here the key point is that with the differentiation operations being explicitly performed using the Leibniz rule Eq. (23) and Eq. (29) have identical structures. Using this one can prove inductively the following:

$$\Gamma^{(n)}(p,0) \equiv \frac{\partial^n G^{-1}(p)}{\partial(-ip_4)^n}, \quad n \geqslant 1. \tag{32}$$

In fact, if it is proven that $\Gamma^{(m)}(p,0) = \partial^m G^{(-1)}(p)/\partial(-ip_4)^m$ for $m \leqslant n-1$, then Eq. (22) and Eq. (28) tell us that $\Gamma^{(n)}(p,0)$ and $\partial^n G^{(-1)}(p)/\partial(-ip_4)^n$ satisfy the same integral equation. From this one concludes that $\Gamma^{(n)}(p,0) = \partial^n G^{(-1)}(p)/\partial(-ip_4)^n$.

Based on Eq. (32) we can obtain the main conclusion in the present work

$$\begin{aligned}\mathcal{G}_0^{-1}[\mu](p) &= G^{-1}(p) + \Gamma^{(1)}(p,0)\mu + \frac{1}{2!}\Gamma^{(2)}(p,0)\mu^2 + \cdots \\ &= G^{-1}(p) + \frac{\partial G^{-1}(p)}{\partial(-ip_4)}\mu + \frac{1}{2!}\frac{\partial^2 G^{-1}(p)}{\partial(-ip_4)^2}\mu^2 + \cdots \\ &= G^{-1}(p) + \frac{\partial G^{-1}(p)}{\partial(p_4)}i\mu + \frac{1}{2!}\frac{\partial^2 G^{-1}(p)}{\partial(p_4)^2} \times (i\mu)^2 + \cdots \\ &= G^{-1}(\vec{p}, p_4 + i\mu) \equiv G^{-1}(\tilde{p}) = i\gamma\cdot\tilde{p}A(\tilde{p}^2) + B(\tilde{p}^2),\end{aligned} \tag{33}$$

where $\tilde{p} = (\vec{p}, p_4 + i\mu)$. This shows that under the rainbow approximation of the DS equation there are only two independent Lorentz structures in the dressed quark propagator at a finite chemical potential. This feature facilitates the numerical calculations considerably.

Here we want to stressed that Eq. (33) only holds for the GCM model and within the radius of convergence of the μ expansion. In the case of real QCD, it should be noted that both the dressed quark gluon vertex and the dressed gluon propagator are chemical potential dependent. In this case, Eq. (33) would fail. Nevertheless, due to the well-known difficulties to deal directly with finite density QCD, we expect that Eq. (33) derived from GCM model is a useful relation for studying the dressed quark propagator at a finite chemical potential.

To avoid the need for a numerical solution of Eq. (12), the author in Ref. [10] provides the following algebraic forms as a better approximation to the realistic numerical solutions of the rainbow DS equation (more detail can be found in Ref. [10]):

$$\sigma_v(p^2) = \frac{1}{2D}\left\{-\bar{m}Ce^{-\frac{p^2}{D}} + \frac{2D^2\left[\frac{p^2}{D} + 2\bar{m}^2 - 1 + e^{-\frac{p^2+2D\bar{m}^2}{D}}\right]}{(p^2+2D\bar{m}^2)^2}\right\},$$

$$\sigma_s(p^2) = \frac{1}{(2D)^{\frac{1}{2}}}\left\{Ce^{-\frac{p^2}{D}} + 4D^2\frac{\left(1-e^{-\frac{b_1p^2}{2D}}\right)\left(1-e^{-\frac{b_3p^2}{2D}}\right)}{b_1b_3p^4} \times \left(b_0 + 2Db_2\frac{1-e^{-\frac{\Lambda p^2}{2D}}}{\Lambda p^2}\right) + \frac{2D\bar{m}^2}{p^2+2D\bar{m}^2} \times \left(1-e^{-\frac{p^2+2D\bar{m}^2}{D}}\right)\right\}, \tag{34}$$

where

$$C = 0.0422, \quad \bar{m} = 0.0111, \quad \Lambda = 10^{-4},$$
$$b_0 = 0.135, \quad b_1 = 2.48, \quad b_2 = 0.502, \quad b_3 = 0.168,$$

with

$$\sigma_v(p^2) = \frac{A(p^2)}{p^2A^2(p^2)+B^2(p^2)},$$
$$\sigma_s(p^2) = \frac{B(p^2)}{p^2A^2(p^2)+B^2(p^2)}.$$

Based on Eqs. (33) and (34), it is easy to obtain the self-energy function of the dressed quark propagator at a finite chemical potential.

