Neutron fraction and neutrino mean free path predictions in relativistic mean field models

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(Received 25 August 2004; published 23 December 2004)

The equation of state (EOS) of dense matter and neutrino mean free path (NMFP) in a neutron star have been studied by using relativistic mean field models motivated by effective field theory. It is found that the models predict too large proton fractions, although one of the models (G2) predicts an acceptable EOS. This is caused by the isovector terms. Except G2, the other two models predict anomalous NMFP's. In order to minimize the anomaly, besides an acceptable EOS, a large M^* is favorable. A model with large M^* retains the regularity in the NMFP even for a small neutron fraction.

DOI: 10.1103/PhysRevC.70.068801 PACS number(s): 26.60.+c, 13.15.+g, 21.30.Fe, 25.30.Pt

The finite-range (FR) (see Refs. [1–4]) and point-coupling (PC) (see Refs. [5–9]) types of relativistic mean field (RMF) models have been quite successful to describe the bulk as well as single particle properties in a wide mass spectrum of nuclei.

The early version of RMF-FR is based on a Lagrangian density which uses nucleon, sigma, omega, and rho mesons as the degrees of freedom with additional cubic and quartic nonlinearities of the sigma meson. For example, NLZ, NL1, NL3, and NL-SH parameter sets belong to this version. Recently, inspired by the effective field and density functional theories for hadrons, a new version of this model (ERMF-FR) has been constructed [3,10]. It has the same terms as the previous RMF-FR but with additional isoscalar and isovector tensor terms and nonlinear terms in the form of sigma, omega, and rho meson combinations. One of the parameter sets of this version is G2. In addition to yielding accurate predictions in finite nuclei and normal nuclear matter [3,4,10], G2 has demanding features like a positive value of the quartic sigma meson coupling constant that leads to the existence of a lower bound in the energy spectrum of this model [11,12] and to the missing zero sound mode in high density symmetric nuclear matter [13]. Moreover, the agreement of the nuclear matter and the neutron matter equation of states (EOS) at high density of G2 with the Dirac-Brueckner-Hartree-Fock calculation [4,11] is better than those of the NL1, NL3, and TM1 models (the standard RMF-FR plus a quartic omega meson interaction).

The difference between RMF-PC and RMF-FR is due to the replacement of mesonic interactions in the FR model by density dependent interactions. It is evident that RMF-PC and RMF-FR have similar qualities in predicting finite nuclei and normal nuclear matter [5,7,9]. This is due to the fact that "finite-range" effects in the RMF-PC model are effectively absorbed by the coupling constants. Therefore in connection with different treatments of the "finite range" in both models, studying the behavior of the PC model at high density should be interesting. In this report, we chose the VA4 parameter set of Ref. [6] (ERMF-PC) because it can be properly extrapolated to the high density and it has also density dependent self- and cross-interactions in the nonlinear terms.

So far the EOS of a neutron star has not been known for sure [14]. However, recently [15] the flow of matter in heavy ion collisions has been used to determine the pressure of nuclear matter with a density from two to five times the nuclear saturation density (ρ_0). Reference [15] has found that these data can be explained only by the variational calculation of Akmal *et al.* [16]. Unfortunately, this interaction cannot be successfully applied to the case of finite nuclei [11]. Reference [11] found that the EOS predicted by G2 is in agreement with data. This result is remarkable, since Ref. [17] states that the minimal requirement for an accurate neutrino mean free path (NMFP) is a correct prediction in the low density limit, as well as the consistency with the corresponding EOS. On the other hand, one should remember that many-body corrections are important but they depend on the model and the approximation of strong interaction used [14,17–25].

According to Refs. [26,27] all RMF-FR models yield lower threshold densities for the direct URCA process than those of variational calculations [16]. In the neutron star cooling model, Migdal *et al.* [28] treated this fact as a fragile point of RMF-FR models. So they disregarded direct URCA in their analysis but Lattimer *et al.* [29] used this fact to develop their direct URCA scenario.

