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Signal of the pion string at high-energy collisions

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We study the possible signals of a pion string associated with the QCD chiral phase transition in LHC Pb-Pb collision at energy $\sqrt{s} = 5.5$ TeV. In terms of the Kibble-Zurek mechanism we discuss the production and evolution of the pion string. The pion string is not topologically stable; it decays into neutral pions and sigma mesons that in turn decay into pions. Our results show that all the neutral pions from the pion string are distributed at the low momentum and the ratio of neutral to charged pions from the pion string violates the isospin symmetry. For the momentum spectra of the total pions, the signal from the sigma particle decay from the pion string will be affected by the large decay width of the sigma significantly.

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I. INTRODUCTION

The formation of topological defects in phase transition is a very generic phenomenon in physics. It can be studied experimentally in different condensed matter systems. It is generally believed that the early evolution of the universe undergoes a sequence of phase transitions, and the produced topological defects in these phase transitions may have observable consequences to the properties of the universe today. For example, the cosmic strings have been suggested as one possible source for the primordial density perturbations that give rise to the large-scale structure of the universe and the temperature fluctuations of the cosmic microwave background (CMB) radiation [1,2]. In particular, in Ref. [3] the effect of the pion string on the primordial magnetic field generation in the early universe has been considered and its cosmological significance is pointed out. However, in this article, we will turn from cosmology to laboratory experiments and attempt to study the possible signals of the pion string in the heavy ion collision experiments that have many similarities with the nonequilibrium phenomena that also take place in heavy ion collisions experiments [4,5].

It is difficult to make experimental tests of our ideas about the formation and evolution of topological defects in cosmology directly. What we can do is to look for analogous processes in experimentally accessible condensed-matter systems. Fortunately, topological defects are formed at phase transitions in certain condensed-matter systems such as superfluids and superconductors, this phenomenon is theoretically very similar to its cosmological counterpart, and we can use this analogy to do "cosmology experiments" [6]. On a more fundamental level, these same experiments can be used to test our understanding of nonequilibrium dynamics of quantum field theories [7].

In relativistic nucleus-nucleus collisions, some phenomena such as those that happen in the Big Bang have been observed, which are called the Little Bang. Thus as hadron momentum spectra correlations provide strong evidence for the existence of the Little Bang: thermal hadron radiation with $T = 90$–100 MeV and strong three-dimensional (Hubble like) expansion with transverse flow velocities 0.5–0.55 c [8]. So the study of the Little Bang from relativistic nucleus-nucleus collisions may construct a bridge between high energy particle physics and the cosmology. Conversely, we are not yet in a position to give an evidence that quark-gluon plasma (QGP) has really been produced. It turns out to be a difficult task to figure out the theoretical picture of QGP even in equilibrium. Beyond the trivial level of the trees and the nonequilibrium properties of the QGP are essentially unknown. There is no unique signal of QGP in the understanding of nucleus-nucleus collisions so far. As pointed out by Rajantie [9], the heavy-ion collisions experiments are so complicated that the reliable and accurate theoretical calculations are needed to confront the experimental results, but our present understanding of the theory is too rudimentary for that.

Consequently, the insight provided by condensed systems experiments is therefore likely to be extremely useful. In particular, it is believed that at a certain value of the beam energy, the QGP produced in the collision cools through a second-order transition point. Pion strings as well as other topological and nontopological strings are expected to be produced [10,11]. An early study on the effects of these strings in the case of heavy ion collisions and in the early universe has been performed [11]; in their article, they speculate that formation and subsequent evolution of the network of these string defects can give rise to inhomogeneous distribution of baryons and also the energy density by using the Kibble mechanism. In this article, we extend their works on the formation and evolution of strings and discuss the possible signal of pion strings during chiral phase transition in the high ion collisions.

So far the theoretical scenario that can be applied to study the formation of topological defects in systems with global symmetry is the Kibble-Zurek mechanism [6,12]. Moreover, according to Pisarski and Wilczek [13] the chiral phase transition is expected to be of the second order for two massless flavors; it is customary then in this article to apply the

Kibble-Zurek mechanism to study the formation and evolution of the pion string during Pb-Pb central collisions at the LHC with energy $\sqrt{s} = 5.5$ TeV.

The remainder of this article is organized as follows. We give a brief review of the pion string in the linear sigma model in Sec. II. It is shown how to use the Kibble-Zurek mechanism to consider the formation of the pion string at LHC in Sec. III. We discuss the evolution and decay of the pion string and their possible observational consequences at LHC in Sec. IV. Sec. V is reserved for summary and discussion.

