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Extraction of two-photon contributions to the proton form factors

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Significant discrepancies have been observed between proton form factors as measured by Rosenbluth separation and polarization transfer techniques. There are indications that this difference may be caused by corrections to the one-photon exchange approximation that are not taken into account in standard radiative correction procedures. In this paper, we constrain the two-photon amplitudes by combining data from Rosenbluth, polarization transfer, and positron-proton scattering measurements. This allows a rough extraction of these two-photon effects in elastic electron-proton scattering and provides an improved extraction of the proton electromagnetic form factors.

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I. INTRODUCTION

Extractions of the proton electromagnetic form factors utilizing the polarization transfer technique show a significant decrease in the ratio of electric to magnetic form factors at large momentum transfers [1,2]. These results contradict a large body of Rosenbluth separation measurements [3–5] that indicate approximate scaling of the form factors, $\mu_p G_E/G_M \approx 1$. This inconsistency leads to a large uncertainty in our knowledge of the proton electromagnetic form factors and could have significant implications for other experiments that rely on similar techniques or assume knowledge of the proton form factors for data interpretation [6–8].

It has been suggested that two-photon exchange contributions could be responsible for the discrepancy between the Rosenbluth, or longitudinal-transverse (L-T), separation and polarization transfer form factors [9]. Calculations of the two-photon exchange diagram suggest that this may indeed be the case [10,11], and there is some evidence for two-photon exchange in comparisons of electron-proton and positron-proton scattering [12]. However, a complete calculation of two-photon exchange must include contributions where the intermediate proton is in an excited state, which are not included in Ref. [10].

In Ref. [11], the contribution from intermediate states is included through two-photon scattering off quarks in the proton, with emission and reabsorption of the partons by the nucleon described in terms of generalized parton distributions. This approach is not expected to be valid at low four-momentum transfers, Q^2, or for small values of the virtual photon polarization, ε, and it yields only half of the effect needed to resolve the discrepancy. For large Q^2 values, this is enough to make any *individual* Rosenbluth measurement consistent (within one standard deviation) with the polarization transfer result, because for any given Q^2 value there is only a two-sigma discrepancy (Fig. 1). However, there is still a clear systematic deviation between the techniques, and a recent high-precision Rosenbluth extraction [5] shows significant deviations (3–5 sigma) from the polarization transfer result, even after applying the corrections of Ref. [11].

The goal of this work is to use the existing data on elastic electron-proton and positron-proton scattering data to estimate the small two-photon amplitudes, and then use these amplitudes to correct the form factors extracted from polarization transfer and Rosenbluth separation measurements. Such an analysis was begun in Ref. [9], but only one of the three two-photon exchange terms was taken into account, and only the effect on the form factor ratio, G_E/G_M, was examined. In this work, we include additional Rosenbluth data and constraints from the limited set of positron measurements. We attempt to extract the values and uncertainties of the two-photon exchange corrections for both G_E, G_M, and G_E/G_M.

The paper is organized as follows: In Section II A we review the formalism of Ref. [9] and give an overview of the procedure we use to separate and constrain the two-photon amplitudes. In Section II B, we use the discrepancy between the Rosenbluth and polarization transfer form factors and the difference between positron-proton and electron-proton scattering to extract the two-photon exchange amplitudes. In Section II C, we parameterize these amplitudes and their uncertainties to determine the correction to the measured proton form factors and the present uncertainties. We conclude in Section III with a discussion of the limitations of such an analysis given the presently available data and discuss future measurement that can improve our knowledge of the two-photon exchange corrections and thus the proton form factors.

II. EXTRACTION OF THE TWO-PHOTON AMPLITUDES

A. Formalism and assumptions of the analysis

Extraction of the higher order contributions from the *e-p* scattering data requires a formalism that goes beyond the one-photon (Born) approximation. Such a formalism was introduced recently by Guichon and Vanderhaeghen [9], and this provides the necessary connection between the cross section and polarization observables. While discussed in terms of two-photon contributions, this formalism includes all terms in the elastic scattering amplitude: vertex corrections; loop corrections (vacuum polarization); soft and hard two-photon contributions; multiphoton exchange; and all terms with just the electron and proton in the final state. In the Born approximation, one obtains two real amplitudes that depend only on the momentum transfer: $G_E(Q^2)$ and $G_M(Q^2)$. In the generalized case, there are three complex amplitudes that depend on both ε and Q^2: $\tilde{G}_E(\varepsilon, Q^2)$, $\tilde{G}_M(\varepsilon, Q^2)$, and $\tilde{F}_3(\varepsilon, Q^2)$. Note that while these amplitudes include all interactions, the corrections beyond the Born terms are generally discussed in terms of two-photon exchange. So we often refer to these here as two-photon exchange corrections, but for this analysis "two-photon" corrections include two-photon exchange, soft multiphoton exchange ("Coulomb distortion"), and all other higher order terms.

To separate the Born and higher order contributions, we break up the generalized form factors into the Born values and the "two-photon" contributions, e.g., $\tilde{G}_E(\varepsilon, Q^2) = G_E(Q^2) + \Delta G_E(\varepsilon, Q^2)$. The amplitude $\tilde{F}_3$ is zero in the Born approximation and comes entirely from higher order terms. Following Ref. [9], we replace $\tilde{F}_3$ with the dimensionless quantity $Y_{2\gamma}$,

$$Y_{2\gamma}(\varepsilon, Q^2) = \mathrm{Re}\left(\frac{\nu \tilde{F}_3(\varepsilon, Q^2)}{M_p^2\,|\tilde{G}_M|}\right), \tag{1}$$

where $\nu = M_p^2\sqrt{(1+\varepsilon)/(1-\varepsilon)}\sqrt{\tau(1+\tau)}$ (equivalent to the definition of ν in Ref. [9]).