Now let us turn to the study of the measure of the dynamical chiral symmetry breaking in the case of nonzero chemical potential. To get a reasonable result for the mixed quark-gluon condensate and vacuum susceptibilities in an effective quark-quark interaction model, the authors in Refs. [11,12] defined the "effective" two-quark condensate as the difference between the "exact" quark propagator (quark propagator in the "Nambu-Goldstone" phase, in which chiral symmetry is dynamically broken and the dressed quarks are confined) and

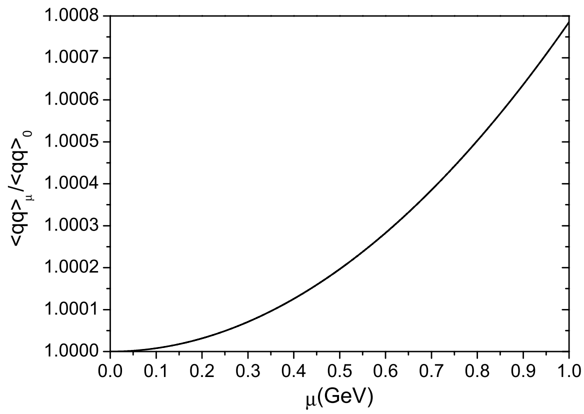

FIG. 1. The ratio $\langle\tilde{0}|\bar{q}q|\tilde{0}\rangle_\mu/\langle\tilde{0}|\bar{q}q|\tilde{0}\rangle_{\mu=0}$ as a function of μ.

the "perturbative" quark propagator (quark propagator in the "Wigner" phase, in which chiral symmetry is not dynamically broken and the dressed quarks are not confined). It can be written as (in the chiral limit and at zero chemical potential):

$$\langle\tilde{0}|\bar{q}q|\tilde{0}\rangle_{\mu=0} \equiv -tr_{DC}\left\{\mathcal{G}_0^{(NG)}[\mu=0]-\mathcal{G}_0^{(W)}[\mu=0]\right\}. \quad (35)$$

It should be noted that Eq. (35) is only valid in an effective quark-quark interaction model (more details can be found in Ref. [13]).

Here we extend the above concept to get a measure of dynamical chiral symmetry breaking in the case of finite μ and obtain the "effective" two-quark condensate with the nonzero μ as follows:

$$\langle\tilde{0}|\bar{q}q|\tilde{0}\rangle_\mu \equiv -tr_{DC}\left\{Re\mathcal{G}_0^{(NG)}[\mu]-Re\mathcal{G}_0^{(W)}[\mu]\right\} = -12\int\frac{dp^4}{(2\pi)^4}Re\{\sigma_s(\tilde{p})\}. \quad (36)$$

Substituting $\mu=0$ into Eq. (36), we have the usual "effective" two-quark condensate in the chiral limit. The calculated ratio $\langle\tilde{0}|\bar{q}q|\tilde{0}\rangle_\mu/\langle\tilde{0}|\bar{q}q|\tilde{0}\rangle_{\mu=0}$ is plotted in Fig. 1. In Fig. 1, we see that $\langle\tilde{0}|\bar{q}q|\tilde{0}\rangle_\mu/\langle\tilde{0}|\bar{q}q|\tilde{0}\rangle_{\mu=0}$ increases with increasing chemical potential μ. This result is a consequence of the necessary momentum dependence of the dressed quark self-energy [4,14–15].

To summarize, based on the rainbow approximation of the Dyson-Schwinger equation and the assumption that the full inverse quark propagator at finite chemical potential is analytic in the neighborhood of $\mu=0$, we can prove that there are only two independent Lorentz structures (instead of four, which comes from general Lorentz structure analysis) in the dressed-quark propagator at nonzero chemical potential and the inverse dressed quark propagator at finite μ can be obtained by making the substitution $p_4 \to p_4+i\mu$ in the dressed quark propagator at $\mu=0$. This feature will considerably facilitate the numerical calculations of the dressed quark propagator at a finite chemical potential. From this the "effective" quark condensates at nonzero chemical potential is analyzed.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China (under Grant Nos. 10425521, 10175033, and 10135030) and the Research Fund for the Doctoral Program of Higher Education (under Grant No. 20030284009).

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