II. THE PION STRING IN QCD

The linear sigma model that serves as a good low-energy effective theory of the QCD was first introduced in the 1960s as a model for pion-nucleon interactions [14] and has attracted much attention recently, especially in studies involving disoriented chiral condensates [4,15]. This model is very well suited to describing the physics of pions in studies of chiral symmetry. In what follows we review the work of Ref. [10], in which it was shown that below the chiral symmetry breaking scale, the linear sigma model admits global vortex line solutions, the pion string. We then attempt to use the Kibble-Zurek mechanism to give out a quantitative description of the formation and evolution of the pion string and their possible observational consequences in heavy ion collisions.

Considering a simple case of QCD with two massless quarks u and d, the Lagrangian of strong interaction is invariant under $SU(2)_L \times SU(2)_R$ chiral transformation

$$\Psi_{L,R} \to \exp(-i\vec{\theta}_{L,R} \cdot \vec{\tau})\Psi_{L,R}, \tag{1}$$

where $\Psi^T_{L,R} = (u, d)_{L,R}$. However, this chiral symmetry does not appear in the low-energy particle spectrum because it is spontaneously broken to the diagonal subgroup formation. Consequently, three Goldstone bosons, the pions, appear and the (constituent) quarks become massive. At low energy, the spontaneous breaking of chiral symmetry can be described by an effective theory, the linear sigma model, which involves the massless pions $\vec{\pi}$ and a massive σ particle. As usual, we introduce the following field:

$$\Phi = \sigma \frac{\tau^0}{2} + i\vec{\pi} \cdot \frac{\vec{\tau}}{2}, \tag{2}$$

where τ^0 is the unity matrix and $\vec{\tau}$ is the Pauli matrices with the normalization condition $\mathrm{Tr}(\tau^a \tau^b) = 2\delta^{ab}$. Under $SU(2)_L \times SU(2)_R$ chiral transformations, Φ transforms as follows:

$$\Phi \to L^+ \Phi R. \tag{3}$$

The renormalizable effective Lagrangian of the linear sigma model can be written as follows:

$$\mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_q, \tag{4}$$

where

$$\mathcal{L}_\Phi = \mathrm{Tr}[(\partial_\mu \Phi)^+ (\partial^\mu \Phi)] - \lambda \left[\mathrm{Tr}(\Phi^+ \Phi) - \frac{f_\pi^2}{2} \right]^2 \tag{5}$$

and

$$\mathcal{L}_q = \overline{\Psi}_L i\gamma^\mu \partial_\mu \Psi_L + \overline{\Psi}_R i\gamma^\mu \partial_\mu \Psi_R - 2g\overline{\Psi}_L \Phi \Psi_R + h.c.. \tag{6}$$

During chiral symmetry breaking, the σ field takes on a nonvanishing vacuum expectation value, which breaks $SU(2)_L \times SU(2)_R$ down to $SU(2)_{L+R}$. It results in a massive sigma particle σ and three massless Goldstone bosons $\vec{\pi}$, as well as giving a mass $m_q = gf_\pi$ to the constituent quarks.[1]

In the previous article, two of us (X.Z. and T.H.) with Brandenberger [10] discovered a type of classical solution, the pion string, in the above linear sigma model. This pion string is very much like the spin vortex produced at the superfluid transition of the liquid ^3He [7]. Similarly to the Z string [17,18] in the standard electroweak model, the pion string is not topologically stable, because any field configuration can be continuously deformed to the trivial vacuum in the QCD sigma model. With finite temperature plasma, however, one of us (Nagasawa) and Brandenberger [19] argued that the pion string can be stabilized. They propose that the interaction of the pion fields with the charged plasma generates a correction to the effective potential and this correction reduces the vacuum manifold S^3 of the zero temperature theory to a lower dimensional submanifold S^1, which makes the pion string stable. Moreover, it has been shown by numerical simulations that semilocal strings, which are also not topologically stable, can be produced at the phase transition [18,20]. In a similar way, pion strings are expected to be produced during the QCD phase transition in the early universe as well as in experiment of the heavy ion collisions. The strings will subsequently decay.

The pion string is a static configuration of the Lagrangian $\mathcal{L}_\Phi$ of Eq. (5). To discuss the pion string, we define the following new fields:

$$\phi = \frac{\sigma + i\pi^0}{\sqrt{2}} \tag{7}$$

and

$$\pi^\pm = \frac{\pi^1 \pm i\pi^2}{\sqrt{2}}. \tag{8}$$

The Lagrangian $\mathcal{L}_\Phi$ now can be rewritten as follows:

$$\mathcal{L}_\Phi = (\partial_\mu \phi^*)(\partial^\mu \phi) + (\partial_\mu \pi^+)(\partial^\mu \pi^-) - \lambda \left(\phi^* \phi + \pi^+ \pi^- - \frac{f_\pi^2}{2} \right)^2. \tag{9}$$

For the static configuration, the energy functional corresponding to the above Lagrangian is given by the following:

$$E = \int d^3x \left[\vec{\nabla}\phi^* \vec{\nabla}\phi + \vec{\nabla}\pi^+ \vec{\nabla}\pi^- + \lambda \left(\phi^* \phi + \pi^+ \pi^- - \frac{f_\pi^2}{2} \right)^2 \right]. \tag{10}$$

[1]The σ field can be used to represent the quark condensate and the order parameter for the chiral phase transition because both exhibit the same behavior under chiral transformations [16], the pions are very light particles and can be considered approximately as massless Goldstone bosons.