Therefore, in this report we will compare the neutron matter prediction at high density from the G2, NLZ, and VA4 models in order to check the result of Ref. [11] and the possibly different predictions from ERMF-PC and ERMF-FR due to the different treatment of the "finite-range effects." Furthermore, the agreement between the G2 EOS with experimental data has motivated us to calculate the NMFP using this model for the direct URCA process. A similar assumption as in Ref. [22] is used, i.e., the ground state of the neutron star is reached once the temperature has fallen below a few MeV. This state is gradually reached from the later stages of the cooling phase. The system is then quite dense and cool so that zero temperature is valid. In this case the direct URCA neutrino-neutron scattering is kinematically possible for low energy neutrinos at and above the threshold density when the proton fraction exceeds $1/9$ [29] or slightly larger if muons are present. Furthermore, the absorption reaction is suppressed. For simplicity, we neglect the random-phase approximation (RPA) correlations.

The effects of self- and cross-interaction terms and the treatment of finite range at high density can be observed by extrapolating the EOS which is presented by the neutron

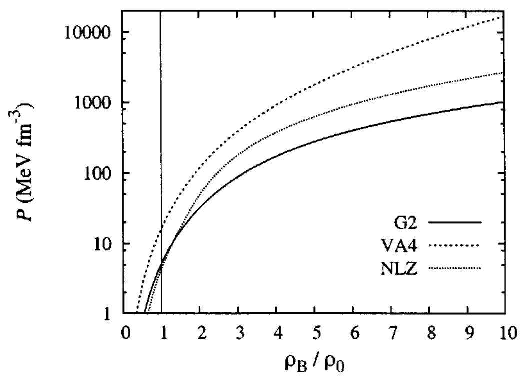

FIG. 1. Equation of states (EOS) of the neutron matter.

matter pressure P and the effective mass M^*, as shown in Figs. 1 and 2, where we compare the results obtained from the G2 [10], NLZ [1], and VA4 [6] models as a function of ρ_B/ρ_0.

It is found that the nuclear matter EOS of VA4 is stiffer than those of NLZ and G2, even for ρ_B less than ρ_0. However, the G2 EOS is softer than the NLZ one at high density but not at low density. This fact emphasizes the result of Ref. [11] that the crucial role of self- and cross-interactions of the meson exchange model is to soften the EOS at high density.

It is shown in Fig. 2 that for $1 \leqslant \rho_B/\rho_0 \leqslant 5$, the effective mass $M^*_{\rm G2} > M^*_{\rm VA4}$, but for $\rho_B/\rho_0 \geqslant 5$ one observes that $M^*_{\rm G2} < M^*_{\rm VA4}$. This indicates that quantitatively M^* depends on the model. We note here that the effective masses of G2 and VA4 depend on self- and cross-interaction terms implicitly. We also note that other mechanisms could also produce a larger M^*, e.g., in the Zimanyi-Moszkowski and linear Hartree-Fock Walecka models [22], where those terms are not present. Although those models give a regular NMFP, they are quite unsuccessful in finite nuclei applications, especially in predicting the single particle spectra of nuclei [30]. Therefore, it is interesting to check whether or not the relation between a large M^* and a regular NMFP also appears in the case of ERMF models.

Now, we calculate the NMFP of the neutron star matter by employing G2, VA4, and NLZ models. Following Refs. [21,22], we start with the neutrino differential scattering cross section

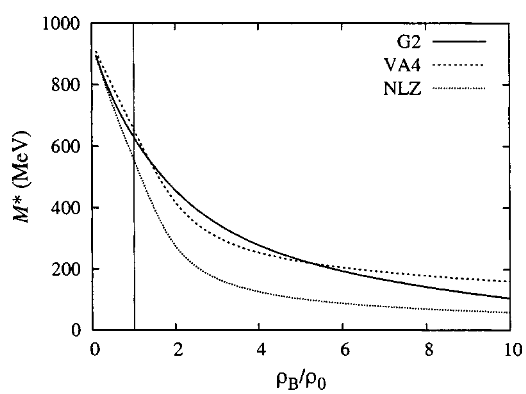

FIG. 2. Effective masses (M^*) of the neutron matter.