We now have the usual Born-level form factors, G_E and G_M, and three two-photon amplitudes: ΔG_E, ΔG_M, and $Y_{2\gamma}$. The first two of these new amplitudes are complex, but since they are expected to be small compared to the Born terms, their main effect comes from their interference with the Born amplitudes. Because the Born amplitudes are real, the contribution from the imaginary part of the two-photon amplitudes should be negligible. Therefore, throughout this paper we neglect the imaginary part of these terms and take $\tilde{G}_E$ and $\tilde{G}_M$ to refer only to the real part of the amplitudes.

The proton form factor ratio, G_E/G_M, has been extracted from cross section and polarization transfer measurements assuming one-photon exchange. In the generalized formalism, the extracted ratio does not yield the true form factor ratio but is a function of these generalized form factors [9],

$$R_{\mathrm{Pol}} = (\tilde{G}_E/\tilde{G}_M) + \left(1 - \frac{2\varepsilon}{1+\varepsilon}\tilde{G}_E/\tilde{G}_M\right)Y_{2\gamma}, \tag{2}$$

$$R_{\text{L-T}}^2 = (\tilde{G}_E/\tilde{G}_M)^2 + 2(\tau + \tilde{G}_E/\tilde{G}_M)Y_{2\gamma}, \tag{3}$$

where $\tau = Q^2/4M_p^2$. Keeping terms up to order α_{EM}, the change to the reduced cross section ($\sigma_r = \tau G_M^2 + \varepsilon G_E^2$ in the Born approximation) is

$$\frac{\Delta\sigma_r}{G_M^2} \approx 2\tau\frac{\Delta G_M}{G_M} + 2\varepsilon\rho^2\frac{\Delta G_E}{G_E} + 2\varepsilon(\tau+\rho)Y_{2\gamma}, \tag{4}$$

where $\rho = G_E/G_M$.

The general procedure for extracting the two-photon amplitudes is as follows: From Eqs. (2) and (3), we see that only the $Y_{2\gamma}$ term leads to a difference between the polarization transfer and L-T form factor ratio, and so this difference will allow us to determine $Y_{2\gamma}$. To obtain the true (Born) form factors we need additional data to constrain the values of ΔG_M and ΔG_E. Because the dominant terms of the two-photon corrections change sign for positron-proton scattering, we can use the existing data for positron-proton scattering as an additional constraint on ΔG_E and ΔG_M, allowing an extraction of the true form factors, G_E and G_M, corrected for two-photon (and multiphoton) exchange contributions. These are the form factors that can be directly connected to the structure of the proton, and that can be compared to lattice calculations or models of the nucleon.

Given the limitations of the existing cross section, polarization transfer, and positron-proton scattering data, we are forced to make some assumptions in the extraction of the two-photon amplitudes. First, we assume that two-photon effects are responsible for all of the discrepancy. Second, we assume that the two-photon amplitudes depend weakly on ε, although we examine the effect of ε-dependence in the error analysis. Finally, we assume that the higher order terms are small compared to the Born amplitudes, and therefore we only consider those contributions that are of order α_{EM} with respect to the Born amplitudes. Specifically, we neglect terms other than the "standard" radiative corrections, two-photon exchange, and soft multiphoton exchange ("Coulomb distortion") which is $\mathcal{O}(\alpha_{EM})$ after resummation.

This last assumption is important for several reasons. We neglected the imaginary part of the two-photon amplitudes because their contribution will be very small as long as the (complex) two-photon amplitudes are small compared to the (real) Born amplitudes. In addition, the two-photon amplitudes contain all higher order diagrams, some of which change sign with the sign of the lepton charge, and some of which do not. The comparison of positron and electron scattering, which we use to constrain ΔG_M, is only sensitive to those terms which change sign. Of those terms not fully treated in the standard radiative correction procedures, the only terms that are of $\mathcal{O}(\alpha_{EM})$ are the two-photon exchange and the Coulomb distortion, both of which depend on the lepton sign. So as long as the terms of order α_{EM}^2 and higher are negligible, the positron data allow us to constraint these amplitudes. One certainly expects this approximation to be valid, as the higher order terms should be suppressed by additional factors of α_{EM}. As we show, all of the data can be explained with two-photon amplitudes that are quite small (<5% of the Born amplitudes), further supporting the initial expectation.

With these assumptions, it is possible to constrain the two-photon contributions to the form factors well enough to extract G_E and G_M, albeit with additional uncertainty due to these two-photon corrections. Further data would allow improved extractions of the two-photon amplitudes, as well as provide better tests of the assumptions used in the analysis.

Finally, one must be careful to remember that for the published data, some radiative corrections have already been applied, and we must take this into account when extracting $Y_{2\gamma}$, ΔG_M, and ΔG_E, which include all such higher order terms. In the analysis of the existing cross section data, two-photon exchange contributions were only partially treated whereas other radiative corrections of $\mathcal{O}(\alpha_{EM})$ were applied. However, except for the well understood bremsstrahlung corrections, all of these terms are included in the two-photon amplitudes ΔG_E and ΔG_M. In principle, the corrections that were applied to the measured cross sections should be removed before using the data to extract these amplitudes. However, it turns out that this is not necessary. The extraction of $Y_{2\gamma}$ comes from the difference between R_{Pol} and $R_{\text{L-T}}$, but R_{Pol} is insensitive to these radiative corrections, whereas $R_{\text{L-T}}$ depends only on the ε-*dependent* corrections to the cross section. Because the loop and vertex corrections are independent of ε [13], they do not modify the value of $R_{\text{L-T}}$ extracted from the data, and so do not change the extracted value of $Y_{2\gamma}$. In the additional terms included in Ref. [14], there is a small ε-dependence in the "Schwinger" correction, but the effect is small and has a negligible impact on the extraction of the two-photon amplitudes.