The time-independent equations of motion are as follows:

$$\nabla^2\phi = 2\lambda\left(\phi^*\phi + \pi^+\pi^- - \frac{f_\pi^2}{2}\right)\phi \tag{11}$$

and

$$\nabla^2\pi^+ = 2\lambda\left(\phi^*\phi + \pi^+\pi^- - \frac{f_\pi^2}{2}\right)\pi^+. \tag{12}$$

The pion string solution with a single winding number extremizing the energy functional in Eq. (10) is given by the following [10]:

$$\phi = \frac{f_\pi}{\sqrt{2}}[1 - \exp(-\mu r)]\exp(i\theta) \tag{13}$$

and

$$\pi^\pm = 0, \tag{14}$$

here the coordinates r and θ are polar coordinates in $x - y$ plane (the string is assumed to lie along the z axis), $\mu^2 = \lambda\frac{89}{144}f_\pi^2$, the energy per unit length of the string is as follows:

$$E = [0.75 + \log(\mu R)]\pi f_\pi^2, \tag{15}$$

where R is introduced as a cutoff because for global symmetry the energy density of the string solution is logarithmically divergent. Generally R is given by the horizon size or the typical separation length between strings. The typical distance between strings can be determined by the string number density at the formation that is based on the the Kibble-Zurek mechanism and the following evolution that is ruled out by the string tension and the interaction between the string and the surrounding matter. Thus the interaction between the strings would not be so significant. In the following numerical calculation, we take $R = O(\text{fm})$, for other parameters we have $\lambda = 9.877$, $f_\pi = 90$ MeV, $m_\pi = 140$ MeV, and $m_\sigma = 400$ MeV [21].

III. THE FORMATION OF THE PION STRINGS AT LHC

The order of the QCD chiral phase transition seems to depend on the mass of the nonstrange u and d quarks, $m_u \approx m_d$, and the mass of the strange quark m_s. At the phase transition temperature on the order of 150 MeV, heavier quark flavors do not play an essential role. In the chiral limit, one can use universality arguments to determine the order of the phase transition. According to universality, the order of the chiral transition in QCD is identical to that in a theory with the same chiral symmetries as QCD, for instance, the $U(N_f)_L \times U(N_f)_R$ linear sigma model for N_f massless quark flavors. This argument was employed by Pisarski and Wilczek [13], who showed that for $N_f = 2$ flavors of massless quarks, the transition can be of second order if the $U(1)_A$ symmetry is explicitly broken by instantons; whereas for three or more massless flavors, the phase transition for the restoration of the $SU(N_f)_R \times SU(N_f)_L$ is first order.

So far, the formation of topological defects has been studied in liquid crystal and superfluid experiments [22], which are systems with global symmetry. It is generally believed that the theoretical scenario that can be applied to determine the defects initially formed immediately after second-order symmetry breaking phase transition in this case is the Kibble-Zurek mechanism. The basic picture of this mechanism is the following. For a second phase transition, after the phase transition, the physical space develops a domainlike structure, with the typical size of the domain being of the order of a relevant correlation length ξ (which depends on the nature of the dynamics of the phase transition). Inside a given domain, the broken phase is roughly uniform, but varies randomly from one domain to the other. The string defects are to be formed in the junction of three or more domains.

It is customary then for us to apply the Kibble-Zurek mechanism to make an estimate of the density of pion strings during Pb-Pb central collisions at the LHC with energy $\sqrt{s} = 5.5$ TeV. At the initial stage of the collision there exists manifestly partons with very large cross section for gluon scattering, so the gluons will reach equilibrium quickly with an initial temperature at about $T_i = 600$ MeV corresponding to the time $t_i = 0.2$ fm [23]. If the entropy of the system is conserved throughout the expansion, using the Bjorken model we have that the thermal freeze out of the fireball occurs at $t_f = 25$ fm when the temperature reaches $T_f = 120$ MeV. The calculation in Ref. [24] shows that the quark-gluon plasma can be formed over very large space-time volumes at the LHC Pb-Pb collisions. The hydrodynamic model predicts that the volume of such a plasma region evolves as follows [25–27]:

$$V(t_f) = V(t_c)\frac{t_f}{t_c}, \tag{16}$$

where $V(t_f) = 2 \times 10^4$ fm^3, whereas evolution of the temperature is given by the following:

$$T(t) = T_i\left(\frac{t_i}{t}\right)^{\frac{1}{3}}. \tag{17}$$

From Eqs. (16) and (17) we obtain that the time when the phase transition occurs at $T_c = 170$ MeV is $t_c \simeq 8.793$ fm and the volume at the freezing out is $V(t_f) = 2 \times 10^4$ fm^3.

