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Parameter-free description of orbital magnetic dipole strength

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The low-lying orbital magnetic dipole strength in even-even nuclei is discussed using a sum-rule approach. It is shown that both the mean excitation energies as well as the summed excitation strengths from the experiment can be described well in heavy nuclei if the moments of inertia and the g factors are substituted by the parameters of the ground-state rotational bands. The influence of the high-lying scissors mode is taken into account explicitly, leading to a successful description of the low-lying mode in heavy deformed nuclei with no free parameters. A quantitative estimate of the gross features of the high-lying mode is deduced. The application of the sum-rule approach to medium-mass nuclei is presented, and the discrepancies with the experimental data are discussed.

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I. INTRODUCTION

The past twenty years have seen great experimental and theoretical efforts to study the orbital magnetic dipole response over a wide mass region [1–3]. The scissors mode, depicting the rotational vibration of the proton and neutron bodies with respect to each other in a deformed nucleus, has been discovered in high-resolution inelastic electron scattering experiments at low momentum transfer [4], only a few years after its theoretical prediction within a semiclassical two-rotor model [5] and the proton-neutron interacting boson model [6]. Nuclear resonance fluorescence (NRF) experiments [7] have measured the fine structure of the mode over the entire $N = 82-126$ major shell [8]. The scissors mode was also found in actinide and fp-shell nuclei [9–12] and in nuclei with an unpaired proton or neutron, see, e.g., Refs. [13–15]. We note in passing that evidence for the scissors mode in the quasicontinuum of excited nuclei has been found [16,17]; some discrepancies with the NRF results seem to exist which we do not discuss here to keep this communication as concise as possible. Investigations of the dynamics of metal clusters [18,19] and Bose-Einstein condensates [20–22] have established this elementary mode as a general phenomenon of deformed quantum systems.

The study of the deformation dependence of the scissors mode strength and excitation energy is critical for an understanding of the structure of the mode. The excitation energies in nuclei of the $N = 82$–126 major shell deduced from the experiments remain approximately constant. This can be attributed to pairing correlations, as was pointed out by Hamamoto and Magnusson [23]. An extension [24] of the schematic approach by Bes and Broglia [25] yields good accord with the data by an empirical fit of the moments of inertia. The second major experimental observation was that the total $B(M1)$ values of the scissors mode are proportional to the square of the deformation parameter [26]. The proportionality between the scissors mode strength and the square of the nuclear deformation parameter could be understood by an analysis of Lo Iudice and Richter [27] based on a phenomenological sum-rule approach with a fixed set of parameters proposed by Lipparini and Stringari [28,29]. A similarly good description was found within the interacting-boson model [30]. It also has been parametrized by a fit to the experimental data [31]. In another sum-rule analysis [32], it was shown that one can account simultaneously for the strength and the energy of the low-lying orbital $M1$ mode simply by using the moments of inertia and g factors of the ground-state rotational bands using one single scaling parameter. In the present communication, we want to work out the consequences of this finding in detail, and we show that the scaling parameter emerges quantitatively correctly from giant resonance parameters. We also investigate the applicability of the sum-rule approach to nuclei outside the $N = 82-126$ major shell.

Microscopic models show different results concerning deformation dependence, fragmentation, and collectivity of the mode, and they depend sensitively on the parameters of the mean field, the residual interaction, and the inclusion of the pairing interaction. We refer to Refs. [1–3] for an overview of the topic and for further references. Most calculations [33–36] predict another orbital magnetic dipole mode at high excitation energy, which should occur at energies above 20 MeV and is associated with the $K = 1$ component of the isovector giant quadrupole resonance (IVGQR). In contrast to its low-energy counterpart, experimental knowledge about the high-lying scissors mode is very limited [1]. The coupled dynamics of the low- and the high-lying mode has been studied recently in a solvable model by Balbutsev and Schuck [37]; this approach, however, fails to describe the data presented below, as pairing is not taken into account.

We present the database in Sec. II, and the sum-rule approach is introduced in Sec. III. The motivation of the moments of inertia of the scissors mode in Sec. IV is followed by the discussion of the results on energy and strength (Sec. V) using the comparison to data in the $N = 82-126$ major shell. A general discussion chapter (Sec. VI) follows.

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II. DATABASE

A. The $N = 82-126$ major shell

For the analysis of the systematics, the mean excitation energies and summed $M1$ strengths have to be extracted from the measured dipole strength distributions. For the present analysis, an excitation energy interval of 2.5–4.0 MeV is chosen in the entire $N = 82-126$ mass region, and we restrict ourselves to even-mass nuclei in this publication. At lower excitation energy, the existence of particle-hole excitations is known [38], whereas for energies above 4 MeV, spin-$M1$ strength dominates [39,40].

The parities of the excited states are determined via the K quantum number using the Alaga rules [41]

$$R_{K=1} = \frac{B(\sigma 1; J; K = 1; 1 \to 2; 0)}{B(\sigma 1; J; K = 1; 1 \to 0; 0)} = 0.5, \qquad (1)$$

$$R_{K=0} = \frac{B(\sigma 1; J; K = 1; 0 \to 2; 0)}{B(\sigma 1; J; K = 1; 0 \to 0; 0)} = 2.0, \qquad (2)$$

where K denotes the projection of the total angular momentum J on the symmetry axis of the deformed nucleus, and $\sigma 1$ represents the multipole character ($M1$, $E1$). Measurements of the linear polarization of the scattered radiation support $\Delta K = 0$ for nearly all electric dipole transitions into the ground state (g.s.). Similarly, practically all $M1$ transitions depopulate states with $K = 1$. Therefore, in the present analysis, states with experimental values of $R \leqslant 1$ are attributed positive parity and those with $R > 1$ negative parity if no direct measurement of the parity was available. States without a decay branch to the ground-state rotational band ($R = 0$) are excluded from the analysis because no conclusion can be made about their structure and parity. In the nuclei ^{194,196}Pt, the states with $R = 0$ have been included, as γ-soft nuclei are expected to exhibit a behavior different from axially symmetric deformed nuclei.

In ^{164}Dy, spin contributions have been found by Frekers and coworkers [39] around 3 MeV. Those are subtracted from the total strength assuming constructive interference. The nucleus ^{164}Er is omitted because for the experiment [42] no highly enriched target material was available and the deduced strength represents a lower limit only. For ^{154}Gd electron scattering, data by Hartmann *et al.* [43] have been used. As a conservative estimate, a linear propagation of errors is assumed since both statistical and systematic errors are important.

The summed $M1$ strengths as well as the mean excitation energies

$$\omega_{M1} = \frac{\sum_i E_{x,i}\, B_i(M1)\uparrow}{\sum_i B_i(M1)\uparrow}, \qquad (3)$$

which have been extracted from this data set, are given in Table I and displayed in Fig. 1. As is evident from Fig. 1(a), the mean excitation energy varies only little over the entire mass range from $A = 140$ through $A = 200$. The strengths systematics [Fig. 1(b)] reflects the deformation dependence of the scissors mode.

TABLE I. Mean excitation energies ω_{M1} and summed $M1$ strengths of the scissors mode for nuclei in the $N = 82-126$ major shell. Also given is the deformation parameter δ.

