Simple parametrization of nucleon form factors

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This Brief Report provides simple parametrizations of the nucleon electromagnetic form factors using functions of Q^2 that are consistent with dimensional scaling at high Q^2. Good fits require only four parameters each for G_{Ep}, G_{Mp}, and G_{Mn} and only two for G_{En}.

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Nucleon electromagnetic form factors are needed for many calculations in nuclear physics. Hence, it would be useful to have a simple parametrization that accurately represents the data over a wide range of Q^2 with reasonable behavior for both $Q^2 \to 0$ and $Q^2 \to \infty$. To obtain reasonable behavior at low Q^2 the power-series representation should involve only even powers of Q. At high Q^2 dimensional scaling rules require $G \propto Q^{-4}$ apart from slowly varying logarithmic corrections that can be ignored safely for most applications. However, the most common parametrizations violate one or both of these conditions. Often one uses the reciprocal of a polynomial in Q [1–3], but then the rms radius cannot be determined because such a parametrization includes unphysical odd powers of Q. This problem can be circumvented using the reciprocal of a polynomial in Q^2, but to obtain good fits for Q^2 in the several $(\mathrm{GeV}/c)^2$ range one must use so many terms that the form factor falls too rapidly at large Q^2 [4]. Yet another parametrization is based upon a continued-fraction expansion in Q^2 [5,6], but the limiting Q^{-4} behavior is usually not enforced because the required parameter constraints become quite cumbersome. In Ref. [7] I proposed a parametrization based upon charge and magnetization densities that was designed to enforce both conditions, but the representations in terms of Fourier-Bessel or Laguerre-Gaussian expansions require a fairly large number of parameters and are somewhat difficult to implement in calculations that are not based upon densities. In this Brief Report, I propose a much simpler parametrization that is suitable for a wide variety of calculations.

Perhaps the simplest parametrization takes the form

$$G(Q^2) \propto \frac{\sum_{k=0}^{n} a_k \tau^k}{1 + \sum_{k=1}^{n+2} b_k \tau^k}, \tag{1}$$

where both numerator and denominator are polynomials in $\tau = Q^2/4m_p^2$ and where the degree of the denominator is larger than that of the numerator to ensure that $G \propto Q^{-4}$ for large Q^2. For magnetic form factors we include a factor of μ on the right-hand side, such that $a_0 \approx 1$ if the data for low Q^2 are normalized accurately. With $n=1$ and $a_0=1$, this parametrization provides excellent fits to G_{Ep}, G_{Mp}/μ_p, and G_{Mn}/μ_n using only four parameters each. However, this approach is less successful for G_{En} because the existing data are still too limited. Therefore, for G_{En} I continue to use the Galster parametrization [8],

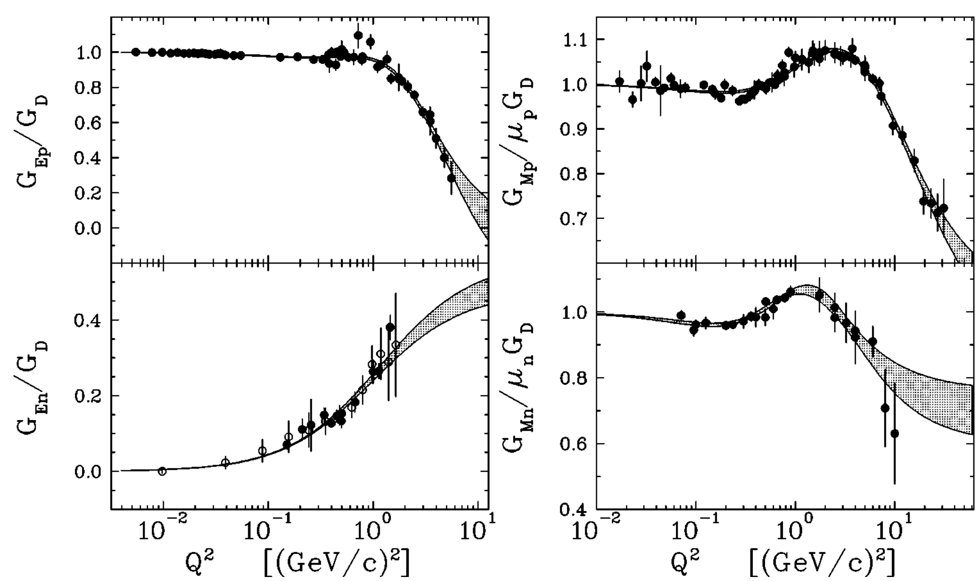

FIG. 1. Fits to nucleon electromagnetic form factors. For G_{En}, data using recoil or target polarization [16–22] are shown as filled circles while data obtained from the deuteron quadrupole form factor [23] are shown as open circles.

TABLE I. Parameters fitted to data for nucleon electromagnetic form factors. The normalization parameter $a_0=1$ was held constant. The second column lists chi-square per datum.