When the LHC Pb-Pb collisions take place, a big fireball is formed in the central region of the collision with the initial temperature around $T(t_i) = 600$ MeV at the time $t_i = 0.2$ fm. The fireball quickly reaches to the equilibrium state and it expands rapidly with the volume and temperature given in Eqs. (16) and (17). During the period when the temperature is higher than $T_c = 170$ MeV (before $t_c \simeq 8.793$ fm), the fireball is in the QGP phase, where the chiral symmetry is unbroken. When the temperature of the fireball decreases down to the critical point $T_c(t_c) = 170$ MeV and its volume is about $V(t_c) \approx 7 \times 10^3$ fm^3, the fireball undergoes a rapid second-order chiral phase transition. At this time the system is in an out-of-equilibrium dynamical state, and the phase with broken chiral symmetry starts to appear due to the fluctuations of the order parameter simultaneously and independently in many separate regions of the expanding fireball. Subsequently during the process with further cooling, these regions grow and merge with each other to realize the new phase with the broken symmetry all over the fireball. At the boundaries where causally disconnected different regions meet, the order parameter field does not necessarily match and a domain structure is formed. This is essentially similar to the process of the defect formation during the cosmological phase transitions in the early universe.

As described by the Kibble-Zurek mechanism, the transition speed can be given by the quench time τ_Q [6,7,12]:

$$\tau_Q = \frac{T_c}{|dT/dt|_{t=t_c}} = 3t_c. \tag{18}$$

From the Ginzburg-Landau theory for the second-order phase transition, the quench time τ_Q can be deduced by the order parameter relaxation time τ with a general form

$$\tau(T) = \tau_0 \left(1 - \frac{T}{T_c}\right)^{-1}, \tag{19}$$

where $\tau_0 \sim \xi_0$ and ξ_0 is the zero temperature-limiting value of the temperature-dependent coherence length $\xi(T)$, and in this article we take $\xi_0 = 1/m_\sigma \simeq 0.49$ fm. When the temperature T is close to T_c, we have the following:

$$\xi(T) = \xi_0 \left(1 - \frac{T}{T_c}\right)^{-\frac{1}{2}}. \tag{20}$$

As the temperature is below T_c the order parameter coherence spreads out with the following velocity:

$$c(T) \sim \frac{\xi}{\tau} = \frac{\xi_0}{\tau_0}\left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}}. \tag{21}$$

The pion strings are expected to be produced at the Zurek freeze-out time t_z when the causally disconnected regions have grown together and the coherence is established in the whole volume. At the Zurek freeze-out temperature $T(t_z) < T_c$, the causal horizon is given by the following:

$$\xi_H(t_z) = \int_0^{t_z} c(T)dt = \frac{\xi_0 \tau_Q}{\tau_0}\left(1 - \frac{T_z}{T_c}\right)^{\frac{3}{2}}. \tag{22}$$

The causal horizon has to be equal to the coherence length $\xi(t_z)$, then from Eqs. (19)–(22) we obtain the following:

$$t_z = t_c + \tau(t_z) = t_c + \sqrt{\tau_0 \tau_Q} \approx 12.4 \text{ fm} \tag{23}$$

and

$$\xi_z = \xi_0(\tau_Q/\tau_0)^{1/4} \simeq \tau_Q^{1/4} \tau_0{}^{3/4} \simeq 1.33 \text{ fm}. \tag{24}$$

Due to the Kibble-Zurek mechanism, a network of pion strings is formed at the Zurek freeze-out time t_z with a typical curvature radius and separation of ξ_z. After that time, the network of the pion string is going to evolve with the fireball expansions until to the freeze-out time t_f.