The other two-photon amplitudes are constrained by the comparison of electron-proton and positron-proton scattering. This is sensitive to those terms that depend on the charge of the lepton: the two-photon exchange and the Coulomb corrections. Because the loop and vertex corrections are identical for positron and electron scattering, their inclusion does not modify the comparison of positron and electron data. Strictly speaking, the contributions due to loop and vertex diagrams are not *corrections* to the generalized form factors; they are included in ΔG_M and ΔG_E. However, the goal is to obtain the true form factors, and applying the loop and vertex to the measured cross sections yields the same result as including them in the two-photon corrections ΔG_E and ΔG_M. Similarly, while soft multiphoton exchange (Coulomb distortion) can play a non-negligible role at low-to-moderate Q^2 values [15], these corrections should not be applied to the data for this analysis, as they are included as part of the higher order corrections (ΔG_M, ΔG_E, and $Y_{2\gamma}$).

B. Extraction of $Y_{2\gamma}$ and $\tilde{G}_M$

To extract $Y_{2\gamma}$, we compare the Rosenbluth extraction of G_E/G_M from a global analysis of cross section data (Fig. 2 of Ref. [6]) to a parameterization of the polarization transfer results and uncertainties, as shown by solid and dotted lines in Fig. 1. We limit ourselves to $0.6 < Q^2 < 6.0$ GeV^2, to

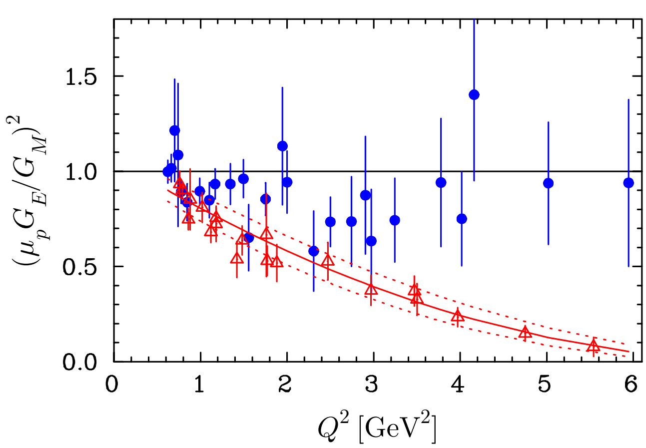

FIG. 1. (Color online) Rosenbluth form factor ratio squared, $R^2_{\text{L-T}}$ (blue circles); polarization transfer ratio squared, R^2_{Pol} (red triangles); and the parameterization of the polarization transfer ratio and uncertainty (solid and dotted red lines) used in the extraction of $Y_{2\gamma}$. The error bars shown on the Rosenbluth extractions include an estimate of the uncertainty in the determination of the normalization of the different data sets in the global analysis.

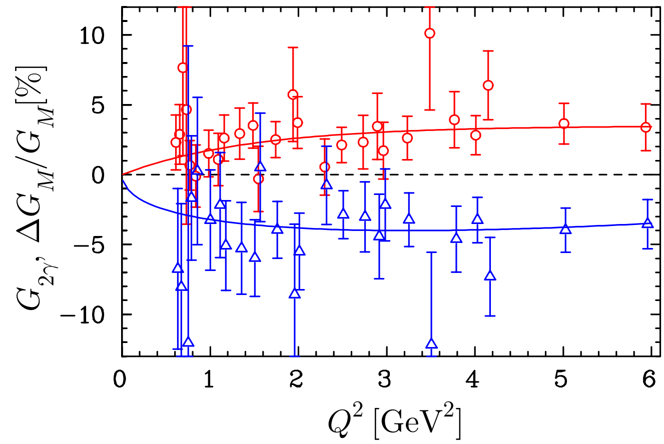

FIG. 2. (Color online) Extracted values of $Y_{2\gamma}$ (red circles) and $\Delta G_M/G_M$ (blue triangles), along with fits to the extracted amplitudes. The fits and parameterized uncertainties are given in the Appendix.

match the Q^2 range of precise polarization transfer data. We then use the difference between $R_{\text{L-T}}$ and R_{Pol} to determine $Y_{2\gamma}$, using R_{Pol} as the approximate value for $\tilde{G}_E/\tilde{G}_M$ in Eqs. (2) and (3). Note that if one instead uses the final value of $\tilde{G}_E/\tilde{G}_M$ extracted from this analysis, the change is negligible. There is no way to extract the ε-dependence of $Y_{2\gamma}$ because we only have a single value of $R_{\text{L-T}}$ at each Q^2 value, taken from the full ε range of the cross-section data, and because the polarization transfer ratios have not been measured at different ε values. However, if the ε-dependence in the amplitudes is large enough to introduce a significant nonlinearity, then the reduced cross section as a function of ε would deviate from the linearity predicted in the one-photon exchange approximation. Although current data are not precise enough to set tight limits on deviations from linearity, the existing data are all consistent with a linear dependence. We therefore assume that the amplitudes are independent of ε and use present limits on nonlinearity from existing Rosenbluth data to estimate the uncertainty in the extracted form factors. Of course, given a model of the ε-dependence, or measurements of the nonlinearities in the two-photon exchange effects, we could incorporate this information on the ε-dependence into the fit to make an improved extraction.