Quantity	χ^2/N	a_1	b_1	b_2	b_3	$r_{\rm rms}$ (fm)	A	B	$\langle r_n^2 \rangle$ (fm^2)
G_{Ep}	0.78	-0.24 ± 0.12	10.98 ± 0.19	12.82 ± 1.1	21.97 ± 6.8	0.863 ± 0.004			
G_{Mp}/μ_p	1.06	0.12 ± 0.04	10.97 ± 0.11	18.86 ± 0.28	6.55 ± 1.2	0.848 ± 0.003			
G_{Mn}/μ_n	0.51	2.33 ± 1.4	14.72 ± 1.7	24.20 ± 9.8	84.1 ± 41	0.907 ± 0.016			
G_{En}	0.80						1.70 ± 0.04	3.30 ± 0.32	-0.112 ± 0.003

$$G_{En}(Q^2) = \frac{A\tau}{1+B\tau} G_D(Q^2), \tag{2}$$

where $G_D=(1+Q^2/\Lambda^2)^{-2}$ with $\Lambda^2=0.71$ (GeV/c)^2 is the dipole form factor. I tried fitting the other three form factors with a variation of the Galster parametrization in which a ratio of polynomials of the same degree multiplied G_D, but that model is much less successful than Eq. (1).

The selection of data remains similar to that of Ref. [7] and emphasizes recoil or target polarization whenever available. Data for G_{Ep} using the Rosenbluth method are omitted for $Q^2>1$ (GeV/c)^2. I selected Refs. [9,10] for G_{Ep}, Refs. [11–14] for G_{Ep}/G_{Mp}, Refs. [2,15] for G_{Mp}, Refs. [16–23] for G_{En}, and Refs. [5,24–28] for G_{Mn}. For G_{En}, $\langle r_n^2\rangle$ from Ref. [29] is included also. These selections differ from my previous analysis by including the most recent data for G_{En} and G_{Mn} and omitting G_{Mn} data from Refs. [30,31] that used the associated-particle technique for the neutron efficiency. See Ref. [7] for a more detailed discussion.

Figure 1 shows the fits to these data and Table I lists the parameters. The quality of these fits, judged visually or by chi-square per datum (χ^2/N), is comparable to the more sophisticated linear expansion analysis of Ref. [7]. For G_{Mn} we now obtain a significantly smaller χ^2/N primarily because experiments using the associated-particle technique were omitted. The error bands were computed using the covariance matrix for nonlinear least squares and are similar in width to the previous bands despite the more restrictive parametrization. However, unlike the previous linear-expansion analysis, the parameters for the present model are highly correlated. Therefore, one cannot reconstruct the error bands using only the standard errors that are listed in Table I based upon diagonal elements of the covariance matrix. Nor can one easily adjust the number of free parameters—even though the parameter uncertainties are relatively large for G_{Mn}, truncating the model at $n=0$ (which removes two parameters) does not provide an adequate fit. Nevertheless, these parametrizations do represent the existing data accurately and offer plausible extrapolations to higher Q^2 that are consistent with dimensional scaling.

It is interesting to observe that this analysis suggests a sign change for G_{Ep} near 15 (GeV/c)^2 and an asymptotic limit for $Q^4G_{Ep}\sim-0.14\pm0.07$ (GeV/c)^4. Recognizing that fits of this kind always underestimate extrapolation uncertainties, we can only conclude that the asymptotic value of Q^4G_{Ep} is likely to be quite small and that the sign may change somewhere beyond 10 (GeV/c)^2. The other three form factors show no evidence for a sign change at large Q^2.

Table I also lists rms radii that are consistent with those in Ref. [7] that use normalization constraints, but the estimated uncertainties are substantially smaller because the present model is much less flexible than the nearly model-independent analysis based upon linear expansion in a complete set of basis functions. Although the present uncertainties are clearly too optimistic, the observation that the radius is largest for G_{Mn} and smallest for G_{Mp} appears to be robust. Note that we have not applied Coulomb corrections to the low Q^2 data, which would produce a small increase in the proton rms radius fitted to G_{Ep} [6,32]. Nor have corrections for Coulomb distortion [33] or for exchange of two hard photons been applied to the proton form factors at large Q^2. The latter are estimated to be on the few percent level for the proton [34–38], but the present data do not permit extraction of two-photon effects without making simplifying assumptions and theoretical calculations still retain considerable model dependence. Furthermore, similar estimates for the neutron are not yet available. Nevertheless, the present parametrization should be able to handle these small adjustments without difficulty in the future.

In conclusion, a simple rational function of Q^2 that is consistent with dimensional scaling at high Q^2 provides excellent fits to the existing data for G_{Ep}, G_{Mp}, and G_{En} while the Galster parametrization fits recent G_{En} data using polarization techniques very well.

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