Nuclide	δ	ω_{M1} (MeV)	$\sum B(M1)$ (μ_N^2)	Ref.
$^{142}_{58}$Ce$_{84}$	0.110	3.00	0.55(4)	[44]
$^{144}_{60}$Nd$_{84}$	0.115	3.15	0.72(5)	[45]
$^{146}_{60}$Nd$_{86}$	0.132	3.46	0.94(17)	[46]
$^{148}_{60}$Nd$_{88}$	0.171	3.49	1.05(24)	[46]
$^{150}_{60}$Nd$_{90}$	0.233	3.12	1.83(27)	[46]
$^{148}_{62}$Sm$_{86}$	0.124	3.07	0.51(12)	[47]
$^{150}_{62}$Sm$_{88}$	0.165	3.18	0.97(17)	[47]
$^{152}_{62}$Sm$_{90}$	0.249	2.97	2.41(33)	[47]
$^{154}_{62}$Sm$_{92}$	0.273	3.26	2.44(38)	[47]
$^{154}_{64}$Gd$_{90}$	0.253	2.91	2.99(62)	[43]
$^{156}_{64}$Gd$_{92}$	0.271	3.06	2.73(56)	[48]
$^{158}_{64}$Gd$_{94}$	0.278	3.10	3.71(59)	[48]
$^{160}_{64}$Gd$_{96}$	0.282	3.11	3.26(51)	[49]
$^{160}_{66}$Dy$_{94}$	0.271	2.87	2.42(30)	[50]
$^{162}_{66}$Dy$_{96}$	0.274	2.93	2.85(22)	[51]
$^{164}_{66}$Dy$_{98}$	0.278	2.97	3.25(43)	[51]
$^{166}_{68}$Er$_{98}$	0.274	2.99	2.55(48)	[42]
$^{168}_{68}$Er$_{100}$	0.274	3.24	3.68(48)	[42]
$^{170}_{68}$Er$_{102}$	0.274	3.22	3.42(69)	[42]
$^{172}_{70}$Yb$_{102}$	0.265	3.03	1.83(49)	[52]
$^{174}_{70}$Yb$_{104}$	0.262	3.15	2.70(88)	[52]
$^{176}_{70}$Yb$_{106}$	0.249	3.33	2.56(97)	[52]
$^{176}_{72}$Hf$_{104}$	0.241	3.28	3.11(27)	[53]
$^{178}_{72}$Hf$_{106}$	0.230	3.21	2.38(36)	[54]
$^{180}_{72}$Hf$_{108}$	0.225	3.19	2.04(28)	[54]
$^{182}_{74}$W$_{108}$	0.208	3.25	1.34(23)	[55]
$^{184}_{74}$W$_{110}$	0.196	3.37	1.04(33)	[55]
$^{186}_{74}$W$_{112}$	0.188	3.19	0.82(21)	[55]
$^{190}_{76}$Os$_{114}$	0.151	2.87	0.85(11)	[56]
$^{192}_{76}$Os$_{116}$	0.143	3.00	0.93(6)	[56]
$^{194}_{78}$Pt$_{116}$	0.125	3.25	1.38(25)	[57]
$^{196}_{78}$Pt$_{118}$	0.115	3.01	0.81(16)	[58,59]

B. Actinide nuclei

Three actinide nuclei, ^{226}Th, ^{236}U, and ^{238}U, have been studied in photon scattering experiments; see Refs. [10,11] as well. The same identification scheme holds as for the nuclei with $82 < N < 126$, but the energy window has been chosen to be 2–3 MeV to account for the lower energy of the scissors mode in the actinides. Table II summarizes the experimental data.

C. Nuclei with $A < 140$

In various nuclei with $A < 140$, low-lying $M1$ strength was detected too. For some fp-shell nuclei, electron

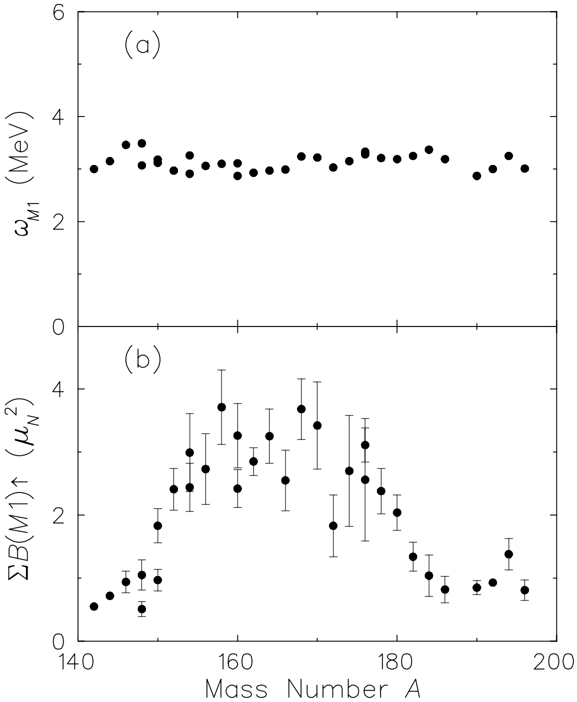

FIG. 1. Mean excitation energies (a) and summed $M1$ strengths (b) of the scissors mode for nuclei in the $N = 82-126$ major shell.

TABLE III. Mean excitation energies ω_{M1} and summed $M1$ strengths of the scissors mode for nuclei with $A < 140$. Also given is the deformation parameter δ.

Nuclide	δ	ω_{M1} (MeV)	$\sum B(M1)$ (μ_N^2)	Ref.
$^{46}_{22}Ti_{24}$	0.256	4.32	0.41(8)	[9,12]
$^{48}_{22}Ti_{26}$	0.221	4.00	0.19(6)	[12,60,64]
$^{50}_{24}Cr_{26}$	0.239	3.63	0.11(2)	[60,65]
$^{56}_{26}Fe_{30}$	0.200	3.45	0.49(8)	[62,66]
$^{66}_{30}Zn_{36}$	0.183	4.30	0.21(6)	[67]
$^{92}_{40}Zr_{52}$	0.092	3.47	0.28(2)	[68]
$^{94}_{42}Mo_{52}$	0.131	3.13	0.48(3)	[69]
$^{108}_{48}Cd_{60}$	0.151	3.45	0.07(2)	[70]
$^{112}_{48}Cd_{64}$	0.159	3.05	0.21(6)	[71]
$^{114}_{48}Cd_{66}$	0.162	2.87	0.18(3)	[72]
$^{122}_{52}Te_{70}$	0.158	3.20	0.83(45)	[73]
$^{124}_{52}Te_{72}$	0.146	3.24	0.35(8)	[74]
$^{126}_{52}Te_{74}$	0.133	3.25	0.67(14)	[73]
$^{130}_{52}Te_{78}$	0.105	3.12	0.13(9)	[73]
$^{134}_{56}Ba_{78}$	0.139	2.99	0.56(9)	[75]
$^{136}_{56}Ba_{80}$	0.111	3.26	0.20(2)	[76]

scattering and proton scattering experiments exist that identify the scissors-mode states in an energy interval close to 4 MeV [60] and allow the spin contributions to be subtracted. Constructive interference has been assumed, which is based on the shell-model description of the scissors mode in the fp-shell nuclei; within this approach, the mode results from recoupling as proposed by Zamick [61]. One might argue about the reliability of extracting the spin strength from the proton scattering data. The uncertainties given in the compilation have been obtained from the values quoted in Refs. [9,60]. For ^{56}Fe no spin contributions have been subtracted as no intermediate-energy forward-angle proton scattering data exist and theory assumes spin contributions to the low-lying $M1$ strength to be comparably small [62].

The decay pattern of specific states can be used to single out mixed-symmetry 1^+ states in nuclei from Zn to ^{108}Cd from a comparison to shell-model or interacting-boson-model predictions. These states typically lie between 3 and 4 MeV. To date, there are no experimental results about possible spin contributions to the $M1$ strength in these nuclei. In the other cadmium, the tellurium, and the open-shell barium isotopes, we have included all dipole excitations between 2.5 and 4.0 MeV where a magnetic character cannot be excluded from measurement or from the systematics of quadrupole-octupole-coupled 1^- states (see, e.g., [63]). The maximum energy in this case is limited by the minimum endpoint energy of the bremsstrahlung spectra used in the NRF experiments. Table III summarizes the values deduced from the experiments in nuclei with $A < 140$.

As is evident from the above description, this last data set is not as homogeneous as for the heavier nuclei, and sensitivity, energy region, and identification scheme vary between the different nuclei in this mass region. However, we tried to perform the data selection in as unbiased a way as possible.

TABLE II. Mean excitation energies ω_{M1} and summed $M1$ strengths of the scissors mode for actinide nuclei. Also given is the deformation parameter δ.