IV. THE EVOLUTION AND DECAY OF THE PION STRING

Similarly to the evolution of the cosmic string in the early universe [3,28,29], when the temperature of the fireball falls down from the Zurek temperature, $T_z \equiv T_i(\frac{t_i}{t_z})^{\frac{1}{3}} = 151.6$ MeV to T_f, the evolution of the string network would obey the following procedure. Initially, at the Zurek time t_z, the pion string has a typical curvature radius and separation of the correlation length ξ_z, which then increases rapidly and eventually approaches a scaling solution in which $\xi(t) \sim t^a$. In the case of the pion string in the heavy ion collision experiments, because the volume of fire ball obeys the following law $V(t) \sim t$, we make the assumption that $a = \frac{1}{3}$ for our case from time t_z up to t_f. Hence, the correlation length of the pion string at time t_f is

$$\xi_f = \xi(t_f) = \xi_z \left(\frac{t_f}{t_z}\right)^{\frac{1}{3}} \simeq 1.69 \text{ fm}. \tag{25}$$

The pion string ceases to evolve at the time t_f and decays because of the fireball disappearance. Hence we have to find out the size and number of loops at the time t_f.

Note that in our situation, closed string loops have a dominant contribution to the total energy of the string. This is because at the freezing out, the string evolution is still ruled out by the frictional force by the surrounding matter so that the free motion of the string is not realized. In addition, the expansion of the system is too rapid that the initial Brownian string distribution will be conserved. Then the initial structure of the string network partially remain and the spatial trajectories of strings are very much complicated. Thus we take the initial pion string network as that of the Brownian one and regard that the distribution of these loops does not change with time, the size of the loops are conformally stretched during the expansion of fireball and a simple scaling can be realized. By using the scale invariance of Brownian string described by Vachaspati and Vilenkin [29], we get the distribution of number density of the pion string with the length between l and $l + dl$ at the freeze-out temperature, T_f:

$$dn(l) = K\xi_f^{-\frac{3}{2}} l^{-\frac{5}{2}} dl, \tag{26}$$

where the parameter K is approximately in the range of $0.01 \sim 0.1$ [7].

Integration of Eq. (21) over dl will result in the total number of the string loops. Note that the string width $r_0 \sim \frac{1}{\mu}$ gives a minimum length of the pion string at time t_f, $l_0 = 2\pi r_0 \simeq 5.6$ fm and the longest string is also constrained by the volume of the system. From Eq. (26) we have the total number of pion strings:

$$N(0) = \frac{2KV(t_f)}{3\xi_f^{\frac{3}{2}} l_0^{\frac{3}{2}}} = 459K. \tag{27}$$

Because K varies from 0.01 to 0.1 the total number of pion strings varies between $N(0) \simeq 4$ and $N(0) \simeq 45$.

In the immediate aftermath of the phase transition, when the temperature is still close to T_c, the string tension remains small and motion of the strings is heavily damped by the frictional effects of the surrounding high-density medium. The mechanism of Nagasawa and Brandenberger implies that pion strings might effectively be stable in this high-density medium until the thermal freeze out time t_f. After that, the string tension approaches its zero-temperature value and the motion of the strings is effectively decoupled from the surrounding medium, there will not be corrections to the effective potential from the thermal bath and pion strings undergo the second phase transition. Even though pion strings undergo the second phase transition under the temperature T_f, pion strings will not decay immediately and can still survive sometime below the freeze-out time t_f, because the pion strings undergo a core phase transition and lose their central structure where the field strength equals to zero but still preserve the winding number of neutral components [30]. For simplicity we assume that pion strings can survive after the decoupling time and all pions that are eventually emitted from pion strings will be completely incoherent with the rest of the pions. According to the work of Ref. [27], the produced pions in the interval $T > T_f$ have time to be thermalized before the freeze-out time, they lead to an enhancement of thermal pions, whereas in the interval $T < T_f$ the produced pions do not get chance to be thermalized, they result in a nonthermal enhancement of pions with low momentum. Thus it is expected that the resultant pion spectrum with the pion string will contain the nonthermal pions that can be taken as the evidence of the pion string.

All pion strings will decay into the sigma particles and neutral pions. To estimate the numbers of the particle produced we notice that for the ansatz Eqs. (13) and (14), the sigma field in Eq. (10) contributes about 50% of the total energy of the string. Due to energy conservation half of the string energy should convert into that carried by the sigma particles. The remaining 50% of the string energy will go to the neutral pions. For global string such as the axion string [31] one expects the mesons produced from the decay of the pion strings with length l have a typical momentum $p \sim 1/l$. Using Eq. (26), we obtain that the total number of sigma particle N_σ emitted from pion strings within a fireball is about 100 ($K = 0.1$), 43 ($K = 0.05$), 22 ($K = 0.03$), and 5 ($K = 0.01$), respectively. And the total number of neutral pions N_{π^0} is about 332 ($K = 0.1$), 148 ($K = 0.05$), 80 ($K = 0.03$), and 19 ($K = 0.01$). As mentioned above it is expected that the eventually resultant pion spectrum will have a nonthermal enhancement at low momentum region because all produced pions from pion strings are distributed at low momentum.