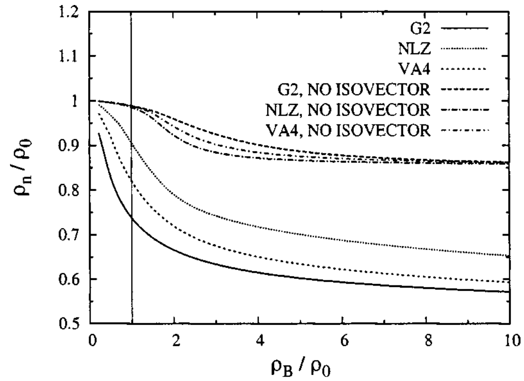

FIG. 3. Neutron fraction in the neutron star matter with and without isovector terms.

$$\frac{1}{V}\frac{d^3\sigma}{d^2\Omega' dE'_\nu} = -\frac{G_F}{32\pi^2}\frac{E'_\nu}{E_\nu}{\rm Im}(L_{\mu\nu}\Pi^{\mu\nu}). \qquad (1)$$

Here E_ν and E'_ν are the initial and final neutrino energies, respectively, $G_F = 1.023 \times 10^{-5}/M^2$ is the weak coupling, and M is the nucleon mass. The neutrino tensor $L_{\mu\nu}$ can be written as

$$L_{\mu\nu} = 8[2k_\mu k_\nu + (k\cdot q)g_{\mu\nu} - (k_\mu q_\nu + q_\mu k_\nu) \mp i\epsilon_{\mu\nu\alpha\beta}k^\alpha q^\beta], \qquad (2)$$

where k is the initial neutrino four-momentum and $q=(q_0,\vec{q})$ is the four-momentum transfer. The polarization tensor $\Pi^{\mu\nu}$, which defines the target particle species, can be written as

$$\Pi^j_{\mu\nu}(q) = -i\int \frac{d^4p}{(2\pi)^4}{\rm Tr}[G^j(p)J^j_\mu G^j(p+q)J^j_\nu], \qquad (3)$$

where $j=n,p,e^-,\mu^-$. $G(p)$ is the target particle propagator and $p=(p_0,\vec{p})$ is the corresponding initial four-momentum. The currents J^j_μ are $\gamma^\mu(C^j_V - C^j_A\gamma_5)$. The explicit forms of $G^j(p)$, C^j_V, and C^j_A of every constituent and also their explanations can be found in Ref. [22]. The NMFP (symbolized by λ) as a function of the initial neutrino energy at a certain density is obtained by integrating the cross section over the time and vector components of the neutrino momentum transfer. As a result we obtain [21,22]

$$\frac{1}{\lambda(E_\nu)} = \int_{q_0}^{2E_\nu - q_0} d|\vec{q}|\int_0^{2E_\nu} dq_0 \frac{|\vec{q}|}{E'_\nu E_\nu}2\pi\frac{1}{V}\frac{d^3\sigma}{d^2\Omega' dE'_\nu}. \qquad (4)$$

Since in our study we assume that the neutron star matter consists only of neutrons, protons, electrons, and muons, the relative fraction of each constituent should be taken into account in the NMFP calculation. The relative fraction is determined by the chemical potential equilibrium and the charge neutrality of the neutron star at zero temperature. The neutron fractions for all models are shown in Fig. 3.

Qualitatively, all parameter sets have similar trend in fraction of each constituent, i.e., when the neutron fraction is decreasing, other constituent (p,e^-,μ^-) fractions are increasing. Quantitatively, isovector terms are responsible for the high proton fraction. G2 has a smaller neutron fraction than

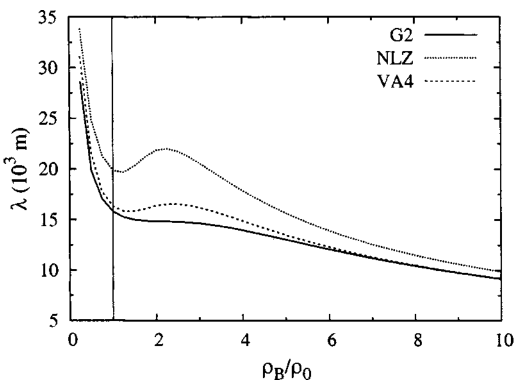

FIG. 4. Neutrino mean free path (NMFP) in the neutron star matter.