Nuclide	δ	ω_{M1} (MeV)	$\sum B(M1)$ (μ_N^2)	Ref.
$^{232}_{90}Th_{142}$	0.216	2.14	2.59(25)	[10]
$^{236}_{92}U_{144}$	0.225	2.37	3.53(53)	[11]
$^{238}_{92}U_{146}$	0.234	2.26	3.19(23)	[10]

III. SUM-RULE APPROACH

In this section, we introduce a sum-rule approach in order to describe the excitation energy and strength of the scissors mode. We use the results of the Lipparini and Stringari analysis [28,29]. Sum rules provide insight into special properties of quantum systems by exploiting commutation relations. A sum rule S_j is defined as the sum of products of the transition strengths $[B_i(\sigma\lambda)]$ by the jth power of the corresponding excitation energies, i.e.,

$$S_j\,[\mathcal{M}(\sigma\lambda)] = \sum_i B_i(\sigma\lambda)\,\omega_i^j, \tag{4}$$

where $\mathcal{M}(\sigma\lambda)$ denotes the field operator characterizing the excitation with multipolarity $\sigma\lambda$. This information can be obtained directly from the measured spectra. If both the structure of the operator as well as the Hamiltonian $\mathcal{H}$ are known, S_j can be calculated from g.s. expectation values of commutators

$$S_{+1}(\mathcal{M}) = \tfrac{1}{2}\langle 0|\,[\mathcal{M},[\mathcal{H},\mathcal{M}]]\,|0\rangle \tag{5}$$

$$S_{-1}(\mathcal{M}) = \tfrac{1}{2}\langle 0|[[\mathcal{X}^{\dagger},\mathcal{H}],\mathcal{X}]|0\rangle, \tag{6}$$

where $\mathcal{X}$ denotes the solution of $[\mathcal{H},\mathcal{X}] = \mathcal{M}$. By combining Eqs. (5) and (6), the energy and strength can be predicted in terms of the sum rules S_{+1} and S_{-1} as

$$\omega = \sqrt{\frac{S_{+1}}{S_{-1}}} \tag{7}$$

$$\sum_i B_i(\sigma\lambda) = \sqrt{S_{+1}S_{-1}}\,. \tag{8}$$

We note that these equations only agree with the experimental summed strength and mean excitation energy [Eq. (3)] when fragmentation can be neglected or in the case where the fragments are distributed symmetrically around the distribution's centroid. The data sets should reasonably fulfill these conditions.

The isovector rotational motion associated with the scissors mode defines the operator

$$\mathcal{M}(M1_{\mathrm{Sc.}}) \propto \sum_i \mathcal{J}_i^x\,\hat{\tau}_i^3 \tag{9}$$

whose commutation relations have been worked out by Lipparini and Stringari [28,29] using a Skyrme-type Hamiltonian. Here, the angular momentum operator perpendicular to the symmetry axis is given by $\mathcal{J}^x$, and the isospin projection is denoted by $\hat{\tau}^3$. As shown in Ref. [28], the inverse energy-weighted sum rule can be related to an isovector moment of inertia, and the linearly energy-weighted sum rule can be related to the excitation energy of the isovector giant dipole resonance (IVGDR). Thus, one obtains for the inversely and linearly energy-weighted sum rules

$$\begin{aligned} S_{+1} &= \omega_{M1}\,B(M1) \\ &= \frac{3}{20\pi}\,r_0^2\,A^{5/3}\,\delta^2\,\omega_D^2\,m_N\,(g_p-g_n)^2\,\left[\mu_N^2\,\mathrm{MeV}\right] \end{aligned} \tag{10}$$

$$\begin{aligned} S_{-1} &= \frac{B(M1)}{\omega_{M1}} \\ &= \frac{3}{16\pi}\,\Theta_{\mathrm{IV}}\,(g_p-g_n)^2\,\left[\mu_N^2\,\mathrm{MeV}^{-1}\right]. \end{aligned} \tag{11}$$

In these equations, $r_0 = 1.15$ fm denotes the nuclear radius constant, A is the mass number, δ is the deformation parameter, and ω_D is the centroid energy of the IVGDR. Furthermore, m_N represents the nucleon mass, and g_p and g_n are the orbital gyromagnetic ratios of the deformed proton and neutron valence bodies. The moment of inertia Θ_{IV} originates from the isovector motion.

IV. MOMENTS OF INERTIA AND g FACTORS

In Eqs. (10) and (11) nearly all quantities are fixed for the individual nucleus by measurements or can be obtained from systematic analyses. However, it is not a priori clear which values should be used for the isovector moment of inertia and the orbital g factors. We shall, for that purpose, inspect the inverse energy-weighted sum rule (11), which is proportional to the product $\Theta_{\mathrm{IV}}(g_p - g_n)^2$, to gain further insight into these quantities. The investigation of this *single* sum rule enables us to identify the moments and then to predict the properties of the scissors mode from both S_{+1} *and* S_{-1}. Since the low-energy spectrum contributes dominantly to S_{-1}, the moments ($g_p - g_n$) and Θ_{IV} can be determined from the experimental data on the low-energy scissors mode without further assumptions.

Within a collective model approach, the g factor of the ground-state (isoscalar) rotational band, g_{IS}, can be expressed in terms of g_p and g_n by [77]

$$g_{\mathrm{IS}} = \frac{1}{2}\,(g_p + g_n) = \frac{\mu(J)}{J}\,, \tag{12}$$

where $\mu(J)$ represents the magnetic moment of a rotational state with angular momentum J. Similarly, one can define a g factor for an *isovector* rotation as

$$g_{\mathrm{IV}} := \tfrac{1}{2}\,(g_p - g_n)\,. \tag{13}$$

One further concludes $g_{\mathrm{IV}} \simeq g_{\mathrm{IS}}$ if the magnetic properties of collective rotations are mainly due to protons, and the contributions of neutrons can be neglected, i.e., $g_n \approx 0$.

While neutrons are not expected to contribute to the orbital magnetic properties of a nuclear collective state, subnucleonic degrees of freedom, such as mesonic exchange currents, might generate a finite contribution. The enhancement of the $E1$ strength beyond the Thomas-Reiche-Kuhn (TRK) sum rule has recently been shown by Bentz and Arima [78] to be proportional to the difference $g_p - g_n$ within a Landau-Migdal approach for nuclear matter. Using the IVGDR parameters of the compilation by Dietrich and Berman [79], one finds that the enhancement of $g_p - g_n$ is at most on the order of 10–15%. We neglect this correction in the following.

The argument for the moment of inertia is analogous to that of the g factors: Let

$$\Theta_{\mathrm{IS}} = \frac{J(J+1)}{2\omega_J} \simeq \frac{3\hbar^2}{\omega_{E2}} \tag{14}$$

be the moment of inertia of the ground-state rotational band, with ω_J the excitation energy of a rotational state with angular momentum J, and ω_{E2} the excitation energy of the 2_1^+ state [80]. The difference between this isoscalar rotation and an isovector motion can be expressed [27] by

$$\Theta_{\mathrm{IV}} = \frac{4NZ}{A^2}\,\Theta_{\mathrm{IS}}, \tag{15}$$

where Z, N, and A are the proton, neutron, and mass number of the nucleus, respectively. The factor $4NZ/A^2$ is approximately constant for all nuclei of the $N = 82{-}126$ major shell and amounts to about 0.96.

If the low-lying $M1$ strength detected in the experiments corresponds to an isovector rotation, the following relation

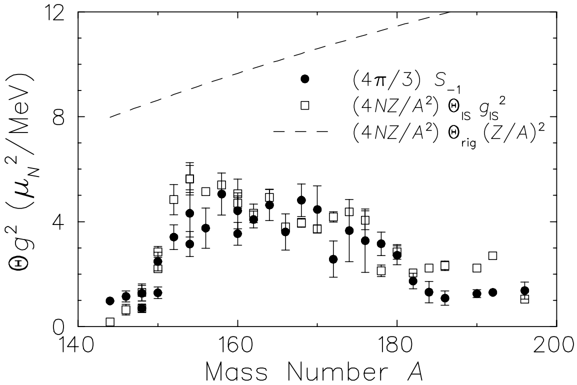

FIG. 2. Comparison of the physical parameters in the description of the scissors mode. Shown are the values for the isovector parameters $\Theta_{IV} g_{IV}^2$ deduced from the inversely energy-weighted sum rule (solid circles), the product $4NZ/A^2 \cdot \Theta_{IS} g_{IS}^2$ (open squares), and the rigid-body values $4NZ/A^2 \cdot \Theta_{rig}(Z/A)^2$ (dashed line). The isovector and isoscalar values agree for most nuclei within the experimental errors.

should hold:

$$\frac{4\pi}{3} S_{-1} \simeq \frac{4NZ}{A^2} \Theta_{IS}\, g_{IS}^2. \tag{16}$$

Figure 2 shows the product $g^2\Theta$ from S_{-1}, i.e., the isovector quantities extracted from the NRF experiments (solid circles) and the measured moments of inertia and g factors for the ground-state band (open squares), multiplied by $4NZ/A^2$. We have concentrated here on the $N = 82{-}126$ major shell, as the identification of the scissors mode fragments is most robust. The g factors have been taken from Ref. [81]. The dashed line shows the expectation for a rigid rotor $\Theta_{rig} = 2/5\, m_N A^{5/3}$—also multiplied by the factor $4NZ/A^2$—with a homogeneous magnetization density $g_{rig} = Z/A$. While the rigid rotor limit clearly overestimates the data, Eq. (16) holds on a 10% level for most nuclei. Some deviations between the sum-rule expectations and the g.s. band are visible in the transitional regions, where the very simple experimental identification scheme is naturally less reliable.