It can be seen that the most of the sigma particles and the neutral pions have a relatively low momentum and these particles are nonthermal particles. The momentum distribution of the sigma from the pion string decay is given by the following:

$$\frac{dN_\sigma(p)}{dp} = \frac{KV(t_f)\sqrt{p}}{D_1\xi_f^{\frac{3}{2}}}, \tag{28}$$

where the normalization factor $D_1 = 0.476$. The momentum distribution of the neutral pions is given by the following:

$$\frac{dN_{\pi^0}(p)}{dp} = \frac{KV(t_f)\sqrt{p}}{D_2\xi_f^{\frac{3}{2}}}, \tag{29}$$

where the normalization factor $D_2 = 0.142$. The averaged momentum of sigma and neutral pion is $\langle p\rangle \simeq 21.1$ MeV. The neutral pions and sigma particles are dominant in the low momentum region, and the momentum distribution of the pions produced at the decay of the pion string can be taken as a distinctive signal of the formation of the pion string in heavy ion collisions.

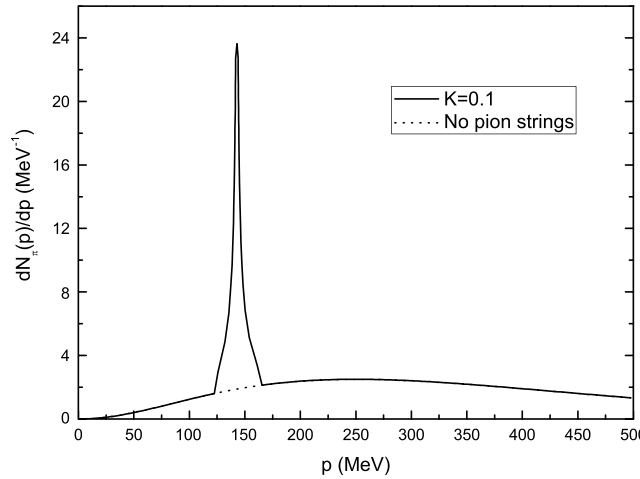

FIG. 1. The momentum spectra of the total pions from the pion string and thermal pions by taking parameter $K = 0.1$ and without considering the large decay width of the sigma. The indirect pions (π^0's, $\pi^\pm$) via σ decay from the pion strings are mostly distributed around $\langle p\rangle \sim 143.2$ MeV.

The sigma particles from pion strings will decay equally into neutral and charged pion mesons.[2] As we know, the σ particle has a large decay width and it is more complicated issue to get the real distribution of the pions from the σ meson. To get the simplified results, we ignore its large decay width and take it as a stable particle. Because the momentum of the sigma can be approximately neglected, for $m_\sigma = 400$ MeV these pions from the sigma decay will have momentum around 143 MeV. In Fig. 1 we plot the distribution of these pions as a function of the momentum together with the thermal pions calculated in Ref. [27]. Numerically there are about 200 nonthermal pions mostly distributed at $p \sim 143$ MeV for $K = 0.1$, (In the following discussion, for simplicity, we always take the parameter $K = 0.1$ for the case of the sigma decay.)

In the above discussion, we get the simplified results by ignoring the large decay width of the sigma and take it as a stable particle. In fact, the σ particle is a broad resonance and have a decay width, then the narrow peak in Fig. 1 will definitely smoothen out. To get a more reality figures, we should take into account the decay width of sigma and redraw the figure. Mass and width of σ particle are somewhat model dependent; in this article we take the Breit-Wigner function as follows [32]:

$$BW_\sigma = \frac{1}{m_\sigma^2 - s - im_\sigma\Gamma_\sigma}, \tag{30}$$

where Γ_σ is a constant. From Eqs. (26), (28), and (30), we plot the distribution of these pions together with the thermal pions as a function of the momentum and the decay width of sigma by taking the parameters as $\Gamma_\sigma = 300$ MeV and

[2]An early study on the effects of pion string in heavy ion collision has been performed in Ref. [11], but they have not quantitatively estimated the numbers of the pions produced from the pion string decay.