Figure 2 shows the extracted values for $Y_{2\gamma}$ as a function of Q^2, along with a fit to these extracted values, given in Eq. (A1) in the Appendix. Guichon and Vanderhaeghen [9] performed a similar extraction of $Y_{2\gamma}$, using fits to R_{Pol} and $R_{\text{L-T}}$. They extracted similar values for $Y_{2\gamma}$, and determined the correction to G_E/G_M assuming that ΔG_E and ΔG_M are negligible. However, while $Y_{2\gamma}$ yields the *difference* between the two techniques, as well as the largest correction to the recoil polarization ratio, comparisons of positron-proton and electron-proton scattering demonstrate that ΔG_M and ΔG_E cannot be neglected [12]. The values of $Y_{2\gamma}$ required to explain the discrepancy yield a 5–8% enhancement of the electron cross section at large ε. Because the dominant term of the two-photon correction changes sign for positron-proton scattering, this would imply a decrease in the positron-proton cross section, and a ratio of positron to electron scattering of $\lesssim 0.9$. Data from Mar *et al.* [16] at large ε and Q^2 yield an average ratio of 1.017 ± 0.024, well above this expectation.

One therefore needs additional input to constrain ΔG_M and ΔG_E. Precise comparisons of positron and electron scattering over a wide range in Q^2 and ε would allow the extraction of these amplitudes, but the positron-proton scattering data above $Q^2 = 1.3$ GeV^2 are limited to small scattering angles, corresponding to $\varepsilon > 0.7$. Because of the very limited ε range, the positron data cannot be used to constrain the ε-*dependence* at large Q^2 values. However, they still provide a useful constraint for the two-photon amplitudes. The positron data at large ε all indicate small two-photon contributions. To be consistent with this data, the contribution of $Y_{2\gamma}$ to the cross section at large ε must be cancelled by the contributions of ΔG_E and ΔG_M. The change in the cross section due to $\Delta G_E/G_E$ is suppressed with respect to the contribution from $\Delta G_M/G_M$ by a factor of $\varepsilon\rho^2/\tau$ [Eq. (4)], which is below 0.15 for $Q^2 = 2$ GeV^2 and below 0.01 for $Q^2 = 5.6$ GeV^2. Therefore, unless $\Delta G_E/G_E$ is much larger than $\Delta G_M/G_M$, the $Y_{2\gamma}$ contribution to the cross section at large ε must be cancelled almost entirely by ΔG_M.

Given the value of $Y_{2\gamma}$, we can determine ΔG_M by requiring that the two-photon contribution to the cross section [Eq. (4)] from $Y_{2\gamma}$ at $\varepsilon = 1$ be cancelled by the contribution from ΔG_M: $\Delta G_M/G_M = -(1+\rho/\tau)Y_{2\gamma}$. Figure 2 shows $\Delta G_M/G_M$ as determined from the above procedure, as well as a fit to these extracted values [Eq. (A2)]. Note that these amplitudes are a few percent of the Born amplitudes, larger than previously believed but still of order α_{EM}, as one would naively expect. To have a significant impact on the positron data, ΔG_E would have to be more than 20% of the Born value, much larger than one would expect for an $\mathcal{O}(\alpha_{EM})$ correction and much larger than the values extracted for $Y_{2\gamma}$ or $\Delta G_M/G_M$.

The remaining two-photon amplitude, ΔG_E, is more difficult to constrain, as it has a much smaller effect on the cross section. However, because both $Y_{2\gamma}$ and ΔG_E yield a correction to the cross section that is proportional to ε, the $\varepsilon \to 0$ limit can be used to extract G_M with minimal uncertainty from $Y_{2\gamma}$ and ΔG_E. So the lack of information on ΔG_E only affects extracted values of G_E/G_M. For this analysis, we take ΔG_E to be zero and use the larger of $Y_{2\gamma}$ and $\Delta G_M/G_M$ as an estimate of the uncertainty of ΔG_E. This yields an additional uncertainty on the extracted value of G_E/G_M of approximately 3–4%, which is smaller than the typical experimental uncertainties from the polarization transfer data.

C. Corrections to the extracted form factors

Having extracted the two-photon amplitudes, we can correct the polarization transfer measurements to yield the true form factor ratio, G_E/G_M. We use the fits to $Y_{2\gamma}$ and ΔG_M [Eqs. (A1) and (A2)] to correct the polarization transfer measurements according to Eq. (2). This yields a correction of approximately 5% at low Q^2 values, growing to 35% at $Q^2 = 5.6$ GeV^2. The fractional uncertainty in these amplitudes is ~50% for large Q^2 values (above 3–4 GeV^2) and increases to 100% for low Q^2 values (≈1 GeV^2). We parameterize the uncertainties in the extraction of $Y_{2\gamma}$, $\Delta G_M/G_M$, and $\Delta G_E/G_E$ [Eqs. (A4), (A5), and (A6)] and use this to determine the uncertainty in the two-photon corrections we apply to the polarization transfer ratio. The dominant uncertainties in the extraction of G_E/G_M are the experimental uncertainties in the polarization transfer measurement (typically 3–15%), the uncertainty in the extracted values of $Y_{2\gamma}$ and ΔG_M (4–12%), and the lack of knowledge of ΔG_E (3–4%). Figures 3 and 4 show the uncorrected and corrected values of G_E/G_M and G_M with the experimental uncertainties shown on the points and the uncertainties related to the two-photon corrections

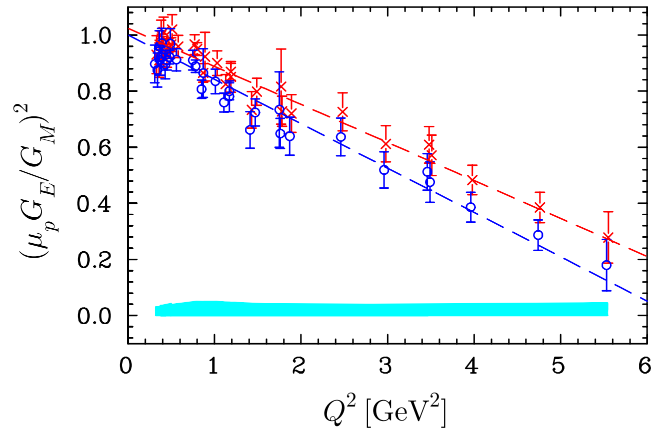

FIG. 3. (Color online) Polarization transfer measurements of $\mu_p G_E/G_M$ as determined using the one-photon exchange approximation (red "×") and after applying the corrections based on the extraction of two-photon contributions as described in the text (blue circles). The error band at the bottom shows the uncertainties associated with the two-photon corrections. The corrected data are well fit by $\mu_p G_E/G_M = 1 - 0.158Q^2$ (bottom dashed line).