V. PARAMETER-FREE DESCRIPTION OF THE SCISSORS MODE

A. Separation of the high-lying orbital $M1$ strength

Since all quantities in the sum rules are fixed, we now attempt to describe the mean excitation energy and summed excitation strength of the mode. However, when dealing with S_{+1} one has to take into account that here contributions from the $K = 1$ component of the IVGQR will dominate and have to be removed in order to compare the parameter-free description to experimental data. We use here the result given by Lipparini and Stringari [28] and define

$$\mathcal{M}(M1_{\mathrm{Sc.}}) \propto \sum_i \mathcal{J}_i^x\, \hat{\tau}_i^3 + \alpha \sum_i \left(\hat{y}_i\, \hat{p}_i^z + \hat{z}_i\, \hat{p}_i^y\right) \hat{\tau}_i^3. \tag{17}$$

The operator is now split into a term from the isovector rotation and a term from the IVGQR containing momenta $\hat{p}^y$, $\hat{p}^z$ and coordinates $\hat{y}$, $\hat{z}$ for the quadrupole vibration.

Evaluating the commutation relations and choosing α in such a way that the low-lying mode is selected [28], S_{+1} takes the form

$$S_{+1} = \frac{3}{5\pi}\, r_0^2\, A^{5/3}\, \delta^2\, \omega_D^2\, m_N\, g_{IS}^2\, \frac{\omega_Q^2}{\omega_Q^2 + 2\omega_D^2}. \tag{18}$$

Here, ω_Q stands for the centroid of the *isoscalar* giant quadrupole resonance (ISGQR). Equation (18) corresponds to Eq. (10) except for the factor

$$\xi := \frac{\omega_Q^2}{\omega_Q^2 + 2\omega_D^2}, \tag{19}$$

which describes the contribution from the IVGQR, using Eq. (13) with $g_{IV} \simeq g_{IS}$.

B. Excitation energy

Combining Eqs. (7), (11), and (18) one obtains for the mean excitation energy of the low-lying scissors mode

$$\omega_{M1} = \frac{2}{\sqrt{15}}\, r_0\, A^{5/6} \sqrt{\frac{A^2}{4NZ}}\, \omega_D \sqrt{m_N\, \omega_{E2}\, \xi}\, \delta. \tag{20}$$

The systematics of the centroid energies are given by Berman and Fultz [82] for the IVGDR and by van der Woude [83] for the ISGQR, correcting for shifts of the $K = 0$ components due to deformation given by Jang [84]. One finds

$$\omega_D \simeq (31.2A^{-1/3} + 20.6A^{-1/6})\,(1 - 0.61\delta)\,\mathrm{MeV}, \tag{21}$$

$$\omega_Q \simeq 64.7A^{-1/3}\,(1 - 0.3\delta)\,\mathrm{MeV}. \tag{22}$$

Figure 3 shows the mean excitation energies extracted from the experiments (solid circles) in comparison to the values (open triangles) predicted by Eq. (20). The agreement is very good; on the average the deviations amount to about 5%. The

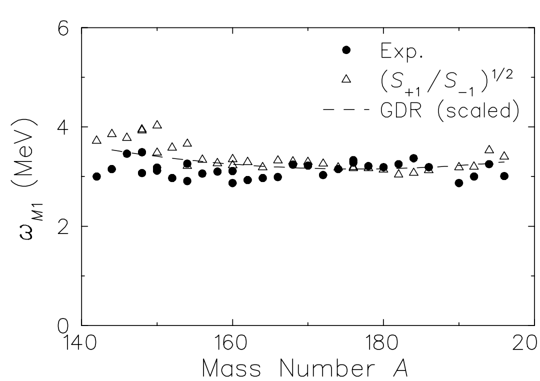

FIG. 3. Mean excitation energy of the scissors mode: experimental values (solid circles) and parameter-free prediction of the sum-rule analysis (open triangles). Error bars are smaller than the symbol size. The deformation dependence of the nuclear moment of inertia leads to the proportionality of the excitation energies of the scissors mode and the IVGDR as indicated by the dashed line.

approximate deformation independence of the energy is well reproduced. This can be understood if one recalls [77] that the nuclear moment of inertia $\Theta_{\rm IS}$ is roughly proportional to, but significantly larger than, the liquid-drop moment of inertia $\Theta_{\rm liq}$

$$\frac{A^2}{4NZ}\,\Theta_{\rm IV} \simeq \Theta_{\rm IS} \simeq \Upsilon\,\Theta_{\rm liq} \simeq \Upsilon\,\Theta_{\rm rig}\,\delta^2, \qquad (23)$$

so that ω_{E2} can be expressed in terms of δ. The average of Υ in the mass region considered here is 7.9 with typical deviations of about 10% (standard deviation). This implies that the excitation energy of the scissors mode is directly proportional to the centroid of the IVGDR and—neglecting the weak variation of ξ with δ—independent from the deformation parameter

$$\omega_{M1} \approx \sqrt{\frac{2}{\Upsilon}\,\xi}\cdot\omega_D, \qquad (24)$$

where we have also suppressed the $(4NZ/A^2)$ term. The good agreement of Eq. (24), shown as a dashed line in Fig. 3, with the experimental data is evident.

C. Transition strength

The low-lying $M1$ strength is derived from

$$\sum B(M1) = \frac{3}{\pi}\sqrt{\frac{3}{20}}\,r_0\,A^{5/6}\sqrt{\frac{4NZ}{A^2}}\,\omega_D\sqrt{\frac{\xi\,m_N}{\omega_{E2}}}\,\delta\,g_{\rm IS}^2, \qquad (25)$$

where we have used Eqs. (16) and (18). Figure 4 depicts the agreement of this sum-rule prediction (open triangles) with the experimental results (solid circles). The strong deformation dependence is generated by the interplay of ω_{E2} and δ. This again becomes clear when inserting Eq. (23) so that

$$\sum B(M1) = \frac{3}{\pi}\sqrt{\frac{\Upsilon}{50}}\,r_0^2\,A^{5/3}\,\omega_D\sqrt{\frac{4NZ}{A^2}}\;m_N\sqrt{\xi}\,g_{\rm IS}^2\,\delta^2. \qquad (26)$$

One obtains the approximate δ^2 dependence of the scissors mode strength originally established in the samarium isotopes by Ziegler *et al.* [26]. The mass dependence (roughly $\propto A^{4/3}$ including the contribution from the excitation energy of the IVGDR) was too weak to be visible in the data of Ref. [26], which refer to small variations of A only.

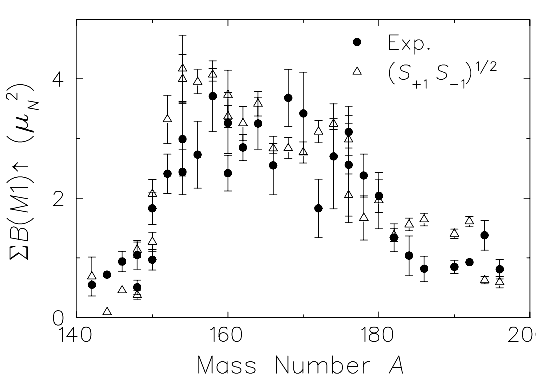

FIG. 4. Summed $M1$ strength of the low-lying orbital magnetic dipole excitations: experimental values (solid circles) and parameter-free description of the sum-rule analysis (open triangles).