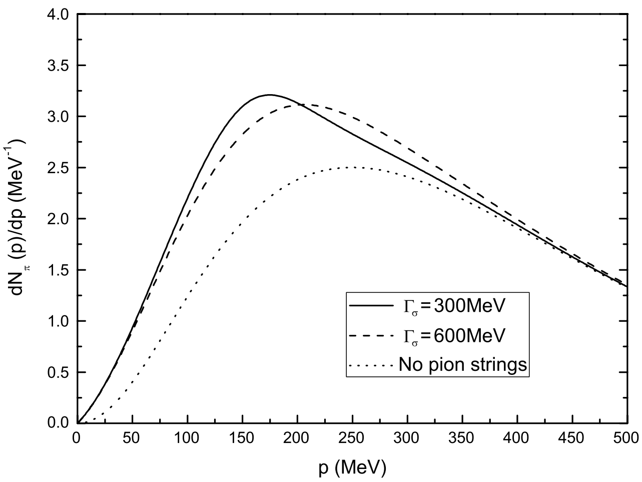

FIG. 2. The momentum spectra of the total pions from the pion string and thermal pions by taking parameters $\Gamma_\sigma = 300\,\text{MeV}$ and $\Gamma_\sigma = 600\,\text{MeV}$ and $K = 0.1$. The indirect pions (π^0's, $\pi^\pm$) via σ decay from the pion strings are broadly distributed from $p \sim 0\,\text{MeV}$ to $p \sim 500$ MeV.

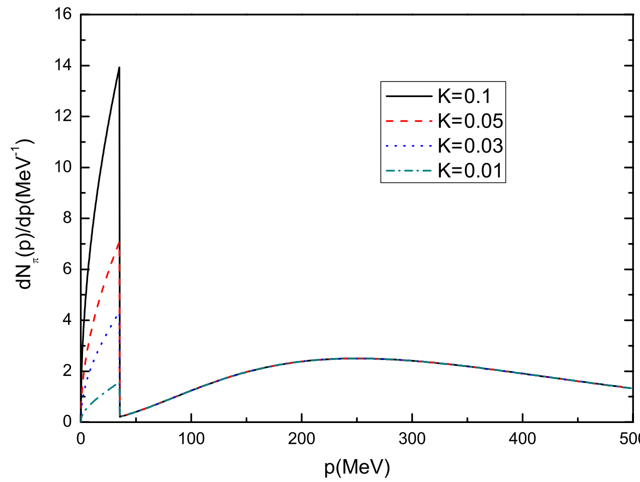

FIG. 3. The momentum spectra of the nonthermal neutral pions (π^0's) emitted from the pion strings together with the thermal pions. The nonthermal pions are mainly distributed around $p \sim 0 - 35\,\text{MeV}$.

$\Gamma_\sigma = 600\,\text{MeV}$ and $K = 0.1$ in Fig. 2. The averaged momentum of indirect pions via sigma decay is $\langle p \rangle \sim 152.5$ MeV and $\langle p \rangle \sim 175.9$ MeV according to $\Gamma_\sigma = 300\,\text{MeV}$ and $\Gamma_\sigma = 600\,\text{MeV}$ respectively. Numerically there are also about 200 nonthermal pions broadly distributed at the momentum region $p \sim 0 - 500$ MeV for $\Gamma_\sigma = 300\,\text{MeV}$ and $\Gamma_\sigma = 600\,\text{MeV}$. From Fig. 2, the narrow peak in Fig. 1 definitely smoothens out and nearly disappears. These nonthermal pions can hardly be distinguished from the thermal pions, so one of the possible signals of pion stings is not significant because of the large decay width of sigma.

Because the string configuration violates the isospin symmetry the direct production of the pions from the pion string decay is only for the neutral pion, not the charged pion.[3] In Fig. 3 we plot these neutral pion distribution together with the thermal pions. Numerically there are about $N_{\pi^0} \sim 20$–300 nonthermal pions distributed in the low momentum region with $\langle p \rangle \sim 21$ MeV. From the Fig. 3, all the neutral pions from pion strings are distributed at the low momentum and the ratio of neutral to charged pions from pion strings violates the isospin symmetry, this can be taken as the possible signal of pion strings.

V. SUMMARY AND DISCUSSION

We have investigated the effects of the pion string in the experiment of the heavy ion collisions. Following the Kibble-Zurek mechanism pion strings are expected to be formed in LHC Pb-Pb collision at energy $\sqrt{s} = 5.5$ TeV, then decay after the freezing out time into pions. These pions are mostly distributed in two separated low-momentum regimes. These effects are expected to be observable and differ from predictions of other models [27,33]. The pion enhancement in a small window around the nonthermal momentum $p_0 \simeq 21.1$ MeV for neutral pions, although for other nonthermal pions, the situation is completely different. If we ignore the large decay width of the sigma and take the sigma as a stable particle, there are the pion enhancement in a small window around the nonthermal momentum $p_0 \simeq 143$ MeV, but actually the sigma has the large decay width, then the peak will smoothen out and almost tend to disappear. So it is difficult to detect this part of nonthermal pions in experiment. All the resultant pion spectrum depend strongly on how long the pion string can survive below the freeze-out time t_f.