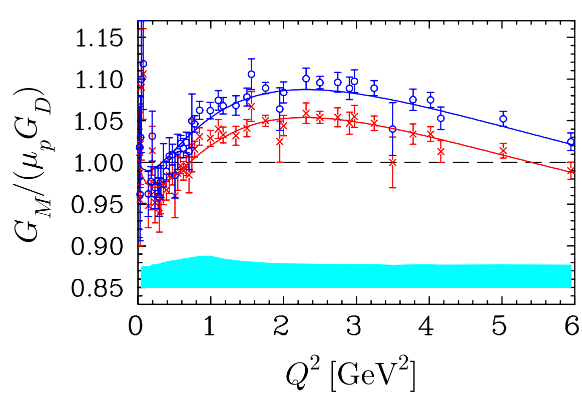

FIG. 4. (Color online) Rosenbluth extraction of $G_M/\mu_p G_D$ as determined using the one-photon exchange approximation (red "×") and after applying the corrections based on the extraction of two-photon contributions as described in the text (blue circles). The error band at the bottom shows the uncertainties associated with the two-photon corrections. The data are compared to fits from Ref. [6], where the bottom curve (red) is the global analysis of the cross-section data and the top curve (blue) includes both cross section and polarization transfer data, assuming a 6%, linear, ε-dependent correction to the cross section data.

shown in the error bar on the bottom. A linear fit to the corrected data yields $\mu_p G_E/G_M = 1 - 0.158Q^2$, with Q^2 in GeV^2, while the uncorrected data yields $\mu_p G_E/G_M = 1 - 0.135(Q^2 - 0.24)$ [3]. The corrected values of G_M are well reproduced by the "polarization" fit of Ref. [6].

Next, we can use the low ε cross sections, where $Y_{2\gamma}$ and ΔG_E have little effect [Eq. (4)], to extract $\tilde{G}_M$. We take the limit as $\varepsilon \to 0$, as in the usual L-T separation, and remove the two-photon contribution, ΔG_M, as determined from the above analysis to yield the corrected value for G_M. The dominant uncertainties are the experimental cross section uncertainties (1–2% uncertainty in G_M) and the uncertainty in ΔG_M (1.5–3%). There is an additional uncertainty (0.5%), coming from the uncertainty in the large ε ratio of positron to electron cross section, which is only known to ~1% from the existing positron data.

In extracting the two-photon amplitudes and the uncertainties in the corrected form factors we assumed that the two-photon amplitudes were independent of ε. Any ε-dependence must be small enough that it does not spoil the observed linearity of the reduced cross sections. However, it could still be large enough to yield a noticeable modification to the extracted value of G_M. If the amplitudes have nonlinearities at low ε, then the value of G_M will have an additional correction. Most of the available Rosenbluth separations at large Q^2 are limited to $\varepsilon \gtrsim 0.2$, and therefore we have to make a significant extrapolation. We can estimate the size of this uncertainty by examining measurements of the linearity of the reduced cross sections that have been performed as tests of the one-photon approximation.

A simple way to parameterize the limit on nonlinearity is to fit the reduced cross sections to a quadratic rather than a linear equation, $\sigma_r = P_0(1 + P_1\varepsilon + P_2\varepsilon^2)$, and use the uncertainty on the ε^2 coefficient, P_2, as an estimate of the possible nonlinear terms. The best linearity limits at high Q^2 come from the SLAC NE11 experiment [17]. Their best measurement, at $Q^2 = 2.5$ GeV^2, yields $P_2 = 0.0 \pm 0.11$. Similar limits ($\delta P_2 = 0.12$–0.2) are set by data at lower Q^2 [18]. In many cases, there are more data points at large ε values, increasing the uncertainty in the extrapolation to $\varepsilon = 0$ beyond what one would estimate from the simple quadratic fit. The simple estimate of the allowed nonlinearities yields possible deviations of 3–4% in the extraction of the cross section at $\varepsilon = 0$, more for cases where the linearity measurements are not as precise or where there is a larger extrapolation to $\varepsilon = 0$. We include a 4% uncertainty on the cross section (2% uncertainty on G_M) due to the possible error in the extrapolation to $\varepsilon = 0$.

To obtain the corrected form factors, we assumed that the discrepancy is fully explained by higher order radiative corrections, specifically two-photon exchange and Coulomb distortion, and had to make some assumptions about the ε-dependence. To the extent that these are valid assumptions (or good approximations), we obtain the correct two-photon amplitudes and can obtain the corrected form factors. One way to test these assumptions is to use the extracted amplitudes to predict the effect of these higher order terms in comparisons of positron-proton and electron-proton scattering. Although we incorporate the $\varepsilon = 1$ constraint of the positron measurements into the extraction, we have not included any other information on the ε-dependence. Unfortunately, data at smaller ε values are limited to low Q^2, where the uncertainties in the two-photon amplitudes are approaching 100%. Although we have to extrapolate to lower Q^2 to make a comparison to the positron data, we do reproduce the trend observed in the positron data.