Since the g.s. nuclear deformation is related to the excitation strength of the 2_1^+ state of the g.s. rotational band, one can deduce from Eq. (26) a proportionality constant between the $M1$ and $E2$ strengths in single-particle units

$$\begin{aligned}\sum B(M1)[\text{W.u.}] \simeq{}& \frac{81}{100\pi}\sqrt{\frac{\Upsilon}{50}}\,r_0^2\,A^{5/3}\,\omega_D\\ &\times m_N\sqrt{\frac{4NZ}{A^2}}\sqrt{\xi}\,g_{\rm IS}^2\\ &\times\frac{B(E2)[\text{W.u.}]}{Z^2}.\end{aligned} \qquad (27)$$

Averaging over the entire mass region, one obtains

$$\sum B(M1)[\text{W.u.}] \simeq 11.3(7)\,\frac{B(E2)[\text{W.u.}]}{Z^2}, \qquad (28)$$

consistent with an empirical fit to the data [31] finding $B(M1) = 10.6\,B(E2)/Z^2$. The uncertainty in the constant of Eq. (28) has been taken to be the standard deviation of the data set, and for the calculation of the average the experimental uncertainties of the g factors and the deformation parameters have been included.

VI. DISCUSSION

A. The $N = 82-126$ major shell

In the following we critically examine the results obtained in the previous sections and some conclusions that can be drawn from them. One problem is the possibility of low-energy orbital $M1$ strength lying above 4 MeV. However, the most recent microscopic calculations from the Tübingen and Dubna groups [35,85] predict only a small fraction of the strength in the energy interval between about 4 and 6 MeV. This would affect the sum rules on a 10% level, well within typical uncertainties of the present approach. In the mass region approaching γ softness, the Alaga rules and their (empirical) relation to the parity of the excited states are certainly less reliable, so the somewhat larger discrepancies between the experimental summed $B(M1)$ strengths and the present results should not be overemphasized.

The good overall agreement of both the predicted energies and strengths with the experimentally observed low-lying magnetic dipole excitations permits us to draw an important conclusion for the parameters: Since ω_{M1} depends on $\Theta_{\rm IV}$ only, but $\sum B(M1)$ depends on both $g_{\rm IV}$ and $\Theta_{\rm IV}$, it is evident that the identity of isoscalar and isovector parameters holds not only for the product $\Theta_{\rm IS}g_{\rm IS}^2 \simeq \Theta_{\rm IV}g_{\rm IV}^2$ but also *individually*, i.e., $\Theta_{\rm IS} \simeq \Theta_{\rm IV}$ and $g_{\rm IS} \simeq g_{\rm IV}$, where we have neglected the factor $4NZ/A^2$, which is close to unity.

In the sum-rule relations given above, both the energy and the strength of the scissors mode depend only on key quantities of collective excitations ($\omega_D, \omega_{E2}, \omega_Q, g_{\rm IS}$). It is therefore natural to conclude that the scissors mode is a collective excitation, a statement that has been disputed [33,35,86]. This conclusion—drawn from the gross properties of the scissors mode—is in agreement with the results of a recent complementary statistical study of the fine structure. In the latter analysis, Poissonian behavior of uncorrelated level spacings has been revealed [87]. Similar findings were reported for collective excitations near the Yrast line in deformed nuclei [88,89].

B. Actinide nuclei

The g factors for ^{232}Th and ^{238}U are tabulated in the compilation by Raghavan [81]; in addition, the g factor of the 2_1^+ state of ^{236}U has been determined in [90] relative to ^{238}U. The sum-rule approach corroborates the lowering of the mean energy of the scissors mode observed in these nuclei. The centroid of the scissors mode in ^{232}Th and ^{236,238}U is predicted at 2.5, 2.6, and 2.6 MeV, respectively, and its summed strengths are expected to be 2.7(5), 5.4(17), and 5.0(8) μ_N^2, respectively. These values need to be compared to the experimental ones listed in Table II. Without showing detailed figures, we note that the sum-rule expectations are in general agreement with the experimental results. Only in ^{238}U is the detected strength somewhat smaller than predicted.

C. $A < 140$ nuclei

Figure 5 displays the experimental data as well as the sum-rule prediction for medium-mass nuclei for both the mean excitation energy (a) and the summed $M1$ strength (b). For the nuclei ^{92}Zr and ^{94}Mo, the g factors for the sum-rule estimate have been taken from [91] and [92], respectively. Throughout the entire mass region, the prediction follows the experimental mean energies, but the sum rule overestimates the measured results by a constant value of about 1 MeV with the exception of ^{92}Zr where the predicted mean energy is actually a little below the experimental value. This finding could possibly be attributed—to some degree—to the limited endpoint energy of part of the photon scattering experiments, and one might argue that some weaker transitions have been below the detection threshold in the electron scattering experiments performed in the lighter masses. The fact that the scaled excitation energy of the IVGDR according to Eq. (24) is closer to the experimental data towards heavier masses might support this assertion (dashed line in Fig. 5).

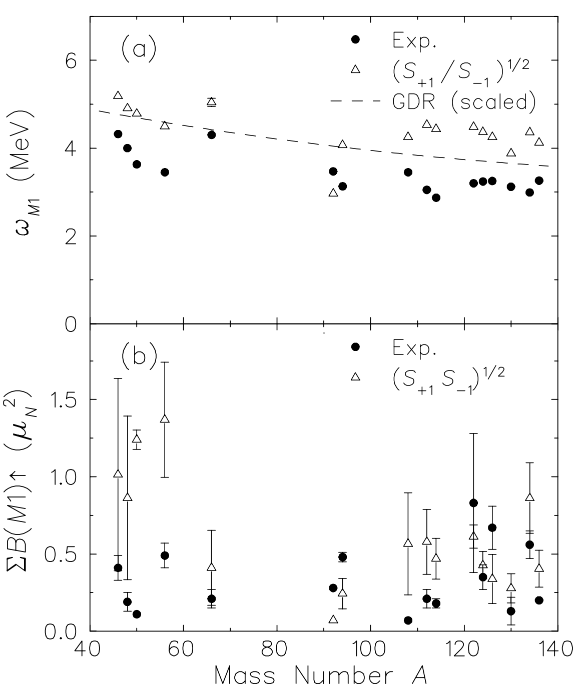

FIG. 5. Comparison of the experimental values (solid circles) and sum-rule predictions (open triangles) for the mean excitation energy (a) and the summed orbital $M1$ strength (b) in nuclei with $A < 140$. The dashed curve indicates again the approximated sum-rule expectation from a scaling of the IVGDR energy according to Eq. (24).

The predicted summed $M1$ strengths scatter around the measured values. While the model roughly follows the experimental data in the $Z > 50$ nuclei, lighter masses show significant deviations: In most nuclei, larger orbital $M1$ strengths are calculated than are deduced from the experiment. The $N = 52$ nuclei ^{92}Zr and ^{94}Mo represent exceptions to this rule so the low experimental strength cannot be attributed to missing strength above 4 MeV in general.

In the work by Guliyev and coworkers [93], the deviations between the experimental data and the results of the sum-rule approach in the Te isotopic chain are traced back to the g factors of the 2_1^+ states, and it is suggested that they might not serve as a measure of the orbital magnetic properties of the scissors mode in weakly deformed nuclei. Such an argument is consistent with microscopic analyses within the quasiparticle RPA [93], the quasiparticle-phonon nuclear model [73], and the interacting-boson model [69].

In lighter nuclei the shell-model calculation of Ref. [62] also predicts some orbital $M1$ strength around 6 MeV for ^{56}Fe. If this strength from the shell-model calculation (KB3G residual interaction, truncation $t = 6$) is included, one obtains a total orbital $M1$ strength of about 0.5 μ_N^2 and a mean excitation energy of 5.2 MeV.

D. Odd-mass nuclei

Another important extension of the present approach would be to include odd-mass nuclei. The extraction of the intrinsic parameters from the measured spectroscopic quantitites for the sum rule approach is, however, challenging. A sum rule within the interacting boson-fermion model by Ginocchio and Leviatan [94] predicts a summed orbital $M1$ strength in deformed odd-A nuclei comparable to that in the even-mass neighbors. So far, complete strength distributions have been reported only for a few cases [14,15,95,96] with some simplifying assumptions to deduce the rough amount of strength hidden in the experimental background of the highly complex NRF spectra. Although the analysis proves to be robust with respect to different experimental data, it appears to fail for cases with comparably little fragmentation of the observed dipole strength as found in ^{163}Dy by Nord *et al.* [15].