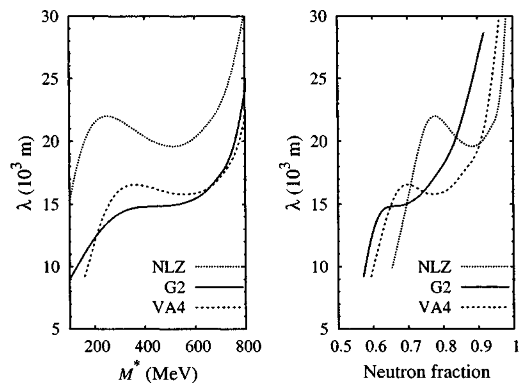

FIG. 5. Neutrino mean free paths (NMFP) in the neutron star matter as functions of M^* and neutron fraction for all parameter sets.

VA4 and NLZ. Therefore, even though G2 has an acceptable EOS, it has a too large proton fraction. This fact leads to such a low threshold density for the direct URCA process. We note that this fact is ruled out by the analysis of the neutron stars cooling data [26,31,32]. Thus, this result indicates that significant improvements in the treatment of the isovector sector of ERMF-FR are urgently required. A variational calculation of Akmal *et al.* [16] allows for a direct URCA process only for $\rho_B/\rho_0 > 5$. The linear Walecka (linear FR) and Zimanyi-Moszkowski (derivative coupling) Hartree-Fock models of Ref. [22] yield a higher critical density for the direct URCA process. Isovector contributions of these models do not drastically change the proton fraction. But, on the other hand, all Hartree-Fock models of Ref. [22] are unable to give a good prediction in finite nuclei, especially in the single particle properties [33,34]. It may be interesting to see also the consistency of their EOS with experimental data [15].

The NMFP for all models can be seen in Fig. 4. Here we use a neutrino energy of $E_\nu = 5$ MeV. In general, from medium to high density, $\lambda_{\rm NLZ}$ is larger than $\lambda_{\rm G2}$ and $\lambda_{\rm VA4}$. In the high density region we clearly see that $\lambda_{\rm G2} \approx \lambda_{\rm VA4}$. The NMFP difference among all models appears to be significant around $1 \leqslant \rho_B/\rho_0 \leqslant 5$ (medium density). For ρ_B/ρ_0 smaller than 1, $\lambda_{\rm VA4} \approx \lambda_{\rm G2} \approx \lambda_{\rm NLZ}$. In other words, in the limit of low density all parameter sets give a similar λ prediction as we expected.

In Fig. 5 we show the dependence of λ with respect to the M^* and neutron fraction. Obviously, NLZ has a maximum NMFP at $M^* \approx 200$ MeV and neutron fraction ≈ 0.75. These lead to a bump in $\lambda_{\rm NLZ}$ as shown in Fig. 4. On the other hand, G2 demonstrates no maximum in M^* and neutron fraction dependences, leading to a smoothly decreasing function of $\lambda_{\rm G2}$ displayed in Fig. 4. For comparison, previous NMFP calculations using all Hartree-Fock models [22] showed also no anomaly. In these models, the predicted NMFP falls off faster than that of the Hartree type model as the density increases.

In conclusion, the EOS and NMFP of ERMF models in the high density states have been studied. It is found that the ERMF-FR and ERMF-PC models have different behaviors at high density and even by using a parameter set that predicts an acceptable EOS, the calculated proton fraction in the neutron star is still too large. The isovector terms are responsible for this. Therefore, improvements in the treatment of the isovector sector of ERMF-FR should be done. Different from the Hartree-Fock calculation of Ref. [22], only the parameter set with an acceptable EOS (G2) has a regular NMFP. In order to minimize the anomalous behavior of λ, a relatively large M^* in RMF models is more favorable. It seems that the relatively large M^* in the ERMF models at high density originates from the presence of the self- and cross interactions in nonlinear terms. The RMF models with relatively large M^* retain their regularities partly or fully even for a small neutron fraction.

The work of A.S. and T.M. has been supported in part by the QUE project.

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