In this article, we have made the assumption that pion strings can survive after the decoupling time, then both the neutral pions and sigma particles emitted from pion strings do not get chance to be thermalized. Conversely, we do not exclude the situation in which part (even all) of pion strings will decay into pions and sigma particles when the time is very close to the decoupling time, and such produced pions from pion strings will be thermalized by the final state interactions and the peak in the pion spectra due to the pion string decay will disappear partly (or completely). However, even though this situation is happened, there still have possible signals of the pion string produced. Then pion string decay can lead to experimentally observable anomlies that are very similar to the DCC (disoriented chiral condensate) decay. It is the ratio of neutral to charged pions, $r = n_0/n_0 + n_{\text{ch}}$, here n_0 is the number of neutral pions, whereas n_{ch} corresponds to charged pions, which is different from what is naively expected ($\frac{1}{3}$) if there are pion strings produced during chiral phase transition.

Therefore, to obtain more reliable conclusions, we need to know more details of the process of the pion string decay, the sigma decay, and behaviors of the fireball at freeze-out time in heavy ion collisions.

[3]The charged pion strings are expected to be also produced; however, according to the mechansim of Nagasawa and Brandenberger the charged pion string will not be able to be stabilized in the plasma, consequently, they will decay away and their decay products will be thermalized before the freezing out.

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[1] A. Rajantie, Contemp. Phys. **44**, 485 (2003).
[2] A. Villenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, 2000).
[3] R. H. Brandenberger and Xin-min Zhang, Phys. Rev. D **59**, 081301 (1999).
[4] K. Rajagopal and F. Wilczek, Nucl. Phys. **B399**, 395 (1993); **B404**, 577 (1993).
[5] K. Rajagopal, arXiv:hep-ph/9504310.
[6] W. H. Zurek, Nature **317**, 505 (1985); Phys. Rep. **276**, 177 (1996).
[7] G. E. Volovik, *The Universe in a Helium Droplet* (Oxford University Press, 2003); V. B. Eltsov, M. Krusius, and G. E. Volovik, cond-mat/9809125.
[8] U. Heinz, Nucl. Phys. **A661**, 140c (1999).
[9] A. Rajantie, Int. J. Mod. Phys. A **17**, 1 (2002).
[10] X. M. Zhang, T. Huang, and R. H. Brandenberger, Phys. Rev. D **58**, 027702 (1998).
[11] A. P. Balachandran, S. Digal, Int. J. Mod. Phys. A **17**, 1149 (2002); A. P. Balachandran and S. Digal, Phys. Rev. D **66**, 034018 (2002).
[12] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
[13] R. D. Pisarski and F. Wilczek, Phys. Rev. D **29**, 338 (1984).
[14] M. Gell-Mann and M. Levy, Nuovo Cim. **16**, 705 (1960).
[15] E. A. Kuraev and Z. K. Silagadze, Acta Phys. Pol. B **34**, 4019 (2003).
[16] M. C. Birse, J. Phys. G **20**, 1537 (1994).
[17] T. Vachaspati, Phys. Rev. Lett. **68**, 1977 (1992); Nucl. Phys. **B397**, 648 (1993).
[18] A. Achucarro and T. Vachaspati, Phys. Rep. **327**, 347 (2000).
[19] M. Nagasawa and R. Brandenberger, Phys. Lett. **B467**, 205 (1999).
[20] A. Achucarro, J. Borrill, and A. Liddle, Phys. Rev. Lett. **82**, 3742 (1999).
[21] D. Röder, J. Ruppert, and D. H. Rischke, Phys. Rev. D **68**, 016003 (2003).
[22] T. W. B. Kibble, Physica C **369**, 87 (2002), and references therein.
[23] U. Heinz, Nucl. Phys. **A685**, 414c (2001).
[24] P. Braun-Munzinger, Nucl. Phys. **A661**, 261c (1999).
[25] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).
[26] C. M. Hung and E. Shuryak, Phys. Rev. C **57**, 1891 (1998); E. Schnedermann and U. Heinz, Phys. Rev. C **47**, 1738 (1993).
[27] P. Zhuang and Z. Yang, Phys. Rev. D **63**, 016004 (2001).
[28] R. H. Brandenberger, Int. J. Mod. Phys. A **9**, 2117 (1994).
[29] T. Vachaspati and A. Vilenkin, Phys. Rev. D **30**, 2036 (1984).
[30] M. Nagasawa and R. Brandenberger, Phys. Rev. D **67**, 043504 (2003).
[31] M. Yamaguchi, M. Kawasaki, and J. Yokoyama, Phys. Rev. Lett. **82**, 4578 (1999).
[32] J. Bai *et al.* (BES Collaboration), High Energy Phys. Nucl. Phys. **28(3)**, 215 (2004).
[33] M. Stephanov, K. Rajagopal, and E. Shuryak, Phys. Rev. D **60**, 114028.