E. Implications on the high-lying scissors mode strength

The high-lying strength was decoupled explicitly from the low-lying $M1$ excitations by the introduction of the factor ξ

which was deduced from the centroids of the IVGDR and the ISGQR. Within a two-state model this factor permits an estimate of the magnitude of the high-lying strength. If B_l, ω_l and B_h, ω_h denote the strengths and energies of the low- and high-lying scissors modes, respectively, the linearly and inversely energy-weighted sum rules can be approximated by

$$S_{+1} \simeq B_h \cdot \omega_h, \; S_{-1} \simeq \frac{B_l}{\omega_l}, \tag{29}$$

where S_{+1} denotes the full linear energy-weighted sum rule according to Eq. (10). However, including the correction introduced in Eq. (18), one finds with

$$\omega_l \simeq \sqrt{\frac{B_h}{B_l}\, \omega_h\, \omega_l\, \xi} \tag{30}$$

a relation for the energy-weighted sums

$$B_h\, \omega_h \approx \frac{1}{\xi}\, B_l\, \omega_l. \tag{31}$$

As already mentioned by Zawischa [2], the energy-weighted sum rule for the high-lying strength was predicted to be a factor of four higher than it is for the low-lying strength. This is also in line with the schematic RPA calculations by Lo Iudice [36] if values for the isovector coupling constant are assumed not to be too large. Using our expressions for the centroid and strength of the low-lying mode, one obtains

$$B_h \approx \frac{3}{5\pi}\, r_0^2\, A^{5/3}\, m_N\, \frac{\omega_D^2}{\omega_h} \delta^2\, g_{\mathrm{IS}}^2\,. \tag{32}$$

The excitation energy of the high-lying strength is taken from the systematics of the IVGQR centroid, $\omega_h \simeq 130\ A^{-1/3}$ MeV [83]. A deformation dependence similar to that of the $0\hbar\omega$ mode is predicted. Within this simple approach one can extract a summed $M1$ strength of $B_h \approx 2\ \mu_N^2$ at high energies for a well deformed nucleus in the $N = 82{-}126$ mass region. This is about 50% of the strength predicted by microscopic calculations [33–35,85], which tend to yield higher values for the low-lying strength as well, but it agrees with the analysis of Ref. [36].

The strengths of both the high-lying and the low-lying mode are proportional to the square of the deformation parameter. But the relation for B_h is independent of both Υ and ξ, i.e., the high-lying strength is unaffected by pairing correlations and admixtures from low-lying states. Due to the absence of Υ and the strict δ^2 dependence, one can formulate the energy-weighted sum for the high-lying strength using the liquid-drop (irrotational) moment of inertia as the mass parameter. This has been pointed out by Lo Iudice and Richter [3,34]. Like the low-lying mode, B_h scales roughly with $A^{4/3}$. This suggests that experiments searching for B_h are most likely to be successful in the strongly deformed lanthanide or actinide nuclei. The experimental task of measuring the high-energy mode is very challenging, as it will be highly fragmented. An analysis of polarized proton scattering on the deformed nucleus ^{154}Sm indicates the presence of the IVGQR [97], from which a strength of $B_h \simeq 3.5(6)\ \mu_N^2$ was derived on the basis of the approach of Ref. [34] in reasonable agreement with microscopic calculations as well as the present model. The centroid of the IVGQR in that nucleus was found to be [1] at 23.4(6) MeV. It was also pointed out in Ref. [1] that from the measured isovector $E2$ strength exhausting 76(11)% of the isovector $E2$ sum rule, the energy-weighted *low-energy* sum rule of Moya de Guerra and Zamick [98] is exhausted on an ~80% level.

VII. CONCLUSION

In summary, we have presented a phenomenological analysis of the scissors mode in even-even nuclei based on sum-rule techniques. Our analysis indicates the near equality of isoscalar and isovector moments of inertia and gyromagnetic ratios for the scissors mode and the ground-state band. By means of this analysis, parameter-free relations for the mean energy and the summed strength have been deduced that are in good accord with the experimental findings in the $N = 82{-}126$ neutron major shell and in the actinides. It is concluded that the scissors mode is a collective excitation because its description depends on collective quantities only.

An application of the sum-rule prediction to scissors mode states in lighter nuclei with mostly smaller deformation parameters is presented. We obtain in most cases larger values for both the mean excitation energy and the summed strength than those detected in the experiments.

Within a two-state model, some simple relations for the predicted high-lying strength have been deduced. From the sum rules, using the same parameters as for the low-energy mode, one expects about 50% of the strength predicted by microscopic model calculations. Experimental efforts to establish the $K = 1$ magnetic dipole component of the IVGQR are needed to finally reach a complete picture of the orbital magnetic dipole strength in nuclei.

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[1] A. Richter, Prog. Part. Nucl. Phys. **34**, 261 (1995).
[2] D. Zawischa, J. Phys. G **24**, 683 (1998).
[3] N. Lo Iudice, Nuovo Cimento **23**, 1 (2000).
[4] D. Bohle, A. Richter, W. Steffen, A. E. L. Dieperink, N. Lo Iudice, F. Palumbo, and O. Scholten, Phys. Lett. **137B**, 27 (1984).
[5] N. Lo Iudice and F. Palumbo, Phys Rev. Lett. **41**, 1532 (1978).
[6] F. Iachello, Nucl. Phys. **A358**, 89c (1981).
[7] U. E. P. Berg, C. Bläsing, J. Drexler, R. D. Heil, U. Kneissl, W. Naatz, R. Ratzek, S. Schennach, R. Stock, T. Weber,