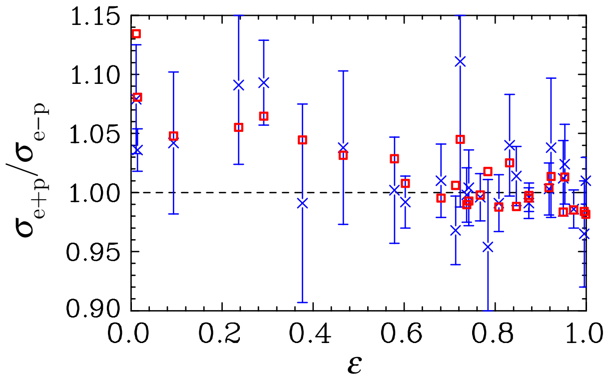

FIG. 5. (Color online) Ratio of positron-proton to electron-proton cross sections (blue "×"), compared to the prediction from the extracted values of $Y_{2\gamma}$ and $\Delta G_M/G_M$.

Figure 5 shows the ratio of positron to electron cross section as a function of ε, along with the predictions based on the fits to the two-photon amplitudes shown in Fig. 2. Although the uncertainties are large, the data are in better agreement with the prediction from the two-photon amplitudes: $\chi^2 = 18.2$ for 28 data points, compared with $\chi^2 = 23.9$ if one assumes $\sigma_{e+p}/\sigma_{e-p} = 1$, i.e., if the two-photon amplitudes are ignored. Although the positron data could be included in the analysis to help constrain the two-photon amplitudes at low Q^2, it is clear that the current data would not significantly modify the values obtained using just the high-ε constraint. Note that the two-photon prediction shown here does not yield exactly unity for $\varepsilon \to 1$. While the individual points for $\Delta G_M/G_M$ from Fig. 2 are determined by requiring that this ratio approaches unity, the parameterizations of the two-photon amplitudes yield slight deviations.

Finally, results have recently become available from a Jefferson Lab measurement that extracts $\mu_p G_E/G_M$ utilizing a modified Rosenbluth separation technique [5]. Values for $\mu_p G_E/G_M$ have been extracted at $Q^2 = 2.64$, 3.20, and at 4.10 GeV^2, and are in agreement with the results from the global analysis shown in Fig. 1 but with significantly smaller uncertainties. These data have not been included in the present analysis, but because they are consistent with the Rosenbluth data that are included, they would have little effect on the extracted amplitudes, although they would decrease the uncertainties in the amplitudes at these intermediate Q^2 values.

III. CONCLUSIONS

It is currently believed that the discrepancy comes from higher order corrections to the Born approximation. If this is true, and we assume a weak ε-dependence of the two-photon amplitudes, we can use the Rosenbluth, polarization transfer, and positron data to constrain the two-photon exchange (and other order α_{EM}) amplitudes. The large correction to the Rosenbluth ratio allows extraction of $Y_{2\gamma}$, which then allows determination of the (smaller) correction to the polarization transfer. Although the $Y_{2\gamma}$ contribution to the polarization transfer ratio is as large as 35% for the largest Q^2 value, the overall trend of a roughly linear decrease in $\mu_p G_E/G_M$ with Q^2 remains. Using the constraint from the two-photon (and multiphoton) exchange, positron cross-section measurements at large ε, we also determine ΔG_M. Although we cannot directly constrain ΔG_E, it has a relatively small effect on the extraction of the form factors, as long as it is not much larger than the other two-photon amplitudes.

Given these constraints on the two-photon amplitudes, we can correct G_E and G_M for two-photon exchange effects, with additional uncertainties associated with these corrections. For G_M, the correction comes from the term ΔG_M, and the uncertainty is dominated by possible ε-independence of the amplitudes, which can lead to deviations from the linear extrapolation to $\varepsilon = 0$. For G_E/G_M, the correction is dominated by $Y_{2\gamma}$ at large Q^2, although ΔG_M also plays a role, especially at low Q^2 values. The uncertainty in G_E/G_M is dominated by the large uncertainties in the $R_{\text{L-T}}$ data, which limit the precision with which we can extract $Y_{2\gamma}$ and ΔG_M. We can use the two-photon amplitudes we extract to provide corrected values of G_E and G_M, but additional uncertainties related to these corrections yield final uncertainties that are 50–100% larger than the experimental uncertainties.

Further experiments are necessary to verify that two-photon exchange is responsible for the observed discrepancy in the form factors and to allow for better extractions of the amplitudes. An experiment has been proposed to measure G_E/G_M using the cross section asymmetry in scattering from a polarized target [19]. This has the same sensitivity to two-photon exchange corrections as the polarization transfer measurements but uses a different experimental technique, thus providing a rigorous check on the systematics of the polarization transfer measurements. Additional positron data at low ε and moderate Q^2 could be used to test the assumption that the discrepancy is fully explained by two-photon exchange and give direct information on the ε-dependence. Such measurements are planned at Novosibirsk [20] and at Jefferson Lab [21]. Additional constraints on the ε-dependence of the two-photon amplitudes can be obtained from measurements of the ε-dependence of the polarization transfer [22].

Additional Rosenbluth measurements, utilizing the improved technique demonstrated by JLab experiment E01-001 [5], can provide an improved extraction of $R_{\text{L-T}}$ at moderate to large Q^2 values, where the uncertainty in the extracted amplitudes is dominated by the uncertainty in the Rosenbluth data. In addition, such measurements could provide much better constraints on the linearity of the reduced cross section and thus on the ε-dependence of the two-photon amplitudes. Such a measurement has been proposed at Jefferson Lab [23], which could reduce the uncertainty in the two-photon amplitudes by a factor of two to three for Q^2 from 1 to 6 GeV^2. Given additional positron-proton scattering data and improved Rosenbluth extractions of G_E/G_M, the two-photon amplitudes can be constrained well enough that the proton

TABLE I. Extracted values of G_M after correcting for two-photon exchange effects. The form factor is given with respect to the dipole form, $G_D = (1 + Q^2/0.71)^{-2}$, with the experimental uncertainties listed first, and the additional uncertainties related to two-photon effects listed second.