H. Wickert, B. Fischer, H. Hollick, and D. Kollewe, Phys. Lett. **149B**, 59 (1984).
[8] U. Kneissl, H. H. Pitz, and A. Zilges, Prog. Part. Nucl. Phys. **37**, 349 (1996).
[9] C. Djalali, N. Marty, M. Morlet, A. Willis, J. C. Jourdain, D. Bohle, U. Hartmann, G. Küchler, A. Richter, G. Caskey, G. M. Crawley, and A. Galonsky, Phys. Lett. **164B**, 269 (1985).
[10] R. D. Heil, H. H. Pitz, U. E. P. Berg, U. Kneissl, K.-D. Hummel, G. Kilgus, D. Bohle, A. Richter, C. Wesselborg, and P. von Brentano, Nucl. Phys. **A476**, 39 (1988).
[11] J. Margraf, A. Degener, H. Friedrichs, R. D. Heil, A. Jung, U. Kneissl, S. Lindenstruth, H. H. Pitz, H. Schacht, U. Seemann, R. Stock, C. Wesselborg, P. von Brentano, and A. Zilges, Phys. Rev. C **42**, 771 (1990).
[12] T. Guhr, H. Diesener, A. Richter, C. W. de Jager, H. de Vries, and P. K. A. de Witt Huberts, Z. Phys. A **336**, 159 (1990).
[13] I. Bauske, J. M. Arias, P. von Brentano, A. Frank, H. Friedrichs, R. D. Heil, R.-D. Herzberg, F. Hoyler, P. Van Isacker, U. Kneissl, J. Margraf, H. H. Pitz, C. Wesselborg, and A. Zilges, Phys. Rev. Lett. **71**, 975 (1993).
[14] J. Enders, N. Huxel, P. von Neumann-Cosel, and A. Richter, Phys. Rev. Lett. **79**, 2010 (1997).
[15] A. Nord, J. Enders, A. E. de Almeida Pinto, D. Belic, P. von Brentano, C. Fransen, U. Kneissl, C. Kohstall, A. Linnemann, P. von Neumann-Cosel, N. Pietralla, H. H. Pitz, A. Richter, F. Stedile, and V. Werner, Phys. Rev. C **67**, 034307 (2003).
[16] M. Krtička, F. Bečvář, J. Honzátko, I. Tomandl, M. Heil, F. Käppeler, R. Reifarth, F. Voss, and K. Wisshak, Phys. Rev. Lett. **92**, 172501 (2004).
[17] A. Schiller, A. Voinov, E. Algin, J. A. Becker, L. A. Bernstein, P. E. Garrett, M. Guttormsen, R. O. Nelson, J. Rekstad, and S. Siem, preprint nucl-ex/0401038 (2004).
[18] E. Lipparini and S. Stringari, Phys. Rev. Lett. **63**, 570 (1989).
[19] V. O. Nesterenko, W. Kleinig, F. F. de Souza Cruz, and N. Lo Iudice, Phys. Rev. Lett. **83**, 57 (1999).
[20] D. Guéry–Odelin and S. Stringari, Phys. Rev. Lett. **83**, 4452 (1999).
[21] O. M. Maragò, S. A. Hopkins, J. Arlt, E. Hodby, G. Hechenblaikner, and C. J. Foot, Phys. Rev. Lett. **84**, 2056 (2000).
[22] M. Cozzini, S. Stringari, V. Bretin, P. Rosenbusch, and J. Dalibard, Phys. Rev. A **67**, 021602(R) (2003).
[23] I. Hamamoto and C. Magnusson, Phys. Lett. B **260**, 6 (1991).
[24] N. Pietralla, P. von Brentano, R.-D. Herzberg, U. Kneissl, N. Lo Iudice, H. Maser, H. H. Pitz, and A. Zilges, Phys. Rev. C **58**, 184 (1998).
[25] D. R. Bes and R. A. Broglia, Phys. Lett. **137B**, 141 (1984).
[26] W. Ziegler, C. Rangacharyulu, A. Richter, and C. Spieler, Phys. Rev. Lett. **65**, 2515 (1990).
[27] N. Lo Iudice and A. Richter, Phys. Lett. B **304**, 193 (1993).
[28] E. Lipparini and S. Stringari, Phys. Lett. **130B**, 139 (1983).
[29] E. Lipparini and S. Stringari, Phys. Rep. **175**, 103 (1989).
[30] P. von Neumann-Cosel, J. N. Ginocchio, H. Bauer, and A. Richter, Phys. Rev. Lett. **75**, 4178 (1995).
[31] N. Pietralla, P. von Brentano, R.-D. Herzberg, U. Kneissl, J. Margraf, H. Maser, H. H. Pitz, and A. Zilges, Phys. Rev. C **52**, R2317 (1995).
[32] J. Enders, H. Kaiser, P. von Neumann-Cosel, C. Rangacharyulu, and A. Richter, Phys. Rev. C **59**, R1851 (1999).
[33] J. Speth and D. Zawischa, Phys. Lett. B **211**, 247 (1988); **219**, 529(E) (1989).
[34] N. Lo Iudice and A. Richter, Phys. Lett. B **228**, 291 (1989).
[35] A. Faessler, Prog. Part. Nucl. Phys. **38**, 195 (1997).
[36] N. Lo Iudice, Phys. Rev. C **57**, 1246 (1998).
[37] E. B. Balbutsev and P. Schuck, Nucl. Phys. **A720**, 293 (2003); **A728**, 471(E) (2003).
[38] S. J. Freeman, R. Chapman, J. L. Durell, M. A. C. Hotchkis, F. Khazaie, J. C. Lisle, J. N. Mo, A. M. Bruce, R. A. Cunningham, P. V. Drumm, D. D. Warner, and J. D. Garrett, Phys. Lett. B **222**, 347 (1989).
[39] D. Frekers, H. J. Wörtche, A. Richter, R. Abegg, R. E. Azuma, A. Celler, C. Chan, T. E. Drake, R. Helmer, K. P. Jackson, J. D. King, C. A. Miller, R. Schubank, M. C. Vetterli, and S. Yen, Phys. Lett. B **244**, 178 (1990).
[40] C. De Coster and K. Heyde, Phys. Rev. Lett. **63**, 2797 (1989).
[41] G. Alaga, K. Alder, A. Bohr, and B. Mottelson, Kgl. Dan. Vidensk. Selsk. Mat.-Fys. Medd. **29** (9), 1 (1955).
[42] H. Maser, S. Lindenstruth, I. Bauske, O. Beck, P. von Brentano, T. Eckert, H. Friedrichs, R. D. Heil, R.-D. Herzberg, A. Jung, U. Kneissl, J. Margraf, N. Pietralla, H. H. Pitz, C. Wesselborg, and A. Zilges, Phys. Rev. C **53**, 2749 (1996).
[43] U. Hartmann, D. Bohle, F. Humbert, and A. Richter, Nucl. Phys. **A499**, 93 (1989).
[44] A. Gade, N. Pietralla, P. von Brentano, D. Belic, C. Fransen, U. Kneissl, C. Kohstall, A. Linnemann, H. H. Pitz, M. Scheck, N. A. Smirnova, F. Stedile, and V. Werner, Phys. Rev. C **69**, 054321 (2004).
[45] T. Eckert, O. Beck, J. Besserer, P. von Brentano, R. Fischer, R.-D. Herzberg, U. Kneissl, J. Margraf, H. Maser, A. Nord, N. Pietralla, H. H. Pitz, S. W. Yates, and A. Zilges, Phys. Rev. C **56**, 1256 (1997).
[46] H. H. Pitz, R. D. Heil, U. Kneissl, S. Lindenstruth, U. Seemann, R. Stock, C. Wesselborg, A. Zilges, P. von Brentano, S. D. Hoblit, and A. M. Nathan, Nucl. Phys. **A509**, 587 (1990).
[47] W. Ziegler, N. Huxel, P. von Neumann-Cosel, C. Rangacharyulu, A. Richter, C. Spieler, C. De Coster, and K. Heyde, Nucl. Phys. **A564**, 366 (1993).
[48] H. H. Pitz, U. E. P. Berg, R. D. Heil, U. Kneissl, R. Stock, C. Wesselborg, and P. von Brentano, Nucl. Phys. **A492**, 411 (1989).
[49] H. Friedrichs, D. Häger, P. von Brentano, R. D. Heil, R.-D. Herzberg, U. Kneissl, J. Margraf, G. Müller, H. H. Pitz, B. Schlitt, M. Schumacher, C. Wesselborg, and A. Zilges, Nucl. Phys. **A567**, 266 (1994).
[50] C. Wesselborg, P. von Brentano, K. O. Zell, R. D. Heil, H. H. Pitz, U. E. P. Berg, U. Kneissl, S. Lindenstruth, U. Seemann, and R. Stock, Phys. Lett. B **207**, 22 (1988).
[51] J. Margraf, T. Eckert, M. Rittner, I. Bauske, O. Beck, U. Kneissl, H. Maser, H. H. Pitz, A. Schiller, P. von Brentano, R. Fischer, R.-D. Herzberg, N. Pietralla, A. Zilges, and H. Friedrichs, Phys. Rev. C **52**, 2429 (1995).
[52] A. Zilges, P. von Brentano, C. Wesselborg, R. D. Heil, U. Kneissl, S. Lindenstruth, H. H. Pitz, U. Seemann, and R. Stock, Nucl. Phys. **A507**, 399 (1990).
[53] M. Scheck, D. Belic, P. von Brentano, J. J. Carroll, C. Fransen, A. Gade, H. von Garrel, U. Kneissl, C. Kohstall, A. Linnemann, N. Pietralla, H. H. Pitz, F. Stedile, R. Toman, and V. Werner, Phys. Rev. C **67**, 064313 (2003).
[54] N. Pietralla, O. Beck, J. Besserer, P. von Brentano, T. Eckert, R. Fischer, C. Fransen, R.-D. Herzberg, D. Jäger, R. V. Jolos,