Q^2 [GeV^2]	$G_M/(\mu_p G_D)$
0.005	$0.751 \pm 0.424 \pm 0.016$
0.011	$0.981 \pm 0.104 \pm 0.021$
0.015	$1.000 \pm 0.074 \pm 0.021$
0.018	$0.973 \pm 0.073 \pm 0.021$
0.022	$1.018 \pm 0.049 \pm 0.022$
0.027	$0.962 \pm 0.056 \pm 0.021$
0.034	$1.030 \pm 0.049 \pm 0.022$
0.045	$1.100 \pm 0.088 \pm 0.024$
0.072	$1.118 \pm 0.055 \pm 0.025$
0.141	$0.962 \pm 0.027 \pm 0.025$
0.179	$0.977 \pm 0.018 \pm 0.026$
0.195	$1.031 \pm 0.030 \pm 0.027$
0.234	$0.970 \pm 0.025 \pm 0.027$
0.273	$0.978 \pm 0.015 \pm 0.028$
0.292	$0.958 \pm 0.023 \pm 0.028$
0.312	$0.977 \pm 0.019 \pm 0.029$
0.350	$0.996 \pm 0.026 \pm 0.030$
0.389	$0.989 \pm 0.012 \pm 0.030$
0.428	$1.007 \pm 0.022 \pm 0.031$
0.473	$1.009 \pm 0.012 \pm 0.032$
0.507	$0.984 \pm 0.027 \pm 0.032$
0.545	$1.013 \pm 0.021 \pm 0.033$
0.584	$1.023 \pm 0.010 \pm 0.034$
0.622	$1.017 \pm 0.011 \pm 0.034$
0.663	$1.024 \pm 0.012 \pm 0.035$
0.701	$1.013 \pm 0.022 \pm 0.035$
0.740	$1.050 \pm 0.032 \pm 0.036$
0.778	$1.046 \pm 0.010 \pm 0.036$
0.846	$1.062 \pm 0.011 \pm 0.037$
0.992	$1.062 \pm 0.010 \pm 0.038$
1.102	$1.074 \pm 0.012 \pm 0.035$
1.168	$1.068 \pm 0.010 \pm 0.034$
1.344	$1.068 \pm 0.012 \pm 0.032$
1.496	$1.078 \pm 0.011 \pm 0.031$
1.557	$1.106 \pm 0.018 \pm 0.031$
1.751	$1.089 \pm 0.007 \pm 0.029$
1.947	$1.064 \pm 0.025 \pm 0.028$
2.000	$1.084 \pm 0.013 \pm 0.028$
2.308	$1.100 \pm 0.013 \pm 0.028$
2.499	$1.096 \pm 0.008 \pm 0.028$
2.743	$1.096 \pm 0.012 \pm 0.028$
2.904	$1.089 \pm 0.014 \pm 0.027$
2.972	$1.097 \pm 0.014 \pm 0.028$
3.243	$1.089 \pm 0.009 \pm 0.027$
3.497	$1.040 \pm 0.032 \pm 0.027$
3.777	$1.075 \pm 0.013 \pm 0.027$
4.018	$1.075 \pm 0.009 \pm 0.027$
4.160	$1.053 \pm 0.014 \pm 0.027$
5.017	$1.052 \pm 0.009 \pm 0.027$
5.945	$1.025 \pm 0.011 \pm 0.027$
7.037	$0.989 \pm 0.015 \pm 0.026$
9.121	$0.943 \pm 0.020 \pm 0.023$

form factors can be determined with uncertainties from the two-photon corrections that are comparable to or smaller than the experimental uncertainties.

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APPENDIX: DETAILS OF THE TWO-PHOTON AMPLITUDE EXTRACTIONS

The fits to the extracted values and uncertainties for $Y_{2\gamma}$, $\Delta G_M/G_M$, and $\Delta G_E/G_E$ are

$$Y_{2\gamma} = 0.035[1 - \exp(-Q^2/1.45)], \tag{A1}$$

$$\Delta G_M/G_M = (0.0124Q^2 - 0.0445\sqrt{Q^2})\%, \tag{A2}$$

$$\Delta G_E/G_E = 0, \tag{A3}$$

$$\delta Y_{2\gamma} = [0.008 + 0.03\exp(-Q^2/0.7) + 0.0015Q^2], \tag{A4}$$

$$\frac{\delta(\Delta G_M/G_M)}{(\Delta G_M/G_M)} = \frac{\delta Y_{2\gamma}}{Y_{2\gamma}}, \tag{A5}$$

$$\delta(\Delta G_E/G_E) = \max(|Y_{2\gamma}|, |(\Delta G_M/G_M)|), \tag{A6}$$

with Q^2 in GeV^2 and with the relative uncertainty on the two-photon corrections limited to 100% at low Q^2 values. At large Q^2 values (above 3–4 GeV^2), the uncertainties in the two-photon amplitudes are ≈40–50%. Below $Q^2 = 1$ GeV^2, the parameterization of the uncertainty on $Y_{2\gamma}$ becomes as large as the value itself, so the uncertainty is taken to be 100% in the analysis. Because $\Delta G_M/G_M$ is derived directly from $Y_{2\gamma}$, the fractional uncertainty on ΔG_M is identical to the fractional uncertainty on $Y_{2\gamma}$. Finally, because we do not have any data with which to constrain ΔG_E, we assume that $\Delta G_E = 0$ and take the uncertainty to be the larger of the other two-photon amplitudes.

These fits and uncertainties are used to determine the correction to G_M as extracted from the cross-section measurements, and the correction to G_E/G_M as extracted from the polarization transfer data, as shown in Fig. 3.

TABLE II. Extracted form factor ratio G_E/G_M from polarization transfer experiments and corresponding value of G_E after applying the two-photon corrections as described in the text. In both cases, the experimental uncertainty is listed first, and the additional uncertainty related to the two-photon effects is listed second.

Ref.	Q^2[GeV^2]	$\mu_p G_E/G_M$ [corrected]	G_E/G_D [corrected]
[24]	0.380	$0.910 \pm 0.053 \pm 0.035$	$0.909 \pm 0.055 \pm 0.040$
	0.500	$0.969 \pm 0.053 \pm 0.043$	$0.979 \pm 0.055 \pm 0.048$
[25]	0.373	$0.961 \pm 0.054 \pm 0.035$	$0.959 \pm 0.056 \pm 0.040$
	0.401	$0.971 \pm 0.053 \pm 0.036$	$0.972 \pm 0.055 \pm 0.041$
	0.441	$0.895 \pm 0.051 \pm 0.036$	$0.899 \pm 0.053 \pm 0.041$
[1]	0.490	$0.921 \pm 0.025 \pm 0.039$	$0.930 \pm 0.028 \pm 0.044$
	0.790	$0.889 \pm 0.023 \pm 0.051$	$0.923 \pm 0.026 \pm 0.056$
	1.180	$0.800 \pm 0.030 \pm 0.047$	$0.854 \pm 0.033 \pm 0.053$
	1.480	$0.724 \pm 0.048 \pm 0.042$	$0.784 \pm 0.052 \pm 0.048$
	1.770	$0.649 \pm 0.054 \pm 0.040$	$0.708 \pm 0.059 \pm 0.046$
	1.880	$0.639 \pm 0.068 \pm 0.039$	$0.699 \pm 0.075 \pm 0.045$
	2.470	$0.637 \pm 0.068 \pm 0.039$	$0.699 \pm 0.075 \pm 0.045$
	2.970	$0.519 \pm 0.064 \pm 0.038$	$0.567 \pm 0.070 \pm 0.043$
	3.470	$0.512 \pm 0.065 \pm 0.039$	$0.555 \pm 0.071 \pm 0.044$
[26]	0.320	$0.897 \pm 0.067 \pm 0.030$	$0.891 \pm 0.068 \pm 0.035$
	0.350	$0.875 \pm 0.061 \pm 0.031$	$0.871 \pm 0.062 \pm 0.036$
	0.390	$0.923 \pm 0.033 \pm 0.034$	$0.923 \pm 0.036 \pm 0.039$
	0.460	$0.910 \pm 0.035 \pm 0.037$	$0.916 \pm 0.037 \pm 0.042$
	0.570	$0.912 \pm 0.040 \pm 0.041$	$0.928 \pm 0.042 \pm 0.046$
	0.760	$0.910 \pm 0.035 \pm 0.047$	$0.943 \pm 0.038 \pm 0.053$
	0.860	$0.807 \pm 0.033 \pm 0.048$	$0.843 \pm 0.036 \pm 0.054$
	0.880	$0.864 \pm 0.087 \pm 0.049$	$0.904 \pm 0.091 \pm 0.055$
	1.020	$0.835 \pm 0.044 \pm 0.051$	$0.883 \pm 0.047 \pm 0.057$
	1.120	$0.759 \pm 0.034 \pm 0.047$	$0.808 \pm 0.037 \pm 0.053$
	1.180	$0.781 \pm 0.055 \pm 0.047$	$0.834 \pm 0.059 \pm 0.053$
	1.420	$0.662 \pm 0.065 \pm 0.041$	$0.715 \pm 0.071 \pm 0.047$
	1.760	$0.734 \pm 0.134 \pm 0.042$	$0.801 \pm 0.146 \pm 0.049$
[2]	3.500	$0.475 \pm 0.072 \pm 0.038$	$0.515 \pm 0.078 \pm 0.043$
	3.970	$0.386 \pm 0.053 \pm 0.039$	$0.414 \pm 0.057 \pm 0.043$
	4.750	$0.287 \pm 0.054 \pm 0.041$	$0.302 \pm 0.057 \pm 0.044$
	5.540	$0.180 \pm 0.092 \pm 0.044$	$0.185 \pm 0.095 \pm 0.046$

Table I gives the corrected values for G_M, determined by taking the uncorrected value of G_M (i.e., $\tilde{G}_M$) from a global analysis of the Rosenbluth data [6] and correcting for the extracted value of ΔG_M [Eq. (A2)]. Additional two-photon uncertainties come from the uncertainty in ΔG_M and a 2% uncertainty due to the possibility of nonlinear terms that modify the extrapolation to $\varepsilon = 0$.

Given G_E/G_M and G_M, we can obtain G_E. However, we have extracted G_E/G_M from the polarization transfer measurements, and G_M from the cross-section measurements, both corrected for the two-photon amplitudes. Because the uncertainty on G_M is much smaller, we extract G_E at the kinematics of the polarization transfer measurements, using the corrected values of G_E/G_M and a fit to the corrected values (and uncertainties) of G_M. Tables I and II give the corrected values for G_M and G_E, relative to the dipole form.

The corrected form factors are well described by the polarization form factor fit to G_M from Ref. [6] (top curve in Fig. 4) and $\mu_p G_E/G_M = 1 - 0.158Q^2$ (bottom curve in Fig. 3). The fit for G_M from Ref. [6] is nearly identical to the best fit to the corrected G_M data; the only noticeable difference is that it is slightly lower (up to 1%) for Q^2 values of 2–4 GeV^2. The fit by Brash *et al*. [27] is 1.5–2.5% below the corrected values of G_M for $Q^2 \gtrsim 1$ GeV^2.

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