U. Kneissl, B. Krischok, J. Margraf, H. Maser, A. Nord, H. H. Pitz, M. Rittner, A. Schiller, and A. Zilges, Nucl. Phys. **A618**, 141 (1997).
[55] R.-D. Herzberg, A. Zilges, P. von Brentano, R. D. Heil, U. Kneissl, J. Margraf, H. H. Pitz, H. Friedrichs, S. Lindenstruth, and C. Wesselborg, Nucl. Phys. **A563**, 445 (1993).
[56] C. Fransen, B. Krischok, O. Beck, J. Besserer, P. von Brentano, T. Eckert, R.-D. Herzberg, U. Kneissl, J. Margraf, H. Maser, A. Nord, N. Pietralla, H. H. Pitz, and A. Zilges, Phys. Rev. C **59**, 2264 (1999).
[57] A. Linnemann, P. von Brentano, J. Eberth, J. Enders, A. Fitzler, C. Fransen, E. Guliyev, R.-D. Herzberg, L. Käubler, A. A. Kuliev, P. von Neumann-Cosel, N. Pietralla, H. Prade, A. Richter, R. Schwengner, H. G. Thomas, D. Weisshaar, and I. Wiedenhöver, Phys. Lett. B **554**, 15 (2003).
[58] P. von Brentano, J. Eberth, J. Enders, L. Esser, R.-D. Herzberg, N. Huxel, H. Meise, P. von Neumann-Cosel, N. Nicolay, N. Pietralla, H. Prade, J. Reif, A. Richter, C. Schlegel, R. Schwengner, S. Skoda, H. G. Thomas, I. Wiedenhöver, G. Winter, and A. Zilges, Phys. Rev. Lett. **76**, 2029 (1996).
[59] N. Pietralla, Dissertation D38, Universität zu Köln, 1996.
[60] A. Willis, M. Morlet, N. Marty, C. Djalali, D. Bohle, H. Diesener, A. Richter, and H. Stein, Nucl. Phys. **A499**, 367 (1989).
[61] L. Zamick, Phys. Rev. C **31**, 1955 (1985).
[62] R. W. Fearick, G. Hartung, K. Langanke, G. Martínez-Pinedo, P. von Neumann-Cosel, and A. Richter, Nucl. Phys. **A727**, 41 (2003).
[63] C. Fransen, O. Beck, P. von Brentano, T. Eckert, R.-D. Herzberg, U. Kneissl, H. Maser, A. Nord, N. Pietralla, H. H. Pitz, and A. Zilges, Phys. Rev. C **57**, 129 (1998).
[64] A. Degener, C. Bläsing, R. D. Heil, A. Jung, U. Kneissl, H. H. Pitz, H. Schacht, S. Schennach, R. Stock, and C. Wesselborg, Nucl. Phys. **A513**, 29 (1990).
[65] J. Enders, P. von Brentano, J. Eberth, R.-D. Herzberg, N. Huxel, H. Lenske, P. von Neumann-Cosel, N. Nicolay, N. Pietralla, H. Prade, J. Reif, A. Richter, C. Schlegel, R. Schwengner, S. Skoda, H. G. Thomas, I. Wiedenhöver, G. Winter, and A. Zilges, Nucl. Phys. **A636**, 139 (1998).
[66] T. Chapuran, R. Starr, R. Vodhanel, and M. K. Brussel, Phys. Rev. C **30**, 54 (1984).
[67] A. Gade, H. Klein, N. Pietralla, and P. von Brentano, Phys. Rev. C **65**, 054311 (2002).
[68] V. Werner, D. Belic, P. von Brentano, C. Fransen, A. Gade, H. von Garrel, J. Jolie, U. Kneissl, C. Kohstall, A. Linnemann, A. F. Lisetskiy, N. Pietralla, H. H. Pitz, M. Scheck, K.-H. Speidel, F. Stedile, and S. W. Yates, Phys. Lett. **B550**, 140 (2002).
[69] N. Pietralla, C. Fransen, D. Belic, P. von Brentano, C. Frießner, U. Kneissl, A. Linnemann, A. Nord, H. H. Pitz, T. Otsuka, I. Schneider, V. Werner, and I. Wiedenhöver, Phys. Rev. Lett. **83**, 1303 (1999).
[70] A. Gade, D. Belic, P. von Brentano, C. Fransen, H. von Garrel, J. Jolie, U. Kneissl, C. Kohstall, A. Linnemann, H. H. Pitz, M. Scheck, F. Stedile, and V. Werner, Phys. Rev. C **67**, 034304 (2003).
[71] H. Lehmann, A. Nord, A. E. de Almeida Pinto, O. Beck, J. Besserer, P. von Brentano, S. Drissi, T. Eckert, R.-D. Herzberg, D. Jäger, J. Jolie, U. Kneissl, J. Margraf, H. Maser, N. Pietralla, and H. H. Pitz, Phys. Rev. C **60**, 024308 (1999).
[72] W. Geiger, Zs. Németh, I. Bauske, P. von Brentano, R. D. Heil, R.-D. Herzberg, U. Kneissl, J. Margraf, H. Maser, N. Pietralla, H. H. Pitz, C. Wesselborg, and A. Zilges, Nucl. Phys. **A580**, 263 (1994).
[73] R. Schwengner, G. Winter, W. Schauer, M. Grinberg, F. Becker, P. von Brentano, J. Eberth, J. Enders, T. von Egidy, R.-D. Herzberg, N. Huxel, L. Käubler, P. von Neumann-Cosel, N. Nicolay, J. Ott, N. Pietralla, H. Prade, S. Raman, J. Reif, A. Richter, C. Schlegel, H. Schnare, T. Servene, S. Skoda, T. Steinhardt, C. Stoyanov, H. G. Thomas, I. Wiedenhöver, and A. Zilges, Nucl. Phys. **A620**, 277 (1997); **A624**, 776(E) (1997).
[74] R. Georgii, P. von Neumann-Cosel, T. von Egidy, M. Grinberg, V. A. Khitrov, J. Ott, P. Prokofjevs, A. Richter, W. Schauer, C. Schlegel, R. Schulz, L. J. Simonova, Ch. Stoyanov, A. M. Sukhovoj, and A. V. Voinov, Phys. Lett. **B351**, 82 (1995).
[75] H. Maser, N. Pietralla, P. von Brentano, R.-D. Herzberg, U. Kneissl, J. Margraf, H. H. Pitz, and A. Zilges, Phys. Rev. C **54**, R2129 (1996).
[76] N. Pietralla, D. Belic, P. von Brentano, C. Fransen, R.-D. Herzberg, U. Kneissl, H. Maser, P. Matschinsky, A. Nord, T. Otsuka, H. H. Pitz, V. Werner, and I. Wiedenhöver, Phys. Rev. C **58**, 796 (1998).
[77] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, MA, 1975), Vol. II.
[78] W. Bentz and A. Arima, Nucl. Phys. **A736**, 93 (2004).
[79] S. S. Dietrich and B. L. Berman, At. Data Nucl. Data Tables **38**, 199 (1988).
[80] S. Raman, C. H. Malarkey, W. T. Milner, C. W. Nestor, Jr., and P. H. Stelson, At. Data Nucl. Data Tables **36**, 1 (1987).
[81] P. Raghavan, At. Data Nucl. Data Tables **42**, 189 (1989).
[82] B. L. Berman and S. C. Fultz, Rev. Mod. Phys. **47**, 713 (1975).
[83] A. van der Woude, in *Electric and Magnetic Giant Resonances in Nuclei*, edited by J. Speth (World Scientific, Singapore, 1991), p. 99.
[84] S. Jang, Nucl. Phys. **A401**, 303 (1983).
[85] V. G. Soloviev, A. V. Sushkov, N. Yu. Shirikova, and N. Lo Iudice, Nucl. Phys. **A600**, 155 (1996).
[86] R. Nojarov and A. Faessler, Z. Phys. A **336**, 151 (1990).
[87] J. Enders, T. Guhr, N. Huxel, P. von Neumann-Cosel, C. Rangacharyulu, and A. Richter, Phys. Lett. B **486**, 273 (2000).
[88] J. F. Shriner, Jr., G. E. Mitchell, and T. von Egidy, Z. Phys. A **338**, 309 (1991).
[89] J. D. Garrett, J. Q. Robinson, A. J. Foglia, and H.-Q. Jin, Phys. Lett. B **392**, 24 (1997).
[90] R. D. Meeker, G. M. Kalvius, B. D. Dunlap, S. L. Ruby, and D. Cohen, Nucl. Phys. **A224**, 429 (1974).
[91] A. E. Stuchbery, N. Benczer-Koller, G. Kumbartzki, and T. J. Mertzimekis, Phys. Rev. C **69**, 044302 (2004).
[92] P. F. Mantica, A. E. Stuchbery, D. E. Groh, J. I. Prisciandaro, and M. P. Robinson, Phys. Rev. C **63**, 034312 (2001).
[93] E. Guliyev, A. A. Kuliev, P. von Neumann-Cosel, and A. Richter, Phys. Lett. B **532**, 173 (2002).
[94] J. N. Ginocchio and A. Leviatan, Phys. Rev. Lett. **79**, 813 (1997).
[95] J. Enders, N. Huxel, U. Kneissl, P. von Neumann-Cosel, H. H. Pitz, and A. Richter, Phys. Rev. C **57**, 996 (1998).
[96] N. Huxel, P. von Brentano, J. Eberth, J. Enders, R.-D. Herzberg, P. von Neumann-Cosel, N. Nicolay, N. Pietralla, H. Prade, C. Rangacharyulu, J. Reif, A. Richter, C. Schlegel, R. Schwengner, S. Skoda, H. G. Thomas, I. Wiedenhöver, G. Winter, and A. Zilges, Nucl. Phys. **A645**, 239 (1999).
[97] H. J. Wörtche, Dissertion D17, Technische Hochschule Darmstadt, 1994.
[98] E. Moya de Guerra and L. Zamick, Phys. Rev. C **47**, 2604 (1993).